Light sterile neutrinos, dark matter, and new resonances in a $U(1)$ extension of the MSSM

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We present ψ'MSSM, a model based on a $U(1)_{\psi'}$ extension of the minimal supersymmetric standard model. The gauge symmetry $U(1)_{\psi'}$, also known as $U(1)_N$, is a linear combination of the $U(1)_\chi$ and $U(1)_\psi$ subgroups of E_6. The model predicts the existence of three sterile neutrinos with masses $\lesssim 0.1$ eV, if the $U(1)_{\psi'}$ breaking scale is of order 10 TeV. Their contribution to the effective number of neutrinos at nucleosynthesis is $\Delta N_\nu \simeq 0.29$. The model can provide a variety of possible cold dark matter candidates including the lightest sterile sneutrino. If the $U(1)_{\psi'}$ breaking scale is increased to 10^3 TeV, the sterile neutrinos, which are stable on account of a Z_2 symmetry, become viable warm dark matter candidates. The observed value of the standard model Higgs boson mass can be obtained with relatively light stop quarks thanks to the D-term contribution from $U(1)_{\psi'}$. The model predicts diquark and diphoton resonances which may be found at an updated LHC. The well-known μ problem is resolved and the observed baryon asymmetry of the universe can be generated via leptogenesis. The breaking of $U(1)_{\psi'}$ produces superconducting strings that may be present in our galaxy. A $U(1)$ R symmetry plays a key role in keeping the proton stable and providing the light sterile neutrinos.

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I. INTRODUCTION

E_6 grand unified theory (GUT) [1] contains two especially interesting maximal subgroups for model building, namely $SU(3)^3$ and $SO(10)\times U(1)_\psi$. Supersymmetric (SUSY) models based on $SU(3)^3$, sometimes referred to as trinification models, have been extensively discussed in the literature. For instance, in SUSY $SU(3)^3$, mechanisms have been proposed to resolve [2] the minimal supersymmetric standard model (MSSM) μ problem or make [3] the proton essentially stable.

The subgroup $SO(10)\times U(1)_\psi$ of E_6 can be decomposed further, via $SU(5)$, to the MSSM gauge symmetry group accompanied by $U(1)_\chi \times U(1)_\psi$ [4,5]. One intriguing combination of these two $U(1)$'s, denoted here as $U(1)_{\psi'}$ (also known as $U(1)_N$ [4] in the literature), is assumed [6] here to be broken at a scale at least an order of magnitude greater than the TeV scale of soft SUSY breaking. We refer to this extension of the MSSM accompanied by $U(1)_{\psi'}$ as ψ'MSSM. The well-known right handed neutrino contained in the matter 16-plet of $SO(10)$ transforms as a singlet under $U(1)_{\psi'}$. This enables the three right handed neutrinos to acquire large masses, so that the standard seesaw scenarios can apply and high scale leptogenesis [7] can be realized [8]. Note that the subscript ψ' reiterates the essential role played by $U(1)_{\psi'}$ in resolving the MSSM μ problem.

Our ψ'MSSM model employs in an essential way a $U(1)$ R symmetry such that dimension five and higher dimensional operators potentially causing proton decay are eliminated. The MSSM μ problem is also resolved and the usual lightest SUSY particle of MSSM remains [9] a compelling dark matter candidate. More intriguingly perhaps, the model predicts that the three $SO(10)$ singlet sterile neutrino matter fields that it contains can only acquire tiny masses, on the order of 0.1 eV or less if $U(1)_{\psi'}$ is broken around 10 TeV. We estimate that for this case the effective number of neutrinos during nucleosynthesis is changed by $\simeq 0.29$. The lightest sterile sneutrino as well as two more particles, which are stable on account of discrete symmetries, can, under certain circumstances, be additional cold dark matter candidates.

If the breaking scale of $U(1)_{\psi'}$ is increased to 10^3 TeV or so, the sterile neutrinos, which happen to be stable on account of a Z_2 symmetry, become plausible candidates for keV scale warm dark matter.

The contribution of the D-term for $U(1)_{\psi'}$ to the mass of the lightest CP-even neutral Higgs boson of the MSSM can be appreciable leading, in the so-called decoupling limit, to the observed value of 125 GeV with relatively light stop quarks.

In addition to the Z' gauge boson associated with the breaking of the $U(1)_{\psi'}$ gauge symmetry, the model predicts

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the existence of diphoton [10] and diquark [11] resonances with masses in the TeV range. A high luminosity or high energy (33 TeV) LHC upgrade may be able to find them. Note that the $U(1)_{\psi'}$ breaking produces superconducting strings [12] which presumably survived inflation and should be present in our galaxy. If the breaking scale is not too high, a 100 TeV collider may be able to make these strings, which definitely would be exciting.

The layout of our paper is as follows. In Sec. II, we introduce the model with its field content, symmetries, and couplings. In Sec. III, we analyze the details of the spontaneous symmetry breaking of $U(1)_{\psi'}$, while in Sec. IV we discuss the spontaneous breaking of the electroweak symmetry. Section V is devoted to the diphoton excess and Sec. VI to the presentation of a numerical example. In Sec. VII, we study the sterile neutrinos. The possible composition of dark matter in the universe is presented in Sec. VIII and our conclusions are summarized in Sec. IX.

II. THE MODEL

We consider a SUSY model based on the gauge group $G_{\rm SM} \times U(1)_{\psi'}$, where $G_{\rm SM} = SU(3)_{\rm c} \times SU(2)_{\rm L} \times U(1)_Y$ is the standard model (SM) gauge group. The GUT-normalized generator $Q_{\psi'}$ of the extra local $U(1)_{\psi'}$ symmetry is given by

$$Q_{\psi'} = \frac{1}{4}(Q_\chi + \sqrt{15}Q_\psi), \tag{1}$$

where Q_χ is the GUT-normalized generator of the $U(1)_\chi$ subgroup of $SO(10)$ which commutes with its $SU(5)$ subgroup and Q_ψ is the GUT-normalized generator of the $U(1)_\psi$ subgroup of E_6 which commutes with its $SO(10)$ subgroup. The $U(1)_{\psi'}$ symmetry is to be spontaneously broken at some scale M and we prefer to implement this breaking by a SUSY generalization of the well-known Brout-Englert-Higgs mechanism.

The important part of the superpotential is

$$\begin{aligned} W = {} & y_u H_u^1 q u^c + y_d H_d^1 q d^c + y_\nu H_u^1 l \nu^c + y_e H_d^1 l e^c \\ & + \frac{1}{2} M_{\nu^c} \nu^c \nu^c + \lambda_\mu^i N H_u^i H_d^i + \kappa S(N\bar{N} - M^2) \\ & + \lambda_D^i N D_i D_i^c + \lambda_q^i D_i q q + \lambda_{q^c}^i D_i^c u^c d^c \\ & + \lambda_L S L \bar{L} + \lambda_{H_d}^\alpha \nu^c \bar{L} H_d^\alpha + \lambda_N^i N_i N_i \frac{\bar{N}^2}{2m_{\rm P}}, \end{aligned} \tag{2}$$

where $m_{\rm P}$ is the reduced Planck mass and y_u, y_d, y_ν, y_e are the Yukawa coupling constants with the family indices suppressed. Here q, u^c, d^c, l, ν^c, e^c are the usual quark and lepton superfields of MSSM including the right handed neutrinos ν^c and H_u^i, H_d^j $(i, j = 1, 2, 3)$ are $SU(2)_{\rm L}$ doublets with hypercharge $Y = 1/2, -1/2$ respectively. The superfields N, $\bar{N}$ constitute a conjugate pair of SM singlets, while S is a gauge singlet. The coupling $\lambda_\mu^{ij} N H_u^i H_d^j$ is diagonalized by appropriate rotations of H_u^i and H_d^j and a discrete Z_2 symmetry under which H_u^α and H_d^α $(\alpha = 2, 3)$ are odd is imposed. Consequently, only H_u^1, H_d^1 couple to quarks and leptons and are the standard electroweak Higgs superfields.

The superfields D_i and D_i^c $(i = 1, 2, 3)$ are color triplets and antitriplets with $Y = -1/3$ and $1/3$ respectively and the coupling $\lambda_D^{ij} N D_i D_j^c$ is diagonalized by appropriate rotations of D_i and D_j^c. The superfields N_i $(i = 1, 2, 3)$ are SM singlets and the coupling $\lambda_N^{ij} N_i N_j \bar{N}^2/2m_{\rm P}$ is again diagonalized by rotating N_i and N_j. We impose an extra Z_2' symmetry under which the N_i's are odd. In order to achieve unification of the MSSM gauge coupling constants, we introduced an extra conjugate pair of $SU(2)_{\rm L}$ doublets L and $\bar{L}$ with $Y = -1/2$ and $1/2$ respectively. These doublets are odd under Z_2 and together with H_d^α and H_u^α $(\alpha = 2, 3)$ form three complete $SU(5)$ multiplets with the color (anti) triplets D_i and D_i^c. Note that the superfields q, u^c, d^c, l, ν^c, e^c, H_u^i, H_d^i, D_i, D_i^c, and N_i form three complete fundamental representations of E_6, while N, $\bar{N}$ and L, $\bar{L}$ are conjugate pairs from incomplete E_6 multiplets.

In Table I, we summarize all the superfields of the model together with their transformation properties under the SM gauge group $G_{\rm SM}$ and their charges under the discrete symmetries Z_2, Z_2', the global R symmetry $U(1)_R$, and the

TABLE I. Superfield content of the model.

Superfields	Representations under $G_{\rm SM}$	Extra Symmetries Z_2	Z_2'	R	$2\sqrt{10}Q_{\psi'}$
	Matter Superfields				
q	$(\mathbf{3}, \mathbf{2}, 1/6)$	+	+	1/2	1
u^c	$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	+	+	1/2	1
d^c	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	+	+	1/2	2
l	$(\mathbf{1}, \mathbf{2}, -1/2)$	+	+	0	2
ν^c	$(\mathbf{1}, \mathbf{1}, 0)$	+	+	1	0
e^c	$(\mathbf{1}, \mathbf{1}, 1)$	+	+	1	1
H_u^α	$(\mathbf{1}, \mathbf{2}, 1/2)$	−	+	1	−2
H_d^α	$(\mathbf{1}, \mathbf{2}, -1/2)$	−	+	1	−3
D_i	$(\mathbf{3}, \mathbf{1}, -1/3)$	+	+	1	−2
D_i^c	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	+	+	1	−3
N_i	$(\mathbf{1}, \mathbf{1}, 0)$	+	−	1	5
	Higgs Superfields				
H_u^1	$(\mathbf{1}, \mathbf{2}, 1/2)$	+	+	1	−2
H_d^1	$(\mathbf{1}, \mathbf{2}, -1/2)$	+	+	1	−3
S	$(\mathbf{1}, \mathbf{1}, 0)$	+	+	2	0
N	$(\mathbf{1}, \mathbf{1}, 0)$	+	+	0	5
$\bar{N}$	$(\mathbf{1}, \mathbf{1}, 0)$	+	+	0	−5
	Extra $SU(2)_{\rm L}$ Doublet Superfields				
L	$(\mathbf{1}, \mathbf{2}, -1/2)$	−	+	0	−3
$\bar{L}$	$(\mathbf{1}, \mathbf{2}, 1/2)$	−	+	0	3

local $U(1)_{\psi'}$ with GUT-normalized charge $Q_{\psi'}$. Note that the discrete symmetries Z_2, Z_2' do not carry $SU(3)_c$ or $SU(2)_L$ anomalies.

The symmetries of the model allow not only the superpotential terms in Eq. (2), but also the following higher order terms (divided by appropriate powers of m_P):

$$\begin{aligned}
&\nu^c H_u^\alpha L N, e^c H_d^\alpha L \bar{N}, H_u^1 H_u^1 ll, H_u^\alpha H_u^\beta ll, H_u^1 H_d^\alpha l\bar{L},\\
&H_u^\alpha H_d^1 l\bar{L}, H_d^1 H_d^1 \bar{L}\,\bar{L}, H_d^\alpha H_d^\beta \bar{L}\,\bar{L}, qu^c qd^c \bar{N}, qu^c e^c l\bar{N},\\
&qd^c \nu^c l\bar{N}, e^c \nu^c LLN, H_u^\alpha qd^c lL, H_u^1 H_u^\alpha lLN,\\
&H_u^1 H_u^1 LLNN, H_u^\alpha H_u^\beta LLNN, H_u^\alpha qu^c l\bar{L}\,\bar{N},\\
&H_d^\alpha qd^c l\bar{L}\,\bar{N}, \nu^c H_d^1 l\bar{L}\,\bar{L}\,\bar{N}, e^c H_u^1 lLLN, qd^c Lqd^c L,\\
&D_i^c u^c u^c \bar{L}\,\bar{L}\,\bar{N}, D_i^c d^c d^c LLN, e^c qd^c lLL, H_u^1 qd^c LLN,\\
&H_d^1 qu^c \bar{L}\,\bar{L}\,\bar{N}, H_d^1 H_d^\alpha l\bar{L}\,\bar{L}\,\bar{L}\,\bar{N}, H_u^\alpha e^c LLLNN,\\
&\nu^c qu^c l\bar{L}\,\bar{L}\,\bar{N}\,\bar{N}, qu^c qu^c \bar{L}\,\bar{L}\,\bar{N}\,\bar{N}, e^c e^c LLLLNN,\\
&H_d^\alpha qu^c l\bar{L}\,\bar{L}\,\bar{L}\,\bar{N}\,\bar{N}\,. \qquad (3)
\end{aligned}$$

Note that all the couplings in Eqs. (2) and (3) can be multiplied by the combinations $N\bar{N}/m_P^2$, $L\bar{L}/m_P^2$, and $\bar{L}l\bar{N}\,\bar{L}\,l\bar{N}/m_P^6$ arbitrarily many times and this exhausts all the possible superpotential couplings compatible with the symmetries of the model.

Assigning baryon number $B = -2/3$ and $2/3$ to the diquark superfields D_i and D_i^c, respectively, we see that the baryon number $U(1)_B$ symmetry is automatically present to all orders in the superpotential and, thus, fast proton decay and other baryon number violating effects are avoided [13].

The fundamental representation of E_6 contains two SM singlets with the quantum numbers of ν^c and N_i. Let us assume that at high energies the gauge symmetry is $G_{\rm SM} \times U(1)_\chi \times U(1)_\psi$. A conjugate pair of Higgs superfields of the type ν^c, $\bar{\nu}^c$ from an incomplete E_6 multiplet can break $U(1)_\chi \times U(1)_\psi$ to $U(1)_{\psi'}$ at a scale of order the GUT scale. So, at lower energies, only the gauge symmetry $G_{\rm SM} \times U(1)_{\psi'}$ of our model survives. The spontaneous breaking of $U(1)_{\psi'}$ at a scale $M \sim 10$ TeV is then achieved by a conjugate pair of Higgs superfields of the type N, $\bar{N}$ from an incomplete E_6 multiplet via the superpotential terms $\kappa S(N\bar{N} - M^2)$. This breaking will generate a network of local superconducting strings. Their string tension, which is determined by the scale M, is relatively small and certainly satisfies the most stringent relevant upper bound from pulsar timing arrays [14]. Note, in passing, that the kinetic mixing of $U(1)_{\psi'}$ and $U(1)_Y$ is negligible—see fourth paper in Ref. [4].

The "bare" MSSM μ term is replaced by a term $\lambda_\mu^1 N H_u^1 H_d^1$, so that the μ term is generated after N acquires a non-zero vacuum expectation value (VEV) $\langle N \rangle$ of order 10 TeV. The same VEV gives masses to the two remaining pairs of $SU(2)_L$ doublets H_u^α, H_d^α ($\alpha = 2, 3$) via the superpotential terms $\lambda_\mu^\alpha N H_u^\alpha H_d^\alpha$ as well as to the diquarks D_i, D_i^c ($i = 1, 2, 3$) via the terms $\lambda_D^i N D_i D_i^c$. The gauge singlet S acquires a VEV $\langle S \rangle$ of order TeV from soft SUSY breaking [15]. (In the SUSY limit the VEV of S is zero.) This VEV generates masses for the extra doublets L, $\bar{L}$ via the term $\lambda_L S L\bar{L}$. Finally, the sterile neutrino fields, which are the fermionic parts of N_i, acquire masses of order 10^{-1} eV or so via the terms $\lambda_N^i N_i N_i \bar{N}^2/2m_P$.

The spontaneous breaking of $U(1)_{\psi'}$ implemented with the fields S, N, $\bar{N}$ delivers, in the exact SUSY limit, four spin zero particles all with the same mass given by $\sqrt{2}\kappa M$. This mass, even for $M \gg 1$ TeV, can be of order TeV by selecting an appropriate value for κ. We should point out though that, depending on the SUSY breaking mechanism, these states may end up with significantly different masses. The diquarks D_i, D_i^c may be found [11] at the LHC.

III. $U(1)_{\psi'}$ BREAKING

We will assume here that the breaking scale of $U(1)_{\psi'}$ is much bigger than the electroweak scale. In this case, the spontaneous breaking of $U(1)_{\psi'}$ is not affected by the electroweak Higgs doublets in any essential way and can be discussed by considering only the superpotential terms

$$\delta W = \kappa S(N\bar{N} - M^2) \qquad (4)$$

in the right-hand side (RHS) of Eq. (2). They give the following scalar potential

$$\begin{aligned}
V = {}&\kappa^2|N\bar{N} - M^2|^2 + \kappa^2|S|^2(|N|^2 + |\bar{N}|^2)\\
&+ (A\kappa S N\bar{N} - (A - 2m_{3/2})\kappa M^2 S + \text{H.c.})\\
&+ m_0^2(|N|^2 + |\bar{N}|^2 + |S|^2) + \text{D-terms}. \qquad (5)
\end{aligned}$$

Here the mass parameter M and the dimensionless coupling constant κ are made real and positive by field rephasing and the scalar components of the superfields are denoted by the same symbol. The parameter $m_{3/2}$ is the gravitino mass, $A \sim m_{3/2}$ is the coefficient of the trilinear soft terms taken real and positive, and $m_0 \sim m_{3/2}$ is the common soft mass of N, $\bar{N}$, and S. We assumed, for definiteness, minimal supergravity. In this case, the coefficients of the trilinear and linear soft terms are related as shown in Eq. (5). Vanishing of the D-terms implies that $|N| = |\bar{N}|$, which yields $\bar{N}^* = e^{i\vartheta}N$, while minimization of the potential requires that $\vartheta = 0$. So, N and $\bar{N}$ can be rotated to the positive real axis by a $U(1)_{\psi'}$ transformation.

We find [15] that the scalar potential in Eq. (5) is minimized at

$$\langle S \rangle = -\frac{m_{3/2}}{\kappa}\left(1 + \sum_{n\geq 1} c_n \left(\frac{m_{3/2}}{M}\right)^n\right) \qquad (6)$$

and

$$\langle N\rangle = \langle \bar{N}\rangle \equiv \frac{N_0}{\sqrt{2}} = M\left(1+\sum_{n\geq 1} d_n\left(\frac{m_{3/2}}{M}\right)^n\right), \quad (7)$$

where c_n, d_n are numerical coefficients of order unity. Assuming that $M \gg m_{3/2}$ and keeping in $\langle S\rangle^2$ and N_0^2 terms up to order $m_{3/2}^2$, these formulas can be approximated as follows:

$$\langle S\rangle \simeq -\frac{m_{3/2}}{\kappa}, \qquad \frac{N_0^2}{2} \simeq M^2 + \frac{Am_{3/2} - m_{3/2}^2 - m_0^2}{\kappa^2}. \quad (8)$$

We should point out that the trilinear and linear soft terms in the second line of Eq. (5) play an important role in our scheme. Substituting N and $\bar{N}$ by their VEVs, these terms yield a linear term in S which, together with the mass term of S, generates [15] a VEV for S of order TeV. It is then obvious that, substituting this VEV of S in the superpotential term $\lambda_L SL\bar{L}$, the superfields L, $\bar{L}$ acquire a mass $m_L = \lambda_L|\langle S\rangle| = \lambda_L m_{3/2}/\kappa$. Moreover, the MSSM μ term is obtained by substituting $\langle N\rangle$ in the superpotential term $\lambda_\mu^1 N H_u^1 H_d^1$ with $\mu = \lambda_\mu^1 N_0/\sqrt{2}$, while H_u^α, H_d^α ($\alpha = 2, 3$) and D_i, D_i^c acquire masses of order TeV from the couplings $\lambda_\mu^\alpha N H_u^\alpha H_d^\alpha$ and $\lambda_D^i N D_i D_i^c$ respectively. Note that, with D_i, D_i^c, L, $\bar{L}$, and H_u^α, H_d^α masses ~TeV, the gauge couplings stay in the perturbative domain for up to four such pairs of color (anti)triplets and $SU(2)_{\rm L}$ doublets.

The mass spectrum of the scalar $S - N - \bar{N}$ system can be constructed by substituting $N = \langle N\rangle + \delta\tilde{N}$ and $\bar{N} = \langle \bar{N}\rangle + \delta\tilde{\bar{N}}$. In the unbroken SUSY limit, we find two complex scalar fields S and $\theta = (\delta\tilde{N} + \delta\tilde{\bar{N}})/\sqrt{2}$ with equal masses $m_S = m_\theta = \sqrt{2}\kappa M$. Soft SUSY breaking can, of course, mix these fields and generate a mass splitting. For example, the trilinear soft term $A\kappa SN\bar{N}$ yields a mass-squared splitting $\pm\sqrt{2}\kappa MA$ with the mass eigenstates now being $(S+\theta^*)/\sqrt{2}$ and $(S-\theta^*)/\sqrt{2}$. This splitting is small for $A \ll \sqrt{2}\kappa M$.

IV. ELECTROWEAK SYMMETRY BREAKING

The standard scalar potential for the radiative electroweak symmetry breaking in MSSM is modified in the present model. A modification originates from the D-term for $U(1)_{\psi'}$:

$$V_D = \frac{g_{\psi'}^2}{80}[-2|H_u|^2 - 3|H_d|^2 + 5(|N|^2 - |\bar{N}|^2)]^2, \quad (9)$$

where $g_{\psi'}$ is the GUT-normalized gauge coupling constant for the $U(1)_{\psi'}$ symmetry and H_u, H_d are the neutral components of the scalar parts of the Higgs $SU(2)_{\rm L}$ doublet superfields H_u^1, H_d^1 respectively. In order to find the leading contribution of this D-term to the electroweak potential, we must integrate out to one loop the heavy degrees of freedom N and $\bar{N}$. To this end, we express these complex scalar fields in terms of the canonically normalized real scalar fields δN, $\delta\bar{N}$, φ, $\bar{\varphi}$ as follows:

$$N = \frac{1}{\sqrt{2}}(N_0 + \delta N)e^{\frac{i\varphi}{N_0}}, \quad \bar{N} = \frac{1}{\sqrt{2}}(N_0 + \delta\bar{N})e^{\frac{i\bar{\varphi}}{N_0}}. \quad (10)$$

Then the combination $|N|^2 - |\bar{N}|^2$, which appears in the D-term in Eq. (9), becomes

$$|N|^2 - |\bar{N}|^2 = \sqrt{2}N_0\eta + \eta\xi, \quad (11)$$

where

$$\eta = \frac{\delta N - \delta\bar{N}}{\sqrt{2}}, \qquad \xi = \frac{\delta N + \delta\bar{N}}{\sqrt{2}} \quad (12)$$

are canonically normalized real scalar fields. The D-term can now be expanded as follows:

$$V_D = \frac{g_{\psi'}^2}{80}[E^2 + 10\sqrt{2}N_0 E\eta + 50N_0^2\eta^2 + \cdots], \quad (13)$$

where $E \equiv -2|H_u|^2 - 3|H_d|^2$. Here we kept only up to quadratic terms in η, ξ, but ignored the mixed quadratic term proportional to $\eta\xi$ since its coefficient is much smaller than the coefficient of the η^2 term assuming that N_0 is much bigger than the electroweak scale.

We see, from Eq. (13), that integrating out the heavy states reduces to the calculation of a path integral over the real scalar field η. To do this, we first need to find the η dependence of the potential V in Eq. (5). So we substitute in this equation N and $\bar{N}$ from Eq. (10). Keeping only η-dependent terms up to the second order and substituting S by its VEV in Eq. (8), we obtain

$$\delta V \simeq \frac{1}{2}\left(-\frac{\kappa^2}{2}N_0^2 + m_{3/2}^2 + m_0^2 + \kappa^2 M^2 + Am_{3/2}\right)\eta^2, \quad (14)$$

which, substituting N_0 from Eq. (8), gives

$$\delta V \simeq m_N^2\eta^2 \quad \text{with} \quad m_N^2 \equiv m_{3/2}^2 + m_0^2. \quad (15)$$

Adding δV to the D-term potential in Eq. (13), we obtain the potential

$$V_\eta = \frac{g_{\psi'}^2}{80}E^2 + \frac{\sqrt{2}g_{\psi'}^2}{8}N_0 E\eta + \left(m_N^2 + \frac{5g_{\psi'}^2}{8}N_0^2\right)\eta^2 + \cdots, \quad (16)$$

which can be given the form

$$V_\eta = \frac{g_{\psi'}^2 E^2}{80}\left(1+\frac{5g_{\psi'}^2 N_0^2}{8m_N^2}\right)^{-1} + \left(m_N^2 + \frac{5g_{\psi'}^2 N_0^2}{8}\right) \times \left(\eta + \frac{g_{\psi'}^2 N_0 E}{8\sqrt{2}\left(m_N^2+\frac{5g_{\psi'}^2 N_0^2}{8}\right)}\right)^2 + \cdots. \tag{17}$$

The path integral

$$\int (d\eta) e^{-iV_\eta \mathcal{V}}, \tag{18}$$

where $\mathcal{V}$ is the spacetime volume, can be readily calculated and, besides an irrelevant overall constant factor, we are left with the term

$$\delta V_D \simeq \frac{g_{\psi'}^2}{80}[2|H_u|^2 + 3|H_d|^2]^2 \left(1+\frac{m_{Z'}^2}{2m_N^2}\right)^{-1} \tag{19}$$

to be added to the usual electroweak symmetry breaking potential. Here $m_{Z'} = \sqrt{5} g_{\psi'} N_0/2$ is the mass of the Z' gauge boson associated with $U(1)_{\psi'}$.

Another modification of the MSSM electroweak potential comes from the integration of the heavy complex field S with mass $\sqrt{2}\kappa M$ in the exact SUSY limit. The cross F-term F_N between the superpotential terms $\kappa S N \bar{N}$ and $\lambda_\mu^1 N H_u^1 H_d^1$ in Eq. (2) together with the mass-squared term of S give

$$2\kappa^2 M^2 |S|^2 + (\kappa S^* \bar{N}^* \tilde{\lambda}_\mu H_u^1 H_d^1 + \text{H.c.}) = \left|\sqrt{2}\kappa M S + \frac{1}{\sqrt{2}}\tilde{\lambda}_\mu H_u^1 H_d^1\right|^2 - \frac{1}{2}\tilde{\lambda}_\mu^2 |H_u^1 H_d^1|^2, \tag{20}$$

where $\tilde{\lambda}_\mu \equiv \lambda_\mu^1$. Integrating out S, we then obtain the extra term

$$-\frac{1}{2}\tilde{\lambda}_\mu^2 |H_u|^2 |H_d|^2 \tag{21}$$

in the electroweak potential. One can show that the integration of all the other heavy fields gives smaller contributions, which we ignore.

Now the potential for the electroweak symmetry breaking as can be derived from the superpotential terms

$$\kappa S(N\bar{N} - M^2) - \tilde{\lambda}_\mu N H_u H_d \tag{22}$$

after substituting the VEVs of S, N, and $\bar{N}$ from Eq. (8) and adding the D-term in Eq. (19) and the term in Eq. (21) is

$$V_{\text{EW}} \simeq m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 - B(H_u H_d + \text{H.c.}) + \lambda_\mu^2 |H_u|^2 |H_d|^2 + \frac{1}{8}(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2 + c(Q_u |H_u|^2 + Q_d |H_d|^2)^2, \tag{23}$$

where $m_{H_u}^2 = \tilde{m}_{H_u}^2 + \mu^2$, $m_{H_d}^2 = \tilde{m}_{H_d}^2 + \mu^2$ with $\tilde{m}_{H_u}$, $\tilde{m}_{H_d}$ being the soft masses of H_u, H_d and $B = \tilde{B} - m_{3/2}$ with $\tilde{B}$ being the coefficient of the soft trilinear term corresponding to the second term in Eq. (22). Here $\lambda_\mu \equiv \tilde{\lambda}_\mu/\sqrt{2}$, g is the $SU(2)_{\text{L}}$ and g' the non-GUT-normalized $U(1)_Y$ gauge coupling constant, $Q_u = 2$, $Q_d = 3$, and

$$c = \frac{g_{\psi'}^2}{80}\left(1+\frac{m_{Z'}^2}{2m_N^2}\right)^{-1}. \tag{24}$$

Note that the potential in Eq. (23) contains the so-called next-to-minimal supersymmetric standard model (NMSSM) term

$$\lambda_\mu^2 |H_u|^2 |H_d|^2. \tag{25}$$

Minimization of the potential in Eq. (23) yields the following relations:

$$\begin{aligned} m_{H_u}^2 &= m_A^2 \cos^2\beta + \frac{1}{2} m_Z^2 \cos 2\beta - \lambda_\mu^2 v^2 \cos^2\beta \\ &\quad - 2c Q_u v^2 (Q_u \sin^2\beta + Q_d \cos^2\beta), \\ m_{H_d}^2 &= m_A^2 \sin^2\beta - \frac{1}{2} m_Z^2 \cos 2\beta - \lambda_\mu^2 v^2 \sin^2\beta \\ &\quad - 2c Q_d v^2 (Q_u \sin^2\beta + Q_d \cos^2\beta). \end{aligned} \tag{26}$$

Here $v^2 = v_u^2 + v_d^2$ with $v_u = \langle H_u \rangle$ and $v_d = \langle H_d \rangle$, $\tan\beta = v_u/v_d$, and the expressions

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2) v^2, m_A^2 = \frac{2B\mu}{\sin 2\beta} \tag{27}$$

for the Z gauge boson mass m_Z and the CP-odd Higgs boson mass m_A are used. Note that the latter is not affected by the extra terms in the potential V_{EW} since they involve only the absolute values of H_u, H_d.

The mass-squared matrix in the CP-even Higgs sector

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22} \end{pmatrix} \tag{28}$$

can be constructed by substituting $H_u = v_u + h_u/\sqrt{2}$ and $H_d = v_d + h_d/\sqrt{2}$ in the RHS of Eq. (23) and keeping only terms quadratic in h_u, h_d. We find

$$\begin{aligned}\mathcal{M}_{11} &= m_{H_u}^2 + \frac{1}{2} m_Z^2 (3\sin^2\beta - \cos^2\beta) + \lambda_\mu^2 v^2 \cos^2\beta \\ &\quad + 2cQ_u v^2 (3Q_u \sin^2\beta + Q_d \cos^2\beta), \\ \mathcal{M}_{12} &= (-m_A^2 - m_Z^2 + 2\lambda_\mu^2 v^2 + 4cQ_u Q_d v^2) \sin\beta\cos\beta, \\ \mathcal{M}_{22} &= m_{H_d}^2 + \frac{1}{2} m_Z^2 (3\cos^2\beta - 2\sin^2\beta) + \lambda_\mu^2 v^2 \sin^2\beta \\ &\quad + 2cQ_d v^2 (3Q_d \cos^2\beta + Q_u \sin^2\beta). \end{aligned} \tag{29}$$

Using the minimization conditions in Eq. (26), $\mathcal{M}_{11}$ and $\mathcal{M}_{22}$ can be cast in the form

$$\begin{aligned}\mathcal{M}_{11} &= m_A^2 \cos^2\beta + (m_Z^2 + 4cQ_u^2 v^2) \sin^2\beta, \\ \mathcal{M}_{22} &= m_A^2 \sin^2\beta + (m_Z^2 + 4cQ_d^2 v^2) \cos^2\beta. \end{aligned} \tag{30}$$

The eigenvalues m_h^2 and m_H^2 of the mass-squared matrix in Eq. (28), which are, respectively, the "tree-level" masses squared of the lightest and heavier neutral CP-even Higgs bosons, can now be constructed:

$$m_{h,H}^2 = \frac{1}{2}\Sigma \mp \sqrt{\frac{1}{4}\Sigma^2 - \Delta} \tag{31}$$

with

$$\begin{aligned}\Sigma &= m_A^2 + m_Z^2 + 4c^2 v^2 (Q_u^2 \sin^2\beta + Q_d^2 \cos^2\beta), \\ \Delta &= m_A^2 m_Z^2 \cos^2 2\beta + 4cv^2 m_A^2 (Q_u \sin^2\beta + Q_d \cos^2\beta)^2 \\ &\quad + \lambda_\mu^2 v^2 m_A^2 \sin^2 2\beta + cv^2 m_Z^2 (Q_u + Q_d)^2 \sin^2 2\beta \\ &\quad + m_Z^2 \lambda_\mu^2 v^2 \sin^2 2\beta - \lambda_\mu^4 v^4 \sin^2 2\beta \\ &\quad - 4cQ_u Q_d \lambda_\mu^2 v^4 \sin^2 2\beta. \end{aligned} \tag{32}$$

Let us note that, here, by tree-level masses we mean the masses without the inclusion of the radiative corrections in MSSM. It is easy to see that m_h^2, in the so-called decoupling limit where $m_A \gg m_Z$, is given by

$$\begin{aligned} m_h^2 &= m_Z^2 \cos^2 2\beta + 4cv^2 (Q_u \sin^2\beta + Q_d \cos^2\beta)^2 \\ &\quad + \lambda_\mu^2 v^2 \sin^2 2\beta. \end{aligned} \tag{33}$$

V. DIPHOTON RESONANCES

The real scalar θ_1 and real pseudoscalar θ_2 components of $\theta = (\delta\tilde{N} + \delta\tilde{\bar{N}})/\sqrt{2} [= (\theta_1 + i\theta_2)/\sqrt{2}]$ with mass $m_\theta = \sqrt{2}\kappa M$ in the exact SUSY limit can be produced at the LHC by gluon fusion via a fermionic D_i, D_i^c loop as indicated in Fig. 1. They can decay into gluons, photons, Z or $W^\pm$ gauge bosons via the same loop diagram as well as a similar fermionic H_u^i, H_d^i loop. The most promising decay channel to search for these resonances is into two photons with the relevant diagrams also shown in Fig. 1.

Applying the results of Ref. [16], the cross section of the diphoton excess is

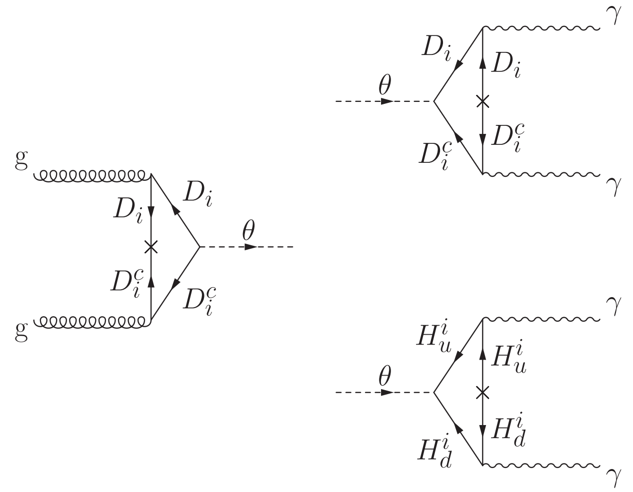

FIG. 1. Production of the complex scalar field θ at the LHC by gluon (g) fusion and its subsequent decay into photons (γ). Solid (dashed) lines represent the fermionic (bosonic) component of the indicated superfields. The arrows depict the chirality of the superfields and the crosses are mass insertions which must be inserted in each of the lines in the loops.

$$\sigma(pp \to \theta_m \to \gamma\gamma) \simeq \frac{C_{gg}}{m_\theta s \Gamma_{\theta_m}} \Gamma(\theta_m \to gg) \Gamma(\theta_m \to \gamma\gamma), \tag{34}$$

where $m = 1, 2$, $C_{gg} \simeq 3163$, $\sqrt{s} \simeq 13$ TeV, Γ_{θ_m} is the total decay width of θ_m, and the decay widths of θ_m to two gluons (g) or two photons (γ) are given by

$$\Gamma(\theta_m \to gg) = \frac{m_\theta^3 \alpha_s^2}{512\pi^3 \langle N \rangle^2} \left(\sum_{i=1}^{3} A_m(x_i) \right)^2, \tag{35}$$

$$\begin{aligned}\Gamma(\theta_m \to \gamma\gamma) &= \frac{m_\theta^3 \alpha_Y^2 \cos^4\theta_W}{9216\pi^3 \langle N \rangle^2} \left[\sum_{i=1}^{3} A_m(x_i) \right. \\ &\quad \left. + \frac{3}{2} \sum_{i=1}^{3} A_m(y_i) \left(1 + \frac{\alpha_2 \tan^2\theta_W}{\alpha_Y} \right) \right]^2. \end{aligned} \tag{36}$$

Here $A_1(x) = 2x[1 + (1 - x)\arcsin^2(1/\sqrt{x})]$, $A_2(x) = 2x\arcsin^2(1/\sqrt{x})$, $x_i = 4m_{D_i}^2/m_\theta^2 > 1$ with $m_{D_i} = \lambda_D^i \langle N \rangle$ being the mass of D_i and D_i^c, $y_i = 4m_{H_i}^2/m_\theta^2 > 1$ with $m_{H_i} = \lambda_\mu^i \langle N \rangle$ being the mass of H_u^i and H_d^i, and α_s, α_Y, and α_2 are the strong, hypercharge, and $SU(2)_{\mathrm{L}}$ fine-structure constants, respectively.

The cross section in Eq. (34) simplifies under the assumption that the spin zero fields θ_m decay predominantly into gluons, namely $\Gamma_{\theta_m} \simeq \Gamma(\theta_m \to gg)$. In this case, one obtains [17]

$$\sigma(pp \to \theta_m \to \gamma\gamma) \simeq 7.3 \times 10^6 \frac{\Gamma(\theta_m \to \gamma\gamma)}{m_\theta} \text{ fb.} \tag{37}$$

For x_i and y_i just above unity, which guarantees that the decay of θ_m to D_i, D_i^c and H_u^i, H_d^i pairs is kinematically blocked, $A_1(x_i)$ and $A_2(y_i)$ are maximized with values $A_1 \simeq 2$ and $A_2 \simeq \pi^2/2$. So we consider this case. It is also more beneficial to consider the decay of the pseudoscalar θ_2 since $A_2(x) > A_1(x)$ for all $x > 1$. Using Eq. (36), we then find that Eq. (37) gives

$$\sigma(pp \to \theta_2 \to \gamma\gamma) \simeq 5.5\left(\frac{m_\theta}{\langle N\rangle}\right)^2 \text{ fb} \simeq 11\kappa^2 \text{ fb}. \qquad (38)$$

In the exact SUSY limit, the complex scalar field θ could decay into a fermionic D_i, D_i^c or H_u^i, H_d^i pair via the superpotential terms $\lambda_D^i N D_i D_i^c$ or $\lambda_\mu^i N H_u^i H_d^i$ if this is kinematically allowed—see Figs. 2(a) and 2(b). It could also decay into a bosonic L, $\bar{L}$ pair via the F-term F_S between the superpotential couplings $\kappa S N \bar{N}$ and $\lambda_L S L \bar{L}$ if this is kinematically allowed—see Fig. 2(c). The decay widths in the three cases are

$$\Gamma_{D^i}^\theta = \frac{(\lambda_D^i)^2}{16\pi} m_\theta, \qquad \Gamma_{H^i}^\theta = \frac{(\lambda_\mu^i)^2}{16\pi} m_\theta, \qquad \Gamma_L^\theta = \frac{(\lambda_L)^2}{8\pi} m_\theta, \qquad (39)$$

respectively, where we assumed that the mass of the relevant D_i, D_i^c, or H_u^i, H_d^i, or L, $\bar{L}$ is much smaller than $m_\theta/2$. Depending on the kinematics the total decay width of the resonance could easily lie in the 100 GeV range. The diphoton, dijet, and diboson decay modes in this case would be subdominant.

Our estimate in Eq. (37) holds provided that the decay widths of θ into a D_i, D_i^c, or H_u^i, H_d^i, or L, $\bar{L}$ pair are subdominant or these decays are kinematically blocked. The latter is achieved for $m_\theta \simeq \sqrt{2}\kappa M < 2m_{D_i} \simeq 2\lambda_D^i M$, $2m_{H_i} \simeq 2\lambda_\mu^i M$, and $2m_L \simeq 2\lambda_L |\langle S\rangle| \simeq 2\lambda_L m_{3/2}/\kappa$, which implies that

$$\kappa \lesssim \sqrt{2}\lambda_D^i, \qquad \sqrt{2}\lambda_\mu^i, \qquad 2\lambda_L \frac{m_{3/2}}{m_\theta}. \qquad (40)$$

Note that the estimate of the maximal cross section of the diphoton excess in Eq. (38) corresponds to saturating the first two of the inequalities in Eq. (40). For simplicity and for not disturbing the MSSM gauge coupling unification, we choose to saturate the third inequality too.

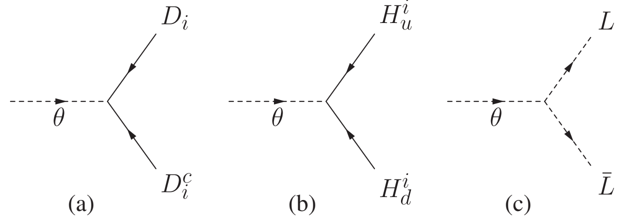

FIG. 2. Decay of the complex scalar field θ into a fermionic D_i, D_i^c (a) or H_u^i, H_d^i (b) pair or a bosonic L, $\bar{L}$ pair (c). The notation is the same as in Fig. 1.

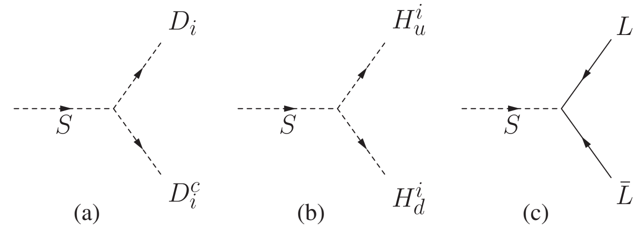

FIG. 3. Decay of the complex scalar field S into a bosonic D_i, D_i^c (a) or H_u^i, H_d^i (b) pair or a fermionic L, $\bar{L}$ pair (c). The notation is the same as in Fig. 1.

The complex scalar field S can decay into a bosonic D_i, D_i^c or H_u^i, H_d^i pair via the F-terms F_N between the superpotential couplings $\kappa S N \bar{N}$ and $\lambda_D^i N D_i D_i^c$ or $\lambda_\mu^i N H_u^i H_d^i$ if this is kinematically allowed—see Figs. 3(a) and 3(b). It could also decay into a fermionic L, $\bar{L}$ pair via the superpotential coupling $\lambda_L S L \bar{L}$ if this is kinematically allowed—see Fig. 3(c). The decay widths $\Gamma_{D^i}^S$, $\Gamma_{H^i}^S$, and Γ_L^S in the three cases are, respectively, equal to the decay widths $\Gamma_{D^i}^\theta$, $\Gamma_{H^i}^\theta$, and Γ_L^θ in Eq. (39). It is obvious that, if the inequalities in Eq. (40) are satisfied so as our estimate of the cross section of the diphoton excess in Eq. (38) to hold, these decay channels of S are also blocked. In this case, S will decay to lighter particles.

Note that, in the exact SUSY limit, the complex scalar field S cannot be produced at the LHC by gluon fusion and, thus, cannot lead to diphoton excess. This would require bosonic D_i, D_i^c loops with mass-squared insertions originating from soft trilinear SUSY breaking terms—for such loops see Ref. [18]. As we already mentioned, the soft SUSY breaking terms generate mixing between the scalar fields S and θ. Consequently, we can have four diphoton resonance states rather than just two from the scalar θ alone. Soft SUSY breaking also gives rise to more diagrams contributing to the diphoton excess. However, our estimate of the cross section of the diphoton excess for exact SUSY is the dominant one provided that the scale of $U(1)_{\psi'}$ breaking is much bigger than the soft SUSY breaking scale. Finally, let us note that demanding that the mass of the Z' gauge boson $m_{Z'} \simeq \sqrt{5} g_{\psi'} M/\sqrt{2} > 3.8$ TeV [19], say, we find that

$$g_{\psi'} M \gtrsim 2.4 \text{ TeV}. \qquad (41)$$

VI. NUMERICAL ANALYSIS

We can show that the gauge coupling constant $g_{\psi'}$ associated with the $U(1)_{\psi'}$ gauge symmetry unifies with the MSSM gauge coupling constants provided that its value at low energies is equal to about 0.45. This value depends very little on the exact value of the diquark, the extra

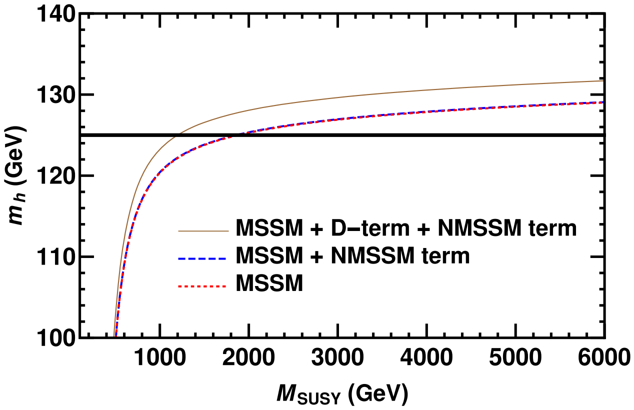

FIG. 4. Higgs boson mass m_h in the decoupling limit and for maximal stop quark mixing versus M_{SUSY} for $M = 10$ TeV, $\tilde{\lambda}_\mu = 0.3$, $\tan\beta = 20$, and $m_{3/2} = 4$ TeV. The dotted (red) curve corresponds to MSSM, the dashed (blue) curve to MSSM plus the NMSSM correction, and the continuous (brown) curve to MSSM plus the D-term and NMSSM corrections. The experimental value of m_h is also depicted by the bold horizontal line.

$SU(2)_{\mathrm{L}}$ doublet, the resonance, and the Z' gauge supermultiplet masses. So the bound in Eq. (41) implies that $M \gtrsim 5.34$ TeV. As an example, we will set $M = 10$ TeV. In addition, we can show that the coupling constants κ and $\tilde{\lambda}_\mu$ remain perturbative up to the GUT scale provided that they are not much bigger than about 0.7. The requirement that the diphoton resonance mass $m_\theta = \sqrt{2}\kappa M$ is bigger than about 4.5 TeV as indicated by the recent CMS results [20], implies that $\kappa \gtrsim 0.32$. In the case where the first two inequalities in Eq. (40) are saturated, we then obtain that $0.5 \gtrsim \lambda_D^i, \lambda_\mu^i \gtrsim 0.22$. For definiteness, we choose $\lambda_D^i \simeq \lambda_\mu^i \simeq 0.3$, which means in particular that $\tilde{\lambda}_\mu \simeq 0.3$. This choice implies that $\kappa \simeq 0.42$, $m_{D_i} \simeq m_{H_i} \simeq 3$ TeV (in particular $\mu \simeq 3$ TeV), $m_\theta \simeq 6$ TeV, and $m_{Z'} \simeq 7.1$ TeV. Saturating the third inequality in Eq. (40), we obtain $m_L \simeq 3$ TeV. Note that, for $\kappa \lesssim 0.7$, the resonance mass remains below 9.9 TeV.

In Fig. 4, we plot the lightest CP-even Higgs boson mass m_h in the decoupling limit versus M_{SUSY}, which is the geometric mean of the stop quark mass eigenvalues. We generally assume maximal stop quark mixing, which maximizes m_h, and include the two-loop radiative corrections to m_h in MSSM using the package SUSYHD [21]. The NMSSM and D-term contributions to m_h are also included from Eq. (33). In this figure, $\tan\beta = 20$ and $m_{3/2} = 4$ TeV. Notice that the NMSSM correction is very small since $\tilde{\lambda}_\mu$ is relatively small. The D-term correction, however, is sizable and allows us to obtain the observed value of m_h with much smaller stop quark masses than the ones required in MSSM or NMSSM. Indeed, the inclusion of the D-term from $U(1)_{\psi'}$ reduces M_{SUSY} from about 1900 GeV to about 1200 GeV. Note, in passing, that λ_L, in this case, is about 0.32.

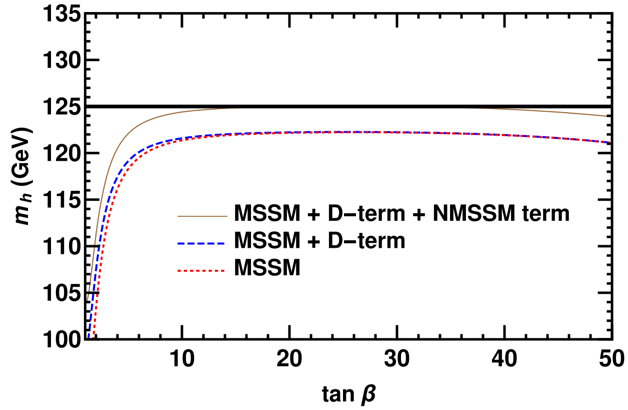

FIG. 5. Higgs boson mass m_h in the decoupling limit and for maximal stop quark mixing versus $\tan\beta$ for $M = 10$ TeV, $\tilde{\lambda}_\mu = 0.3$, $M_{\mathrm{SUSY}} = 1200$ GeV, and $m_{3/2} = 4$ TeV. The notation is the same as in Fig. 4.

In Fig. 5, we plot m_h in the decoupling limit and for maximal stop quark mixing versus $\tan\beta$ for $M = 10$ TeV, $\tilde{\lambda}_\mu = 0.3$, $M_{\mathrm{SUSY}} = 1200$ GeV, and $m_{3/2} = 4$ TeV. We see that the experimental value of m_h is achieved at $\tan\beta = 20$ as it should consistently with Fig. 4. However, as one can see from Fig. 5, the observed m_h can be practically obtained in a wide range of $\tan\beta$'s. Note that, without the inclusion of the D-term contribution from $U(1)_{\psi'}$, the Higgs boson mass remains well below its observed value for all the values of $\tan\beta$. This again shows the crucial role of the D-term for obtaining the observed value of m_h with relatively low stop quark masses. Finally, we notice that, for larger $\tan\beta$'s, m_h decreases as $\tan\beta$ increases in all three cases depicted in this figure. This is due to the relatively large value of μ.

In Fig. 6, we depict m_h under the same assumptions versus $m_{3/2}$ for $M = 10$ TeV, $\tilde{\lambda}_\mu = 0.3$, $\tan\beta = 20$, and $M_{\mathrm{SUSY}} = 1200$ GeV. The observed Higgs boson mass is

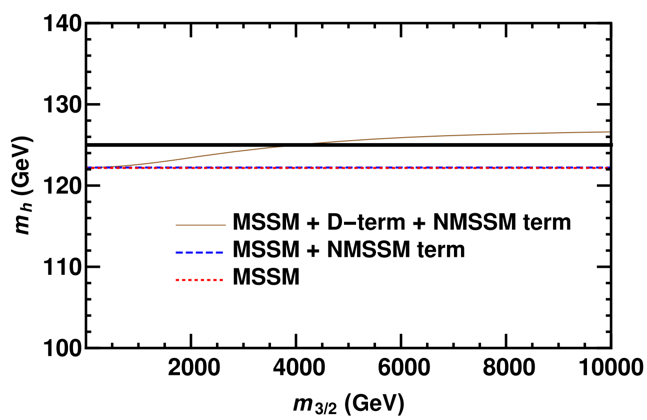

FIG. 6. Higgs boson mass m_h in the decoupling limit and for maximal stop quark mixing versus $m_{3/2}$ for $M = 10$ TeV, $\tilde{\lambda}_\mu = 0.3$, $\tan\beta = 20$, and $M_{\mathrm{SUSY}} = 1200$ GeV. The notation is the same as in Fig. 4.

obtained at $m_{3/2} = 4$ TeV consistently with Figs. 4 and 5. We see again that, without the D-term, m_h remains well below its observed value for all $m_{3/2}$'s. We also observe that, without the D-term, m_h is independent from the value of $m_{3/2}$ as it should.

In the present numerical example, the cross section of the diphoton excess in Eq. (38) turns out to be equal to 1.94 fb. Needless to say that higher cross sections can be obtained for higher values of κ. The diphoton resonance mass, as already discussed, is equal to 6 TeV and the diquark masses about 3 TeV. In conclusion, we see that our model can predict diphoton and diquark resonances which hopefully can be observed in future experiments.

VII. STERILE NEUTRINOS

After the spontaneous breaking of the $U(1)_{\psi'}$ symmetry, the fermionic components of the three superfields N_i, which are SM singlets, acquire masses $m_{N_i} \simeq \lambda_N^i M^2/m_{\rm P}$ via the last superpotential coupling in Eq. (2). These masses can be $\lesssim 0.1$ eV for $M \sim 10$ TeV and these fermionic fields, which are stable on account of the Z_2' symmetry in Table I, can act as sterile neutrinos.

In the early universe, the sterile neutrinos are kept in equilibrium via reactions of the sort $N_i \bar{N}_i \leftrightarrow$ a pair of SM particles or $N_i +$ a SM particle $\leftrightarrow N_i +$ a SM particle. These reactions proceed via a s- or t-channel exchange of a Z' gauge boson. The thermal average $\langle \sigma v \rangle$, where σ is the corresponding cross section and v the relative velocity of the annihilating particles, is estimated to be of order T^2/M^4 with T being the cosmic temperature. The interaction rate per sterile neutrino is then given by

$$\Gamma_{N_i} = n\langle \sigma v \rangle \sim \frac{T^5}{M^4}, \tag{42}$$

where $n \sim T^3$ is the number density of massless particles in thermal equilibrium. The decoupling temperature $T_{\rm D}$ of sterile neutrinos is estimated from the condition

$$\Gamma_{N_i} \sim H \sim \frac{T^2}{m_{\rm P}}, \tag{43}$$

where H is the Hubble parameter. This condition implies that

$$T_{\rm D} \sim M\left(\frac{M}{m_{\rm P}}\right)^{\frac{1}{3}}. \tag{44}$$

Here we followed the same strategy as the one used for estimating the SM neutrino decoupling temperature via processes involving weak gauge boson exchange. In the case of ordinary neutrinos, however, the scale M should be identified with the electroweak scale, which is of order 100 GeV, and the decoupling temperature turns out to be of order 1 MeV. From Eq. (44), we see that $T_{\rm D}$ scales like $M^{4/3}$. So, in our case and for $M \simeq 10$ TeV, $T_{\rm D}$ is expected to be of order 460 MeV, which is well above the critical temperature for the QCD transition.

The effective number of massless degrees of freedom in equilibrium right after the decoupling of sterile neutrinos is 61.75. At $T \sim 1$ MeV and just before the decoupling of the SM neutrinos, this number is reduced to 10.75. So, due to entropy conservation in each comoving volume, the temperature of ordinary neutrinos T_ν is raised relative to the temperature of the sterile neutrinos T_N by a factor $(61.75/10.75)^{1/3}$. Consequently, the contribution of the three sterile neutrinos to the effective number of neutrinos at big bang nucleosynthesis is

$$\Delta N_\nu = 3 \times \left(\frac{10.75}{61.75}\right)^{\frac{4}{3}} \simeq 0.29. \tag{45}$$

This result is perfectly compatible with the Planck satellite bound [22] on the effective number of massless neutrinos

$$N_\nu = 3.15 \pm 0.23. \tag{46}$$

Note that although the derivation of our estimate in Eq. (45) is somewhat rough, we believe that the result is quite accurate. This is due to the fact that the effective number of massless degrees of freedom in equilibrium right after the decoupling of sterile neutrinos does not change if $T_{\rm D}$ varies between the critical temperature of the QCD transition, which is about 200 MeV, and the mass of the charm quark $m_c \simeq 1270$ MeV. Also, a more accurate determination of the decoupling temperature of ordinary neutrinos does not change the effective number of massless degrees of freedom in equilibrium just before this temperature is reached.

VIII. DARK MATTER

The scalar component of the superfield N_i, which is expected to have mass of order $m_{3/2}$, can decay into a fermionic N_i and a particle-sparticle pair via a Z' gaugino exchange provided that this is kinematically allowed. A necessary (but not sufficient) condition for this decay to be possible is that there exist sparticles which are lighter than the scalar N_i. Note that, as a consequence of the unbroken discrete symmetry Z_2', the decay products of the scalar N_i should necessarily contain an odd number of N_j superfields.

If the decay of the lightest scalar N_i (denoted as $\hat{N}$) is kinematically blocked, this particle can contribute to the cold dark matter in the universe. In the early universe, the scalar $\hat{N}$ is kept in equilibrium since, for example, a pair of these scalars can annihilate into a pair of SM particles via a Z' gauge boson exchange. The thermal average $\langle \sigma v \rangle$ in this case and for s-wave annihilation is expected to be

$$\langle \sigma v \rangle \sim \frac{m_{\hat{N}}^2}{M^4}, \qquad (47)$$

where $m_{\hat{N}}$ is the mass of the scalar $\hat{N}$.

Following the standard analysis of Ref. [23], we can estimate the freeze-out temperature T_{f} of the sterile sneutrino $\hat{N}$ as well as its relic abundance $\Omega_{\hat{N}} h^2$ in the universe. To this end, we take $M \simeq 5.34$ TeV, which saturates the lower bound on $m_{Z'}$ [19] mentioned in Sec. V. The requirement that $\Omega_{\hat{N}} h^2$ equals the cold dark matter abundance $\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$ from the Planck satellite data [24] then implies that $m_{\hat{N}} \simeq 1.25$ TeV. The freeze-out temperature T_{f} in this case is about 51 GeV and the corresponding number of massless degrees of freedom 86.25. Higher values of M require even higher values of $m_{\hat{N}}$. So we see that the SUSY spectrum is pushed up considerably if the decay of the lightest sterile sneutrino is kinematically blocked and this particle contributes to the cold dark matter of the universe.

The model possesses an accidental lepton parity symmetry Z_2^{lp} under which the superfields l, e^c, ν^c, L, $\bar{L}$ are odd. Combining this symmetry with the baryon parity Z_2^{bp} subgroup of $U(1)_B$ under which q, u^c, d^c are odd, we obtain a matter parity symmetry Z_2^{mp} under which q, u^c, d^c, l, e^c, ν^c, L, $\bar{L}$ are odd. A discrete R-parity can then be generated if we combine this symmetry with fermion parity. The bosonic q, u^c, d^c, l, e^c, ν^c, L, $\bar{L}$ and the fermionic H_u^i, H_d^i, D_i, D_i^c, N_i, S, N, $\bar{N}$ are odd under this R-parity. Note that the decay products of these particles with the exception, of course, of the fermionic N_i cannot contain a single N_i because of the Z_2' symmetry. Also, they cannot contain a single L, $\bar{L}$, H_u^α, H_d^α except, of course, for the decay products of the bosonic L, $\bar{L}$ and fermionic H_u^α, H_d^α themselves as a consequence of the Z_2 symmetry. The S, N, $\bar{N}$ fermions can decay into a Higgs boson-Higgsino pair, while the D_i, D_i^c fermions can decay into a quark-squark pair. So all the particles with negative R-parity, except the bosonic L, $\bar{L}$ and the fermionic H_u^α, H_d^α, N_i end up yielding the usual stable lightest sparticle of MSSM which can, in principle, participate in the cold dark matter of the universe.

The possible fate of the N_i superfields has been already discussed. The Z_2 symmetry and R-parity imply that the lightest state in the bosonic L, $\bar{L}$ and fermionic H_u^α, H_d^α, or in the fermionic L, $\bar{L}$ and bosonic H_u^α, H_d^α, which is hopefully neutral, is stable. We thus have two more candidates for cold dark matter. Their relic abundances in the universe depend on details. However, if their masses are large, these abundances can be negligible. Finally, let us mention that, if the breaking scale $<N>$ of $U(1)_{\psi'}$ is increased to about 10^3 TeV, the sterile neutrinos become plausible candidates for keV scale warm dark matter (for a recent review see Ref. [25]). In conclusion, we see that the model possesses many possible candidates for the composition of dark matter.

IX. SUMMARY

We have explored the implications of appending a $U(1)$ gauge symmetry to the MSSM gauge group $SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times U(1)_Y$. This $U(1)$ symmetry, referred to here as $U(1)_{\psi'}$, arises from a linear combination of $U(1)_\chi$ and $U(1)_\psi$ contained in E_6. The three matter 27-plets in E_6 give rise to three $SO(10)$ singlet fermions N_i, called sterile neutrinos, which are prevented from acquiring masses via renormalizable couplings by a combination of symmetries, especially a $U(1)$ R symmetry. Thus, for a relatively low ($\sim$10 TeV or so) breaking scale of $U(1)_{\psi'}$, these fermionic N_i's, the lightest of which happens to be stable, only acquire tiny masses $\lesssim$0.1 eV and their contribution as fractional cosmic neutrinos during nucleosynthesis has been estimated. The lightest sterile sneutrino as well as two more particles, which are stable on account of discrete symmetries, can, under certain circumstances, be cold dark matter candidates in addition to the usual lightest sparticle of MSSM. Note that the breaking of $U(1)_{\psi'}$ at suitably higher energies, of order 10^3 TeV or so, would yield keV scale masses for the fermionic N_i's and thus transform them into plausible warm dark matter candidates. The D-term for $U(1)_{\psi'}$ can contribute appreciably to the mass of the lightest neutral CP-even MSSM Higgs boson. Consequently, the observed value of this mass can be obtained in the decoupling limit with relatively light stop quarks. The spontaneous breaking of $U(1)_{\psi'}$ yields superconducting cosmic strings which presumably were not inflated away. The model also predicts the existence of diquark and diphoton resonances which may be found at the LHC or its future upgrades. The MSSM μ problem is naturally resolved. The right handed neutrinos can acquire large masses, which allows the standard seesaw mechanism and the leptogenesis scenario to be realized. Baryon number is conserved to all orders in perturbation theory rendering a stable proton.

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