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Naturally realized two dark Z's near the electroweak scale

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Chiral representations are the key to obtaining light fermions from some ultraviolet completed theories. The well-known chiral example is one family set of 15 chiral fields in the standard model. We find a new chiral theory $\mathrm{SU}(2)_{\mathrm{dark}} \times \mathrm{U}(1)_Q$ with 16 chiral fields, which does not have any gauge and gravitational anomalies. The group $\mathrm{SU}(2)_{\mathrm{dark}} \times \mathrm{U}(1)_Q$ may belong to the dark sector, and we present a derivation of the spectrum from the $\mathrm{E}_8 \times \mathrm{E}_8'$ heterotic string. Necessarily, there appear two degrees at low energy: two dark-Z''s, or a dark-Z' plus a dark photon. Being chiral, there is a chance to probe this theory at TeV accelerators. Since the model belongs to the dark sector, the way to probe it is through the kinetic mixing.

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I. INTRODUCTION

In particle physics, there has been a deep question known as the gauge hierarchy problem: "How do the Standard Model (SM) fermions appear at such a small electroweak scale, compared to an ultraviolet completed scale, the Planck mass M_{P}, or the grand unification (GUT) scale M_{GUT}?" Two issues in the hierarchy problem are as follows: (i) obtaining massless SM particles at the ultraviolet completed scale, and (ii) rendering the electroweak scale masses to the SM fermions. The first issue is resolved by the profound and simple requirement, a chiral theory at the ultraviolet completed scale [1]. The second issue is the method obtaining the vacuum expectation value (VEV) of the Higgs field at the electroweak scale $v_{\mathrm{ew}} \simeq 246$ GeV, a kind of TeV scale, for which the most well-known example is supersymmetry (SUSY) [2].

In this paper, we propose that any particles appearing at the TeV scale, for a detection possibility at the LHC, must satisfy condition (i). The best known example is a spinor representation in the $\mathrm{SO}(4n+2)$ GUT models [3]. Orbifolding in extra dimensions presents a possibility of massless particles, as shown in a simple field theoretic orbifold [4]. But, the orbifold compactification in string theory is the prototype example [5], providing a simple geometrical interpretation. Note, however, that fermionic constructions [6] and Gepner models [7] have also been used in four-dimensional (4D) phenomenology from string. In these 4D constructions, it was necessary to check whether vectorlike representations of exotically charged particles, which appear quite often, are present or not present as studied in Refs. [8].

Anyway, condition (i) is the basic requirement we satisfy at low-energy effective theory in four dimensions. To realize condition (ii), model parameters are required to be known in detail, and hence we do not discuss it here except by pointing out several mass scales in particle physics. The SM chiral theory, realized in nature, describes the electroweak scale physics successfully. So, we anticipate that if a natural chiral model is found, then it might have a great chance to be realized in nature. Since any new particle has not been detected at the LHC so far, a new particle in the new chiral theory, which interacts with the SM sector extremely feebly, must be in the dark sector. Here, the dark sector is not introduced just for explaining cold dark matter (CDM) of the Universe. The well-known CDM examples, "invisible" axions [9], and weaky interacting massive particles [10], belong to the visible sector. On the other hand, the heterotic $\mathrm{E}_8 \times \mathrm{E}_8'$ string [11] implies a possibility of a dark sector from E_8'. If the dark sector introduces CDM, then it is just a bonus. In fact, a dark sector for various possibilities of CDM particles has been introduced earlier [12] to account for the excess of positron spectra. Even though the dark sector interacts with the SM sector extremely feebly, it can be probed by the kinetic mixing terms [13] of two SM gauge bosons, photon and Z. Since the dark sector does not carry the SM weak hypercharge, the charge raising and lowering gauge bosons in the dark sector cannot have kinetic mixings with $W^{\pm}$ of the SM.

In the dark sector, a simple chiral theory is shown to be $\mathrm{SU}(2)_{\mathrm{dark}} \times \mathrm{U}(1)_Q$, which does not have any gravitational and gauge anomalies. Because the rank of this gauge group is 2, we will have two Z' gauge bosons at low energy. This minimal model will be called two dark Z model (TDZ). The first new particles observed at the CERN SPS proton-antiproton collider were $W^{\pm}$ and Z [14]. This is because it is relatively easy to identify leptons at high-energy colliders. With this new chiral model, therefore, we expect that the first new particles expected at the LHC are two dark Z's.

In Sec. II, we present the minimal chiral model. In Sec. III, the kinetic mixing in the $\mathrm{SU}(2)_{\mathrm{dark}} \times \mathrm{U}(1)_Q$ is discussed. In Sec. IV, it is shown that the minimal chiral model is derivable from a string compactification. Section V is a conclusion. In Appendix, a SUSY scenario based on the hidden sector $\mathrm{SU}(5)'$ from $\mathbf{Z}_{12-I}$ orbifold compactification [15] is discussed.

II. MINIMAL CHIRAL MODEL

If a new gauge boson is discovered at a terrestrial observatory using the SM particle beams, it may be from a chiral theory. A pure U(1) gauge theory can survive down to the TeV scale, but we need matter Higgs fields to render it mass. Thus, a pure gauge theory is completely decoupled from the LHC machine. Furthermore, to couple to the SM particles, we need some matter fields possessing both quantum numbers of the SM and this hypothetical U(1). Then, it is not a pure gauge theory.

Sometimes, a chiral extension of the SM, $\mathrm{SU}(2)_L\times \mathrm{SU}(2)_R\times \mathrm{U}(1)$, is considered as a low-energy model [16]. Its subgroup $\mathrm{SU}(2)_L\times \mathrm{U}(1)_Y\times \mathrm{U}(1)_{B-L}$ may be considered as the simplest extension of the SM. Our dark sector, however, does not include these extensions because the SM fermions carry $B-L$ charges that form a vectorlike representation of $\mathrm{U}(1)_{B-L}$. We will not allow vectorlike representations. Another simple extension with a strongly interacting effective extra U(1) in the dark sector has been studied [17] to probe self-interacting dark matter.

With these caveats, we consider a new chiral theory. A chiral theory near the electroweak scale should not have gravitational and gauge anomalies. First, consider the rank-1 gauge groups. If we consider an SU(2), it is not possible to have a chiral theory because there must be an even number of doublets [18]. With only one $\mathrm{U}(1)_Y$ group, the absence of gravitational anomaly requires $\mathrm{Tr}\,Y=0$ and the absence of gauge anomaly in addition requires $\mathrm{Tr}\,Y^3=0$. For example, even though two charged fields $Y=+1$ and -1 do not have these anomalies, the model is not allowed in our framework because it is vectorlike. But if we use the Y of the SM, these two conditions are satisfied.[1] Second, let us consider the rank-2 gauge groups,

SU(3): Vectorlike representations, hence not allowed,

$\mathrm{SU}(2)\times\mathrm{SU}(2)$: No chiral theory with even number of doublets,

$\mathrm{U}(1)\times\mathrm{U}(1)'$: Six conditions for the absence of anomalies, $\{\mathrm{Tr}Y,\mathrm{Tr}Y',\mathrm{Tr}Y^3,\mathrm{Tr}Y'^3,\mathrm{Tr}YY'^2,\mathrm{Tr}Y^2Y'\}=0$, and

$$\mathrm{SU}(2)\times\mathrm{U}(1):\ \text{Two conditions with doublets and singlets,}\quad \{\mathrm{Tr}Y,\mathrm{Tr}Y^3\}=0. \tag{1}$$

Thus, the simplest case is $\mathrm{SU}(2)_{\rm dark}\times\mathrm{U}(1)_Q$, and at least two dark-$Z'$'s are predicted at low energy. Two conditions in (1) for $N(=\text{even})$ doublets and $2N$ singlets are

$$\sum_{i=1}^{4N} Q_i=0, \tag{2}$$

and

$$\sum_{i=1}^{4N} Q_i^3=\left(\sum_{i=1}^{4N}Q_i\right)\left(\sum_{i=1}^{4N}Q_i^2-\sum_{i\neq j}^{4N}Q_iQ_j\right)+3\sum_{i\neq j\neq k}^{4N}Q_iQ_jQ_k=0. \tag{3}$$

Condition (2) satisfies Eq. (3) if the term $\sum_{i\neq j\neq k}$ is vanishing. The number of terms in this sum is

$$\binom{4N}{3}, \tag{4}$$

which is very large. So, a complete search is more involved. The well-known chiral theory, satisfying (3), is the SM, $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$.

Here, we present a simpler one $\mathrm{SU}(2)_{\rm dark}\times\mathrm{U}(1)_Q$, satisfying the conditions in (2) and (3), with the following fermions:

$$\begin{aligned} &Q=\frac{1}{2}:\ \ell_i=\begin{pmatrix}E_i\\ N_i\end{pmatrix}_{\frac{+1}{2}},\qquad \begin{matrix}E^c_{i,-1}\\ N^c_{i,0}\end{matrix}\qquad (i=1,2,3),\\ &Q=-\frac{3}{2}:\ \mathcal{L}=\begin{pmatrix}\mathcal{E}\\ \mathcal{F}\end{pmatrix}_{\frac{-3}{2}},\qquad \begin{matrix}\mathcal{E}^c_{,+1}\\ \mathcal{F}^c_{,+2}\end{matrix}, \end{aligned} \tag{5}$$

where the subscripts denote the Q charges. There are four doublets without the SU(2) anomaly [18]. One set, one of $E^c_{i,-1}$ and $\mathcal{E}^c_{,+1}$, forms a vectorlike pair, but we keep them to provide masses for all the particles after breaking $\mathrm{SU}(2)_{\rm dark}\times\mathrm{U}(1)_Q$. In Eq. (5), there appear 16 left-handed chiral fields,[2] and they do not introduce any gravitational and gauge anomalies. It is interesting to observe that there appear 16 chiral fields as in the spinor representation **16** in SO(10). Equation (5) realizes the TDZ model.

To break the rank-2 gauge group $\mathrm{SU}(2)_{\rm dark}\times\mathrm{U}(1)_Q$ completely and to give fermions masses, let us introduce two doublets[3] and a singlet of scalars,

$$\Phi_u=\begin{pmatrix}\phi_u^+\\ \phi_u^0\end{pmatrix}_{Q=\frac{+1}{2}},\quad \Phi_d=\begin{pmatrix}\phi_d^0\\ \phi_d^-\end{pmatrix}_{Q=\frac{-1}{2}},\quad S_{Q=2}, \tag{6}$$

where their VEVs are[4]

[1] In addition, the SM requires $\mathrm{Tr}\,Y=0$ for quarks and leptons separately, and also additional conditions for the absence of non-Abelian gauge anomalies.

[2] Note that the SM has 45 chiral fields.

[3] With supersymmetry, we need two doublets to make all chiral fields massive.

[4] Choosing the $Q=2$ singlet for breaking U(1) is just for an illustration.

$$\langle \phi^0_{u,d} \rangle = \frac{V_{u,d}}{\sqrt{2}}, \qquad \langle S \rangle = \frac{V_S}{\sqrt{2}}. \tag{7}$$

Then, masses of two dark-Z''s are

$$M^2_{Z'_1} = (g_2^2 + g_Q^2) V_D^2 = \frac{g_2^2}{\cos^2\theta} V_D^2, \tag{8}$$

$$M^2_{Z'_2} = (2g_Q)^2 \frac{V_S^2}{2} = 2 g_Q^2 V_S^2 = 2 g_2^2 \tan^2\theta V_S^2, \tag{9}$$

where g_2 and g_Q are the $SU(2)_{\rm dark}$ and $U(1)_Q$ couplings, respectively, $\tan\theta \equiv g_Q/g_2$, and $V_D^2 = V_u^2 + V_d^2$. Thus, the mass ratio of two dark-Z' masses is

$$r = \sqrt{2} \left| \frac{V_S}{V_D} \sin\theta \right|. \tag{10}$$

If $V_S \to 0$, Z'_2 may be called a dark photon, which is included in our terminology TDZ. This estimate will be used in Appendix.

III. THE KINETIC MIXING

If multiple dark-U(1) gauge bosons are present, they can mix with the SM photon, most probably via kinetic mixings as suggested in [13]. Since the rank of the $SU(2)_{\rm dark} \times U(1)_Q$ gauge group is 2, there are two dark-Z''s and we summarize their kinetic mixing with a photon.[5] These arise via loops between photon and dark-photon through an intermediate particle(s) χ that carries both the electromagnetic and dark charges. After a proper diagonalization procedure of the kinetic energy terms, the electromagnetic charge of χ can be millicharged, $\mathcal{O}(\alpha/2\pi)e$. In the heterotic $E_8 \times E'_8$ string model, the extra E'_8 gauge group may contain dark photons that will be called dark-Z''s, leading to the kinetic mixing of $\mathcal{O}(\alpha/2\pi)$ [19]. Indeed, an explicit model for this kind from string compactification exists in the literature [20,21].

The intermediate $\mathcal{O}(\mathrm{MeV})$ millicharged particles have not been ruled out by observations in the previous study [22–24]. For the discovery possibility at the LHC, we consider the electroweak scale dark-Z''s.

Consider three Abelian gauge groups $U(1)_{\rm QED}$ and $U(1)_i (i = 1, 2)$. The kinetic mixing of $U(1)_{\rm QED}$ and $U(1)_i$ dark-Z''s are parametrized by, following the notation of [21,23],

$$\mathcal{L} = -\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - \frac{1}{4}\hat{X}^1_{\mu\nu}\hat{X}^{1\,\mu\nu} - \frac{1}{4}\hat{X}^2_{\mu\nu}\hat{X}^{2\mu\nu} - \frac{\xi_1}{2}\hat{F}_{\mu\nu}\hat{X}^{1\,\mu\nu} - \frac{\xi_2}{2}\hat{F}_{\mu\nu}\hat{X}^{2\,\mu\nu} - \frac{\xi_{12}}{2}\hat{X}^1_{\mu\nu}\hat{X}^{2\,\mu\nu}, \tag{11}$$

[5] Their mixing with the Z boson is omitted here.

where $\hat{A}_\mu(\hat{X}^i{}_\mu)$ is the $U(1)_{\rm QED}$ [dark-$U(1)^i$] gauge boson, and its field strength tensor is $\hat{F}_{\mu\nu}(\hat{X}^i_{\mu\nu})$. The kinetic mixings are parametrized by ξ's, which are generically allowed by the gauge invariance and the Lorentz symmetry. In the low-energy effective theory, ξ's are considered to be completely arbitrary parameters. An ultraviolet-completed theory is expected to generate the kinetic mixing parameters. The usual diagonalization procedure of these kinetic terms leads to the relation,

$$\begin{pmatrix} A_\mu \\ X^1_\mu \\ X^2_\mu \end{pmatrix} = \begin{pmatrix} B_{11} & 0 & 0 \\ \frac{\xi_1 - \xi_2\xi_{12}}{\sqrt{1-\xi_{12}^2}} & \sqrt{1-\xi_{12}^2} & 0 \\ \xi_2 & \xi_{12} & 1 \end{pmatrix} \begin{pmatrix} \hat{A}_\mu \\ \hat{X}^1_\mu \\ \hat{X}^2_\mu \end{pmatrix}, \tag{12}$$

where

$$B_{11} = \sqrt{1 - \frac{(\xi_1 - \xi_2)^2 + 2\xi_1\xi_2(1-\xi_{12})}{1-\xi_{12}^2}} \tag{13}$$

and we obtain

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X^1_{\mu\nu}X^{1\mu\nu} - \frac{1}{4}X^2_{\mu\nu}X^{2\mu\nu}, \tag{14}$$

where the new field strengths are $F_{\mu\nu}, X^1_{\mu\nu}$, and $X^2_{\mu\nu}$. The photon corresponds to A_μ and dark-Z''s correspond to $X^i_\mu (i = 1, 2)$. If the dark-Z''s are exactly massless, there exists an SO(3) symmetry in the $A_\mu - X^i_\mu$ field space.

Using the above SO(3) symmetry, let us take the following simple interaction Lagrangian of a SM fermion with a photon in the original basis as

$$\mathcal{L} = \bar{\psi}(Q\hat{e}\gamma^\mu)\psi\hat{A}_\mu. \tag{15}$$

Note that in this basis there is no direct interaction between the electron and the hidden sector gauge boson $\hat{X}$. If there exists a hidden sector Dirac fermion χ with the $U(1)_{\rm ex}$ charge Q_χ, its interaction with the hidden sector gauge boson is simply represented by

$$\mathcal{L} = \bar{\chi}(\hat{e}_i^{\rm ex} Q_i^{\rm ex}\gamma^\mu)\chi\hat{X}^i_\mu, \tag{16}$$

where $\hat{e}_{\rm ex}$ can be different from $\hat{e}$ in general. In this case, there is also no direct interaction between the hidden fermion and the visible sector gauge boson $\hat{A}_\mu$. We can recast the Lagrangian (15) in the transformed basis A_μ and X_μ,

$$\mathcal{L} = \bar{\psi}\left(\frac{\sqrt{1-\xi_{12}^2}}{\sqrt{1-\xi_1^2-\xi_{12}^2+2\xi_1\xi_2\xi_{12}}} Q\hat{e} \right)\gamma^\mu\psi A_\mu, \tag{17}$$

where we used the inverse of (12),

$$\frac{1}{\mathrm{Det}}\begin{pmatrix} \sqrt{1-\xi_{12}^2}, & 0, & 0 \\ -\frac{\xi_1-\xi_2\xi_{12}}{\sqrt{1-\xi_{12}^2}}, & B_{11}, & 0 \\ -\frac{\xi_2-\xi_1\xi_{12}}{\sqrt{1-\xi_{12}^2}}, & -\xi_{12}B_{11}, & B_{11}\sqrt{1-\xi_{12}^2} \end{pmatrix} \tag{18}$$

with

$$\mathrm{Det}=\sqrt{1-\xi_1^2-\xi_2^2-\xi_{12}^2+2\xi_1\xi_2\xi_{12}}.$$

Here, one notices that the standard model fermion has a coupling only to the visible sector gauge boson A_μ even after changing the basis of the gauge bosons. However, the coupling constant $\hat{e}$ is modified to e as suggested in Eq. (17). Similarly, we derive the following couplings for χ:

$$\begin{aligned}\mathcal{L}=\bar{\chi}\gamma^\mu\Bigg[&\frac{\hat{e}_1^{\mathrm{ex}}}{\mathrm{Det}}\left(B_{11}X_\mu^1-\frac{\xi_1-\xi_2\xi_{12}}{\sqrt{1-\xi_{12}^2}}A_\mu\right)Q_1^{\mathrm{ex}}\\ &+\frac{\hat{e}_2^{\mathrm{ex}}}{\mathrm{Det}}\Bigg(B_{11}\sqrt{1-\xi_{12}^2}X_\mu^2-B_{11}\xi_{12}X_\mu^1\\ &-\frac{\xi_2-\xi_1\xi_{12}}{\sqrt{1-\xi_{12}^2}}A_\mu\Bigg)Q_2^{\mathrm{ex}}\Bigg]\chi.\end{aligned} \tag{19}$$

In this basis, the hidden sector matter field χ now can couple to the visible sector gauge boson A_μ with the couplings $-\hat{e}_1^{\mathrm{ex}}Q_1^{\mathrm{ex}}(\xi_1-\xi_2\xi_{12})/\sqrt{1-\xi_{12}^2}/\mathrm{Det}$ to the mass eigenstate X_μ^1 and $\hat{e}_2^{\mathrm{ex}}Q_2^{\mathrm{ex}}(\xi_2-\xi_1\xi_{12})/\sqrt{1-\xi_{12}^2}/\mathrm{Det}$ to the mass eigenstate X_μ^2. In terms of the aforementioned SO(3) symmetry, it simply means the mismatch between the gauge couplings of the electron and other fermions. Thus, we can set the physical hidden sector coupling e_{ex} as $e_{\mathrm{ex}}\equiv\hat{e}_{\mathrm{ex}}$, and we define the coupling of the field χ to the visible sector gauge boson A_μ, introducing the millicharge parameter ε_i, as $\varepsilon_i e$ such that

$$e=\frac{\hat{e}}{\sqrt{1-\xi_1^2-\xi_2^2}},$$

$$\varepsilon_1=-\frac{\hat{e}_1^{\mathrm{ex}}}{e}\frac{\xi_1-\xi_2\xi_{12}}{\sqrt{1-\xi_1^2-\xi_2^2-\xi_{12}^2+2\xi_1\xi_2\xi_{12}}}\approx-\frac{\hat{e}_1^{\mathrm{ex}}}{e}\xi_1, \tag{20}$$

$$\varepsilon_2=-\frac{\hat{e}_2^{\mathrm{ex}}}{e}\frac{\xi_2-\xi_1\xi_{12}}{\sqrt{1-\xi_1^2-\xi_2^2-\xi_{12}^2+2\xi_1\xi_2\xi_{12}}}\approx-\frac{\hat{e}_2^{\mathrm{ex}}}{e}\xi_2. \tag{21}$$

Note in general that $e\neq e_i^{\mathrm{ex}}$. Since $\xi_{i,12}\simeq\mathcal{O}(\varepsilon_i e/e_i^{\mathrm{ex}})$ is expected to be small, the condition $\xi_{i,12}<1$ gives $\alpha_i^{\mathrm{ex}}/\alpha>\varepsilon^2$. From a fundamental theory, one can calculate the ratio e_{ex}/e in principle, which is possible with the detailed knowledge of the compactification radius [25].

Similarly, one can calculate the mixing of the SM Z boson with dark-Z''s, which, however, is not presented here.

IV. FROM A STRING MODEL

In this section, we derive the minimal chiral model (5) discussed in Sec. II from a string theory. The $\mathrm{E}_8\times\mathrm{E}_8'$ heterotic string model compactified on the $\mathbf{Z}_{12-I}$ orbifold gives the flipped $\mathrm{SU(5)_{flip}}$ times $\mathrm{SU(5)'}\times\mathrm{SU(2)'}$ with several extra U(1)'s [15]. Here, the factor $\mathrm{SU(5)_{flip}}$ contains a gauge group U(1): $\mathrm{SU(5)}\times\mathrm{U(1)}_X$. The first important U(1) gauge group is $\mathrm{U(1)}_X$ in $\mathrm{SU(5)}\times\mathrm{U(1)}_X$, which is free of any gauge anomaly. The second is the anomalous $\mathrm{U(1)_{anom}}$. Except these two U(1) factors, $\mathrm{U(1)}_X$ and $\mathrm{U(1)_{anom}}$, the non-Abelian gauge group is $\mathrm{SU(5)}\times\mathrm{SU(5)'}\times\mathrm{SU(2)'}$. Note that the charges of $\mathrm{U(1)}_X$ and $\mathrm{U(1)_{anom}}$ are[6]

$$X=(-2,-2,-2,-2,-2;0^3)(0^8)', \tag{22}$$

$$\begin{aligned}Q'_{\mathrm{anom}}&=84Q_1+147Q_2-42Q_3-63Q_5-9Q_6\\ &=18(0^5;56,98,-28)(3,3,3,3,-9;21,21,-45)',\end{aligned} \tag{23}$$

where

$$\begin{aligned}Q_1&=(0^5;12,0,0)(0^8)',\\ Q_2&=(0^5;0,12,0)(0^8)',\\ Q_3&=(0^5;0,0,12)(0^8)',\\ Q_4&=(0^8)(0^4,0;12,-12,0)',\\ Q_5&=(0^8)(0^4,0;-6,-6,12)',\\ Q_6&=(0^8)(-6,-6,-6,-6,18;0,0,6)'.\end{aligned}$$

With the second line of (23), the sum of entries of $(\cdots)'$ is zero, which implies that it commutes with our SU(2)′ raising/lowering shift $(0^8)(\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2})'$. This definition also commutes with a set of raising/lowering generators of SU(5)′, in particular with $(0^8)(\frac{\pm1}{2},\frac{\mp1}{2},\frac{\mp1}{2},\frac{\mp1}{2},\frac{\mp1}{2},\frac{\pm1}{2},\frac{\pm1}{2},\frac{\pm1}{2})'$. For an easier calculation, we can use a simpler form; i.e., we can use any other linear combination Q_{anom} in terms of Q'_{anom} of (23) and anomaly free U(1) charges belonging to $\mathrm{E}_8\times\mathrm{E}_8'$, $18(-3,-3,-3,-3,+9,0,0,+3)'$, which will not change $\mathrm{Tr}Q_{\mathrm{anom}}$ and $\mathrm{Tr}(Q_{\mathrm{anom}}Q_aQ_b)$ where Q_a and Q_b are anomaly free gauge charges.[7] With this process, we use the following Q_{anom} in the tables:

$$\begin{aligned}Q_{\mathrm{anom}}&=63(0^5;16,28,-8)(0^5;6,6,-12)'\\ &=42\left\{2Q_1+\frac{7}{2}Q_2-Q_3-\frac{3}{2}Q_5\right\}.\end{aligned} \tag{24}$$

[6] For the definition, see Ref. [26].

[7] With this new definition, $c_{a\gamma\gamma}$ presented in [26,27] remains intact even though the individual entries are changed.

TABLE I. The $SU(5)'$ representations. The bold entries are $Q_{\rm anom}/63$.

Sect.	States	$SU(5)'$	Multiplicity	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label
T_1^0	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\underline{-10000};\frac{1}{4}\frac{1}{4}\frac{1}{2})'$	$\overline{\mathbf{10}}'_0$	1	−2	−2	−2	0	+3	+9	−567(**−9**)	T'_1
	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\underline{\frac{1}{2}\frac{1}{2}\frac{-1}{2}\frac{-1}{2}}\frac{1}{2};\frac{-1}{4}\frac{-1}{4}0)'$										
T_1^0	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\underline{-10000};\frac{1}{4}\frac{1}{4}\frac{1}{2})'$	$(\mathbf{5}',\mathbf{2}')_0$	1	−2	−2	−2	0	+3	+9	−567(**−9**)	F'_1
	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{-1}{2};\frac{-1}{4}\frac{-1}{4}0)'$										
	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\underline{\frac{1}{2}\frac{-1}{2}\frac{-1}{2}\frac{-1}{2}}\frac{-1}{2};\frac{-1}{4}\frac{-1}{4}0)'$										
	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(00001;\frac{-3}{4}\frac{-3}{4}\frac{-1}{2})'$										
T_1^0	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(\underline{\frac{-1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}\frac{-1}{2};\frac{-1}{4}\frac{-1}{4}0)'$	$\bar{\mathbf{5}}'_0$	1	−2	−2	−2	0	+3	−15	−567(**−9**)	F'_2
	$(00000;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0000-1;\frac{1}{4}\frac{1}{4}\frac{1}{2})'$										
T_1^+	$(\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6};\frac{1}{3}\frac{-1}{3}0)(\underline{\frac{-5}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}}\frac{1}{2};\frac{1}{12}\frac{-1}{4}0)'$	$\bar{\mathbf{5}}'_{-5/3}$	1	+4	−4	0	+4	+1	+11	−315(**−5**)	F'_3
	$(\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6};\frac{1}{3}\frac{-1}{3}0)(\frac{-1}{3}\frac{-1}{3}\frac{-1}{3}\frac{-1}{3}0;\frac{7}{12}\frac{1}{4}\frac{1}{2})'$										
T_4^+	$(\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6};\frac{-1}{6}\frac{1}{6}\frac{1}{2})(\underline{\frac{2}{3}\frac{-1}{3}\frac{-1}{3}\frac{-1}{3}}0;\frac{1}{3}00)'$	$\mathbf{5}'_{-5/3}$	3	−2	+2	+6	+4	−2	+2	0(**0**)	F'_4
	$(\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6};\frac{-1}{6}\frac{1}{6}\frac{1}{2})(\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{2};\frac{-1}{6}\frac{-1}{2}\frac{-1}{2})'$										
T_4^-	$(\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6};\frac{-1}{6}\frac{-1}{2}\frac{1}{6})(\underline{\frac{-2}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}}0;\frac{-1}{3}00)'$	$\bar{\mathbf{5}}'_{5/3}$	3	−2	−6	+2	−4	+2	−2	−1260(**−20**)	F'_5
	$(\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6};\frac{-1}{6}\frac{-1}{2}\frac{1}{6})(\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{2};\frac{1}{6}\frac{1}{2}\frac{1}{2})'$										
T_7^-	$(\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6};\frac{1}{3}0\frac{-1}{3})(\underline{\frac{5}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}}\frac{-1}{2};\frac{-1}{12}\frac{1}{4}0)'$	$\mathbf{5}'_{5/3}$	1	+4	0	−4	−4	−1	−11	+567(+**9**)	F'_6
	$(\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6}\frac{-1}{6};\frac{1}{3}0\frac{-1}{3})(\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}0;\frac{-7}{12}\frac{-1}{4}\frac{-1}{2})'$										

In the orbifold compactification, frequently there appears an anomalous $U(1)_A$ gauge field $\tilde{A}_\mu$ from a subgroup of $E_8\times E_8'$ [28]. The charge of this anomalous $U(1)_A$ is given in Eq. (24). In addition, the anomaly cancellation in ten dimensions (10D) requires the so-called Green-Schwarz term in terms of the second rank antisymmetric-tensor field $B_{MN}(M,N=1,2,\ldots,10)$ [29]. The 10D B_{MN} always introduces a model-independent (MI) axion $a_{\rm MI}$ in 4D, $\partial_\mu a_{\rm MI}\propto\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}(\mu,\text{etc.}=1,2,3,4)$ where $H^{\nu\rho\sigma}$ is the field strength of $B^{\rho\sigma}$ [30]. The anomalous U(1) gauge boson absorbs the MI axion to become massive, and there results a global symmetry $U(1)_{\rm anom}$ below the compactification scale. More phenomenologically, $U(1)_{\rm anom}$ can be suggested for a plausible flavor symmetry [26]. The global symmetry $U(1)_{\rm anom}$ is good for a Peccei-Quinn symmetry [31] toward "invisible" axions at the intermediate scale $M_{\rm int}$ [9]. Except for the two U(1)'s, Eqs. (22) and (24), all U(1)'s are assumed to be broken at a high-energy scale, much above $M_{\rm int}$. In more detail, it works as follows. Suppose that five U(1) charges out of $Q_{1,\ldots,6}$ are broken, and there is only one gauge symmetry remaining, which we identify as $U(1)_{\rm anom}$. Now, we can consider two continuous parameters, one is the MI-axion direction and the other the phase of $U(1)_{\rm anom}$ transformation. Out of two continuous directions, only one phase or pseudoscalar is absorbed by the $U(1)_{\rm anom}$ gauge boson, and one continuous direction survives. The remaining continuous degree corresponds to a global symmetry, which is called the 't Hooft mechanism [32]: "If both a gauge symmetry and a global symmetry are broken by one scalar VEV, the gauge symmetry is broken and a global symmetry is surviving." The resulting global charge is a linear combination of the original gauge and global charges. Even though we obtain a global symmetry $U(1)_{\rm anom}$, it is obtained from the original two gauge symmetries, one from the two-index antisymmetric tensor gauge field B_{MN} in 10D and the other the $U(1)_{\rm anom}$ subgroup of $E_8\times E_8'$ given in Eq. (24).

Here, the primed groups $SU(5)'\times SU(2)'$ are the hidden sector non-Abelian gauge groups. The hidden sector representations under $SU(5)'\times SU(2)'$ are given in Tables I and II [26]. After removing vectorlike representations from Tables I and II, we obtain

$$(\overline{\mathbf{10}}',\mathbf{1}'),(\mathbf{5}',\mathbf{2}'),(\bar{\mathbf{5}}',\mathbf{1}'),(\mathbf{1}',\mathbf{2}'),\quad\text{under}$$
$$SU(5)'\times SU(2)',\to\Psi_{AB},\Phi^{A\alpha},\psi_A,\phi^\alpha,\tag{25}$$

where the tensor notation is used in the second line with the $SU(5)'$ index $A=\{1,2,3,4,5\}$ and the $SU(2)'$ index $\alpha=\{1,2\}$. The representations in (25) do not lead to an $SU(5)'$ anomaly. Let two SU(2) subgroup indices of $SU(5)'$ be $i=\{1,2\}$ and $I=\{4,5\}$ so that the five $SU(5)'$ indices split into

$$\{A\}\equiv\{i,3,I\}.\tag{26}$$

By the VEV of $\Phi^{A\alpha}\equiv(\mathbf{5}',\mathbf{2}')$,

$$\langle\Phi^{A=3,\alpha=2}\rangle=V_1,\tag{27}$$

TABLE II. The $SU(2)'$ representations with the convention of Table I. We listed only the upper component of $SU(2)'$ from which the lower component can be obtained by applying T^- of $SU(2)'$.

Sect.	States	$SU(2)'$	Multiplicity	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label
T_1^0	$(00000;\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6})(\underline{1000}0;\tfrac{1}{4}\tfrac{1}{4}\tfrac{1}{2})'$ $(00000;\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6})(00000;\tfrac{-3}{4}\tfrac{-3}{4}\tfrac{-1}{2})'$	$(\mathbf{5}',\mathbf{2}')_0$	1	−2	−2	−2	0	+3	−3	$-567(\mathbf{-9})$	F_1'
T_1^0	$(00000;\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6})(00001;\tfrac{1}{4}\tfrac{1}{4}\tfrac{1}{2})'$	$\mathbf{2}'_0$	1	−2	−2	−2	0	+3	+21	$-567(\mathbf{-9})$	D_2
T_1^+	$(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6};\tfrac{1}{3}\tfrac{-1}{3}0)(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{2};\tfrac{1}{12}\tfrac{3}{4}0)'$	$\mathbf{2}'_{-5/3}$	1	+4	−4	0	−8	−5	+5	$+63(\mathbf{+1})$	D_3
T_1^-	$(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6};\tfrac{-2}{3}0\tfrac{-1}{3})(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}0;\tfrac{-1}{12}\tfrac{1}{4}\tfrac{1}{2})'$	$\mathbf{2}'_{5/3}$	1	−8	0	−4	−4	+5	−5	$-819(\mathbf{-13})$	D_4
T_1^-	$(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6};\tfrac{1}{3}0\tfrac{2}{3})(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}0;\tfrac{-1}{12}\tfrac{1}{4}\tfrac{1}{2})'$	$\mathbf{2}'_{5/3}$	1	+4	0	+8	−4	+5	−5	$-315(\mathbf{-5})$	D_5
T_2^+	$(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6};\tfrac{1}{6}\tfrac{-1}{6}\tfrac{1}{2})(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}0;\tfrac{1}{6}\tfrac{1}{2}0)'$	$\mathbf{2}'_{5/3}$	1	+2	−2	+6	−4	−4	−8	$-126(\mathbf{-2})$	D_6
T_2^-	$(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6};\tfrac{1}{6}\tfrac{-1}{2}\tfrac{-1}{6})(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{2};\tfrac{1}{3}0\tfrac{1}{2})'$	$\mathbf{2}'_{-5/3}$	1	+2	−6	−2	+4	+4	+8	$-882(\mathbf{-14})$	D_7
T_4^+	$(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6};\tfrac{-1}{6}\tfrac{1}{6}\tfrac{1}{2})(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{2};\tfrac{-1}{6}\tfrac{1}{2}\tfrac{1}{2})'$	$\mathbf{2}'_{-5/3}$	2	−2	+2	+6	−8	+4	+8	$-378(\mathbf{-6})$	$2D_8$
T_4^-	$(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6};\tfrac{-1}{6}\tfrac{-1}{2}\tfrac{1}{6})(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}0;\tfrac{2}{3}00)'$	$\mathbf{2}'_{5/3}$	2	−2	−6	+2	+8	−4	−8	$-882(\mathbf{-14})$	$2D_9$
T_7^+	$(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6};\tfrac{1}{3}\tfrac{2}{3}0)(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{2};\tfrac{7}{12}\tfrac{1}{4}0)'$	$\mathbf{2}'_{-5/3}$	1	+4	+8	0	+4	−5	+5	$+1827(\mathbf{+29})$	D_{10}
T_7^+	$(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6};\tfrac{-2}{3}\tfrac{-1}{3}0)(\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{6}\tfrac{1}{2};\tfrac{7}{12}\tfrac{1}{4}0)'$	$\mathbf{2}'_{-5/3}$	1	−8	−4	0	+4	−5	+5	$-945(\mathbf{-15})$	D_{11}
T_7^-	$(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6};\tfrac{1}{3}0\tfrac{-1}{3})(\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{6}\tfrac{-1}{2};\tfrac{-1}{12}\tfrac{-3}{4}0)'$	$\mathbf{2}'_{5/3}$	1	+4	0	−4	+8	+5	−5	$+189(\mathbf{+3})$	D_{12}

we obtain a group containing two $SU(2)\times U(1)'$ subgroups from non-Abelian factors $SU(5)'\times SU(2)'$, i.e., a rank-4 subgroup from the rank-5 non-Abelian group, which is denoted as

$$SU(2)_1\times U(1)_1\times SU(2)_2\times U(1)_2, \tag{28}$$

where the index i is for $SU(2)_1$ and the index I is for $SU(2)_2$. In fact, the VEV (27) breaks the rank-5 $SU(5)'\times SU(2)'$ down to rank-4 $SU(4)'\times U(1)'$. The rank-3 $SU(4)'$ is further broken down to rank-2 $SU(2)_1\times SU(2)_2$ by the VEV in the direction

$$\langle\overline{\mathbf{10}}{}'_0\rangle = V_2\colon \begin{pmatrix} 0, & 0, & 0, & 0, & 0\\ 0, & 0, & 0, & 0, & 0\\ 0, & 0, & 0, & 0, & 0\\ 0, & 0, & 0, & 0, & V_2\\ 0, & 0, & 0, & -V_2, & 0 \end{pmatrix}. \tag{29}$$

Summarizing the above discussion, the rank-5 $SU(5)'\times SU(2)'$ is broken down to a rank-3 group by V_1 and V_2,

$$SU(2)_1\times SU(2)_2\times U(1), \tag{30}$$

where

$$Q = \begin{pmatrix} \frac{-1}{2}, & 0, & 0, & 0, & 0\\ 0, & \frac{-1}{2}, & 0, & 0, & 0\\ 0, & 0, & 1, & 0, & 0\\ 0, & 0, & 0, & 0, & 0\\ 0, & 0, & 0, & 0, & 0 \end{pmatrix} \otimes \begin{pmatrix} +1, & 0\\ 0, & -1 \end{pmatrix}$$
$$= Y_1\otimes \begin{pmatrix} +1, & 0\\ 0, & -1 \end{pmatrix} \tag{31}$$

and

$$Y_1 = \begin{pmatrix} \frac{-1}{2}, & 0, & 0, & 0, & 0\\ 0, & \frac{-1}{2}, & 0, & 0, & 0\\ 0, & 0, & 1, & 0, & 0\\ 0, & 0, & 0, & 0, & 0\\ 0, & 0, & 0, & 0, & 0 \end{pmatrix},$$
$$Y_2 = \begin{pmatrix} \frac{+1}{3}, & 0, & 0, & 0, & 0\\ 0, & \frac{+1}{3}, & 0, & 0, & 0\\ 0, & 0, & \frac{+1}{3}, & 0, & 0\\ 0, & 0, & 0, & \frac{-1}{2}, & 0\\ 0, & 0, & 0, & 0, & \frac{-1}{2} \end{pmatrix}. \tag{32}$$

Thus, V_2 breaks Y_2, which does not participate in Q of Eq. (31). $SU(2)_1$ and $SU(2)_2$ generators are

$$T^i = \begin{pmatrix} (2\times 2)^i, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & \mathbf{0} \end{pmatrix}, \qquad T^I = \begin{pmatrix} \mathbf{0}, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & (2\times 2)^I \end{pmatrix}. \tag{33}$$

The $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_Q$ quantum numbers are

$$\begin{aligned} &\Psi_{ij} \oplus \Psi^{i3} \oplus \Psi_{iJ} \oplus \Psi^{I3} \oplus \Psi_{IJ} = (\mathbf{1},\mathbf{1})_{+1} \oplus (\mathbf{2},\mathbf{1})_{\frac{-1}{2}} \oplus (\mathbf{2},\mathbf{2})_{\frac{+1}{2}} \oplus (\mathbf{1},\mathbf{2})_{-1} \oplus (\mathbf{1},\mathbf{1})_0^{(a)}, \\ &\Phi^{i\alpha} \oplus \Phi^{3\alpha} \oplus \Phi^{I\alpha} = (\mathbf{2},\mathbf{1})_{\frac{+1}{2}} + (\mathbf{2},\mathbf{1})_{\frac{-3}{2}} + (\mathbf{1},\mathbf{1})_{+2} + (\mathbf{1},\mathbf{1})_0^{(b)} + (\mathbf{1},\mathbf{2})_{+1} + (\mathbf{1},\mathbf{2})_{-1}, \\ &\psi_A = \psi_i \oplus \psi_3 \oplus \psi_I = (\mathbf{2},\mathbf{1})_{\frac{+1}{2}} \oplus (\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{2})_0, \\ &\phi^\alpha = (\mathbf{1},\mathbf{1})_{+1} + (\mathbf{1},\mathbf{1})_{-1}, \end{aligned} \tag{34}$$

where several colored pairs form vectorlike representations. Removing the green and blue vectorlike pairs, and one combination of the red pair $\mathbf{1}_{0,A} = (1/\sqrt{2})[(\mathbf{1},\mathbf{1})_0^{(a)} - (\mathbf{1},\mathbf{1})_0^{(b)}]$ where $S(A)$ represents the (anti)symmetric combination, we obtain

$$(\mathbf{1},\mathbf{1})_{+1} \oplus (\mathbf{2},\mathbf{2})_{\frac{+1}{2}} \oplus (\mathbf{1},\mathbf{2})_{-1} \oplus (\mathbf{1},\mathbf{1})_{0,S} \oplus (\mathbf{2},\mathbf{1})_{\frac{-3}{2}} \oplus (\mathbf{1},\mathbf{1})_{+2} \oplus (\mathbf{2},\mathbf{1})_{\frac{+1}{2}} \oplus (\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{2})_0. \tag{35}$$

Now, let us break[8] $\mathrm{SU}(2)_2$ by the VEV $\langle(\mathbf{1},\mathbf{2})_0\rangle$ that does not carry the Q charge. So, the surviving gauge group is $\mathrm{SU}(2)_{\text{dark}} \times \mathrm{U}(1)_Q$ where $\mathrm{SU}(2)_{\text{dark}}$ is $\mathrm{SU}(2)_1$. Then, the $\mathrm{SU}(2)_{\text{dark}} \times \mathrm{U}(1)_Q$ representations result,

$$\begin{aligned} &\mathbf{1}_{+1} \oplus 2\cdot \mathbf{2}_{\frac{+1}{2}} \oplus 2\cdot \mathbf{1}_{-1} \oplus \mathbf{1}_0 \oplus \mathbf{2}_{\frac{-3}{2}} \oplus \mathbf{1}_{+2} \oplus \mathbf{2}_{\frac{+1}{2}} \\ &\quad \oplus \mathbf{1}_{-1} \oplus 2\cdot \mathbf{1}_0, \end{aligned} \tag{36}$$

which are exactly those appearing in Eq. (5).

Considering only the low-energy SUSY, we have shown that the minimal chiral model is derivable from a string compactification. So, it will be useful if the SUSY scenario is consistent with the unification of gauge coupling constants. Since there are so many unknown parameters in this study, we deferred a brief discussion on the SUSY scenario to Appendix.

V. CONCLUSION

We obtained a new chiral model with the gauge group $\mathrm{SU}(2)_{\text{dark}} \times \mathrm{U}(1)_Q$ without any gauge and gravitational anomalies. This gauge group may belong to the dark sector. We also derived this chiral spectrum from a compactification of the heterotic string. This new chiral theory has a chance to be found at TeV scale accelerators through the kinetic mixing effects. Necessarily, there appear two degrees at low energy: two dark-Z''s, or a dark-Z' plus a dark photon [if $V_S = 0$ in Eq. (9)].

[8]In Appendix, we do not break $\mathrm{SU}(2)_2$.

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APPENDIX: HIDDEN SECTOR SU(5)′

1. Mass scales

Toward a suggestion for an ultraviolet completed theory, we discuss at which scales symmetry breakings are introduced. First, we need one confining force for dynamical SUSY breaking [33,34]. The confining non-Abelian gauge group at the intermediate scale is chosen as $\mathrm{SU}(5)'$. Around the same scale, $\mathrm{SU}(5)'$ is broken down to $\mathrm{SU}(4)'$ and at a somewhat lower scale to $\mathrm{SU}(2)_1$ by the condensation of matter superfield, breaking $\mathrm{SU}(2)_2$. Because $\mathrm{SU}(2)_2$ is neutral under the $\mathrm{SU}(2)_{\text{dark}} \times \mathrm{U}(1)_Q$ transformation, the discussion leading to the minimal model is intact. A rough sketch of related scales is shown in Fig. 1.

The confining superfields in Eq. (35) are

$$(\mathbf{2}, A^\alpha)_{\frac{+1}{2}} \oplus (\mathbf{1}, B^\alpha)_{-1} \oplus (\mathbf{1}, C^\alpha)_0, \tag{A1}$$

where $\alpha = \{1,2\}$ counts the number of color degrees of $\mathrm{SU}(2)_2$. The anomaly matching conditions [35] must lead to the following composite states under $\mathrm{SU}(2)_{\text{dark}} \times \mathrm{U}(1)_Q$:

$$2\cdot \mathbf{D}_{\frac{+1}{2}} \oplus 2\cdot \mathbf{S}_{-1}, \tag{A2}$$

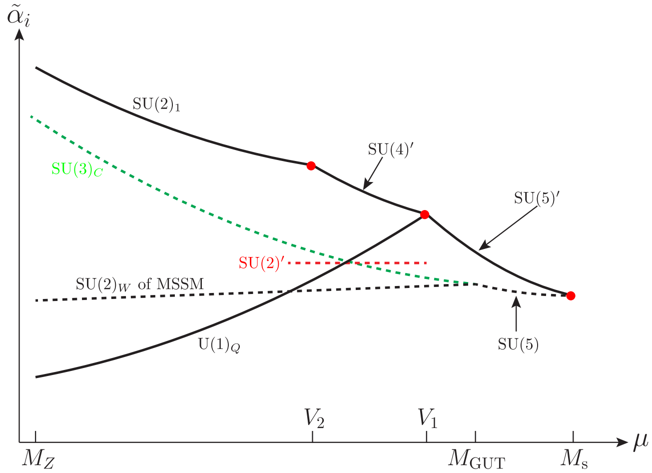

FIG. 1. The red dashed line is for SU(2)$'$.

where the composite states $\mathbf{D}$ and $\mathbf{S}$ are $\mathrm{SU(2)_{dark}\times U(1)}_Q$ doublets and singlets, respectively, composed of A, B, and C degrees,

$$\mathbf{D}_{\frac{+1}{2}} \propto \epsilon_{\alpha\beta}(\mathbf{2},A^\alpha)_{\frac{+1}{2}}(\mathbf{1},C^\beta)_0,$$
$$\mathbf{S}_{-1} \propto \epsilon_{\alpha\beta}(\mathbf{1},B^\alpha)_{\frac{+1}{2}}(\mathbf{1},C^\beta)_0. \tag{A3}$$

Even though $\mathrm{SU(2)}_2$ is smaller than the color $\mathrm{SU(3)}_C$, it can confine at the intermediate scale if SU(5)$'$ and SU(4)$'$ run between the GUT scale and the intermediate scale. So, the SU(4)$'$ breaking VEV V_2, Eq. (29), is around the intermediate scale.

$$\begin{aligned}&\mathrm{SU(5)'\times SU(2)'}|_{V_1<M_{\rm GUT}}\\&\to \mathrm{SU(4)'\times U(1)'}|_{V_2}\\&\to \mathrm{SU(2)_1\times SU(2)_2\times U(1)}_Q|_{M_{\rm int}}\\&\to \mathrm{SU(2)_{dark}\times U(1)}_Q,\end{aligned} \tag{A4}$$

where $V_1 < M_{\rm GUT}$. From the compactification scale down to $M_{\rm GUT}$, SU(5)$'$ runs more steeply than SU(2)$'$, which is illustrated as the separate couplings at V_1 in Fig. 1.

In the radiative breaking of the SM gauge group in the MSSM, the large top quark Yukawa coupling plays a crucial role. To break $\mathrm{SU(2)_{dark}\times U(1)}_Q$, near the electroweak scale, we need some large Yukawa coupling(s) involving $\mathbf{2}_{\frac{+1}{2}}$'s, $\mathbf{2}_{\frac{-3}{2}}$, $\mathbf{1}_{-1}$'s, and $\mathbf{1}_{+2}$ in Eq. (36).

2. Running of couplings

For a rough guess of the coupling constants, we use just one loop evolution equations. With the mass order of Fig. 1, we have the following running of gauge couplings[9]:

[9]Spectra of $\mathrm{SU(5)\times U(1)}_X$ are counted from Ref. [26]. The gauge group $\mathrm{U(1)}_Q$, which is not the anomalous U(1), survives down to the TeV scale.

$$\mathrm{SU(5)_{flip}}: \frac{1}{g_5^2(M_{\rm GUT})} = \frac{1}{g_5^2(M_{\rm st})} + \frac{1}{8\pi^2}(-3\cdot5+12)\ln\frac{M_{\rm st}}{M_{\rm GUT}}, \tag{A5}$$

$$\mathrm{SU(5)'}: \frac{1}{\tilde g_4^2(V_1)} = \frac{1}{\tilde g_5^2(M_{\rm st})} + \frac{1}{8\pi^2}(-3\cdot5+3)\ln\frac{M_{\rm st}}{V_1}, \tag{A6}$$

$$\mathrm{SU(4)'}: \frac{1}{\tilde g_4^2(V_2)} = \frac{1}{\tilde g_4^2(V_1)} + \frac{1}{8\pi^2}(-3\cdot4+4)\ln\frac{V_1}{V_2}, \tag{A7}$$

$$\mathrm{SU(2)_1}: \frac{1}{\tilde g_2^2(M_Z)} = \frac{1}{\tilde g_2^2(V_1)} + \frac{1}{8\pi^2}(-3\cdot2+4)\ln\frac{V_1}{M_Z}, \tag{A8}$$

$$\mathrm{U(1)}_Q: \frac{1}{\tilde g_Q^2(M_Z)} = \frac{1}{\tilde g_Q^2(V_1)} + \frac{1}{8\pi^2}(+4)\ln\frac{V_1}{M_Z}, \tag{A9}$$

where SU(5)$'$ couplings are tilded. In Fig. 1, we also sketched the running of the $\mathrm{SU(2)}_W$ [$\mathrm{SU(3)}_C$] coupling as the (green) dashed line. From the observed value of α_2 at $\mu = M_Z$ for $\sin^2\theta_W(M_Z)\simeq 0.23$ [36], we obtain its GUT scale value. Identifying this as the SU(5)$'$ coupling at $M_{\rm GUT}$, we estimate the couplings as sketched in Fig. 1. We assume that SU(4)$'$ gauginos condense at $\approx V_1$,

$$\langle \tilde G_A^B \tilde G_B^A\rangle \neq 0. \tag{A10}$$

Then, between V_1 and V_2, we consider the group SU(4)$'$. At V_2, $\tilde\alpha_2$ has not reached an order one value, but $\langle\Psi_{IJ}\rangle$ can be developed at V_2. The representation $\mathbf{6}$ of SU(4)$'$ has a larger Casimir operator $\frac{5}{2}$ than that of the fundamental representation $\frac{15}{8}$. So, we expect that there appear composites $\mathbf{D}_{\frac{+1}{2}}$ and $\mathbf{S}_{-1}$ discussed in this appendix.

Using the electroweak coupling at M_Z, $\alpha_2\simeq 3.38\times10^{-2}$ [37], we obtain its evolution to $M_{\rm GUT}$, $\alpha_2(M_{\rm GUT})\simeq 0.0412$ where $M_{\rm GUT}=2.5\times10^{16}$ GeV is used. At the hypothetical string scale $M_s\simeq 0.7\times10^{18}$ GeV, we obtain the coupling $\alpha_5(M_s)\simeq 0.0389$, which is equated to $\tilde\alpha_5(M_s)$. Now, we can run the hidden sector couplings down from M_s. Suppose that gaugino condensation occurs at $M_{\rm cond}=10^{13}$ GeV. With $V_1 = M_{\rm GUT}$,

$$\mathrm{SU(5)'}: \tilde\alpha_5(M_{\rm GUT})\simeq 0.0517, \quad \text{for } M_{\rm GUT}=2.5\times10^{16}\ \text{GeV}. \tag{A11}$$

For SU(5), the Casimir of the adjoint representation is 25/12 times larger than that of the fundamental representation. So, gauginos couple more strongly than the fundamentals. The SU(4)$'$ coupling at V_2 is

$$\mathrm{SU(4)'}: \tilde\alpha_4(V_2)\simeq 0.1066, \quad \text{if } V_2 = 10^{13}\ \text{GeV}. \tag{A12}$$

Let us equate (A12) as the $\mathrm{SU(2)}_1$ coupling at V_2. Then, the $\mathrm{SU(2)}_1$ and $\mathrm{U(1)}_Q$ couplings at M_Z are

$$\mathrm{SU}(2)_1: \ \tilde{\alpha}_2(M_Z) \simeq 0.743, \\ \mathrm{U}(1)_Q: \ \tilde{\alpha}_Q(M_Z) \simeq 0.0251, \tag{A13}$$

such that the mixing angle is $|\sin\theta|_{M_Z} \simeq \sqrt{\tilde{\alpha}_Q/(\tilde{\alpha}_2+\tilde{\alpha}_Q)}|_{M_Z} \simeq 0.213$. At M_Z, $\tilde{\alpha}_2(M_Z)$ is much larger than the electroweak coupling $\alpha_2(M_Z)$. If the VEV $\langle \mathbf{2}_{1/2}\rangle$ has the same order as V_D of Eq. (8), then we obtain dark-Z'_1 mass at the electroweak scale. Actually, the dark-Z' masses depend on the parameters, the mixing angle, and the VEVs given in Eqs. (8) and (9).

We also note that there exists a possible superpotential term,

$$\Psi_{IJ}\Phi^{I\alpha}\Phi^{J\beta}\epsilon_{\alpha\beta} \sim (\mathbf{1},\mathbf{1})_0\,(\mathbf{1},\mathbf{2})_{+1}(\mathbf{1},\mathbf{2})_{-1} \tag{A14}$$

which may allow $\langle\Psi_{IJ}\rangle = V_2$ by the condensation of $\langle(\mathbf{1},\mathbf{2})_{+1}(\mathbf{1},\mathbf{2})_{-1}\rangle$.

3. Confinement of $\mathrm{SU}(2)_1$

The large $\mathrm{SU}(2)_1$ coupling in Eq. (A13) suggests a possibility that $\mathrm{SU}(2)_1$ confines around the electroweak scale. Let us consider four doublets of Eq. (5) as

$$D_1 = \begin{pmatrix} P_1 \\ N_1 \end{pmatrix}_{L,\frac{+1}{2}}, \qquad D_2 = \begin{pmatrix} P_2 \\ N_2 \end{pmatrix}_{L,\frac{+1}{2}}, \\ \bar{D}_1 = \begin{pmatrix} P_2 \\ N_2 \end{pmatrix}_{R,\frac{-1}{2}}, \qquad \bar{\mathcal{D}}_2 = \begin{pmatrix} P_2 \\ N_2 \end{pmatrix}_{R,\frac{+3}{2}}, \tag{A15}$$

where two $\mathrm{SU}(2)_1$ doublets are represented as R-handed chiral fields and the subscripts are the $\mathrm{U}(1)_Q$ charges. Below the $\mathrm{SU}(2)_1$ confinement scale, we consider the following condensations:

$$\langle \bar{D}_1 D_1\rangle = V_D, \qquad \langle \bar{\mathcal{D}}_2 D_2\rangle = V_S, \tag{A16}$$

and use the mass ratio presented in Eq. (10). When $\tilde{\alpha}_2$ becomes order 1 at the scale μ_2, let us assume that $\mathrm{SU}(2)_1$ confines. The condensation scale is guessed as $\mu_2/2$, following the estimate of the QCD condensation scale $\langle\bar{u}u\rangle \approx 1$ GeV/3 where $\alpha_C(1\text{ GeV}) \simeq O(1)$. $\tilde{\alpha}_2$ becomes order 1 at $\mu_2 \simeq 22.2$ GeV. In this setup, we estimate the masses of two dark-Z''s as [38]

$$M_1 \sim \frac{\tilde{g}_2(\mu_2)}{2}\frac{\mu_2}{2} \gtrsim 20\text{ GeV}, \quad M_2 \simeq \sqrt{2}\sin\theta\frac{V_S}{V_D} \sim 6\text{ GeV}, \tag{A17}$$

where we used $V_S = V_D$ and $\sin\theta = 0.2$. Note, however, that our estimate is very primitive because we used $V_1 = M_{\mathrm{GUT}}$, one-loop running for gauge coupling evolution, followed the hypothetical SUSY breaking, and a naive chiral symmetry breaking below the $\mathrm{SU}(2)_1$ confinement scale. Nevertheless, this crude estimate has led to two electroweak scale dark-Z''s.

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