PHYSICAL REVIEW D **96,** 056011 (2017)

Rapidity dependence of transverse-momentum multiplicity correlations

Adam Bzdak*
AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, 30-059 Kraków, Poland

(Received 27 June 2017; published 18 September 2017)

Following previous work [A. Bzdak and D. Teaney, Longitudinal fluctuations of the fireball density in heavy-ion collisions, Phys. Rev. C **87**, 024906 (2013)], we propose to analyze the rapidity dependence of transverse momentum and transverse-momentum multiplicity correlations. We demonstrate that the orthogonal polynomial expansion of the latter has the potential to discriminate between models of particle production.

DOI: 10.1103/PhysRevD.96.056011

I. INTRODUCTION

One of the central problems in high-energy hadronic collisions is to understand the longitudinal structure of systems created in proton-proton (p + p), proton-nucleus (p + A) and nucleus-nucleus (A + A) collisions.

Not long ago, it was argued that an event-by-event long-range fluctuation of the fireball rapidity distribution results in rather peculiar two-particle and multiparticle rapidity correlations [1,2]. Recent measurement by the ATLAS collaboration at the LHC [3] revealed new and rather unexpected scaling results on asymmetric rapidity fluctuations in p + p, p + A and A + A interactions. Recently, this problem has drawn a noticeable theoretical [4–11] and experimental [3,12,13] interest, see also [14–23] for recent related studies.

To summarize the main idea, the single-particle rapidity distribution in each event, $N(y)$, can be written as

$$\frac{N(y)}{\langle N(y)\rangle} = 1 + a_0 + a_1 y + \cdots, \tag{1}$$

where a_0 describes the rapidity independent fluctuation of the fireball. a_1 represents the fluctuating long-range forward-backward rapidity asymmetry.[1] This coefficient can be driven for example by the difference in the number of left- and right-going sources of particles, e.g., wounded nucleons [24,25]. $\langle N(y)\rangle$ is the average rapidity distribution in a given centrality class. By definition $\langle a_i\rangle = 0$.

It is straightforward to calculate the two-particle rapidity correlation [1]

$$\frac{C(y_1,y_2)}{\langle N(y_1)\rangle\langle N(y_2)\rangle} = \langle a_0^2\rangle + \langle a_1^2\rangle y_1 y_2 + \cdots \tag{2}$$

where[2]

$$C(y_1,y_2) = \langle N(y_1)N(y_2)\rangle - \langle N(y_1)\rangle\langle N(y_2)\rangle. \tag{3}$$

As seen in Eq. (2), the long-range fluctuation of the fireball rapidity distribution, parametrized by fluctuating a_i, results in rather nontrivial correlations. The first term corresponds to a well-known rapidity independent multiplicity fluctuation, and it can be driven by, e.g., the impact parameter or volume fluctuation. The second term, $\sim y_1 y_2$, is related to the fluctuating forward-backward asymmetry in rapidity. In the wounded nucleon model [24,25] $\langle a_1^2\rangle \sim \langle (w_L - w_R)^2\rangle$, where $w_{L(R)}$ is the number of left(right)-going wounded nucleons [1]. Recently, the ATLAS collaboration observed $\langle a_1^2\rangle y_1 y_2$ in the two-particle rapidity correlation functions measured in p + p, p + Pb and Pb + Pb collisions [3]. They found that at a given event multiplicity N_{ch}, $\langle a_1^2\rangle$ approximately scales with $1/N_{\mathrm{ch}}$ and numerically is very similar for all colliding systems.[3] This surprising result still calls for a quantitative explanation.

II. TRANSVERSE-MOMENTUM MULTIPLICITY CORRELATIONS

It is proposed here to analyze, in a similar way, the rapidity dependence of transverse momentum and especially transverse-momentum multiplicity correlations.

Analogously to Eq. (1) we have

$$\frac{P_t(y)}{\langle P_t(y)\rangle} = 1 + b_0 + b_1 y + \cdots, \tag{4}$$

where $P_t(y)$ is the average (in one event) transverse momentum of particles in a given rapidity bin y

*bzdak@fis.agh.edu.pl

[1] We are not interested in statistical fluctuations, which can also generate nonzero values of a_i. These are removed by measuring correlation functions.

[2] For clarity we skip $\langle a_0 a_1\rangle$, which vanishes in symmetric (e.g., p + p) collisions.

[3] One would expect rather different results in, e.g., peripheral Pb + Pb and ultracentral p + p collisions ($N_{\mathrm{ch}} \sim 150$ [3]). The ATLAS result suggests that the number of particle sources (at a given N_{ch}) and their fluctuations are actually similar in all measured systems.

$$P_t(y) = \frac{1}{N}\sum_{i=1}^{N} p_t^{(i)}, \quad (5)$$

where N is the number of particles (in a given event) at y. Here $p_t^{(i)}$ is the transverse momentum magnitude of the ith particle. $\langle P_t(y)\rangle$ is the average of $P_t(y)$ over many events in a given centrality class. We note that $\langle b_i\rangle = 0$, in close analogy to the a_i coefficients.

The transverse momentum correlation function (studied extensively in the literature for rather different reasons, see, e.g., [26–28]) reads

$$\frac{C_{[P,P]}(y_1,y_2)}{\langle P_t(y_1)\rangle\langle P_t(y_2)\rangle} = \langle b_0^2\rangle + \langle b_1^2\rangle y_1 y_2 + \cdots, \quad (6)$$

where

$$C_{[P,P]}(y_1,y_2) \equiv \langle P_t(y_1)P_t(y_2)\rangle - \langle P_t(y_1)\rangle\langle P_t(y_2)\rangle. \quad (7)$$

The first term in Eq. (6) describes an event-by-event rapidity independent transverse momentum fluctuation. This could be driven for example by an event-by-event long-range multiplicity fluctuation (if event multiplicity is correlated with P_t). The second term describes the forward-backward rapidity asymmetric transverse momentum fluctuation. A possible source of this effect is the forward-backward fireball multiplicity fluctuation.

It would be especially interesting to measure an event-by-event relation between a_i and b_i coefficients. In order to do this, one can construct a simple correlation function

$$C_{[N,P]}(y_1,y_2) \equiv \langle N(y_1)P_t(y_2)\rangle - \langle N(y_1)\rangle\langle P_t(y_2)\rangle, \quad (8)$$

witch correlates multiplicity and transverse momentum, see, e.g., [28]. This results in

$$\frac{C_{[N,P]}(y_1,y_2)}{\langle N(y_1)\rangle\langle P_t(y_2)\rangle} = \langle a_0 b_0\rangle + \langle a_1 b_1\rangle y_1 y_2 + \cdots. \quad (9)$$

The meaning of mixed coefficients $\langle a_i b_k\rangle$ is easy to understand. The first term describes the relation between rapidity independent fluctuation of multiplicity and transverse momentum. The second term is particularly interesting and describes how rapidity asymmetry in multiplicity is related to rapidity asymmetry of transverse momentum. If the particle multiplicity and P_t are not correlated then $\langle a_i b_k\rangle = \langle a_i\rangle\langle b_k\rangle = 0$.

In general, the above correlation functions can be expanded in terms of the orthogonal polynomials [1]. For example,

$$\frac{C_{[N,P]}(y_1,y_2)}{\langle N(y_1)\rangle\langle P_t(y_2)\rangle} = \sum_{i,k}\langle a_i b_i\rangle T_i(y_1)T_k(y_2), \quad (10)$$

with T_i being, e.g., the Chebyshev or the Legendre polynomials [1,4], and analogously for Eqs. (2) and (6).

III. DISCUSSION AND CONCLUSIONS

Several comments are in order.

Consider a set of events with $a_1 > 0$, i.e., the fireball multiplicity is larger for positive y, $N(y) \sim a_1 y$. The question is what is the rapidity dependence of the transverse momentum in this case. If P_t is also larger for positive y then $b_1 > 0$ and thus $\langle a_1 b_1\rangle > 0$. This scenario is expected in a typical hydrodynamical framework, see, e.g., [29].

For example, in the color glass condensate (CGC) framework [30,31] one could expect a rather different conclusion. Consider a proton-proton event, where the two protons are characterized by different saturation scales, Q_1 and Q_2. The importance of such fluctuations was recently discussed in Refs. [9,32–35]. Here $Q_1^2 = Q_{0,1}^2 e^{+\lambda y}$ and $Q_2^2 = Q_{0,2}^2 e^{-\lambda y}$ with $\lambda \sim 0.3$, see, e.g., [36]. We choose $Q_{0,1} > Q_{0,2}$ so that in a given rapidity interval, say $|y| < 2$, $Q_1 > Q_2$, resulting in rapidity asymmetric $N(y)$. In this case [29,37]

$$N(y) \sim S_t Q_2^2[2 + \ln(Q_1^2/Q_2^2)], \quad (11)$$

$$P_t(y) \sim \frac{2Q_1 - \frac{2}{3}Q_2}{1 + \ln(Q_1/Q_2)}, \quad (12)$$

that is, in CGC the multiplicity is driven by the smaller scale in contrast to the transverse momentum controlled by the larger one [37]. Since $Q_1^2 \sim e^{+\lambda y}$ and $Q_2^2 \sim e^{-\lambda y}$, the multiplicity and the transverse momentum rapidity asymmetries have different signs. If $N(y)$ is growing with rapidity, then $P_t(y)$ is decreasing with y. Consequently $a_1 \gtrless 0$ means $b_1 \lesseqgtr 0$ and $\langle a_1 b_1\rangle < 0$. Clearly, this observation should be treated with caution and more detailed calculations are warranted, see, e.g., [38,39]. The sole purpose of this exercise was to demonstrate that the sign of $\langle a_1 b_1\rangle$ is not at all obvious, and could potentially discriminate between different models of particle production.

As discussed earlier, the ATLAS collaboration reported a surprising scaling of $\langle a_1^2\rangle$ in $\mathrm{p+p}$, $\mathrm{p+Pb}$ and $\mathrm{Pb+Pb}$ collisions [3]. At a given event multiplicity N_{ch}, $\langle a_1^2\rangle$ scales with $1/N_{\mathrm{ch}}$ and is quantitatively very similar for all three systems. It would be very interesting to see if $\langle b_1^2\rangle$ and $\langle a_1 b_1\rangle$ satisfy similar scaling.

Obviously, it would be also desired to study higher order correlation functions [2,40].

An alternative way to analyze the above correlation functions is the principal component analysis, discussed in Ref. [41].

In conclusion, it is proposed to analyze the rapidity dependence of transverse momentum and in particular transverse-momentum multiplicity correlation functions using the orthogonal polynomial expansion. A careful study of the coefficients $\langle a_i^2\rangle$, $\langle b_i^2\rangle$ and $\langle a_i b_k\rangle$ could potentially discriminate between different models of particle production, and reveal detailed information on the longitudinal structure of systems created in $\mathrm{p+p}$, $\mathrm{p+A}$ and $\mathrm{A+A}$ collisions.

ACKNOWLEDGMENTS

We thank Piotr Bożek and Volker Koch for useful comments. This work is supported by the Ministry of Science and Higher Education (MNiSW) and by the National Science Centre, Grant No. DEC-2014/15/B/ST2/00175, and in part by DEC-2013/09/B/ST2/00497.

[1] A. Bzdak and D. Teaney, Longitudinal fluctuations of the fireball density in heavy-ion collisions, Phys. Rev. C **87**, 024906 (2013).
[2] A. Bzdak and P. Bozek, Multiparticle long-range rapidity correlations from fluctuation of the fireball longitudinal shape, Phys. Rev. C **93**, 024903 (2016).
[3] M. Aaboud *et al.* (ATLAS Collaboration), Measurement of forward-backward multiplicity correlations in lead-lead, proton-lead and proton-proton collisions with the ATLAS detector, Phys. Rev. C **95**, 064914 (2017).
[4] J. Jia, S. Radhakrishnan, and M. Zhou, Forward-backward multiplicity fluctuation and longitudinal harmonics in high-energy nuclear collisions, Phys. Rev. C **93**, 044905 (2016).
[5] P. Bozek, W. Broniowski, and A. Olszewski, Two-particle correlations in pseudorapidity in a hydrodynamic model, Phys. Rev. C **92**, 054913 (2015).
[6] A. Monnai and B. Schenke, Pseudorapidity correlations in heavy ion collisions from viscous fluid dynamics, Phys. Lett. B **752**, 317 (2016).
[7] W. Broniowski and P. Bozek, Simple model for rapidity fluctuations in the initial state of ultrarelativistic heavy-ion collisions, Phys. Rev. C **93**, 064910 (2016).
[8] B. Schenke and S. Schlichting, 3D glasma initial state for relativistic heavy ion collisions, Phys. Rev. C **94**, 044907 (2016).
[9] A. Bzdak and K. Dusling, Probing proton fluctuations with asymmetric rapidity correlations, Phys. Rev. C **93**, 031901 (2016); Saturation scale fluctuations and multiparticle rapidity correlations Phys. Rev. C **94**, 044918 (2016).
[10] W. Ke, J. S. Moreland, J. E. Bernhard, and S. A. Bass, Constraints on rapidity-dependent initial conditions from charged particle pseudorapidity densities and two-particle correlations, arXiv:1610.08490.
[11] R. He, J. Qian, and L. Huo, Analyzing fluctuations in rapidity with two-particle correlations and a toy granular model, arXiv:1702.03137.
[12] S. Jowzaee (STAR Collaboration), Rapidity correlations in the RHIC beam energy scan, at the XXVI international conference on ultrarelativistic heavy-ion collisions (Quark Matter 2017), Chicago, 2017.
[13] S. Oh (ALICE Collaboration), Azimuthal correlations of longitudinal structure at midrapidity in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with ALICE, at the XXVI international conference on ultrarelativistic heavy-ion collisions (Quark Matter 2017), Chicago, 2017.
[14] P. Bozek, W. Broniowski, and J. Moreira, Torqued fireballs in relativistic heavy-ion collisions, Phys. Rev. C **83**, 034911 (2011).
[15] A. Bialas and K. Zalewski, Multibin long-range correlations, Nucl. Phys. **A860**, 56 (2011).
[16] A. Bialas, A. Bzdak, and K. Zalewski, Hidden asymmetry and long range rapidity correlations, Phys. Lett. B **710**, 332 (2012).
[17] L. P. Csernai, G. Eyyubova, and V. K. Magas, New method for measuring longitudinal fluctuations and directed flow in ultrarelativistic heavy ion reactions, Phys. Rev. C **86**, 024912 (2012); Erratum, Phys. Rev. C **88**, 019902(E) (2013).
[18] L. Pang, Q. Wang, and X. N. Wang, Effects of initial flow velocity fluctuation in event-by-event $(3+1)$D hydrodynamics, Phys. Rev. C **86**, 024911 (2012).
[19] V. Vovchenko, D. Anchishkin, and L. P. Csernai, Longitudinal fluctuations of the center of mass of the participants in heavy-ion collisions, Phys. Rev. C **88**, 014901 (2013).
[20] A. Olszewski and W. Broniowski, Multibin correlations in a superposition approach to relativistic heavy-ion collisions, Phys. Rev. C **92**, 024913 (2015).
[21] J. Casalderrey-Solana, M. P. Heller, D. Mateos, and W. van der Schee, Longitudinal Coherence in a Holographic Model of Asymmetric Collisions, Phys. Rev. Lett. **112**, 221602 (2014).
[22] V. Vechernin, Forward-backward correlations between multiplicities in windows separated in azimuth and rapidity, Nucl. Phys. **A939**, 21 (2015).
[23] L. G. Pang, H. Petersen, G. Y. Qin, V. Roy, and X. N. Wang, Decorrelation of anisotropic flow along the longitudinal direction, Eur. Phys. J. A **52**, 97 (2016).
[24] A. Bialas, M. Bleszynski, and W. Czyz, Multiplicity distributions in nucleus-nucleus collisions at high energies, Nucl. Phys. **B111**, 461 (1976).
[25] A. Bialas and W. Czyz, Wounded nucleon model and deuteron-gold collisions at RHIC, Acta Phys. Pol. B **36**, 905 (2005).
[26] S. Gavin and M. Abdel-Aziz, Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions, Phys. Rev. Lett. **97**, 162302 (2006).
[27] S. Gavin, G. Moschelli, and C. Zin, Rapidity correlation structure in nuclear collisions, Phys. Rev. C **94**, 024921 (2016).

[28] M. A. Braun, R. S. Kolevatov, C. Pajares, and V. V. Vechernin, Correlations between multiplicities and average transverse momentum in the percolating color strings approach, Eur. Phys. J. C **32**, 535 (2004).

[29] P. Bozek, A. Bzdak, and V. Skokov, The rapidity dependence of the average transverse momentum in $p + Pb$ collisions at the LHC: The color glass condensate versus hydrodynamics, Phys. Lett. B **728**, 662 (2014).

[30] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, The color glass condensate, Annu. Rev. Nucl. Part. Sci. **60**, 463 (2010).

[31] J. P. Blaizot, High gluon densities in heavy ions collisions, Rep. Prog. Phys. **80**, 032301 (2017).

[32] C. Marquet, G. Soyez, and B. W. Xiao, On the probability distribution of the stochastic saturation scale in QCD, Phys. Lett. B **639**, 635 (2006).

[33] L. McLerran and M. Praszalowicz, Fluctuations and the rapidity dependence of charged particles spectra in fixed centrality bins in p*A* collisions, Ann. Phys. (Berlin) **372**, 215 (2016).

[34] L. McLerran and P. Tribedy, Intrinsic fluctuations of the proton saturation momentum scale in high multiplicity $p + p$ collisions, Nucl. Phys. **A945**, 216 (2016).

[35] H. Mntysaari and B. Schenke, Revealing proton shape fluctuations with incoherent diffraction at high energy, Phys. Rev. D **94**, 034042 (2016).

[36] M. Praszalowicz and A. Francuz, Geometrical scaling in inelastic inclusive particle production at the LHC, Phys. Rev. D **92**, 074036 (2015).

[37] A. Dumitru and L. D. McLerran, How protons shatter colored glass, Nucl. Phys. **A700**, 492 (2002).

[38] F. O. Duraes, A. V. Giannini, V. P. Goncalves, and F. S. Navarra, Rapidity dependence of the average transverse momentum in hadronic collisions, Phys. Rev. C **94**, 024917 (2016).

[39] K. Deja and K. Kutak, The rapidity dependence of the average transverse momentum in $p + p$ and $p + Pb$ collisions–revisited, Phys. Rev. D **95**, 114027 (2017).

[40] P. Di Francesco, M. Guilbaud, M. Luzum, and J. Y. Ollitrault, Systematic procedure for analyzing cumulants at any order, Phys. Rev. C **95**, 044911 (2017).

[41] R. S. Bhalerao, J. Y. Ollitrault, S. Pal, and D. Teaney, Principal Component Analysis of Event-By-Event Fluctuations, Phys. Rev. Lett. **114**, 152301 (2015).