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Simple criterium for *CP* conservation in the most general 2HDM

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We find a set of necessary and sufficient conditions for the *CP* conservation in the most general 2HDM in terms of observable quantities. This set contains two simple and relatively easily testable conditions instead of the more complex conditions usually discussed.

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I. INTRODUCTION

CP violation is one of the important yet not well understood aspect of the fundamental physics. The modern LHC data [1–4] allow us to conclude that the observed particle $h(125)$ is a Higgs boson with spin-*CP* parity 0^{++}—but only under the assumption that this particle has a definite parity. Generally it can may happen that it does not possess a definite parity, and this indeed happens in many models. In this case the data give no information about $h(125)$ parity [5].

In the standard model the *CP* violation is described by the Cabbibo-Kobayashi-Maskawa matrix, but its origin remains unclear. The extension of SM with two Higgs doublets, called the two Higgs doublet model (2HDM), has been introduced in 1974 with the main aim to provide an extra source of *CP* violation [6]. Later, many variants of 2HDM were considered with different features, providing different physical realizations. Each of these variants is described by Lagrangian with many parameters. The choice of the set of these parameters, describing the same physical reality (the basis choice in the parameter space) is ambiguous. In nonminimal models such as 2HDM the current situation, with the properties of the observed Higgs boson resembling those of the SM Higgs (*SM-like scenario* [7–10] or *alignment limit* [11]), can be described by different nonequivalent sets of parameters.

We consider in the paper two types of problems which appear in the study of *CP* violation in these models.

(A) Let us have a variant of 2HDM, constructed as a model for the description of some set of phenomena.
How to know whether the CP symmetry is violated or not in this model without detailed calculations of each particular effect? Here a basis independent recipe is desirable.

(B) Consider 2HDM as an approximation to the description of the Nature. How do we know whether the *CP* violation is described by this approximation, or whether the observed *CP* violation is an effect of yet another, weaker interaction.
How to check the presence of CP violation in the experiments with particles that appear in 2HDM?

II. 2HDM

The 2HDM describes a system of two spinless isospinor fields ϕ_1, ϕ_2 with hypercharge $Y = 1$. The most general form of the 2HDM potential is as follows

$$\begin{aligned} V = {} & \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ & + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \\ & + [\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{H.c.}] \\ & - \frac{m_{11}^2}{2}(\phi_1^\dagger\phi_1) - \frac{m_{22}^2}{2}(\phi_2^\dagger\phi_2) - \left[\frac{m_{12}^2}{2}(\phi_1^\dagger\phi_2) + \text{H.c.}\right]. \end{aligned} \tag{1}$$

The potential parameters are restricted by the requirement that the potential should be positive at large quasiclassical values of ϕ_i (*positivity constraints*). We assume also that these coefficients are not too big so that one can use estimates based on the lowest nontrivial approximation of the perturbation theory.

After Electroweak Symmetry Breaking the 2HDM contains 3 neutral Higgs bosons $h_a \equiv h_{1,2,3}$ (in general with indefinite *CP* parity) and the charged Higgs bosons $H^\pm$, with the masses M_a and $M_\pm$, respectively (*the numbering of h_a is independent of the ordering of masses M_a*).

A. Reparametrization freedom

2HDM describes system of two fields with identical quantum numbers. Therefore, its description in terms of original fields ϕ_i or in terms of their linear superpositions ϕ_i' are equivalent; this statement verbalizes the *reparameterization*

*Deceased

(RPa) freedom of the model. The RPa group consists of RPa transformations $\hat{\mathfrak{F}}$ of the form:

$$(\phi_1'\phi_2') = \hat{\mathcal{F}}_{\rm gen}(\theta,\tau,\rho)(\phi_1\phi_2),\\ \hat{\mathcal{F}}_{\rm gen} = e^{i\rho_0}(\cos\theta e^{i\rho/2}\sin\theta e^{i(\tau+\rho/2)}\\ -\sin\theta e^{-i(\tau+\rho/2)}\cos\theta e^{-i\rho/2}). \tag{2}$$

This transformation induces a transformation of the parameters of the Lagrangian in such a way that the new Lagrangian, written in terms of fields ϕ_i', describes the same physical content. We refer to these different choices as different RPa bases. A subgroup of the RPa group—the *rephasing* group RPh—describes a freedom of adjusting the relative phase of the fields ϕ_i.

Transformation (2) is parametrized by angles θ, ρ, τ and ρ_0. The parameter ρ_0 describes an overall phase transformation of the fields and can be ignored since it does not affect the parameters of the potential. The parameter ρ describes the RPh symmetry transformation of system.

The $U(1)_{\rm EM}$ symmetry preserving ground state of this system is given by a global minimum of the potential and reads

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2}\end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi}/\sqrt{2}\end{pmatrix}, \tag{3}$$

with the standard parametrization $\tan\beta = v_2/v_1$, or $v_1 = v\cos\beta$, $v_2 = v\sin\beta$.

B. The Higgs basis

We use below the RPa basis with $v_2 = 0$ (the Higgs, or Georgi, basis [12]), in which the 2HDM potential can be written in the form [13]

$$V_{HB} = M_\pm^2(\Phi_2^\dagger\Phi_2) + \frac{\Lambda_1}{2}\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)^2 + \frac{\Lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2\\ + \Lambda_3\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)(\Phi_2^\dagger\Phi_2) + \Lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ + \left[\frac{\Lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \Lambda_6\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)(\Phi_1^\dagger\Phi_2)\\ + \Lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + {\rm H.c.}\right]. \tag{4}$$

For this basis we use capital letters to denote the fields and the parameters of potential, Φ_i and Λ_j, respectively.

III. RELATIVE COUPLINGS

In the discussion below we use the relative couplings for each neutral Higgs boson[1] h_a (in 2HDM $a = 1, 2, 3$):

$$\chi_a^P = \frac{g_a^P}{g_{\rm SM}^P}, \qquad \chi_a^\pm = \frac{g(H^+H^-h_a)}{2M_\pm^2/v},\\ \chi_a^{H^+W^-} = \frac{g(H^+W^-h_a)}{M_W/v}. \tag{5}$$

The quantities χ_a^P (where $P = V(W,Z), q = t, b, \ldots, \ell = \tau, \ldots$) are the ratios of the couplings of h_a with the fundamental particles P to the corresponding couplings for the would-be SM Higgs boson with mass M_a. The other relative couplings describe the interactions of h_a with the charged Higgs boson $H^\pm$. Couplings χ_a^V and $\chi_a^\pm$ are real due to Hermiticity of Lagrangian, and are directly measurable. Couplings $\chi_a^{H^+W^-}$, χ_a^q and χ_a^ℓ are generally complex. Besides, one can note that [13]

$$\chi_{ab}^Z \equiv \frac{g(Zh_ah_b)}{M_Z/v} = -\varepsilon_{abc}\chi_c^V. \tag{6}$$

There are useful sum rules among these couplings, namely

$$(a)\ \sum_a(\chi_a^V)^2 = 1, \qquad (b)\ (\chi_a^V)^2 + |\chi_a^{H^+W^-}|^2 = 1. \tag{7}$$

Both the real and imaginary parts of the Yukawa couplings χ_a^q and χ_a^ℓ can be measured in principle using distributions of Higgs bosons decay products in $h_a \to \bar{q}q$, $h_a \to \bar{\ell}\ell$. The absolute value of the coupling $\chi_a^{H^+W^-}$ is fixed by the sum rule (7) b) and is well measurable.

The unitarity of the rotation matrix describing transition from components of fields ϕ_i to the physical Higgs fields h_a allows to obtain the following relations for couplings $\chi_a^{H^+W^-}$ (the factor $e^{i\rho}$ represents the rephasing freedom in the Higgs basis):

$$\chi_1^{H^+W^-} \equiv (\chi_1^{H^-W^+})^* = -e^{i\rho}\frac{\chi_1^V\chi_2^V - i\chi_3^V}{\sqrt{1-(\chi_2^V)^2}},\\ \chi_2^{H^+W^-} \equiv (\chi_2^{H^-W^+})^* = e^{i\rho}\sqrt{1-(\chi_2^V)^2},\\ \chi_3^{H^+W^-} \equiv (\chi_3^{H^-W^+})^* = -e^{i\rho}\frac{\chi_2^V\chi_3^V + i\chi_1^V}{\sqrt{1-(\chi_2^V)^2}}. \tag{8}$$

Note, that here we discuss couplings which appear in the Lagrangian. Radiative corrections (RC) change these couplings; however in most cases these corrections are small and therefore the corresponding observables differ weakly from those presented in the Lagranian. In this sense we treat the latter ones as being measurable. Therefore the identities (6)–(8) are valid with accuracy to RC.

When describing interactions of the Higgs bosons with the gauge bosons $V = W, Z$, one should distinguish interactions of different Lorentz structure: the vectorial $h_aV_\mu V^\mu$, tensor $h_aV_{\mu\nu}V^{\mu\nu}$ and axial-tensor $h_a\tilde{V}_{\mu\nu}V^{\mu\nu}$ interactions, with the coupling constants g_a^V, g_{aT}^V and $\tilde{g}_{aT}^V$, respectively (here $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\tilde{V}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}V^{\alpha\beta}$).

[1] We omit the adjective "relative" below.

These interactions can be separated in the experiment from each other by the study of angular correlations in the decays like $h_a \to ZZ \to e^+e^-\mu^+\mu^-$ [14]. In this paper we concentrate on vectorial couplings appearing in Lagrangian (and modified weakly by the RC).

In the 2HDM tensor and axial-tensor interactions appear only due to RC (mainly from t-loops). In most cases they give very small contributions to the observed decays rates of h_a and are hardly observable.

The axial-tensor interaction can imitate CP violation. This interaction was studied in Ref. [14] for CP conserved variant of 2HDM, in which one of neutral Higgs bosons is pseudoscalar A. It was found that the corresponding decay $A \to e^+e^-\mu^+\mu^-$, etc. can be observable only in a very narrow range of parameters ($M_A \approx 2m_t$, low $\tan\beta$). If it is realized, the study of angular correlation of leptons allows to distinguish CP odd nature of the initial state (as it is done currently at the LHC in the study of CP properties of the Higgs boson [1–4]) without violation of CP symmetry.

IV. A MINIMAL COMPLETE SET OF OBSERVABLES IN 2HDM

In Ref. [13] a *minimal complete* set of *directly measurable quantities (observables)* defining the 2HDM was found ($a = 1, 2, 3$), namely:

$$\begin{aligned}&\text{v.e.v. of Higgs field } v = 246\ \text{GeV},\\ &\text{masses of Higgs bosons } M_a, M_\pm,\\ &2\ (\text{out of } 3)\ \text{couplings } \chi_a^V,\\ &3\ \text{couplings } \chi_a^\pm,\\ &\text{quartic coupling } g\,(H^+H^-H^+H^-).\end{aligned} \tag{9}$$

In the most general 2HDM, these 11 observables are independent from each other. In particular variants of the 2HDM, additional relations among these parameters may appear.

The parameters of the potential in the Higgs basis are expressed through these observables and free parameter ρ [it appears in $\Lambda_{5,6,7}$ via couplings $\chi_a^{H^+W^-}$ given in Eq. (8)]:

$$\begin{aligned}&\Lambda_1 = \sum_a (\chi_a^V)^2 M_a^2/v^2; \quad \Lambda_5 = \sum_a (\chi_a^{H^-W^+})^2 M_a^2/v^2;\\ &\Lambda_4 = \left(\sum_a M_a^2 - M_\pm^2\right)/v^2 - \Lambda_1; \quad \Lambda_3 = 2(M_\pm^2/v^2)\sum_a \chi_a^V \chi_a^\pm;\\ &\Lambda_6 = \sum_a \chi_a^V \chi_a^{H^-W^+} M_a^2/v^2; \quad \Lambda_7 = 2(M_\pm^2/v^2)\sum_a \chi_a^{H^-W^+}\chi_a^\pm;\\ &\Lambda_2 = 2g(H^+H^-H^+H^-).\end{aligned} \tag{10}$$

The parameters of the potential (1) with the values of $\tan\beta$ and ξ defined in (3) are obtained from parameters (10) with the aid of transformation (2) in the following form

$$(\phi_1\phi_2) = \hat{\mathcal{F}}_{HB}(\Phi_1\Phi_2), \qquad \hat{\mathcal{F}}_{HB} = \hat{\mathcal{F}}_{\text{gen}}(\theta = -\beta, \tau = \xi, -\rho). \tag{11}$$

V. CONDITIONS FOR A CP CONSERVATION

In the Higgs models like 2HDM neutral scalar particles coincide with their antiparticles. Therefore, in such models one can discuss the P-parity violation, but not C-parity. When in addition we consider fermions, the P-parity violation is transformed to the CP violation (see e.g. [15–17]).

A. Basic facts

The CP symmetry is conserved in the 2HDM and similar models containing spinless bosons if[2]

- Each observable physical neutral spinless boson has definite P-parity (12a)
- There are no P-violating interactions between these bosons. (12b)

In the 2HDM we denote neutral spinless P-even bosons as h_1, h_2 (they are called often as h and H) and P-odd boson as h_3 (it is called often as A). The condition (12b) means the absence of the interactions[3] $(i, j, k = 1, 2)$

$$h_i h_j h_3, \qquad h_3h_3h_3, \qquad h_ih_jh_kh_3, \qquad h_ih_3h_3h_3. \tag{13}$$

It is well known that in the 2HDM the CP conservation holds if

There exists the RPa basis in which :

- all parameters of potential are real, (14a)
- relative phase (3) $\xi = 0$. (14b)

It is worth mentioning that the condition (14a) forbids the explicit CP violation while conditions (14a) and (14b) together forbid the spontaneous CP violation.

The statement (12) only describes CP conservation, but does not provide a criterium for CP conservation in the considered model. The description (14) is RPa basis-dependent. Below we discuss both (A) and (B) setups of our problem, presented in the Introduction.

Many authors consider solution of problem (A) as a necessary step in solving problem (B). Our approach is

[2]Note that CP violation originating from fermions and (or) vector bosons, appears in the interactions of spinless particles via radiative correction.

[3]Similar conditions were discussed in Ref. [18].

different: we look for a solution to problem (B), with a solution to problem (A) appearing as a by-product.

B. Method of the *CP*-odd basis-independent invariants

Many authors found the basis independent criterium for *CP* violation or conservation in terms of parameters of the Higgs potential; this corresponds to problem (A) stated in the Introduction. For this goal they constructed the RPa basis-invariant *CP*-odd combinations of parameters of the potential and as a condition for *CP* conservation they demanded vanishing of all these invariants. For 2HDM three such invariants $Im\mathcal{J}_{1,2,3}$ were found in Refs. [19,20], the procedure for construction of such invariants for multi-Higgs models is developed in Ref. [21].

To solve the corresponding problem (B) the invariants [19,20] for 2HDM were expressed via measurable quantities in Refs. [22]. In the terms of quantities (9), the corresponding conditions for the *CP* conservation read as

$$Im\mathcal{J}_1 = \sum_{i,j,k}\varepsilon_{ijk}\frac{2M_i^2M_\pm^2}{v^4}\chi_i^V\chi_k^V\chi_j^\pm = 0,$$
$$Im\mathcal{J}_2 = 2\chi_1^V\chi_2^V\chi_3^V\sum_{i,j,k}\varepsilon_{ijk}\frac{M_i^4M_k^2}{v^6} = 0,$$
$$Im\mathcal{J}_{30} = 4\sum_{i,j,k}\varepsilon_{ijk}\mathcal{T}_i\chi_j^V\chi_k^\pm = 0,$$
$$\text{where } \mathcal{T}_i = fr(M_\pm^2\chi_i^\pm + M_i^2\chi_i^V)M_i^2M_\pm^2v^6. \qquad (15)$$

Note that in this approach there are four *CP*-odd invariants but one should check vanishing only of two of them (see e.g. [17]). Since the choice of these two invariants is not fixed from beginning, the presented set contains three conditions, instead of necessary two. In our opinion the equations (15) are too complicated and their experimental verification, discussed in [19,20,22,23], requires very complex procedure.

C. A direct criterium for *CP* conservation

In the direct method we approach directly problem (B) mentioned above. We start with a description of *CP* conservation (12) and use only observables, which by definition are basis independent.

In Refs. [24,25] we proposed conditions for *CP* conservation, based only on one condition[4] (12a), without checking up of condition (12b), in the form

$$\prod_a\chi_a^V = 0, \qquad \prod_a\chi_a^\pm = 0. \qquad (16)$$

Below we simplify these conditions and prove that the set of new conditions is *necessary and sufficient*.

(i) The direct criterium

In the *CP* conserving 2HDM, all h_a should have definite P-parity. In particular, one of them is P-odd, while two others are P-even (12a). Therefore, the *necessary condition* for a *CP* conservation is an existence of one neutral Higgs boson (we denote it h_3), which doesn't couple to the *CP*-even states VV and H^+H^-. So it reads:

$$\boxed{\begin{array}{c}\text{There exists an neutral Higgs boson } h_3 \text{ for}\\ \text{which vectorial coupling } g_3^V \equiv g(h_3VV) = 0\\ \text{and } g_3^\pm \equiv g(h_3H^+H^-) = 0.\end{array}} \qquad (17)$$

Now, one has to check condition (12b). In order to do this, we substitute Eqs. (17), (8) into (10), choosing $\rho = 0$. The choice $\rho = 0$ together with constraint $\chi_3^V = 0$ in (8) makes both $\chi_{1,2}^{H^+W^-}$ real while $\chi_3^{H^+W^-}$ is imaginary. Next, we insert these $\chi_a^{H^+W^-}$ into (10). The parameter Λ_5 contains real negative quantity $(\chi_3^{H^+W^-})^2$ while in Λ_6 and Λ_7 this imaginary $\chi_3^{H^+W^-}$ is multiplied by χ_3^V or $\chi_3^\pm$, which are zeros (17). Hence, with this choice all parameters of potential Λ_a in the Higgs basis are real. Therefore, in view of the statement (14), the *CP*-symmetry of model is not violated. In particular, the *CP* violating interactions (13) do not appear. (Besides, it is easy to check that conditions (17) ensure compliance of conditions (15) and first two conditions (16)). Therefore we conclude that

$$\boxed{\begin{array}{c}\text{the conditions (17) are necessary and sufficient}\\ \text{for establishing } CP \text{ conservation in the boson}\\ \text{sector of 2HDM (problem B).}\end{array}}$$

Note that Eq. (10), after substitution of Eq. (17) together with Eq. (11), can be treated as a solution of problem (A).

(ii) A discussion of the direct criterium. In view of identity (6), the first condition (17) can be written also in the form $g(Zh_1h_2) = 0$. In some cases, checking this condition in the latter form may be more convenient. Such form of criterium for *CP* nonviolation[5] was discussed in Ref. [26].

Besides, the conditions for *CP* conservation written via the *CP*-odd invariants (15) are fulfilled

[4]A particular version of such approach was used in [26].

[5]Let us recall that such condition is insufficient to establish *CP* conservation, since it does not guarantee the fulfillment of the condition (12b), i.e., nonappearance of *CP* odd vertices (13).

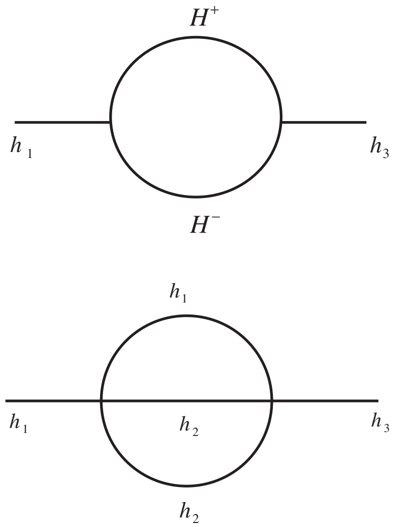

FIG. 1. Mixing provided by the H^+H^- loop (up) and by one of *CP* violated vertices (13) (down).

if some masses M_a are degenerate and some special relations among observable couplings χ_a^V and $\chi_a^\pm$ take place. This combination can be treated as an alternative form of our necessary and sufficient condition in this particular case. Note, that if $g(h_3H^+H^-) \neq 0$, the loop diagrams of Fig. 1 mix the Higgs states with different "bare" *CP* parity, which results in *CP* violation.

VI. POSSIBILITIES FOR A VERIFICATION

The verification of the *CP* conservation requires observation of all scalars of the model. In the SM-like scenario realized in Nature this looks difficult (see, e.g., [25]). Moreover, one should check if some measurable quantities are equal to zero. In any case, these measurements cannot be performed with a high accuracy. From this point of view the proposal to change a direct criterium to a condition for a nonobservation of decay $h_3 \to h_1 Z$, etc. given in [26] looks attractive. Nevertheless, one can not hope for a high accuracy in testing *CP* conservation in the 2HDM. Let us remind that the possible observation of weak enough decay $h_3 \to ZZ \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ do not contradict *CP* conservation. If this decay is observed, the correlations between momenta of leptons should be studied. The *CP* conservation for h_3 will be confirmed if these correlations show axial-tensor nature of this interaction, i.e., pseudoscalar nature of h_3.

If in the future the experiments show, within the definite experimental uncertainties, an agreement with criteria (17), the important problem arises for the further studies, namely *whether one can expect that the more accurate measurements will show violation of CP in our 2HDM, i. e. violation of criteria* (17), *or the observed CP violation is in fact an effect beyond approximation given by 2HDM, and some new weaker interactions should be implemented in the description of Nature.*

This situation is similar to that in atomic physics. Atomic interaction (QED) conserves P-parity. Parity nonconservation appears at the smallest distances due to next level weak interaction. It is observed as a small effect in rare atomic transitions.

VII. CONCLUSIONS

We present here a compact set of necessary and sufficient conditions for *CP* conservation in 2HDM (17), which are common for all mechanisms of *CP* violation. We prove that the verification of *CP* conservation in 2HDM requires to measure two simple and relatively easily testable observables instead of more complex conditions with *CP*-odd invariants (15) discussed by many authors [19,20,22,23].

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Note added.—Maria Krawczyk passed away on May 24 2017, while we were editing this paper.

APPENDIX: NECESSARY CONDITION FOR *CP* CONSERVATION IN MULTI-HIGGS DOUBLET MODEL

The criterium of *CP* conservation in the multi-Higgs doublet models—nHDM are also of interest. Here, the method of *CP*-odd invariants allows to construct many equations, which can be used for obtaining conditions for *CP* conservation. Both complete set of these equations and their expressions via measurable quantities are absent up to now (see, e.g., [21]).

The direct method used above allows us to formulate for the nHDM simple necessary conditions for the *CP* conservation.

After Electroweak Symmetry Breaking the nHDM contains $2n-1$ neutral Higgs bosons h_a, generally with indefinite *CP* parity, and $n-1$ charged Higgs boson $H_b^\pm$, with masses M_a, $M_{b\pm}$, respectively. The couplings h_aVV obey the first sum rule (7). In the case of *CP* conservation one can split spinless neutral particles h_a into two groups: P-even $h_1, \ldots, h_n$ and P-odd $h_{n+1}, \ldots, h_{2n-1}$. Similarly to (17), the condition for a *CP* conservation in the nHDM can be written as

$$\boxed{\begin{array}{c}\text{There exist}\, n-1\, \text{neutral Higgs bosons}\, h_c,\ \text{for}\\ \text{which}\, g(h_cVV)=0,\ g(h_cH_b^+H_b^-)=0\\ \text{with}\quad n+1\le c\le 2n-1,\ 1\le b\le n-1.\end{array}} \tag{A1}$$

These $n(n-1)$ conditions are *necessary* for *CP* conservation. We do not know now whether these conditions are *sufficient* or not.

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