

Trimaximal μ - τ reflection symmetry

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The μ - τ reflection symmetry $(\nu_e, \nu_\mu, \nu_\tau) \rightarrow (\bar{\nu}_e, \bar{\nu}_\tau, \bar{\nu}_\mu)$ and the TM1 mixing (a Pontecorvo–Maki–Nakagawa–Sakata matrix with the first column fixed to the tribimaximal form) are both well compatible with experiments. If both approaches are simultaneously assumed, all lepton mixing parameters except for θ_{13} are predicted. In particular, one expects maximal CP violation ($|\delta| = 90^\circ$), maximal atmospheric mixing ($\theta_{23} = 45^\circ$), a slightly less-than-tribimaximal solar mixing angle ($\theta_{12} \approx 34^\circ$), as well as values of 0 or π for the two Majorana phases. We study the renormalization stability of this highly predictive framework when neutrino mass is described by an effective Weinberg operator and by the type I seesaw mechanism, both in the standard model and with supersymmetry.

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The structure of neutrino mixing, the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, is considered as an important clue for possible underlying symmetries of the three generations of fermions in the standard model (SM). Many discrete flavor symmetries have been proposed in trying to understand the observed mixing—see, e.g., the reviews [1–5]. In particular, it had long been speculated that the neutrino mixing could be tribimaximal (TBM) [6–10], which could originate from non-Abelian discrete symmetries such as A_4 and S_4 . However, the TBM mixing predicts zero θ_{13} which has been excluded by reactor neutrino experiments [11–13].

It is well understood that $\theta_{13} = 0$ in TBM is attributed to μ - τ symmetry [14], which is defined as the invariance of the neutrino mass terms under the interchange of ν_μ and ν_τ . Therefore in the light of nonzero θ_{13} , breaking the μ - τ symmetry has been considered and extensively studied in many references in the past. However, there is a variation of the μ - τ symmetry which does not require any breaking and is still well compatible with experiments. It is called μ - τ reflection symmetry [9,15–18], which attaches the CP transformation to the interchange of ν_μ and ν_τ ,

$$\nu_e \rightarrow \bar{\nu}_e, \quad \nu_\mu \rightarrow \bar{\nu}_\tau, \quad \nu_\tau \rightarrow \bar{\nu}_\mu. \quad (1)$$

The μ - τ reflection symmetry allows nonzero θ_{13} and predicts $\theta_{23} = 45^\circ$ and $\delta = \pm 90^\circ$. Consequently it has aroused a lot of interest recently [19–33]. To generate TBM mixing the μ - τ symmetry determines the third column of this mixing matrix and there is another \mathbb{Z}_2 symmetry that is responsible for the first or second column [34–36]. Those \mathbb{Z}_2 symmetries are assumed to be “residual symmetries”, after the full flavor group is broken. They could be accidental or subgroups of the full flavor group. If the μ - τ symmetry is replaced with μ - τ reflection symmetry, then we get a variation of TBM with its first or second

column fixed and at the same time we will have nonzero θ_{13} , $\theta_{23} = 45^\circ$, and $\delta = \pm 90^\circ$. We study the consequences of this assumption in this paper. General deviations of the TBM mixing with some part being fixed have been discussed in many references [37–46] and the case that the first/second column is fixed is usually referred to as TM1/TM2 mixing, respectively [40]. In the TBM mixing, $\theta_{12} = \sin^{-1} \frac{1}{\sqrt{3}} \approx 35.3^\circ$ is a little higher than the global best-fit value $\theta_{12}^{\text{exp}} = 33.56^{+0.77}_{-0.75}$ [47], while in TM1 or TM2 it deviates from 35.3° with a lower or a higher value, respectively [40]:

$$\begin{aligned} \theta_{12}^{\text{TM1}} &= \cos^{-1} \left(\frac{\sqrt{2}}{\sqrt{3} \cos \theta_{13}} \right) \approx 34.2^\circ, \\ \theta_{12}^{\text{TM2}} &= \sin^{-1} \left(\frac{1}{\sqrt{3} \cos \theta_{13}} \right) \approx 35.8^\circ. \end{aligned} \quad (2)$$

Since θ_{12}^{TM1} is well compatible with θ_{12}^{exp} while θ_{12}^{TM2} is disfavored at about 3σ , in this paper we will consider TM1 only.

When the TM1 symmetry,¹ and μ - τ reflection symmetry are imposed on the neutrino mass terms simultaneously, all the PMNS parameters except for θ_{13} are predicted (in addition to the predictions mentioned above, the two Majorana phases are 0 or π). In the near future, this framework can be tested not only by a precision measurement of θ_{12} and θ_{23} , but also by the confirmation of a maximal Dirac CP phase $|\delta| = \pi/2$, for which hints have recently appeared in T2K [48,49]. Besides, its predictions on Majorana phases could be verified in neutrinoless double beta decay ($0\nu\beta\beta$) experiments [50].

Note that both the TM1 symmetry and μ - τ reflection symmetry may be residual symmetries of a larger flavor

¹For simplicity, we will refer to the symmetry responsible for the TM1 mixing as the TM1 symmetry in this paper.

symmetry broken at a high energy scale. Since μ - τ reflection symmetry is essentially a generalized CP symmetry, looking for a horizontal flavor symmetry that contains it as a subsymmetry is more interesting and also more complicated. This is an active subject of ongoing research and some non-Abelian discrete groups in semidirect product form, such as $A_4 \rtimes \mathbb{Z}_2^{CP}$, $S_4 \rtimes \mathbb{Z}_2^{CP}$, $\Delta(6n^2) \rtimes \mathbb{Z}_2^{CP}$ can be the origin of the mixing scheme that we study here [51–54].

It is most likely that the predictions of TM1 and μ - τ reflection symmetries are exact only at the scale where the horizontal flavor symmetry breaks into these residual symmetries. When going to lower energy scales these predictions will unavoidably receive corrections from renormalization group (RG) running [55]. Therefore in this paper, we will also study the RG corrections on the predictions from the joined TM1 and μ - τ reflection symmetry. We consider the case in which neutrino mass is described by the effective Weinberg operator, as well as by the most popular realization of this operator, the type I seesaw [56–59]. Both the SM and the (minimal supersymmetric standard model) MSSM are assumed.

The remainder of the paper is organized as follows. In Sec. II, we introduce the TM1 symmetry and the μ - τ reflection symmetry, and study the phenomenology if both are simultaneously present. Then we study the RG running effects on the PMNS parameters in the cases we mentioned above, presented in Sec. III. Finally we summarize our result and conclude in Sec. IV.

II. TRIMAXIMAL μ - τ REFLECTION SYMMETRY

The TM1 mixing and its symmetry as well as model-building aspects have been studied in many references (see e.g., [40,44–46,60–62]). In the following we denote the TM1 symmetry as $\mathbb{Z}_2^{\text{TM1}}$. The μ - τ reflection symmetry was originally proposed in Refs. [9,15–18] and later extensively studied in, e.g., [19–23,25–33]. It can be regarded as a generalized CP symmetry [63,64] so we use \mathbb{Z}_2^{CP} to denote it.² Although both symmetries as well as their phenomenology have been extensively studied in the literature, their combination which provides a very effective description of the neutrino mixing data with only one free parameter, has attracted much less attention. Therefore in this section, we will discuss the theoretical and phenomenological aspects of this combination.

The explicit transformations of $\mathbb{Z}_2^{\text{TM1}}$ and \mathbb{Z}_2^{CP} in the flavor basis are given as

$$\mathbb{Z}_2^{\text{TM1}}: \nu \rightarrow R^{\text{TM1}} \nu, \quad (3)$$

$$\mathbb{Z}_2^{\text{CP}}: \nu \rightarrow R^{\mu\tau} \bar{\nu}, \quad (4)$$

²We prefer the symbol \mathbb{Z}_2^{CP} to $\mathbb{Z}_2^{\mu\tau}$ for the μ - τ reflection symmetry because the latter is widely used for the μ - τ symmetry without CP transformation.

where $\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)^T$ and the two matrices R^{TM1} and $R^{\mu\tau}$ are

$$R^{\text{TM1}} \equiv -\frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}, \quad (5)$$

$$R^{\mu\tau} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (6)$$

The matrix R^{TM1} has been derived in, e.g., Ref. [35] while the form of $R^{\mu\tau}$ is obvious according to the meaning of interchanging the μ and τ flavor. Since the μ - τ reflection symmetry is essentially a generalized CP symmetry, it is necessary to check the consistency condition of flavor symmetry and CP symmetry [65]:

$$R^{\text{TM1}} R^{\mu\tau} = R^{\mu\tau} (R^{\text{TM1}})^*. \quad (7)$$

The neutrino mass terms

$$\mathcal{L} \supset -\nu_\alpha M_{\alpha\beta}^\nu \nu_\beta + \text{H.c.}, \quad (8)$$

should be invariant under the transformations in Eqs. (3) and (4). Therefore, the mass matrix M^ν should satisfy

$$(R^{\text{TM1}})^T M^\nu R^{\text{TM1}} = M^\nu, \quad (9)$$

$$(R^{\mu\tau})^T M^\nu R^{\mu\tau} = (M^\nu)^*. \quad (10)$$

The above two equations can be broken down into equations in terms of the entries of M^ν , so one can obtain explicit constraints on those:

$$M_{11}^\nu, M_{23}^\nu = \text{real}, \quad (11)$$

$$M_{12}^\nu = (M_{13}^\nu)^*, \quad (12)$$

$$M_{22}^\nu = (M_{33}^\nu)^*, \quad (13)$$

$$\text{Im}(M_{22}^\nu) = 2\text{Im}(M_{23}^\nu), \quad (14)$$

$$M_{11}^\nu = \sum_i \text{Re}(M_{i2}^\nu). \quad (15)$$

The above equations are equivalent to Eqs. (9) and (10), which means they are sufficient and necessary conditions for Eq. (8) being invariant under the transformations. With the above constraints, M^ν can be parametrized by four real parameters r , $x_{1,2}$, and y :

$$M^\nu = \begin{pmatrix} r + x_1 + x_2 & x_1 & x_1 \\ x_1 & x_2 & r \\ x_1 & r & x_2 \end{pmatrix} + iy \begin{pmatrix} 0 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & 2 \end{pmatrix}. \quad (16)$$

As one can check, Eq. (16) is the most general mass matrix that satisfied Eqs. (9) and (10). The mass matrix contains only four real parameters; those are the three neutrinos masses and one degree of freedom for the PMNS matrix. As we will show later, this degree of freedom is just θ_{13} . Therefore, the mass matrix with the form in Eq. (16) is highly predictive. It predicts all the parameters except for θ_{13} in the PMNS matrix, including two mixing angles (θ_{12}, θ_{23}), one Dirac phase δ and two Majorana phases (α_{21}, α_{31}).

The mass matrix is diagonalized by

$$(U^\nu)^T M^\nu U^\nu = \text{diag}(m'_1, m'_2, m'_3). \quad (17)$$

For M^ν in Eq. (16), due to the residual symmetries, U^ν can be analytically solved:

$$U^\nu = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2}c & \sqrt{2}s \\ 1 & -\sqrt{2}c - i\sqrt{3}s & i\sqrt{3}c - \sqrt{2}s \\ 1 & -\sqrt{2}c + i\sqrt{3}s & -i\sqrt{3}c - \sqrt{2}s \end{pmatrix}, \quad (18)$$

where $(s, c) = (\sin \theta, \cos \theta)$ are given by

$$s = \sqrt{\frac{\Delta + x_1 - 2x_2}{2\Delta}}, \quad c = \text{sign}(y) \sqrt{1 - \frac{x_1 - 2x_2 + \Delta}{2\Delta}}, \quad (19)$$

and

$$(m'_1, m'_2, m'_3) = \left(r + 2x_1 + x_2, r + \frac{\Delta}{2} - \frac{x_1}{2}, r - \frac{\Delta}{2} - \frac{x_1}{2} \right), \quad (20)$$

$$\Delta \equiv \sqrt{24y^2 + (x_1 - 2x_2)^2}. \quad (21)$$

Here $\text{sign}(y)$ implies that we have taken $c = \sqrt{1 - s^2}$ for positive y and $c = -\sqrt{1 - s^2}$ for negative y . Note that (m'_1, m'_2, m'_3) computed from Eq. (20) are not necessarily positive (but always real), so they may be different from the neutrino masses by some minus signs.

Comparing the above result to the standard parametrization of the PMNS matrix

$$U_{\text{PMNS}} = U \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}), \quad (22)$$

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ -e^{i\delta}c_{12}c_{23}s_{13} + s_{12}s_{23} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}, \quad (23)$$

we can extract the predictions on all the PMNS parameters. It turns out that the predictions differ for positive and negative y . Next we will discuss both cases:

(i) Positive y ($y > 0$)

If $y > 0$, then $c = \sqrt{1 - s^2}$ is positive. We extract some phases from U^ν so that

$$\text{diag}(1, -e^{i\beta}, e^{-i\beta})U^\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\sqrt{1-s^2}}{\sqrt{3}} & \frac{is}{\sqrt{3}} \\ \frac{2i\sqrt{3}s-3\sqrt{2-2s^2}}{6\sqrt{3-s^2}} & \frac{6+is\sqrt{6-6s^2}}{6\sqrt{3-s^2}} & \frac{\sqrt{3-s^2}}{\sqrt{6}} \\ \frac{2i\sqrt{3}s+3\sqrt{2-2s^2}}{6\sqrt{3-s^2}} & \frac{-6+is\sqrt{6-6s^2}}{6\sqrt{3-s^2}} & \frac{\sqrt{3-s^2}}{\sqrt{6}} \end{pmatrix} \text{diag}(1, 1, -i), \quad (24)$$

has the same phase convention as the standard parametrization, which requires

$$\beta = \arg(\sqrt{3}c - i\sqrt{2}s). \quad (25)$$

Comparing Eq. (24) to Eqs. (23) and (22), we get

$$\theta_{23} = 45^\circ, \quad \delta = -90^\circ, \quad c_{12} = \sqrt{\frac{2}{3}} \frac{1}{c_{13}}. \quad (26)$$

If $m'_{1,2,3} \geq 0$, then the Majorana phases should be $(1, e^{ia_{21}/2}, e^{ia_{31}/2}) = (1, 1, -i)$. However, $m'_{1,2,3}$ could be negative, which can be converted to positive by further adding some phases to the right-hand side of Eq. (24). Therefore the actual Majorana phases depend on the signs of $m'_{1,2,3}$:

$$(1, e^{ia_{21}/2}, e^{ia_{31}/2}) = \left(1, \sqrt{\text{sign}(m'_2/m'_1)}, -i\sqrt{\text{sign}(m'_3/m'_1)}\right). \quad (27)$$

(ii) Negative y ($y < 0$)

If $y < 0$, then $c = -\sqrt{1-s^2}$ is negative so we need to remove the minus sign of the 12-entry of (18). Therefore Eqs. (24) and (25) are modified to

$$\begin{aligned} & \text{diag}(1, -e^{i\beta}, e^{-i\beta})U^\nu \\ &= \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\sqrt{1-s^2}}{\sqrt{3}} & -\frac{is}{\sqrt{3}} \\ \frac{-2i\sqrt{3}s-3\sqrt{2-2s^2}}{6\sqrt{3-s^2}} & \frac{6-is\sqrt{6-6s^2}}{6\sqrt{3-s^2}} & \frac{\sqrt{3-s^2}}{\sqrt{6}} \\ \frac{-2i\sqrt{3}s+3\sqrt{2-2s^2}}{6\sqrt{3-s^2}} & \frac{-6-is\sqrt{6-6s^2}}{6\sqrt{3-s^2}} & \frac{\sqrt{3-s^2}}{\sqrt{6}} \end{pmatrix} \\ & \times \text{diag}(1, -1, i), \end{aligned} \quad (28)$$

where now

$$\beta = \arg(-\sqrt{3}c + i\sqrt{2}s). \quad (29)$$

In this case, comparing with the standard parametrization of the PMNS matrix we have

$$\theta_{23} = 45^\circ, \quad \delta = 90^\circ, \quad c_{12} = \sqrt{\frac{2}{3}} \frac{1}{c_{13}}, \quad (30)$$

and the Majorana phases are

$$(1, e^{ia_{21}/2}, e^{ia_{31}/2}) = \left(1, \sqrt{\text{sign}(m'_2/m'_1)}, i\sqrt{\text{sign}(m'_3/m'_1)}\right).$$

As a summary, we have

$$\theta_{23} = 45^\circ, \quad \theta_{12} = \cos^{-1}\left(\sqrt{\frac{2}{3}} \frac{1}{c_{13}}\right) \approx 34.2^\circ, \quad (31)$$

if the experimental value $\theta_{13} \approx 9^\circ$ is taken as an input, and

$$\delta = \pm 90^\circ, \quad \alpha_{21} = \frac{\pi}{2} \pm \frac{\pi}{2}, \quad \alpha_{31} = \frac{\pi}{2} \pm \frac{\pi}{2}, \quad (32)$$

where the positive/negative signs depending on the signs of y and (m'_1, m'_2, m'_3) computed from Eq. (20). Here the Majorana phases are predicted to be either 0 or π . There have been many studies [66–74] on the option to measure

the Majorana phases with upcoming neutrinoless double beta decay ($0\nu\beta\beta$) experiments. It was demonstrated in particular that expected nuclear and experimental uncertainties allow us in principle to measure the phases, or at least constrain them nontrivially. The actual physical observable for $0\nu\beta\beta$ is the effective mass $|M_{ee}|$, which has significant dependence on the Majorana phases. For the inverted mass ordering, $|M_{ee}|$ is always nonzero, which necessarily leads to $0\nu\beta\beta$ at some level. For the normal mass ordering, it is well known that $|M_{ee}|$ can be zero for very small neutrino mass; however, $|M_{ee}| = 0$ does not mean that $0\nu\beta\beta$ experiments tell us nothing about the Majorana phases. As it has been noticed in Refs. [67,75], this case still gives some constraints on the Majorana phases. In the scenario of this work, the relation between $|M_{ee}|$ and the Majorana phases is more explicit because all the neutrino parameters except for the lightest neutrino mass m_L have been determined by symmetries or by experiments, enabling us to compute $|M_{ee}|$ explicitly, as shown in Fig. 1. Note that in this scenario, $|M_{ee}| < 10^{-3}$ eV is possible only if $\alpha_{21} = \pi$. So if the future experiments push the upper bound of $|M_{ee}|$ down to 10^{-3} eV and still do not observe $0\nu\beta\beta$ decay, then we can draw the conclusion that $\alpha_{21} = \pi$.

We can confront the predictions of the mixing scheme with current data [47]. First we study the predictions of TM1 mixing, namely the first column of the PMNS matrix being $(\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{6}}, \sqrt{\frac{1}{6}})^T$. The χ^2 -function is defined as

$$\chi^2 = \sum \frac{(x_i - x_i^0)^2}{\sigma_i^2}, \quad (33)$$

where x_i^0 represents the data of the i th experimental observable, σ_i the corresponding 1σ absolute error, and

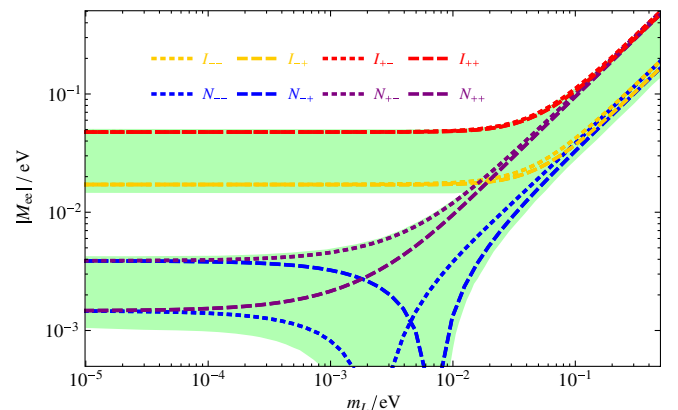


FIG. 1. Prediction on the effective mass $|M_{ee}|$, according to Eqs. (31) and (32). In the notation $N_{\pm\pm}$ ($I_{\pm\pm}$), N/I standards for normal/inverted mass ordering respectively and the subscripts are the signs of two Majorana phases $e^{ia_{21}}$ and $e^{ia_{31}}$ (α_{21}, α_{31} are always 0 or π in this model). The light green region is the bound from the global fit, taken from [76].

x_i the prediction of the model. For the normal ordering, TM1 has a χ^2 -minimum of $1.14(=0.063+0.000+1.058+0.0223)$ at the values $\theta_{13}=8.5^\circ$ and $\theta_{23}=41.6^\circ$. The numbers in brackets denote the contributions of θ_{13} , θ_{23} , θ_{12} , and δ to the total value. In case of an inverted ordering, the χ^2 -minimum is $1.20(=0.006+0.000+1.056+0.143)$ at the values $\theta_{13}=8.5^\circ$ and $\theta_{23}=50.0^\circ$. Note that TM1 has two free parameters. Combining TM1 with μ - τ reflection symmetry, which in total has only one free parameter, gives for the normal ordering a χ^2 -minimum of $3.88(=0.063+2.730+1.058+0.0308)$ at the value $\theta_{13}=8.5^\circ$. In the inverted ordering, the χ^2 -minimum is $5.76(=0.006+4.672+1.056+0.0234)$ at the value $\theta_{13}=8.5^\circ$.

III. RG CORRECTIONS

The residual symmetries we discussed in the previous section may appear at a very high energy scale, which we refer to as the flavor symmetry scale. Due to radiative corrections, the predictions at the flavor symmetry scale may be modified at the low energy scale, at which they are confronted with experimental measurements. If there is no new physics between the two scales, the corrections can be computed without many unknown parameters involved. However, it is also possible that some new physics appear in the middle so that the RG corrections would depend on more unknown parameters. For example, in the type I seesaw mechanism, the masses of right-handed neutrinos could be below the flavor symmetry scale; in this case the RG corrections would also depend on the masses of right-handed neutrinos.

A. RG running based on the Weinberg operator

To avoid the dependence on too many parameters, we will first focus on the case that all other new physics scales are above the flavor symmetry scale. In this case, the calculation will be based on the RGE of the SM extended by the Weinberg operator,

$$\mathcal{L} \supset \frac{1}{4} \kappa_{\alpha\beta} (\tilde{H}^\dagger L_\alpha) (\tilde{H}^\dagger L_\beta) + \text{H.c.}, \quad (34)$$

where L is the lepton doublet and H the Higgs doublet. After electroweak symmetry breaking $\langle \tilde{H} \rangle = (v/\sqrt{2}, 0)^T$, the neutrino mass matrix is given by

$$M_{\alpha\beta}^\nu = -\frac{v^2}{4} \kappa_{\alpha\beta}. \quad (35)$$

Constrained by the residual symmetries, M^ν depends on four parameters (r, x_1, x_2, y) in Eq. (16). Those parameters are actually highly constrained by neutrino oscillation measurements on the two mass-squared differences

$$\delta m^2 \equiv m_2^2 - m_1^2, \quad \Delta m^2 \equiv m_3^2 - \frac{m_1^2 + m_2^2}{2}, \quad (36)$$

and $\sin \theta_{13}$. In this section we will fix them at the best-fit values [47,77] as the result of our calculation varies very little within experimental uncertainties. If the lightest neutrino mass m_L is also known, then (r, x_1, x_2, y) can be determined by $(\theta_{13}, \delta m^2, \Delta m^2, m_L)$. In Sec. II we have demonstrated how to compute $(\theta_{13}, \delta m^2, \Delta m^2, m_L)$ for given values of (r, x_1, x_2, y) . Determining (r, x_1, x_2, y) from experimental values of $(\theta_{13}, \delta m^2, \Delta m^2, m_L)$ is then of course also possible.

However there are some positive/negative signs one needs to choose in determining (r, x_1, x_2, y) . The first one is the sign of Δm^2 , known as the neutrino mass ordering. Both the normal (NO, $\Delta m^2 > 0$) and the inverted ordering (IO, $\Delta m^2 < 0$) should be taken into consideration. The next one is the sign of the Dirac phase δ . The μ - τ reflection symmetry only predicts $|\delta| = 90^\circ$ but both $+90^\circ$ and -90° are possible. Besides, as summarized in Eq. (32), the two Majorana phases take values of $\frac{\pi}{2} \pm \frac{\pi}{2}$, where we have to choose between the positive/negative signs.

Therefore, there are four positive/negative signs (and thus 16 physically inequivalent cases) relevant in determining (r, x_1, x_2, y) . However, as it can be seen from the mass matrix, for $\delta = +90^\circ$ and -90° , the mass matrix in one case is simply the complex conjugate of the other, so we only need to study one of the two cases. Actually, the result of RG running of both cases shows that the radiative corrections on both cases are the same except that for δ it differs by a minus sign. This reduces the 16 cases to 8 cases in our analysis. In addition, the case of positive $\delta = +90^\circ$ is disfavored by current global fits. For simplicity, we refer to the 8 cases as $N_{\pm\pm}$ and $I_{\pm\pm}$ where N/I stands for the normal/inverted ordering and the two \pm stand for the signs of $e^{i\alpha_{21}}$ and $e^{i\alpha_{31}}$, respectively.

We solve the RGEs using the code REAP [78] and compute the RG corrections. The results are presented in Fig. 2 for N_{++} and Fig. 3 for all the 8 cases. We set the flavor symmetry scale at $\Lambda = 10^{14}$ GeV. Actually as shown in Fig. 2 the RG corrections depend linearly on $\log \Lambda$, so if Λ is changed to another value Λ' , the RG corrections can be evaluated correspondingly by simply multiplying a factor of $\log \Lambda' / \log \Lambda$. Another parameter that may have significant effect is the lightest neutrino mass m_L . In Fig. 3 we show the RG corrections for different values of m_L by green, yellow, orange, and red points, corresponding to $m_L = (1, 20, 40, 60)$ meV respectively³. We assume here that strong limits on the neutrino mass scale from cosmology are valid [79] and simply note that the effect of running roughly scales with m_L for values larger than 60 meV.

As shown in Fig. 3, typically the corrections to θ_{12} , θ_{13} , θ_{23} , δ , α_{21} , α_{31} are about 0.1, 0.001, 0.005, 0.1, 0.05, 0.1 degrees respectively, except for some cases where due

³We do not take $m_L = 0$ here because for $m_L = 0$, the RG corrections are almost the same as $m_L = 1$ meV except for the Majorana phases which are not well defined when $m_L = 0$.

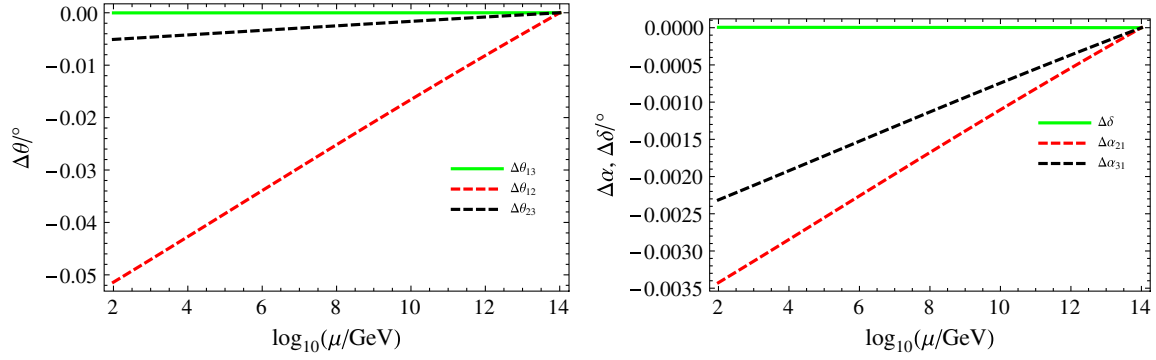


FIG. 2. RG running of the mixing angles (left panel) and the Dirac/Majorana phases (right panel) in the SM for the normal hierarchy and $m_L = 0.05$ eV.

to some cancellations the RG corrections are suppressed. To understand the cancellation, we take θ_{12} as example, for which the analytic expression reads [80]

$$\frac{d\theta_{12}}{d\ln\mu} = -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} s_{23}^2 \frac{|m_1 + m_2 e^{i\alpha_{21}}|^2}{\delta m^2} + \mathcal{O}(\theta_{13}). \quad (37)$$

Here y_τ is the tau-lepton Yukawa coupling. The plot for θ_{12} in Fig. 3 shows that the corrections in the four cases $N_{\pm\pm}$ and $I_{\pm\pm}$ are suppressed, which can be understood from Eq. (37): the correction is proportional to $|m_1 + m_2 e^{i\alpha_{21}}|^2$, which can be small if $e^{i\alpha_{21}} = -1$ and $m_1 \approx m_2$. The latter always happens in the inverted ordering and in the normal ordering when the smallest mass m_1 approaches $\sqrt{\delta m^2}$.

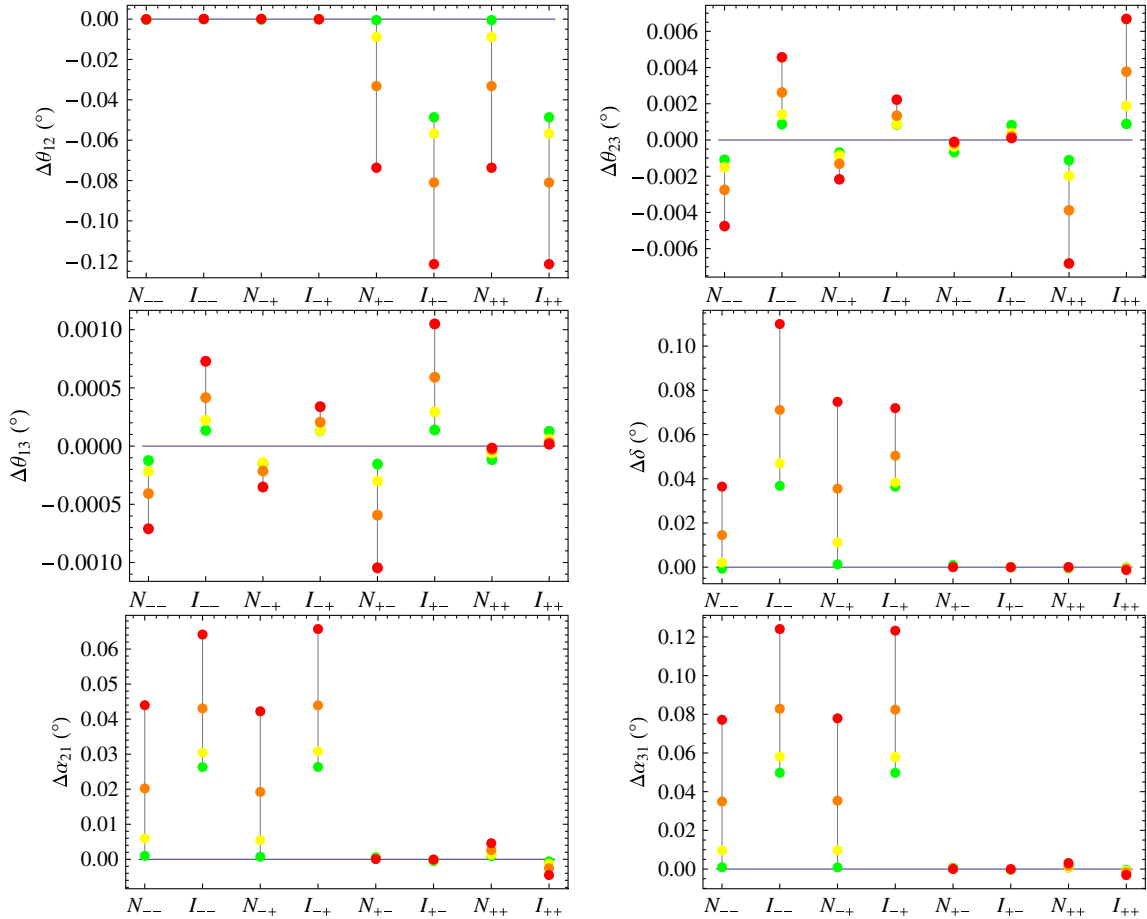


FIG. 3. RG corrections for all the 8 cases, N/I for normal/inverted hierarchy and $+/-$ for $e^{i\alpha} = +1/-1$ where α stands for the two Majorana phases. The result depends on the lightest neutrino mass, which is set at (1,20,40,60) meV for points colored from green to red.

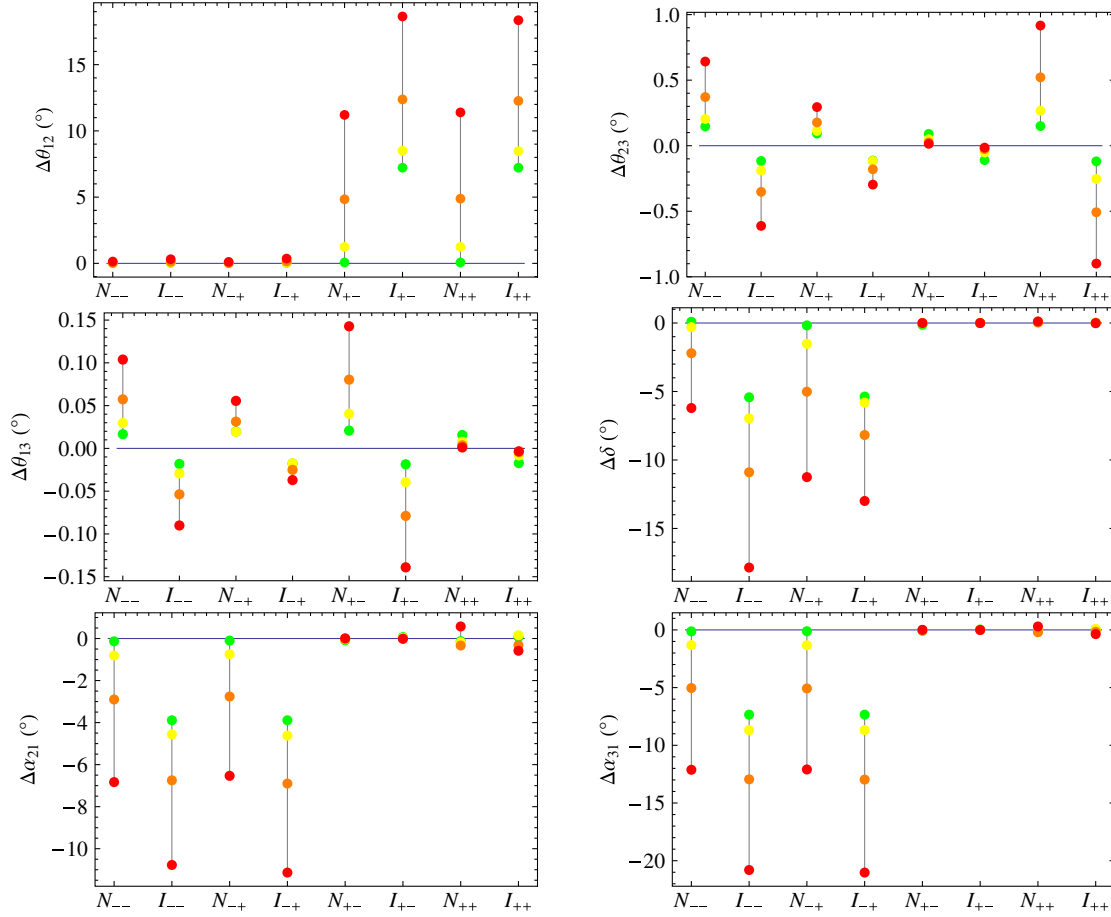


FIG. 4. Same as Fig. 3 except that the SM is replaced with the MSSM for $\tan\beta = 20$.

Except for some cases with cancellations, the RG corrections generally increase when m_L increases. This behavior is very common regarding small perturbations to the mass matrix, which has been studied in Ref. [81] from a more general point of view. The reason is because for larger m_L , the mass spectrum is closer to the quasidegenerate situation, where the PMNS mixing becomes unstable when the mass matrix suffers perturbations. Besides, among the three mixing angles, θ_{12} generally receives the largest correction (except for cancellations); this is because the gap between m_1 and m_2 is much smaller than that of m_1 and m_3 or m_2 and m_3 .

Since all the corrections are at the order of or even lower than 0.1° , we can draw the usual conclusion that in the context of the SM with the Weinberg operator only, the RG corrections are negligible when compared with current and near future experimental measurements. As is well known, if we replace the SM with the MSSM, then according to Ref. [80] the RG corrections to the neutrino mixing would be amplified by a factor of $\tan^2\beta$. To illustrate this effect, we compute the RG corrections again in the context of the MSSM with $\tan\beta = 20$, and the result is shown in Fig. 4. As one can see, the RG corrections in the MSSM with large $\tan\beta$

are significantly enhanced to measurable values compared to Fig. 3.

B. RG running based on type I seesaw

In this section, we consider new physics that appears below the flavor symmetry scale. The Weinberg operator itself is UV incomplete and is usually believed to be a low-energy effective operator. Here we consider the type I seesaw realization of this operator only. Heavy right-handed neutrinos N_i ($i = 1, 2, \dots$) are integrated out to generate the Weinberg operator. We consider the scenario that the right-handed neutrino masses (or the seesaw scale) are lower than the flavor symmetry scale. So at the flavor symmetry scale, we should consider the symmetry of the following Lagrangian instead of the Weinberg operator,

$$\mathcal{L} \supset -y_{ij} N_i \tilde{H}^\dagger L_j - \frac{1}{2} N_i M_{ij} N_j + \text{H.c.} \quad (38)$$

Next we need to specify the transformation rules of $\mathbb{Z}_2^{\text{TM1}}$ and \mathbb{Z}_2^{CP} for the right-handed neutrinos. This depends on how we assign the right-handed neutrinos to the representations of the flavor symmetry, which is rather model-dependent. For simplicity, we assume that the number of

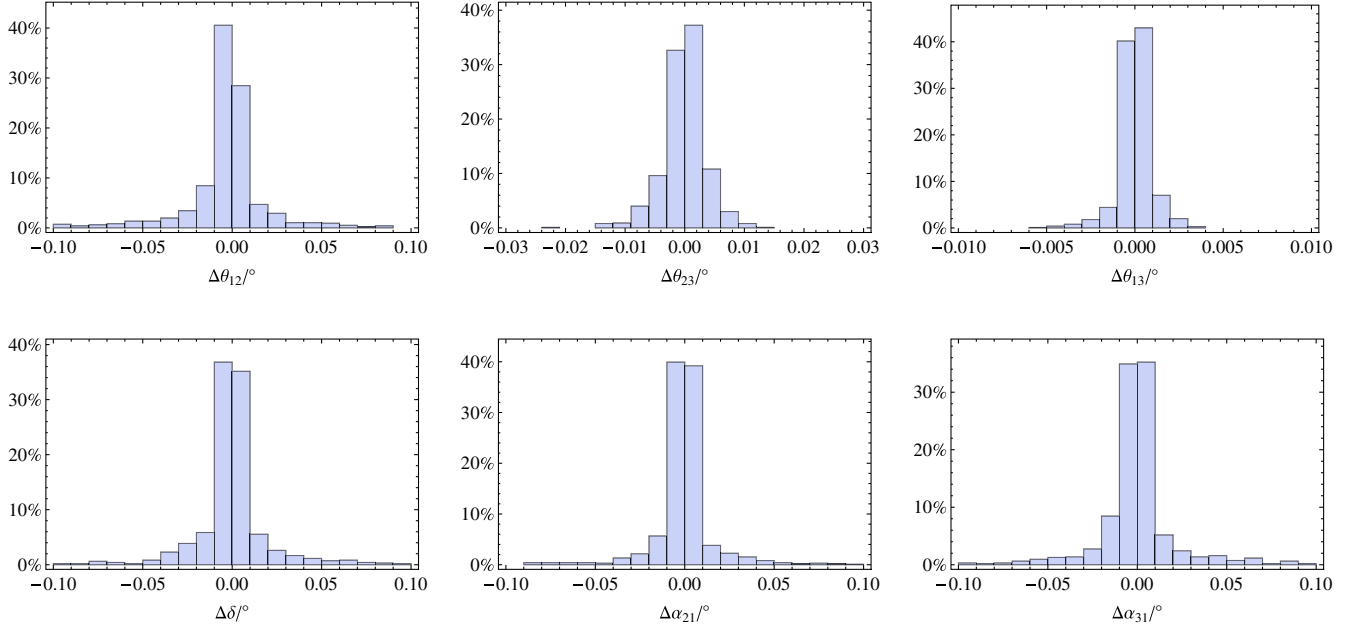


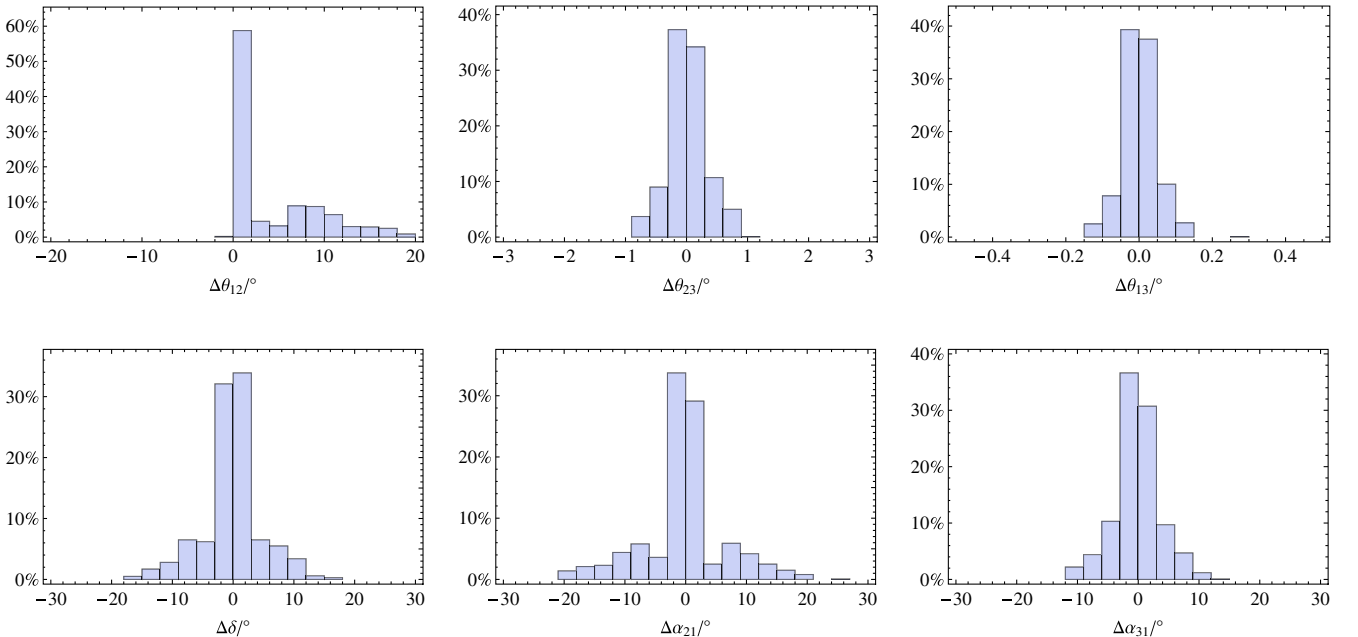
FIG. 5. The distributions of RG correction in the SM extended by the type I seesaw.

right-handed neutrinos is three and that they have the same transformation rule as the left-handed neutrinos. As a result, both the Dirac mass matrix m_D and the heavy Majorana matrix M will be in the form of Eq. (16). As one can check explicitly, if both m_D and M are in the form of Eq. (16), then the light-neutrino mass matrix

$$M^\nu = -m_D^T M^{-1} m_D \quad (39)$$

is also of the form in Eq. (16). As we have discussed, each matrix of the form (16) contains four real parameters

thus in the Lagrangian (38) we have 8 free parameters. The tree-level predictions in Eqs. (31) and (32) are independent of the values of these parameters. However, the RG corrections inevitably depend on these parameters. As we have argued, when some new physics such as the right-handed neutrinos appears below the flavor symmetry scale, the RG corrections would usually depend on many unknown parameters, which makes it difficult to evaluate the RG corrections exactly. To understand generally how large the RG corrections would be, we adopt random scattering in the allowed parameter

FIG. 6. The distributions of RG correction in the MSSM with $\tan\beta = 20$ extended by the type I seesaw.

space rather than focus on some specific parameter settings.

We randomly generate 1000 samples with right-handed neutrino masses M_1, M_2, M_3 distributed from 10^6 GeV to 10^{13} GeV and the lightest neutrino mass m_L from 1 meV to 60 meV. Rectangular distributions are used for $\log M_{1,2,3}$ and m_L . The positive/negative signs of Δm^2 and Dirac/Majorana phases are also chosen randomly. The Yukawa couplings can be computed once (M_1, M_2, M_3) and (m_1, m_2, m_3) have been set. We again use the code REAP [78], which automatically integrates out the heavy right-handed neutrinos when the energy scale goes below their masses.

The results are presented in Fig. 5, where we can see most RG corrections are distributed in small ranges, e.g., $\Delta\theta_{12}$, $\Delta\theta_{23}$, and $\Delta\theta_{13}$ are most likely less than 0.05° , 0.01° , and 0.005° respectively. So generally, the deviations are similar to the results in Fig. 3 where right-handed neutrinos are not introduced. However, large corrections are also possible. We do not find any significant cutoff of the deviations when the number of samples are increased, though the distributions above remain almost the same. This implies the RG corrections could be very large, but would require fine-tuning in the parameter space. For example, when the number of samples is increased to 10^4 , we find only two samples with $|\Delta\theta_{23}| > 3^\circ$. Therefore, we can draw the conclusion that generally the RG corrections in the type I seesaw scenario are of similar magnitude as with the Weinberg operator only.

Again, the RG corrections can be significantly amplified within supersymmetric scenarios. We compute the RG corrections in the MSSM extended by the type I seesaw with $\tan\beta = 20$. The result is shown in Fig. 6 where we can see that compared to Fig. 5, the RG corrections in the MSSM with large $\tan\beta$ are significantly enhanced by up to two orders of magnitude to measurable values.

IV. CONCLUSION

Combining μ - τ reflection and TM1 symmetry leads to a very predictive framework. We have shown in Sec. II that it

not only can accommodate nonzero θ_{13} but also predicts all other PMNS parameters, including all CP phases ($\delta = \pm\pi/2$ and the Majorana phases are 0 or π).

With these symmetries, the neutrino mass matrix can be constrained to the form (16) containing only four real parameters. Given the experimental values of θ_{13} , δm^2 , and Δm^2 as input, the mass matrix can be exactly reconstructed for a fixed value of the smallest mass m_L and several choices of positive/negative signs. Therefore, for the SM extended by the Weinberg operator, the RG corrections can be exactly evaluated as the only free parameter is m_L .

We have computed the RG corrections to the scenario, which are in agreement with known results, namely that in the SM they are typically small, but can be enhanced to measurable values within supersymmetric scenarios and within explicit multiscale scenarios such as the type I seesaw mechanism.

In summary, the mixing scheme we propose here is very well compatible with data and addresses the closeness of δ with $-\pi/2$, of θ_{23} with $\pi/4$ and that $\sin^2\theta_{12}$ is slightly less than $1/3$. If future data confirms those special values of the mixing parameters, the proposed scheme seems an attractive approach to the description of lepton mixing. On the other hand, some deviations could occur in the future, which could either be explained by RG corrections if the deviations are small, or exclude this mixing scheme if they are large. One particularly noteworthy example is the deviation of θ_{23} from 45° , which was recently hinted by the NOVA measurement [82] $\theta_{23} = 39.5_{-1.3}^{+1.7}$ or $52.2_{-1.8}^{+1.3}$. Such a large deviation ($\gtrsim 5^\circ$) if confirmed by future data, would exclude this mixing scheme embedded in the simple scenarios considered in this paper.

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