Strong and radiative decays of excited vector mesons and predictions for a new $\phi(1930)$ resonance

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We study the phenomenology of two nonets of excited vector mesons, $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$, which (roughly) correspond to radially excited 2^3S_1 and to orbitally excited 1^3D_1 vector mesons. We evaluate the strong and radiative decays of these mesons into pseudoscalar and ground-state vector mesons by using an effective relativistic QFT model based on flavor symmetry. We compare decay widths and branching ratios with various experimental results listed in the PDG. An overall agreement of theory with experiment reinforces the standard quark-antiquark assignment of the resonances mentioned above. Predictions for not-yet-measured quantities are also made. In particular, we shall also make predictions for the not-yet-discovered $s\bar{s}$ state of the 1^3D_1 nonet, denoted as $\phi(???)$. Its mass can be estimated to be about 1930 MeV; hence we shall call this putative state $\phi(1930)$. Its main decays are into $KK^*(892)$ (about 200 MeV) and KK (about 100 MeV). Since this state couples also to $\gamma\eta$, it can be searched for in the near future in the photoproduction-based experiments GlueX and CLAS12 at Jefferson Lab.

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I. INTRODUCTION

Strong interactions are described by quantum chromodynamics (QCD). While the degrees of freedom of the QCD Lagrangian are elementary “colored” quarks and gluons, the physical spectrum listed in the PDG [1] consists of “white" hadrons. Hadrons are, in general, bound states of quarks and gluons and are further classified into mesons (bosonic hadrons) and baryons (fermionic hadrons).

The vast majority of mesons consist of “conventional” quark-antiquark states (see the results of the quark model in Ref. [2]), but nowadays experimental evidence for nonconventional mesons is mounting, e.g. Refs. [3,4]. Some known resonances in the low-energy sector might be predominately four-quark states, e.g. Ref. [5], or gluonic, e.g. Ref. [6]; in the energy region of charmonium and bottomonium masses many resonances, the so-called X, Y and Z states, have been found [7]. Similarly, conventional baryons are three-quark states [8], but pentaquark states have been also recently discovered [9].

In this work we concentrate on the light mesonic sector by studying the phenomenology (in particular, strong and radiative decays) of two types of excited quark-antiquark vector mesons. The firm understanding of conventional $\bar{q}q$ states is necessary to look for further nonconventional states with the same quantum numbers. Moreover, the general status of excited states is rather poorly understood and improvement is needed; see the quark model review in the PDG [10]. Excited vector mesons (for some previous theoretical studies on them, see Refs. [11–15]) are especially interesting in this context for various reasons:

(i) The ground-state vector mesons $\{\rho(770), K^*(892), \phi(1020), \omega(782)\}$ are very well known and represent an excellent example of an ideal $\bar{q}q$ nonet. The nonrelativistic quantum numbers are $(n, L, S) = (1, 0, 1)$ (n being the principal quantum number, L the spacial angular momentum, and S the spin); hence, in the nonrelativistic spectroscopic notation one has $n\ ^{2S+1}L_J = 1\ ^3S_1$ and in the relativistic notation $J^{PC} = 1^{--}$. Considering that the nonet of the pseudoscalar meson is special due to the importance of spontaneous symmetry breaking and the axial anomaly, one may say that ground-state vector mesons are the lightest ideal $\bar{q}q$ objects.

(ii) Two types of excited vector mesons have been experimentally measured. Although data need improvement, there is enough information on masses and decays to undertake a systematic analysis. One nonet corresponds (predominantly) to the radial excitation, with the quantum numbers $(n, L, S) = (2, 0, 1)$ (spectroscopic notation 2^3S_1, relativistic notation $J^{PC} = 1^{--}$); the associated states are $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$. [Note that there are only two nonets of states with $n = 2$ listed in the PDG. Besides vector mesons, a tentative assignment is done also for excited pseudoscalar mesons with $(n, L, S) = (2, 0, 0)$; for details and references see the recent work in Ref. [16].] The second nonet corresponds (predominantly) to the orbital excitation; i.e. it has quantum numbers $(n, L, S) = (1, 2, 1)$ (spectroscopic notation 1^3D_1, relativistic notation still

$J^{PC} = 1^{--}$). The associated states are $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$. It should be stressed from the very beginning that the physical resonances listed above do not correspond exactly to the mentioned nonrelativistic quark-antiquark configurations, but can arise from mixing of them (moreover, a dressing of meson-meson pairs also takes place). Hence we should regard the previous assignment as indicating the predominant contribution.

(iii) The state $\phi(???)$ belonging to the nonet of orbitally excited 1^3D_1 states has not yet been observed. It is the last missing state of that nonet, hence the empty space in Table II of Ref. [10]. Its mass can be estimated by the mass differences between the two nonets described above, obtaining $m_{\phi(???)} - m_{\phi(1680)} \simeq m_{\rho(1700)} - m_{\rho(1450)} \simeq 250$ MeV. Thus we expect that $\phi(???)$ has a mass of about 1930 MeV and we therefore postulate the existence of a new resonance, denoted $\phi(1930)$ (note that the value 1930 MeV is not far from the old quark model prediction of 1880 MeV [2]).

In this work, we study the decays of the excited vector mesons $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$ by using a quantum field theoretical (QFT) approach whose d.o.f. are mesons. The fields entering in our model correspond to these resonances and hence roughly, but not exactly, to the quark-antiquark configurations 2^3S_1 and 1^3D_1 predicted by the quark model (as discussed above mixing is possible). Indeed, a direct link of our vector fields to an underlying microscopic wave function of the quarks is not possible and also not necessary (details later on). The Lagrangian of the model is constructed under the requirement of flavor symmetry (i.e. the invariance under the rotation of the quarks u, d, and s, a well-proven approximate symmetry of QCD), as well as invariance under parity P and charge conjugation C. Indeed, flavor symmetry is present in various approaches of low-energy QCD, such as chiral perturbation theory [17,18], linear sigma models [19–23], and also Bethe-Salpeter approaches [24,25]. Namely, even after spontaneous symmetry breaking (SSB), flavor symmetry is still manifest. The use of flavor symmetric mesonic QFT models, on which our approach is based, has proven to be successful in the past in studies on decays of mesonic multiplets, such as the well-known tensor mesons [26], the pseudovector mesons [27], the pseudotensor mesons [28], and also the more difficult scalar mesons (in which the scalar glueball leaks in) [29,30]. Here, we shall follow the very same methodology of those works.

To be more specific, we write a Lagrangian that contains the two nonets of excited vector mesons previously mentioned as well as their main decay products: the ground-state vector mesons and pseudoscalar mesons. Moreover, in a second step, we shall also include the photon for radiative decays. When dominant terms in the large-N_c expansion are kept, the number of parameter of our Lagrangian is rather small: four parameters (two for each nonet) which allow us to calculate a large number of decay rates (about 64 decay rates, 32 per each nonet). As a result, we can check the status of the quark-antiquark assignment for these states and make predictions for decay rates which are poorly known or have not yet been measured. In the PDG there are also many branching ratios which can be compared to our results. We shall find that the overall interpretation as $\bar{q}q$ states is satisfactory, but some points deserve further clarification and more data are needed (fortunately, the experiments GlueX and CLAS12 start operation soon). Then, as a last important step, we will make predictions for the yet-undiscovered $\bar{s}s$ state $\phi(???) \equiv \phi(1930)$.

The paper is organized as follows. In Sec. II we present the fields of the model, the Lagrangian, and the theoretical expressions for the decay widths, as well as the determination of the parameters. Then, in Sec. III we present the results (separately for each excited nonet) of the decay widths and compare them to the experimental results. In particular, we shall also discuss various decay ratios. In Sec. IV we summarize our results and present our outlook, in particular in connection with the putative and yet-undiscovered state $\phi(1930)$. Various details are relegated to the appendixes.

II. THE MODEL

We use an effective relativistic quantum field theoretical model based on flavor symmetry. The d.o.f. are mesonic fields, which correspond to quark-antiquark states. In this section, we first present the fields of the model and their assignment; then we show the Lagrangian and finally the expressions of the strong and radiative decay widths.

A. The fields of the model

As a first step, we introduce four nonets of mesons, which are given in terms of matrices. Intuitively, each matrix corresponds to the following quark-antiquark content:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix}. \tag{1}$$

Although strictly speaking only the relativistic notation J^{PC} is relevant in a relativistic QFT treatment, we shall also keep track of the nonrelativistic notation for these multiplets, which should be heuristically understood as the dominant contribution in the (in our approach invisible) microscopic wave function of the quark and the antiquark pairs. In this way, a link with the quark model results—although approximate—is possible and allows for a better intuitive understanding of these states. The explicit form of the matrices for the nonet of pseudoscalar mesons, the nonet of ground-state vector mesons, and two nonets of excited vector mesons read

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}; \qquad V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega^\mu+\rho^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} & K_i^{\mu\star +} \\ \rho^{\mu -} & \frac{\omega^\mu-\rho^{\mu 0}}{\sqrt{2}} & K^{\mu\star 0} \\ K^{\mu\star -} & \bar{K}^{\mu\star 0} & \phi^\mu \end{pmatrix}; \tag{2}$$

$$V_E^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_E^\mu+\rho_E^{\mu 0}}{\sqrt{2}} & \rho_E^{\mu +} & K_E^{\mu\star +} \\ \rho_E^{\mu -} & \frac{\omega_E^\mu-\rho_E^{\mu 0}}{\sqrt{2}} & K_E^{\mu\star 0} \\ K_E^{\mu\star -} & \bar{K}_E^{\mu\star 0} & \phi_E^\mu \end{pmatrix}; \qquad V_D^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_D^\mu+\rho_D^{\mu 0}}{\sqrt{2}} & \rho_D^{\mu +} & K_D^{\mu\star +} \\ \rho_D^{\mu -} & \frac{\omega_D^\mu-\rho_D^{\mu 0}}{\sqrt{2}} & K_D^{\mu\star 0} \\ K_D^{\mu\star -} & \bar{K}_D^{\mu\star 0} & \phi_D^\mu \end{pmatrix}. \tag{3}$$

(For the explicit quark-antiquark microscopic current leading to these fields, see Ref. [28]). The matrix P describes the nonet of pseudoscalar mesons corresponding to the states $\{\pi, K, \eta \equiv \eta(547), \eta' \equiv \eta'(958)\}$. Namely, the two fields denoted as η_N and η_S in Eq. (2) mix and generate the physical fields η and η':

$$\eta_N = \eta\cos\theta_P - \eta'\sin\theta_P \quad \text{and} \quad \eta_S = \eta\sin\theta_P - \eta'\cos\theta_P. \tag{4}$$

The mixing angle θ_P is set to $-42°$ [31]. Using other values in the range $(-40°, -45°)$, e.g. Refs. [22,32,33], would affect only marginally our results. In the nonrelativistic notation, this nonet corresponds (predominantly) to $(n, L, S) = (1, 0, 0)$, and hence to $n\ ^{2S+1}L_J = 1\ ^1S_0$. The relativistic notation is $J^{PC} = 0^{-+}$.

The nonet of ground-state vector mesons, denoted as V^μ, is associated to the resonances $\{\rho(770), K^*(892), \phi(1020), \omega(782)\}$. Actually, also the bare fields $\omega = \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $\phi = \bar{s}s$ entering in Eq. (2) mix, but the mixing is sufficiently small to be neglected (about $-3°$ [10]); then, $\omega(782)$ is regarded as purely nonstrange and $\phi(1020)$ as purely strange. The nonrelativistic quantum numbers are $(n, L, S) = (1, 0, 1)$, and hence $n\ ^{2S+1}L_J = 1\ ^3S_1$. Relativistically, $J^{PC} = 1^{--}$.

Finally, we turn to the excited vector mesons. The matrix V_E^μ describes the first nonet of excited vector mesons that we assign to the states $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$. As for vector mesons, we neglect the isoscalar mixing; hence $\omega(1420)$ is purely nonstrange $[\sqrt{1/2}(\bar{u}u + \bar{d}d)]$ and $\phi(1680)$ purely strange, $\bar{s}s$. This nonet corresponds to (predominantly but not identically) radially excited vector mesons with the nonrelativistic quantum numbers $(n,L,S) = (2,0,1)$, and hence $n\ ^{2S+1}L_J = 2\ ^3S_1$. Relativistically, $J^{PC} = 1^{--}$ (just as the ground-state vector mesons). Note that the state $K^*(1410)$ has been clearly seen in the recent lattice studies of Ref. [34] while $\omega(1420)$ and $\phi(1680)$ have been observed in the lattice studies of Refs. [35,36].

The matrix V_D^μ describes the second nonet of excited vectors: $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$. Also here, the isoscalar mixing is neglected. Nonrelativistically, this nonet corresponds (always predominantly) to orbitally excited vector mesons with $(n, L, S) = (1, 2, 1)$, and hence $n\ ^{2S+1}L_J = 1\ ^3D_1$. Relativistically, $J^{PC} = 1^{--}$. The $\bar{s}s$ state in the 1^3D_1 nonet, denoted as $\phi(???)$, has not yet been experimentally seen. By using our model, we make predictions for this resonance. In order to include this state into our calculation, we have to estimate its mass. In Table I we present the masses of the known members of the two nonets. It is visible that the mass difference between radially and orbitally excited vector mesons are approximately the same for all states. The reason for that is the same type of strong dynamics describing these mesons and their masses.

Hence, we can estimate the mass of $\phi(???)$ as

$$m_{\phi(???)} \simeq (m_{\phi(1680)} + 250 \pm 20)\ \text{MeV} = 1930 \pm 20\ \text{MeV}. \tag{5}$$

From now on we shall call this hypothetical state

$$\phi(???) \equiv \phi(1930).$$

All the fields have precise transformations under parity P, charge conjugation C, and flavor symmetry $U(3)_V$, which are summarized in Table II.

B. The Lagrangian

The Lagrangian of the model is obtained by properly coupling the matrices listed above and by requiring invariance under P, C, and $U(3)_V$. It explicitly reads

TABLE I. Mass differences between the members of the two nonets of excited vector mesons.

V_E	$\rho(1450)$	$K^*(1410)$	$\omega(1420)$	$\phi(1680)$
V_D	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$	$\phi(???)$
Difference	250 MeV	270 MeV	230 MeV	?

TABLE II. Transformation properties of the nonets under charge, parity, and flavor transformations. Notice that the parity transformation for vector states is obtained by lowering the Lorentz index.

	Parity (P)	Charge conjugation (C)	Flavor $[U(3)_V]$
P	$-P(t,-\vec{x})$	P^t	$UPU^\dagger$
V^μ	$V_\mu(t,-\vec{x})$	$-(V^\mu)^t$	$UV^\mu U^\dagger$
V_E^μ	$V_{E,\mu}(t,-\vec{x})$	$-(V_E^\mu)^t$	$UV_E^\mu U^\dagger$
V_D^μ	$V_{D,\mu}(t,-\vec{x})$	$-(V_D^\mu)^t$	$UV_D^\mu U^\dagger$

$$\mathcal{L} = \mathcal{L}_{EPP} + \mathcal{L}_{DPP} + \mathcal{L}_{EVP} + \mathcal{L}_{DVP} \tag{6}$$

where

$$\mathcal{L}_{EPP} = ig_{EPP}Tr([\partial^\mu P, V_{E,\mu}]P),$$
$$\mathcal{L}_{DPP} = ig_{DPP}Tr([\partial^\mu P, V_{D,\mu}]P), \tag{7}$$

$$\mathcal{L}_{EVP} = g_{EVP}Tr(\tilde{V}_E^{\mu\nu}\{V_{\mu\nu}, P\}),$$
$$\mathcal{L}_{DVP} = g_{DVP}Tr(\tilde{V}_D^{\mu\nu}\{V_{\mu\nu}, P\}). \tag{8}$$

The terms contain various processes: $\mathcal{L}_{EPP}$ describes the decay $V_E \to PP$, $\mathcal{L}_{DPP}$ the decay $V_D \to PP$, $\mathcal{L}_{EVP}$ the decay $V_E \to VP$, and finally $\mathcal{L}_{DVP}$ the decay $V_D \to VP$. The notation $[A,B] = AB - BA$ stands for the usual commutator and $\{A,B\} = AB + BA$ for the anticommutator. Moreover, the dual fields have been defined in the standard way:

$$\tilde{V}_E^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha V_{E,\beta} - \partial_\beta V_{E,\alpha}), \tag{9}$$

$$\tilde{V}_D^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha V_{D,\beta} - \partial_\beta V_{D,\alpha}). \tag{10}$$

For every term of the Lagrangian there is a corresponding coupling constant: g_{EPP}, g_{DPP}, g_{EVP}, and g_{DVP}; hence the model contains four parameters. To fix them we use some of the experimental data taken from PDG; see Sec. III. A. The extended forms of the interaction terms are presented in Appendix A.

Finally, we shall also study the radiative decays of the type $V \to \gamma P$. To this end, we need to perform the following replacement of the vector field strength tensor as (see e.g. Ref. [37])

$$V_{\mu\nu} \to V_{\mu\nu} + \frac{e_0}{g_\rho}QF_{\mu\nu}, \tag{11}$$

where $F_{\mu\nu}$ is the field strength tensor for photons; $e_0 = \sqrt{4\pi\alpha}$ (with $\alpha \approx 1/137$) is the electric charge of the proton; $g_\rho = 5.5 \pm 0.5$ is the $\rho\pi\pi$ coupling constant; and $Q = \text{diag}\{2/3, -1/3, -1/3\}$ is the matrix with the charges of the quarks. Note that radiative decays do not necessitate any new parameter. For more details, see Appendix B.

Next, we discuss three important theoretical aspects and further developments/improvements of our model.

(i) Large-N_c suppressed terms. In our framework, all the nonets are interpreted as $\bar{q}q$ states; hence the coupling constants g_{EPP}, g_{DPP}, g_{EVP}, g_{DVP} scale as $1/\sqrt{N_c}$ and are dominant in the large-N_c expansion [38] (for a review, see Ref. [39]). For instance, concerning the decay, the $\tilde{V}_E^{\mu\nu} \to VP$ further flavor-symmetric terms which are suppressed in the large-N_c limit have the form

$$Tr(\tilde{V}_E^{\mu\nu})Tr(V_{\mu\nu}P) \quad \text{and} \quad Tr(\tilde{V}_E^{\mu\nu})Tr(V_{\mu\nu})Tr(P) \tag{12}$$

(and similarly, for $\tilde{V}_D^{\mu\nu} \to VP$). The corresponding coupling constants are proportional to $1/N_c^{3/2}$ and $1/N_c^{5/2}$, respectively. At the present level of accuracy of the experimental data, these terms can be safely neglected. Once more precise data become available, they can be included. Note that such terms do not exist for $V_E \to PP$ and $V_D \to PP$ because of the anticommutator (in turn, this is the reason why $\omega_{E,D} \to \pi\pi$ and $\phi_{E,D} \to \pi\pi$ vanish).

(ii) Flavor-breaking terms. There are terms which explicitly break flavor symmetry, such as

$$iTr(\tilde{\lambda}[\partial^\mu P, V_{E,\mu}]P), \tag{13}$$

where $\tilde{\lambda} \propto \text{diag}\{0, m_d - m_u, m_s - m_u\}$ is proportional to the mass differences. Typically, $m_d - m_u$ can be safely neglected, but $m_s - m_u$ can be non-negligible (in Refs. [29,30] there is a contribution of about 10% from such a term). Also in this case, these effects are not taken into account here because the precision of data would not allow us to constrain them. Note that one could also write down terms which break flavor symmetry and are subleading in the large-N_c expansions, but they are doubly suppressed and neglected in the present work.

(iii) Mixing between excited vectors. Within our treatment, the mesonic fields correspond directly to the physical fields. Indeed, one could start with two nonets of fields $V^\mu_{E,\text{bare}}$ and $V^\mu_{D,\text{bare}}$ which correspond to purely radially excited (2^3S_1) and orbitally excited (1^3D_1) mesons, respectively. In this case, one would write down a Lagrangian analogous to the one of Eqs. (7)–(8), where however the couplings should be named differently: g^{bare}_{EPP}, g^{bare}_{DPP}, g^{bare}_{EVP}, g^{bare}_{DVP}. In addition, one should add the mixing term between the bare configurations:

$$\delta_{\rm mix}{\rm Tr}[V_{E,{\rm bare},\mu}V^{\mu}_{D,{\rm bare}}]. \tag{14}$$

Then, one should perform the usual $O(2)$ rotation:

$$V_{E,\mu} = V_{E,{\rm bare},\mu}\cos\theta_{ED} + V^{\mu}_{D,{\rm bare}}\sin\theta_{ED}, \tag{15}$$

$$V_{D,\mu} = -V_{E,bare,\mu}\sin\theta_{ED} + V^{\mu}_{D,bare}\cos\theta_{ED}. \tag{16}$$

The previous equations are valid in the flavor limit because all the members of the nonet rotate with the same mixing angle. In terms of the physical $V_{E,\mu}$ and $V_{D,\mu}$ no mixing is present. For instance, the coupling constant g_{EPP} reads in terms of the bare couplings as

$$g_{EPP} = g^{bare}_{EPP}\cos\theta_{ED} + g^{bare}_{EPP}\sin\theta_{ED}; \tag{17}$$

similar relations hold for the other coupling constants. The important point is that, once we have performed the rotation, the fields V_E and V_D correspond to the physical ones and the mixing angle θ_{ED} cannot be determined as long as flavor symmetry is valid (in fact, it completely disappears from all physical quantities). This is why, by no loss of generality, we did not include any mixing term in our Lagrangian of Eqs. (7)–(8). Thus, while we cannot state that the physical fields have a certain microscopic wave function, our analysis is more general in the sense that the fields of our model are already a mixture of the bare quark-model configurations. Even though we do not expect θ_{ED} to be large, our analysis is independent of its precise value. Indeed, the only way to render the mixing of bare configurations visible is to include the violations of flavor symmetry discussed above in point (ii). In that way, different mixing angles emerge and one cannot "rotate away" the mixing. Such deviations are anyhow expected to be small, as the splitting of the masses in Table II shows (similar mass differences between the corresponding members of the multiplets).

In conclusion, the (here neglected) effects (i), (ii), and (iii) show how to potentially improve the present model in a systematic way. This is left as work for the future.

C. Decay widths: Theoretical expressions

The tree-level decay widths for a resonance $R = V_E \equiv E$ or $R = V_D \equiv D$ can be calculated by performing a standard QFT calculation. The results for the three channels that we consider [pseudoscalar-pseudoscalar (PP), vector-pseudoscalar (VP), and photon-pseudoscalar (γP)] read explicitly

$$\Gamma_{R\to PP} = s_{RPP}\frac{|\vec{k}|^3}{6\pi m_R^2}\left(\frac{g_{RPP}}{2}\lambda_{RPP}\right)^2, \tag{18}$$

$$\Gamma_{R\to VP} = s_{RVP}\frac{|\vec{k}|^3}{12\pi}\left(\frac{g_{RVP}}{2}\lambda_{RVP}\right)^2, \tag{19}$$

$$\Gamma_{R\to\gamma P} = \frac{|\vec{k}|^3}{12\pi}\left(\frac{g_{RVP}}{2}\frac{e_0}{g_\rho}\lambda_{R\gamma P}\right)^2, \tag{20}$$

where

$$|\vec{k}| = \frac{\sqrt{m_R^4 + (m_A^2 - m_B^2)^2 - 2(m_A^2 + m_B^2)m_R^2}}{2m_R} \tag{21}$$

is the modulus of the three-momentum of the outgoing particles. Moreover, m_R refers to the mass of the decaying resonance, while m_A and m_B to the masses of decay products. In Tables III, IV, and V we report the flavor degeneracy coefficients s_{RPP}, s_{RVP} as well as the Clebsch-Gordan coefficients λ_{RPP}, λ_{RVP}, $\lambda_{R\gamma P}$ arising from the explicit evaluation of traces (see Appendix A).

The inclusion of loops and the evaluation of the positions of the poles would allow us to go beyond tree level; this is also left as an outlook. For the decays that we will examine, the ratio "width"/"mass" (Γ/M) is safely below 1, ensuring that loop corrections do not change much the tree-level results [40]. Yet, the study of the pole trajectories and the

TABLE III. All symmetry factors and the amplitude's coefficients of Eq. (18). They can be extracted from the Lagrangian of Eq. (7), whose expanded form is presented in Appendix A.

Decay channel		Symmetry factor, Eq. (18)	Amplitude, Eq. (18)
$V_E \to PP$	$V_D \to PP$	$s_{EPP} = s_{DPP}$	$\lambda_{EPP} = \lambda_{DPP}$
$\rho(1450) \to \bar{K}K$	$\rho(1700) \to \bar{K}K$	2	$\frac{1}{2}$
$\rho(1450) \to \pi\pi$	$\rho(1700) \to \pi\pi$	1	1
$K^*(1410) \to K\pi$	$K^*(1680) \to K\pi$	3	$\frac{1}{2}$
$K^*(1410) \to K\eta$	$K^*(1680) \to K\eta$	1	$\frac{1}{2}(\cos\theta_p - \sqrt{2}\sin\theta_p)$
$K^*(1410) \to K\eta'$	$K^*(1680) \to K\eta'$	1	$\frac{1}{2}(\sqrt{2}\cos\theta_p + \sin\theta_p)$
$\omega(1420) \to \bar{K}K$	$\omega(1650) \to \bar{K}K$	2	$\frac{1}{2}$
$\phi(1680) \to \bar{K}K$	$\phi(1930) \to \bar{K}K$	2	$\frac{1}{\sqrt{2}}$

TABLE IV. All symmetry factors and the amplitude's coefficients of Eq. (19). They can be extracted from the Lagrangian of Eq. (8), whose expanded form is presented in Appendix A.

Decay channel		Symmetry factor, Eq. (19)	Amplitude, Eq. (19)
$V_E \to VP$	$V_D \to VP$	$s_{EVP} = s_{DVP}$	$\lambda_{EVP} = \lambda s_{DVP}$
$\rho(1450) \to \omega\pi$	$\rho(1700) \to \omega\pi$	1	$\frac{1}{2}$
$\rho(1450) \to K^*(892)K$	$\rho(1700) \to K^*(892)K$	4	$\frac{1}{4}$
$\rho(1450) \to \rho(770)\eta$	$\rho(1700) \to \rho(770)\eta$	1	$\frac{1}{2}\cos\theta_p$
$\rho(1450) \to \rho(770)\eta'$	$\rho(1700) \to \rho(770)\eta'$	1	$\frac{1}{2}\sin\theta_p$
$K^*(1410) \to K\rho$	$K^*(1680) \to K\rho$	3	$\frac{1}{4}$
$K^*(1410) \to K\phi$	$K^*(1680) \to K\phi$	1	$\frac{1}{2\sqrt{2}}$
$K^*(1410) \to K\omega$	$K^*(1680) \to K\omega$	1	$\frac{1}{4}$
$K^*(1410) \to K^*(892)\pi$	$K^*(1680) \to K^*(892)\pi$	3	$\frac{1}{4}$
$K^*(1410) \to K^*(892)\eta$	$K^*(1680) \to K^*(892)\eta$	1	$\frac{1}{4}(\cos\theta_p + \sqrt{2}\sin\theta_p)$
$K^*(1410) \to K^*(892)\eta'$	$K^*(1680) \to K^*(892)\eta'$	2	$\frac{1}{4}(\sqrt{2}\cos\theta_p - \sin\theta_p)$
$\omega(1420) \to \rho\pi$	$\omega(1650) \to \rho\pi$	3	$\frac{1}{2}$
$\omega(1420) \to K^*(892)K$	$\omega(1650) \to K^*(892)K$	4	$\frac{1}{4}$
$\omega(1420) \to \omega(782)\eta$	$\omega(1650) \to \omega(782)\eta$	1	$\frac{1}{2}\cos\theta_p$
$\omega(1420) \to \omega(782)\eta'$	$\omega(1650) \to \omega(782)\eta'$	1	$\frac{1}{2}\sin\theta_p$
$\phi(1680) \to K\bar{K}^*$	$\phi(1930) \to K\bar{K}^*$	4	$\frac{1}{2\sqrt{2}}$
$\phi(1680) \to \phi(1020)\eta$	$\phi(1930) \to \phi(1020)\eta$	1	$\frac{1}{\sqrt{2}}\sin\theta_p$
$\phi(1680) \to \phi(1020)\eta'$	$\phi(1930) \to \phi(1020)\eta'$	1	$\frac{1}{\sqrt{2}}\cos\theta_p$

TABLE V. Amplitude's coefficients of Eq. (20) extracted from Eq. (8), together with the shift of Eq. (11).

Decay channel		Amplitude, Eq. (20)
$V_E \to \gamma P$	$V_D \to \gamma P$	$\lambda_{E\gamma P} = \lambda_{D\gamma P}$
$\rho(1450) \to \gamma\pi$	$\rho(1700) \to \gamma\pi$	$\frac{1}{6}$
$\rho(1450) \to \gamma\eta$	$\rho(1700) \to \gamma\eta$	$\frac{1}{2}\cos\theta_p$
$\rho(1450) \to \gamma\eta'$	$\rho(1700) \to \gamma\eta'$	$\frac{1}{2}\sin\theta_p$
$K^*(1410) \to \gamma K$	$K^*(1680) \to \gamma K$	$\frac{1}{3}$
$\omega(1420) \to \gamma\pi$	$\omega(1650) \to \gamma\pi$	$\frac{1}{2}$
$\omega(1420) \to \gamma\eta$	$\omega(1650) \to \gamma\eta$	$\frac{1}{6}\cos\theta_p$
$\omega(1420) \to \gamma\eta'$	$\omega(1650) \to \gamma\eta'$	$\frac{1}{6}\cos\theta_p$
$\phi(1680) \to \gamma\eta$	$\phi(1930) \to \gamma\eta$	$\frac{1}{3}\sin\theta_p$
$\phi(1680) \to \gamma\eta'$	$\phi(1930) \to \gamma\eta'$	$\frac{1}{3}\cos\theta_p$

eventual generation of additional poles (a typical QFT phenomenon that occurs when the coupling strength is strong enough, e.g. Refs. [41–44]) are surely interesting and could be addressed within our framework at a later stage.

III. RESULTS

In this section we present the results. First, in Sec. 3. A we determine the parameters of the model by using some selected data from PDG. Then, in Sec. III. B we concentrate on the results for the decays of the states $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and in Sec. III. C for the states $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$. In both cases we shall present summarizing tables and compare to additional ratios quoted in the PDG. When referring to a particular experiment, we shall use the notation of the PDG (first author and year in which the corresponding publication appeared).

A. Determination of the coupling constants

In order to determine the coupling constants one has to choose four well-known experimental values (two for each nonet). At present a full fit to all experimental values does not seem like the best procedure. In some cases, some observables were measured by a single experiment; in other cases different experimental results are not compatible with each other. Hence, the choice of four rather stable experimental results seems like the best strategy to fix our four parameters. Later on, it will be possible to compare the results to partial widths and to quite many ratios between partial widths; see Secs. III. B and III. C.

Concerning g_{EPP} and g_{EVP}, we use the following experimental data taken from PDG [1]:

$$\Gamma^{\text{exp}}_{K^*(1410)\to K\pi} = 15.3 \pm 3.3 \text{ MeV} \tag{22}$$

$$\Gamma^{\text{tot,exp}}_{\phi(1680)} = 150 \pm 50 \text{ MeV}. \tag{23}$$

We do so because the decay $K^*(1410) \to K\pi$ is well known and the width of the rather narrow resonance $\phi(1680)$ is the sum of a few decay channels, all of them also described by our model: $\phi(1680) \to K^*(892)K$, $\phi(1680) \to \phi(1020)\eta$, and $\phi(1680) \to \bar{K}K$. Moreover, $\phi(1680) \to K^*(892)K$ is reported by the PDG to be dominant: this property fits very well with our results (details in the next subsection). Upon minimizing the function

$$F_E(g_{EPP}, g_{EVP}) = \left(\frac{\Gamma_{K^*(1410\to K\pi)} - \Gamma^{\rm exp}_{K^*(1410)\to K\pi}}{\delta\Gamma^{\rm exp}_{K^*(1410)\to K\pi}}\right)^2 + \left(\frac{\Gamma_{\phi(1680)\to K^*(892)K} + \Gamma_{\phi(1680)\to\phi(1020)\eta} + \Gamma_{\phi(1680)\to\bar{K}K} - \Gamma^{\rm tot,exp}_{\phi(1680)}}{\delta\Gamma^{\rm tot,exp}_{\phi(1680)}}\right)^2 \tag{24}$$

we obtain

$$g_{EPP} = 3.66 \pm 0.4 \quad \text{and} \quad g_{EVP} = 18.4 \pm 3.8. \tag{25}$$

Similarly, in order to determine the coupling constants g_{DPP} and g_{DVP} we need to choose two experimental values. For consistency, we use the results of the experiments Aston 84 [45] and Aston 88 [46] in connection to the rather well-known resonance $K^*(1680)$. The values that we will use are also in agreement with the fit of PDG; see below. The first quantity that we use is the $K\rho$ to $K\pi$ ratio

$$\left.\frac{\Gamma_{K^*(1680)\to K\rho}}{\Gamma_{K^*(1680)\to K\pi}}\right|_{\rm exp} = 1.2 \pm 0.4 \text{ by Aston 84 [51]}, \tag{26}$$

which basically fixes the ratio g_{DVP}/g_{DPP}. Note that the fit done by the PDG [1] reads $0.81^{+0.14}_{-0.09}$ and is compatible with Aston 84. Next, we use the decay width $\Gamma_{K^*(1680)\to K\pi}$, obtained by the following two quantities:

$$\left.\frac{\Gamma_{K^*(1680)\to K\pi}}{\Gamma^{\rm tot}_{K^*(1680)}}\right|_{\rm exp} = 0.388 \pm 0.036 \quad \text{and}$$
$$\Gamma^{\rm tot}_{K^*(1680)}\big|_{\rm exp} = 205 \pm 50 \text{ MeV by Aston 88 [45]}, \tag{27}$$

out of which

$$\Gamma_{K^*(1680)\to K\pi}\big|_{\rm exp} = 79 \pm 21 \text{ MeV from Aston 88 [45]}. \tag{28}$$

Note that the PDG quotes the fit $\Gamma_{K^*(1680)\to K\pi}/\Gamma^{\rm tot}_{K^*(1680)} = 0.387 \pm 0.026$, which is basically the value determined in Aston 88. The full width quoted by the PDG reads 322 ± 110 MeV (as average) and is also compatible with Aston 88. Finally, by using Eqs. (26) and (28) and performing the standard minimization of

$$F_D(g_{DPP}, g_{DVP}) = \left(\frac{\frac{\Gamma_{K^*(1680)\to K\rho}}{\Gamma_{K^*(1680)\to K\pi}} - \left(\frac{\Gamma_{K^*(1680)\to K\rho}}{\Gamma_{K^*(1680)\to K\pi}}\right)^{\rm exp}}{\delta\left(\frac{\Gamma_{K^*(1680)\to K\rho}}{\Gamma_{K^*(1680)\to K\pi}}\right)}\right)^2 + \left(\frac{\Gamma_{K^*(1680)\to K\pi} - \Gamma^{\rm exp}_{K^*(1680)\to K\pi}}{\delta\Gamma_{K^*(1680)\to K\pi}}\right)^2 \tag{29}$$

we obtain

$$g_{DPP} = 7.15 \pm 0.94 \quad \text{and} \quad g_{DVP} = 16.5 \pm 3.5. \tag{30}$$

For further details on the used approach for the determination of the parameters and for the subsequent error propagation, see Appendix C. Note that in calculating the errors of the coupling constants of Eqs. (25) and (30) we did not consider the uncertainties on the masses.

B. Results for the nonet $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$

In Tables VI, VII, and VIII we report the results for the decays of $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ into PP, VP and γP pairs. An overall agreement of theory with data is visible: theoretically large decays are clearly seen in experiments, while theoretically small decays were generally not seen. These results show that the understanding of this nonet as a regular $\bar{q}q$ is quite stable.

TABLE VI. Decays widths of (predominantly) radially excited vector mesons into two pseudoscalar mesons ($V_E \to PP$).

Decay process $V_E \to PP$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \to \bar{K}K$	6.6 ± 1.4	$< 6.7 \pm 1.0$ by Donnachie 91 [49]
$\rho(1450) \to \pi\pi$	30.8 ± 6.7	$\sim 27 \pm 4$, seen by Clegg 94 [47]
$K^*(1410) \to K\pi$	15.3 ± 3.3	15.3 ± 3.3 by PDG
$K^*(1410) \to K\eta$	6.9 ± 1.5	Not listed in PDG
$K^*(1410) \to K\eta'$	≈ 0	Not listed in PDG
$\omega(1420) \to \bar{K}K$	5.9 ± 1.3	Not listed in PDG
$\phi(1680) \to \bar{K}K$	19.8 ± 4.3	Seen by Buon 82 [54]

TABLE VII. Decays widths of (predominantly) radially excited vector mesons into a pseudoscalar meson and a ground-state vector meson ($V_E \to VP$).

Decay process $V_E \to VP$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \to \omega\pi$	74.7 ± 31.0	$\sim 84 \pm 13$ seen by Clegg 94 [47]
$\rho(1450) \to K^*(892)K$	6.7 ± 2.8	Possibly seen by Coan 04 [62]
$\rho(1450) \to \rho(770)\eta$	9.3 ± 3.9	$<16.0 \pm 2.4$ by Donnachie 91 [49]
$\rho(1450) \to \rho(770)\eta'$	≈ 0	Not listed in PDG
$K^*(1410) \to K\rho$	12.0 ± 5.0	$<16.2 \pm 1.5$ by PDG
$K^*(1410) \to K\phi$	≈ 0	Not listed in PDG
$K^*(1410) \to K\omega$	3.7 ± 1.5	Not listed in PDG
$K^*(1410) \to K^*(892)\pi$	28.8 ± 12.0	$>93 \pm 8$ by PDG
$K^*(1410) \to K^*(892)\eta$	≈ 0	Not listed in PDG
$K^*(1410) \to K^*(892)\eta'$	≈ 0	Not listed in PDG
$\omega(1420) \to \rho\pi$	196 ± 81	Dominant, $\Gamma_{\rm tot} = (180 - 250)$ by PDG
$\omega(1420) \to K^*(892)K$	2.3 ± 1.0	Not listed in PDG
$\omega(1420) \to \omega(782)\eta$	4.9 ± 2.0	Not listed in PDG
$\omega(1420) \to \omega(782)\eta'$	≈ 0	Not listed in PDG
$\phi(1680) \to K\bar{K}^*$	110 ± 46	Dominant, $\Gamma_{\rm tot} = 150 \pm 50$ by PDG
$\phi(1680) \to \phi(1020)\eta$	12.2 ± 5.1	Seen by Achasov 14 [63]
$\phi(1680) \to \phi(1020)\eta'$	≈ 0	Not listed in PDG

TABLE VIII. Decay widths of (predominantly) radially excited vector mesons into a photon and a pseudoscalar meson ($V_E \to \gamma P$).

Decay process $V_E \to \gamma P$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \to \gamma\pi$	0.072 ± 0.042	Not listed
$\rho(1450) \to \gamma\eta$	0.23 ± 0.14	$\sim$0.2–1.5 (see text)
$\rho(1450) \to \gamma\eta'$	0.056 ± 0.033	Not listed
$K^*(1410) \to \gamma K$	0.18 ± 0.11	< 0.0529 MeV seen by PDG and Alavi-Harati 02B [64]
$\omega(1420) \to \gamma\pi$	0.60 ± 0.36	1.90 ± 0.75 (see text)
$\omega(1420) \to \gamma\eta$	0.023 ± 0.014	Not listed
$\omega(1420) \to \gamma\eta'$	0.0050 ± 0.0030	Not listed
$\phi(1680) \to \gamma\eta$	0.14 ± 0.09	Seen
$\phi(1680) \to \gamma\eta'$	0.076 ± 0.045	Not listed

Yet, there are some quantities which deserve more detailed comments; see below. Namely, besides partial widths, the PDG reports experimental results about ratios of different decay channels.

(i) Resonance $\rho(1450)$, strong decays. Concerning $\rho(1450)$, various ratios can be checked. The $\pi\pi/\omega\pi$ ratio was determined by Clegg 94 in Ref. [47]:

$$\left.\frac{\Gamma_{\rho(1450)\to\pi\pi}}{\Gamma_{\rho(1450)\to\omega\pi}}\right|_{\rm exp} \sim 0.32\,\text{by Clegg 94 [46]}. \qquad (31)$$

The corresponding theoretical 0.41 ± 0.20 agrees well with Clegg 94. Note that this ratio depends on the ratio of coupling constant g_{EPP}/g_{EVP} and therefore represents an independent confirmation of this quantity. Along the same line, the ratio

$$\left.\frac{\Gamma_{\rho(1450)\to\pi\pi}}{\Gamma_{\rho(1450)\to\eta\rho}}\right|_{\rm exp} = 1.3 \pm 0.4\,\text{Aulchenko 15 [47]} \qquad (32)$$

is in qualitative agreement with the theoretical value of 3.3 ± 1.6. Moving on, the upper limit for the ratio

$$\left.\frac{\Gamma_{\rho(1450)\to KK}}{\Gamma_{\rho(1450)\to\omega\pi}}\right|_{\rm exp} < 0.08\,\text{Donnachie 91 [48]} \qquad (33)$$

is also compatible with the theoretical value of 0.088 ± 0.043. In summary, $g_{EPP}/g_{EVP} \simeq 1/5$ is in good agreement with various experimental results of $\rho(1450)$. Next, we consider

$$\left.\frac{\Gamma_{\rho(1450)\to\eta\rho}}{\Gamma_{\rho(1450)\to\omega\pi}}\right|_{\rm exp} = \begin{cases} 0.081 \pm 0.020\,\text{Aulchenko 15 [47]} \\ \sim 0.21\,\text{Donnachie 91 [48]} \\ >2\,\text{Fukui 91} \end{cases} \qquad (34)$$

which shall be compared to our theoretical result of ≈ 0.12. This value is in good agreement with the latest determination of Aulchenko 15 [48] and also with Donnachie 91 [49]. On the contrary, the lower limit of Fukui 91 [50] is in disagreement with the other experiments as well as with theory. Here, the theoretical result is parameter independent: this ratio is purely fixed by flavor symmetry and phase space. Due to the fact that in Sec. III. A we used only the errors of the decay width to determine the parameter's errors, no theoretical error for this ratio can be determined. Of course, the uncertainties of other quantities (such as the masses) and the validity of the employed Lagrangians (see the discussion in Sec. II. B) induce an error for this quantity (with the estimate of about 10%–20%, which is the expected precision of our effective model). The very same comment will apply to all the ratios of the type PP/PP and PV/PV, which are independent of the coupling constants of our approach.

As a concluding remark, the interpretation of $\rho(1450)$ as an excited ρ meson is in good agreement with the present data of various experiments. In this respect, a lighter resonance $\rho(1290)$ (see e.g. Refs. [14,15,51]) is not needed in the $\bar{q}q$ assignment. Eventually, such a lighter resonance can emerge as a companion pole [41–43] once loops are included.

(ii) Resonance $K^*(1410)$, strong decays. The resonance $K^*(1410)$ is well established, both experimentally and on the lattice [34]. Its decay into $K\pi$ reported in Table VI turns out to be correct because this branching ratio was used to fix the parameters (see Sec. III. A). However, the decay $K^*(1410) \to K^*(892)\pi$ is too small when compared to the summary of the PDG: roughly, an overestimation of a factor 3 is present. We now discuss the ratios of $K^*(1410)$. We start with

$$\left.\frac{\Gamma_{K^*(1410)\to\rho K}}{\Gamma_{K^*(1410)\to K^*(892)\pi}}\right|_{\rm exp} < 0.17 \text{ Aston 84 [51]}, \qquad (35)$$

which should be compared to ≈ 0.42, which is too large. This ratio is fixed by flavor symmetry and is independent of the parameters; hence a disagreement is quite surprising and should be clarified in the future. Next, we consider

$$\left.\frac{\Gamma_{K^*(1410)\to\pi K}}{\Gamma_{K^*(1410)\to K^*(892)\pi}}\right|_{\rm exp} < 0.16 \text{ Aston 84 [51]}, \qquad (36)$$

which should be compared with 0.53 ± 0.26. Indeed, this is the seed of the disagreement concerning the decay $K^*(1410) \to K^*(892)\pi$ of Table VII. Namely, the value by Aston 84 has been used to set the upper limit quoted in the PDG.

(iii) Resonance $\omega(1420)$, strong decays. The decays of the resonance $\omega(1420)$ are in agreement with data. The decay $\omega(1420) \to K^*(892)K$ is theoretically the largest one (in agreement with the PDG, which classifies this decay as dominant). All the other decay rates are quite small (few MeV) and were not yet discovered experimentally (hence, our results are predictions). From the 2017 update of the PDG one can use the values [52–60]

$$\left.\frac{\Gamma_{\omega(1420)\to\omega\eta}}{\Gamma^{\rm tot}_{\omega(1420)}}\frac{\Gamma_{\omega(1420)\to e^+e^-}}{\Gamma^{\rm total}_{\omega(1420)}}\right|_{\rm exp} = (1.6^{+0.09}_{-0.07}) \times 10^{-8} \text{ by Achasov 16B [52]} \qquad (37)$$

$$\left.\frac{\Gamma_{\omega(1420)\to\rho\pi}}{\Gamma^{\rm tot}_{\omega(1420)}}\frac{\Gamma_{\omega(1420)\to e^+e^-}}{\Gamma^{\rm total}_{\omega(1420)}}\right|_{\rm exp} = (0.73 \pm 0.08) \times 10^{-6} \text{ by Aulchenko 15A [53]} \qquad (38)$$

to extract (in the numerator the average of errors is used)

$$\left.\frac{\Gamma_{\omega(1420)\to\omega\eta}}{\Gamma_{\omega(1420)\to\rho\pi}}\right|_{\rm exp} = \frac{(1.6 \pm 0.08) \times 10^{-8}}{(0.73 \pm 0.08) \times 10^{-6}} = 0.021 \pm 0.001. \qquad (39)$$

This result compares very well to our theoretical result ≈ 0.025. [Note that for the quantity in Eq. (37) other values are listed in the PDG: they are all compatible with Eq. (37) and with each other.]

(iv) Resonance $\phi(1680)$, strong decays. The partial decay widths of the resonance $\phi(1680)$ fits reported in Tables VI and VII show that the channel $\phi(1680) \to KK^*(892)$ is dominant, in agreement with the PDG quote. Moreover, the following ratios can be experimentally obtained and both of them are compatible with theory:

$$\left.\frac{\Gamma_{\phi(1680)\to K\bar{K}}}{\Gamma_{\phi(1680)\to K^*(892)K}}\right|_{\rm exp} = 0.07 \pm 0.01 \text{ Buon 82 [54]} \qquad (40)$$

is in good agreement with the theoretical value (dependent on the ratio of couplings g_{EPP}/g_{EVP}) of 0.18 ± 0.09; similarly, the parameter independent ratio

$$\left.\frac{\Gamma_{\phi(1680)\to\eta\phi}}{\Gamma_{\phi(1680)\to K^*(892)K}}\right|_{\rm exp} = 0.07 \pm 0.01 \text{ Aubert 08S [55]} \qquad (41)$$

agrees with the theoretical value of ≈ 0.11.

(iv) Radiative decays. As a last step, we discuss also the decay rates of excited vector mesons into photon and pseudoscalar mesons. These radiative decays were determined by using "vector meson dominance" without the need for any new parameter. The radiative decays of $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ are still poorly determined experimentally, but the theoretically predicted sizable decays were seen in experiments. In two cases, numerical values can be extracted (in other cases, our theoretical results represent predictions). The decay $\rho(1450) \to \gamma\eta$ can be estimated by using

$$\Gamma_{\rho(1450)\to\gamma\eta} \frac{\Gamma_{\rho(1450)\to e^+e^-}}{\Gamma^{\text{total}}_{\rho(1450)}}\bigg|_{\text{exp}} = 2.2 \pm 0.5 \pm 0.3 \text{ eV by Akhmetshin 01B [56]}, \quad (42)$$

and

$$\Gamma_{\rho(1450)\to\pi\pi} \frac{\Gamma_{\rho(1450)\to e^+e^-}}{\Gamma^{\text{total}}_{\rho(1450)}}\bigg|_{\text{exp}} = \begin{cases} 0.12 \text{ keV by Diekmann 88 [57]} \\ 0.027^{+0.015}_{-0.010} \text{ keV by Kurdadze 83 [58]} \end{cases}. \quad (43)$$

as well as $\Gamma_{\rho(1450)\to\pi\pi} \approx 84$ MeV (see Clegg 94 [47]). One obtains

$$\Gamma_{\rho(1450)\to\gamma\eta}\big|_{\text{exp}} \approx \begin{cases} 1.5 \text{ MeV} \\ 0.2 \text{ MeV} \end{cases}, \quad (44)$$

which is reported in Table VIII. The error unfortunately cannot be determined since Clegg 94 did not report any and there are no other determinations of $\Gamma_{\rho(1450)\to\pi\pi}$. Nevertheless, the qualitative agreement of the second determination with the theoretical result (0.23 ± 0.14 MeV) is rather promising. Similarly, concerning $\omega(1420) \to \gamma\pi^0$ we use

$$\frac{\Gamma_{\omega(1420)\to\gamma\pi^0}}{\Gamma^{\text{total}}_{\omega(1420)}} \frac{\Gamma_{\omega(1420)\to e^+e^-}}{\Gamma^{\text{total}}_{\omega(1420)}}\bigg|_{\text{exp}} = 2.03^{+0.70}_{-0.75} \times 10^{-8} \text{ by Akhmetshin 05 [59]} \quad (45)$$

together with

$$\frac{\Gamma_{\omega(1420)\to e^+e^-}}{\Gamma^{\text{total}}_{\omega(1420)}}\bigg|_{\text{exp}} = (23 \pm 1) \times 10^{-7} \text{ by Henner 02 [60]} \quad (46)$$

to obtain

$$\Gamma_{\omega(1420)\to\gamma\pi^0}\big|_{\text{exp}} = (1.9 \pm 0.75) \text{ MeV}. \quad (47)$$

[Note that the ratio (46) was also estimated by Achasov 03D [61] to be 6.6, but without errors, therefore we chose to use Henner 02. Moreover, the value 6.6 combined with Eq. (38) would deliver a ratio $\Gamma_{\omega(1420)\to\rho\pi}/\Gamma^{\text{tot}}_{\omega(1420)}$ larger than 1, a result which is not consistent.] Also in this case, the agreement with the theoretical result $\Gamma_{\omega(1420)\to\gamma\pi^0} = 0.60 \pm 0.36$ MeV is interesting. Quite remarkably, the decay $\omega(1420) \to \gamma\pi^0$ is the strongest radiative decay that the theory predicts and correspondingly there is a sizable value that can be extracted by the present experimental information.

Finally, the upper limit of decay $K^*(1410) \to K\gamma$ as quoted by the PDG (determined in the work of Alavi-Harati 02B [64]) is 0.052 MeV; this value is compatible with the present theoretical result. In future experiments it should be possible to determine this quantity. The decay width $\phi(1680) \to \gamma\eta$ has been also seen experimentally by Achasov 14 [63], but no value is reported.

Summarizing, the overall agreement of theory with data is stable and confirms that the first nonet of excited vectors $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ is a standard $\bar{q}q$ nonet, predominantly corresponding to the first radial excitation of vector mesons. Future experimental results are expected to come and further checks will be possible in the near (GlueX and CLAS12) and less near (PANDA) future.

C. Results for the nonet $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$

In Tables IX, X, and XI we report the results for the second nonet of excited vector mesons $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$. With some exceptions to be discussed later on, there is also in this case an overall qualitative agreement of theory with data. One may therefore conclude that the assignment of these mesons to a nonet of orbitally excited vector mesons is viable. Next, we concentrate on the detailed description of the results and to the comparison of numerous ratios listed in the PDG.

TABLE IX. Decays widths of (predominantly) orbitally excited vector mesons into two pseudoscalar mesons ($V_D \to PP$).

Decay process $V_D \to PP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \to \bar{K}K$	40 ± 11	$8.3^{+10}_{-8.3}$ MeV (see text)
$\rho(1700) \to \pi\pi$	140 ± 37	75 ± 30 by Becker 79 [65]
$K^*(1680) \to K\pi$	82 ± 22	125 ± 43 by PDG
$K^*(1680) \to K\eta$	52 ± 14	Not listed in PDG
$K^*(1680) \to K\eta'$	0.72 ± 0.02	Not listed in PDG
$\omega(1650) \to \bar{K}K$	37 ± 10	Not listed in PDG
$\phi(1930) \to \bar{K}K$	104 ± 28	Resonance not yet known

TABLE X. Decays widths of (predominantly) orbitally excited vector mesons into a pseudoscalar meson and a ground-state vector meson ($V_D \to VP$).

Decay process $V_D \to VP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \to \omega\pi$	140 ± 59	Seen (see text)
$\rho(1700) \to K^*(892)K$	56 ± 23	83 ± 66 MeV (see text)
$\rho(1700) \to \rho\eta$	41 ± 17	68 ± 42 MeV (see text)
$\rho(1700) \to \rho\eta'$	≈ 0	Not listed in PDG
$K^*(1680) \to K\rho$	64 ± 27	101 ± 35 by PDG
$K^*(1680) \to K\phi$	13 ± 6	Not listed in PDG
$K^*(1680) \to K\omega$	21 ± 9	Not listed in PDG
$K^*(1680) \to K^*(892)\pi$	81 ± 34	96 ± 33 by PDG
$K^*(1680) \to K^*(892)\eta$	0.5 ± 0.2	Not listed in PDG
$K^*(1680) \to K^*(892)\eta'$	≈ 0	Not listed in PDG
$\omega(1650) \to \rho\pi$	370 ± 156	~ 205, 154 ± 44, ~ 273, 120 ± 18 (see text)
$\omega(1650) \to K^*(892)K$	42 ± 18	Not listed in PDG
$\omega(1650) \to \omega(782)\eta$	32 ± 13	~ 100, 56 ± 30 (see text)
$\omega(1650) \to \omega(782)\eta'$	≈ 0	Not listed in PDG
$\phi(1930) \to K\bar{K}^*$	260 ± 109	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta$	67 ± 28	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta'$	≈ 0	Resonance not yet known

(i) Resonance $\rho(1700)$, strong decays. The total width of the resonance $\rho(1700)$ as resulting from our theoretical analysis reads 417 ± 147, which is in agreement with the PDG estimate of 250 ± 100 MeV. The theoretical results show a slight overestimation of PP decays. Namely, while the $\pi\pi$ channel is in agreement with Becker 79 [65] as reported in Table IX, there are additional measurements of this channel in older experiments [66–70]:

$$\Gamma_{\rho(1700)\to\pi\pi}|_{\rm exp} = \begin{cases} 56 \pm 29\,\text{Martin 78C [65]} \\ 75 \pm 32\,\text{Froggatt 77 [66]}\,; \\ 63 \pm 30\,\text{Hyams 73 [67]} \end{cases} \tag{48}$$

overall it looks compatible, but a new experimental determination would be useful.

For the KK channel, we combine

$$\left.\frac{\Gamma_{\rho(1700)\to KK}}{\Gamma_{\rho(1700)\to 2(\pi^+\pi^-)}}\right|_{\rm exp} = 0.015 \pm 0.010\,\text{Delcourt 81B [68]}, \tag{49}$$

$$\left.\frac{\Gamma_{\rho(1700)\to\pi\pi}}{\Gamma_{\rho(1700)\to 2(\pi^+\pi^-)}}\right|_{\rm exp} = 0.13 \pm 0.05\,\text{Aston 80 [69]}, \tag{50}$$

and

$$\left.\frac{\Gamma_{\rho(1700)\to\pi\pi}}{\Gamma^{\rm tot}_{\rho(1700)}}\right|_{\rm exp} = 0.287^{+0.043}_{-0.042} \pm 0.05\,\text{Becker 79 [64]}, \tag{51}$$

in order to obtain

$$\Gamma_{\rho(1700)\to KK}|_{\rm exp} = 8.3^{+10.4}_{-8.3}\ \text{MeV}. \tag{52}$$

This is the result reported in Table IX. It is smaller than the theoretical result, but large errors are present.

In a similar way, we use

$$\left.\frac{\Gamma_{\rho(1700)\to KK^*(892)}}{\Gamma_{\rho(1700)\to 2(\pi^+\pi^-)}}\right|_{\rm exp} = 0.15 \pm 0.03\,\text{Delcourt 81B [68]} \tag{53}$$

to obtain

$$\Gamma_{\rho(1700)\to KK^*(892)}|_{\rm exp} = 83 \pm 66\ \text{MeV} \tag{54}$$

TABLE XI. Decay widths of (predominantly) orbitally excited vector mesons into a photon and a pseudoscalar meson ($V_D \to \gamma P$).

Decay process $V_D \to \gamma P$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \to \gamma\pi$	0.095 ± 0.058	Not listed
$\rho(1700) \to \gamma\eta$	0.35 ± 0.21	Not listed
$\rho(1700) \to \gamma\eta'$	0.13 ± 0.08	Not listed
$K^*(1680) \to \gamma K$	0.30 ± 0.18	Not listed
$\omega(1650) \to \gamma\pi$	0.78 ± 0.47	Not listed
$\omega(1650) \to \gamma\eta$	0.035 ± 0.021	Not listed
$\omega(1650) \to \gamma\eta'$	0.012 ± 0.007	Not listed
$\phi(1930) \to \gamma\eta$	0.19 ± 0.12	Resonance not yet known
$\phi(1930) \to \gamma\eta'$	0.13 ± 0.08	Resonance not yet known

as reported in Table X. Although the resulting experimental error is very large, the result is compatible with the theoretical value of 56 ± 23 MeV. Note that the decay mode $\rho(1700) \to KK^*(892)$ was also possibly seen by Coan 04 [62] and clearly seen in radiative decays by Bizot 80 [71] and Delcourt 81B [69], but one cannot use those results to obtain an independent determination of this partial width.

The last decay that can be determined along the same procedure is $\rho(1700) \to \rho\eta$. Upon using [72]

$$\left.\frac{\Gamma_{\rho(1700)\to\rho\eta}}{\Gamma_{\rho(1700)\to 2(\pi^+\pi^-)}}\right|_{\mathrm{exp}} = 0.123 \pm 0.027\,\text{Delcourt 82 [72]}, \tag{55}$$

we derive

$$\Gamma_{\rho(1700)\to\rho\eta}\big|_{\mathrm{exp}} = 68 \pm 42\ \text{MeV}, \tag{56}$$

as reported in Table X. Again, the error is large, but the value fits well with theory. [For completeness, it should be also stressed that $\Gamma_{\rho(1700)\to\rho\eta}/\Gamma^{\mathrm{tot}}_{\rho(1700)}$ was determined to be < 0.04 by Donnachie 87B [73], out of which $\Gamma_{\rho(1700)\to\rho\eta} < 10 \pm 4$. This result is not compatible with theory or with Eq. (56).]

We now turn to the discussion of ratios. To this end, we use the reported decay widths involving the dilepton pair e^+e^-, $\Gamma_{\rho(1700)\to MM}\cdot\frac{\Gamma_{\rho(1700)\to e^+e^-}}{\Gamma^{\mathrm{tot}}_{\rho(1700)}}$, and decay ratios involving $2(\pi^+\pi^-)$, $\frac{\Gamma_{\rho(1700)\to MM}}{\Gamma_{\rho(1700)\to 2(\pi^+\pi^-)}}$, where MM is a certain meson-meson channel (PP or VP). We first study PP/PP ratios, then PP/PV, and finally PV/PV.

Concerning the (parameter independent) $\pi\pi/KK$ ratio, we have [first two ratios from e^+e^-, third one from $2(\pi^+\pi^-)$]

$$\frac{\Gamma_{\rho(1700)\to\pi\pi}}{\Gamma_{\rho(1700)\to KK}} = \begin{cases} \sim 3.7\,\text{Diekman 88 [57]} + \text{Bizot 80 [71]} \\ 0.83 \pm 0.82\,\text{Kurdadze 83 [58]} + \text{Bizot 80 [71]}, \\ 8.7 \pm 6.7\,\text{Aston 80 [69]} + \text{Delcourt 81B [68]} \end{cases} \tag{57}$$

which shall be compared to the theoretical result of ≈ 3.5, which is in rough agreement with experiment (especially with the first determination above).

Concerning $\pi\pi/\pi\rho$, one obtains [first two ratios from e^+e^-, third one from $2(\pi^+\pi^-)$]

$$\left.\frac{\Gamma_{\rho(1700)\to\pi\pi}}{\Gamma_{\rho(1700)\to\eta\rho}}\right|_{\mathrm{exp}} = \begin{cases} \sim 18\,\text{Diekmann 88 [57]} + \text{Antonelli 88 [74]} \\ 4.1 \pm 2.7\,\text{Kurdadze 83 [58]} + \text{Antonelli 88 [74]}. \\ 1.1 \pm 0.47\,\text{Aston 80 [69]} + \text{Delcourt 82 [72]} \end{cases} \tag{58}$$

The theoretical result 3.4 ± 1.1 fits quite well with the second entry. There is also agreement with the last experimental result quoted above. On the contrary, the first entry (without error) is not compatible with theory or with the other experimental values.

Next, for the $KK/\eta\rho$ ratio [first result from e^+e^-, second from $2(\pi^+\pi^-)$] we get

$$\left.\frac{\Gamma_{\rho(1700)\to KK}}{\Gamma_{\rho(1700)\to\eta\rho}}\right|_{\mathrm{exp}} = \begin{cases} 5.0 \pm 4.7\,\text{Bizot 80 [71]} + \text{Antonelli 88 [74]} \\ 0.12 \pm 0.09\,\text{Delcourt 81B [68]} + \text{Antonelli 88 [74]} \end{cases}. \tag{59}$$

The corresponding theoretical width of 0.98 ± 0.33 is compatible with the first entry (due to the larger experimental errors) but not with the second. Also in this case, the experimental values do not agree with each other.

Going further, we discuss the ratio $KK/K^*(892)K$, which is quite problematic [first result from e^+e^-, second from $2(\pi^+\pi^-)$, third one listed in the PDG]:

$$\left.\frac{\Gamma_{\rho(1700)\to KK}}{\Gamma_{\rho(1700)\to K^*(892)K}}\right|_{\mathrm{exp}} = \begin{cases} 0.11 \pm 0.10\,\text{Bizot 80 [71]} \\ 0.10 \pm 0.07\,\text{Delcourt 81B [68]}. \\ 0.052 \pm 0.026\,\text{Buon 82 [54]} \end{cases} \tag{60}$$

The corresponding theoretical result reads 0.71 ± 0.24. Hence, we have a mismatch of the listed experimental ratios with our value. The reason for this mismatch is easy to understand: ratios of the type PP/VP depend solely on the ratio of coupling constants g_{DPP}/g_{DVP}, which is fixed by Eq. (26) in which the results of Aston 84 [45] and Aston 88 [46] are used (see Sec. III. A). There is no way to bring those experimental results and the ones of Eq. (60) in agreement with each other. Since the ratio in Eq. (26) seems to be based on a quite solid result, we tend to believe that a new determination of $KK/K^*(892)K$ is necessary.

Finally, we turn to the (parameter independent) $K^*(892)K/\eta\rho$ ratio [first result from e^+e^-, second from $2(\pi^+\pi^-)$]: [74]

$$\left.\frac{\Gamma_{\rho(1700)\to K^*(892)K}}{\Gamma_{\rho(1700)\to\eta\rho}}\right|_{\exp} = \begin{cases} 43\pm21\,\text{Bizot 80 [71]}+\text{Antonelli 88 [74]} \\ 1.22\pm0.27\,\text{Delcourt 81B [68]}+\text{Delcourt 82 [72]} \end{cases}. \tag{61}$$

Unfortunately, the two values are not consistent with each other. The theoretical result that reads ≈ 1.37 fits quite well with the second entry. This result shows that $\Gamma_{\rho(1700)\to K^*(892)K}$ is possibly overestimated by the quantity $\Gamma_{\rho(1700)\to K^*(892)K}\cdot\frac{\Gamma_{\rho(1700)\to e^+e^-}}{\Gamma^{\rm tot}_{\rho(1700)}}=0.305\pm0.071$ reported by Bizot 80 [71]. A smaller value of the latter would lead to a better agreement with our theoretical results. This comment also applies for the disagreement with the ratio reported in Eq. (60).

(ii) Resonance $K^*(1680)$, strong decays. We now discuss the resonance $K^*(1680)$, which is experimentally rather well known. The decay widths fit well with the experiment (in agreement with the fact that we used one of them to fix the strength of the parameters; see Sec. III. A). In addition, we can study two ratios.

The $K\pi/K^*(892)\pi$ ratio is determined by the PDG and Aston 84 (both entries bold) as

$$\left.\frac{\Gamma_{K^*(1680)\to K\pi}}{\Gamma_{K^*(1680)\to K^*(892)\pi}}\right|_{\exp} = \begin{cases} 1.30^{+0.23}_{-0.14}\ \text{fit by PDG} \\ 2.8\pm1.1\ \text{by Aston 84 [51]} \end{cases}. \tag{62}$$

The two values do not agree well with each other, but due to large errors they are not incompatible. The theoretical result 1.01 ± 0.34 fits well with the PDG value.

The second ratio, $K\rho/K^*(892)\pi$, reads [75]

$$\left.\frac{\Gamma_{K^*(1680)\to K\rho}}{\Gamma_{K^*(1680)\to K^*(892)\pi}}\right|_{\exp} = \begin{cases} 1.05^{+0.27}_{-0.11}\ \text{fit by PDG} \\ 0.97\pm0.09^{+0.30}_{-0.10}\ \text{by Aston 87 [75]} \end{cases}. \tag{63}$$

The theoretical value ≈ 0.79 fits well with both entries.

(iii) Resonance $\omega(1650)$, strong decays. The dominant decay of $\omega(1650)$ is represented by the mode $\omega(1650)\to\rho\pi$. The qualitative picture agrees well with the theory (see Table X). Yet, the theoretical result has a very large error. The decay width $\Gamma_{\omega(1650)\to\rho\pi}$ can be determined by using the ratios

$$\left.\frac{\Gamma_{\omega(1650)\to\rho\pi}}{\Gamma^{\rm tot}_{\omega(1650)}}\right|_{\exp} = \begin{cases} \sim0.65\ \text{Achasov 03D [61]} \\ 0.380\pm0.014\ \text{Henner 02 [60]} \end{cases} \tag{64}$$

together with $\Gamma^{\rm tot}_{\omega(1650)}=315\pm35$ MeV [1], finding

$$\Gamma_{\omega(1650)\to\rho\pi}|_{\exp} = \begin{cases} \sim205\ \text{MeV Achasov 03D [61]} \\ 120\pm18\ \text{MeV Henner 02 [60]} \end{cases}. \tag{65}$$

The theoretical result is in agreement with the upper determination but overestimates the latter. (The latter value would point to a smaller value of the parameter g_{DVP}.) Yet, other determinations can be obtained by using

$$\frac{\Gamma_{\omega(1650)\to\rho\pi}}{\Gamma^{\rm tot}_{\omega(1650)}}\frac{\Gamma_{\omega(1650)\to e^+e^-}}{\Gamma^{\rm tot}_{\omega(1650)}} = 1.56\pm0.23\ \text{Aulchenko 15A [53]} \tag{66}$$

together with

$$\left.\frac{\Gamma_{\omega(1650)\to e^+e^-}}{\Gamma^{\rm tot}_{\omega(1650)}}\right|_{\exp} = \begin{cases} \sim18\ \text{Achasov 03D [61]} \\ 32\pm1\ \text{Henner 02 [60]} \end{cases} \tag{67}$$

out of which

$$\Gamma_{\omega(1650)\to\rho\pi} = \begin{cases} \sim273\ \text{MeV, Achasov 03D [61]}+\text{Aulchenko 15A [53]} \\ 154\pm44\ \text{Henner 02 [60]}+\text{Aulchenko 15A [53]} \end{cases}. \tag{68}$$

The results still have large errors and a clear outcome is difficult to assess. Surely, this decay width is large and is the dominant decay channel of $\omega(1650)$.

Following the same procedure, by using [76]

$$\left.\frac{\Gamma_{\omega(1650)\to\omega\eta}}{\Gamma^{\rm tot}_{\omega(1650)}}\frac{\Gamma_{\omega(1650)\to e^+e^-}}{\Gamma^{\rm tot}_{\omega(1650)}}\right|_{\exp} = 0.57\pm0.06\ \text{Aubert 06D [76]}, \tag{69}$$

we obtain

$$\Gamma_{\omega(1650)\to\omega\eta}\big|_{\mathrm{exp}} = \begin{cases} \sim 100\ \mathrm{MeV}, & \text{Achasov 03D [61] + Aubert 06D [76]} \\ 56 \pm 30\ \mathrm{MeV}, & \text{Henner 02 [60] + Aubert 06D [76]} \end{cases}, \tag{70}$$

which shall be compared with the theoretical result of 32 ± 13. Hence, it fits quite well with the second.

Finally, we can use Eqs. (66) and (69) to determine the ratio

$$\frac{\Gamma_{\omega(1650)\to\omega\eta}}{\Gamma_{\omega(1650)\to\rho\pi}}\bigg|_{\mathrm{exp}} = 0.365 \pm 0.054 \tag{71}$$

which is somewhat larger than the theoretical value ≈ 0.086.

(iv) Putative resonance $\phi(1930)$, strong decays. This resonance has not been found yet. The results of Tables IX and X are therefore predictions. For the reader's convenience we summarize them in Table XII. Hopefully, it will be possible to measure this state in the upcoming studies of GlueX and CLAS12 at Jefferson Lab. We comment further on this possibility in the conclusions.

(v) Radiative decays. The results are reported in Table XI. Experimentally, they were not yet seen. The magnitude of these decay widths is similar to the one of the lighter nonet of excited vector mesons; compare with Table VIII. In particular, the largest decay rate is $\omega(1650) \to \gamma\pi$, in agreement with the fact that $\omega(1650) \to \rho\pi$ is dominant. In general, these radiative decays seem quite interesting and important for the future studies of this nonet.

TABLE XII. Summary table for the putative state $\phi(1930)$.

Meson $\phi(1930)$	
Quark composition	$\approx s\bar{s}$
Old spectroscopy notation	(Predominantly) $n^{2S+1}L_J = 1^3D_1$
n	(Predominantly) 1
S	(Predominantly) $1\uparrow\uparrow$
L	(Predominantly) 2
J^{PC}	1^{--}
Mass	$\approx 1930 \pm 40$ MeV
Decays	
Decay channel	Decay width (MeV)
$\phi(1930) \to \bar{K}K$	104 ± 28
$\phi(1930) \to K\bar{K}^*$	260 ± 109
$\phi(1930) \to \Phi(1020)\eta$	67 ± 28
$\phi(1930) \to \Phi(1020)\eta'$	≈ 0
$\phi(1930) \to \gamma\eta$	0.19 ± 0.12
$\phi(1930) \to \gamma\eta'$	0.13 ± 0.08

Finally, the nonet $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$ is well compatible with a nonet of excited vector mesons, predominantly corresponding to orbitally excited vector states. However, the errors of the theoretical results are quite large and some experimental results are not yet fully in agreement with each other. Hence, even if the qualitative picture is quite satisfactory, there is room for quantitative improvements. Moreover, the experimental determination of radiative decays and the measurement of the yet-missing state $\phi(1930)$ represent a useful test to fully establish the nature of this nonet.

IV. DISCUSSIONS AND CONCLUSIONS

In this work we have studied the strong and radiative decays of the vector mesons $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$ by using a flavor-invariant QFT Lagrangian approach. This Lagrangian contains four coupling constants, corresponding to the dominant interaction terms in the large-N_c expansion, that have been determined by using four well-known experimental quantities. Then, we have compared our results to the averages and fits of the PDG as well as to selected experiments listed therein; see Tables VI–XI. Moreover, we have studied a large number of ratios for which an experimental counterpart was measured or could be deduced by combining present data. In summary, the assignment of these mesonic states to (predominantly) radially excited and to orbitally excited vector mesons works well. Typically, the dominant decays seen in experiment are also the leading ones in theory, while those decays which were not yet seen in experiment are generally quite small theoretically. In the future, it will be possible to further test our theoretical approach by measuring those decays which are not yet listed in PDG. In some cases, some decay ratios which were measured by more than one experiment are not in agreement with each other. Also along this direction, future determinations will be useful.

Besides strong decays, we have also calculated radiative decays of the type $R \to \gamma P$ by using vector meson dominance. For the lighter nonet $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ some radiative decays have been measured (in a couple of cases even the corresponding decay width can be determined from existing data); for the heavier nonet $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$, no experimental results exist at present. Hence, our results are

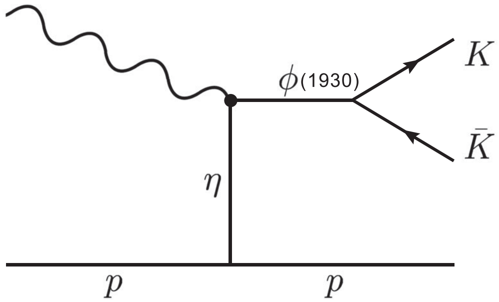

FIG. 1. Feynman diagram for the process of Eq. (72).

predictions. The study of radiative decays of excited vector mesons seems quite promising in future experimental activities.

One important outcome of our approach concerns the predictions for a novel ϕ state, belonging to the heavier nonet (predominantly orbitally excited vector mesons). By comparison with the mass differences between the two nonets, we have estimated that the mass is about 1930 MeV; hence we have called this state $\phi(1930)$. This mass is not far from the quark model prediction of 1890 MeV [2] and is also compatible with the lattice result of Ref. [35] in which the mass of this predominantly $\bar{s}s$ state is about 1950 MeV. In Table XII we summarize the results for the putative state $\phi(1930)$. Note that the KK decay is about 100 MeV [2], which turns out to be similar to the results of the quark model. On the contrary, the mode $\phi(1930) \to K^*(892)K$ is quite different: the quark model predicts 50 MeV, sizably smaller than the result reported in Table XII. (Even if the errors are large, one can conclude that in our approach the corresponding partial decay width is larger than 100 MeV. In general, we predict that $\Gamma_{\phi(1930)\to K^*K} > \Gamma_{\phi(1930)\to KK}$.)

The putative state $\phi(1930)$ is quite broad, thus making its discovery more difficult. Yet, a dedicated search by using partial wave analysis could reveal the existence of this state. In general, the very promising GlueX [77–79] and CLAS12 [80] experiments take place in the near future. The process in Fig. 1,

$$\gamma + p \to K^+ + K^- + p, \gamma + p \to K^0 + \bar{K}^0 + p, \quad (72)$$

is an example of a process that can be studied at GlueX and CLAS12. Quite remarkably, each mesonic vertex is contained in the present paper: $\phi(1930)\gamma\eta$ and $\phi(1930)KK$. An important outlook of the present work is a dedicated study of this reaction. For the baryonic part, one can use a well-defined hadronic model containing baryons and their interactions with mesons (in particular, the coupling of the nucleons to the η meson is necessary), as for instance the extended linear sigma model (eLSM) based on the mirror assignment presented in Refs. [81–83]. The analogous diagram in which $\phi(1930)$ couples to $K^*(892)K$,

$$\gamma + p \to K^{*+}(892) + K^- + p \to K^+ + K^- + \pi^0 + p, \quad (73)$$

also takes place (together with analogous isospin related reactions) and should be studied in the same context.

Other experiments are important as well. In principle, the resonance $\phi(1930)$ should also be contained in the data of *BABAR* [84], in which the reaction $e^+e^- \to K^+K^-$ was studied. Namely, in this reaction all the vector mesons $\rho^0(1450)$, $\omega(1420)$, $\phi(1680)$, $\rho^0(1700)$, $\omega(1650)$, $\phi(???) \equiv \phi(1930)$ enter and an interference of different amplitudes takes place (for a recent analysis, see Ref. [85] and references therein). Similarly, BESIII can also study a similar reaction, but typically this experiment focused on the range of energy above 2 GeV.

In the future, PANDA will be a leading experiment for spectroscopy [86]. While the energy in the center of mass will be too high to create excited vector mesons in a fusion process, it will be very possible to produce excited vector mesons together with light mesons (such as pions and kaons).

On the theoretical side, we enumerated the possible straightforward improvements of our approach: the systematic inclusion of large-N_c and flavor-symmetry violating terms. With new and more precise data, this study can be easily performed. A further outlook consists in the extension of the model by using chiral symmetry. Here, it was not needed because we did not link the excited vector mesons to chiral partners. For instance, the chiral partners of the orbitally excited vector mesons are the rather well-known pseudovector mesons $\{b_1(1235), K_{1,B}, h_1(1170), h_1(1380)\}$ (for the corresponding mathematical setup, see Ref. [87]). Chiral symmetry can help to relate the decays of this nonet to the decays of $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) \equiv \phi(1930)\}$. The same extension to radially excited vector mesons is more difficult because the corresponding chiral partners, a nonet of excited axial-vector states, has not yet been experimentally discovered.

In conclusion, the theoretical and experimental study of the two nonets of excited vector mesons is an interesting subject of low-energy spectroscopy. While the qualitative picture seems clear, further measurements, with special attention on radiative decays, are needed to fully establish the nature of these states. Moreover, the discovery of $\phi(1930)$ would represent a nice confirmation of the quark-antiquark picture also for excited states in the low-energy domain.

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APPENDIX A: EXTENDED FORM OF THE LAGRANGIAN

In this appendix we present the extended expressions of the Lagrangian introduced in Sec. II. B, Eq. (6).

The Lagrangian terms of our model $\mathcal{L}_{iPP}$ with $i = E$, D presented in Sec. II, Eq. (7), are given by

$$\begin{aligned}
\mathcal{L}_{iPP} &= ig_{iPP}\mathrm{Tr}[[\partial^\mu P, V_{i,\mu}]P] \\
&= \frac{ig_{iPP}}{4}\{K_\mu^{*0}((\partial^\mu\bar{K}^0)\pi^0 - \bar{K}^0(\partial^\mu\pi^0) - \sqrt{2}(\partial^\mu K^-)\pi^+ + \sqrt{2}K^-(\partial^\mu\pi^+) \\
&\quad + (\partial^\mu\eta_N)\bar{K}^0 - \eta_N(\partial^\mu\bar{K}^0) + \sqrt{2}\eta_S(\partial^\mu\bar{K}^0) - \sqrt{2}(\partial^\mu\eta_S)\bar{K}^0) \\
&\quad + \bar{K}_\mu^{*0}(K^0(\partial^\mu\pi^0) - (\partial^\mu K^0)\pi^0 - \sqrt{2}K^+(\partial^\mu\pi^-) + \sqrt{2}(\partial^\mu K^+)\pi^- \\
&\quad + \eta_N(\partial^\mu K^0) - (\partial^\mu\eta_N)K^0 - \sqrt{2}\eta_S(\partial^\mu K^0) + \sqrt{2}(\partial^\mu\eta_S)K^0) \\
&\quad + K_\mu^{*-}((\partial^\mu K^+)\pi^0 - K^+(\partial^\mu\pi^0) - \sqrt{2}K^0(\partial^\mu\pi^+) + \sqrt{2}(\partial^\mu K^0)\pi^+ \\
&\quad + \eta_N(\partial^\mu K^+) - (\partial^\mu\eta_N)K^+ - \sqrt{2}\eta_S(\partial^\mu K^+) + \sqrt{2}(\partial^\mu\eta_S)K^+) \\
&\quad + K_\mu^{*+}(K^-(\partial^\mu\pi^0) - (\partial^\mu K^-)\pi^0 - \sqrt{2}(\partial^\mu\bar{K}^0)\pi^- + \sqrt{2}\bar{K}^0(\partial^\mu\pi^-) \\
&\quad + (\partial^\mu\eta_N)K^- - \eta_N(\partial^\mu K^-) + \sqrt{2}\eta_S(\partial^\mu K^-) - \sqrt{2}(\partial^\mu\eta_S)K^-) \\
&\quad + \rho_\mu^0(\bar{K}^0(\partial^\mu K^0) - (\partial^\mu\bar{K}^0)K^0 + K^+(\partial^\mu K^-) - (\partial^\mu K^+)K^- + 2\pi^+(\partial^\mu\pi^-) - 2(\partial^\mu\pi^+)\pi^-) \\
&\quad + \rho_\mu^-(\sqrt{2}K^+(\partial^\mu\bar{K}^0) - \sqrt{2}(\partial^\mu K^+)\bar{K}^0 + 2\pi^0(\partial^\mu\pi^+) - 2(\partial^\mu\pi^0)\pi^+) \\
&\quad + \rho_\mu^+(\sqrt{2}K^0(\partial^\mu K^-) - \sqrt{2}(\partial^\mu K^0)K^- + 2(\partial^\mu\pi^0)\pi^- - 2\pi^0(\partial^\mu\pi^-)) \\
&\quad + \omega(K^0(\partial^\mu\bar{K}^0) - (\partial^\mu K^0)\bar{K}^0 + K^+(\partial^\mu K^-) - (\partial^\mu K^+)K^-) \\
&\quad + \sqrt{2}\phi((\partial^\mu K^0)\bar{K}^0 - K^0(\partial^\mu\bar{K}^0) - K^+(\partial^\mu K^-) + (\partial^\mu K^+)K^-)\}. \qquad \text{(A1)}
\end{aligned}$$

We recall that for $i = E$ the states correspond to $\{\rho, K^*, \phi, \omega\} = \{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and for $i = D$ to $\{\rho, K^*, \phi, \omega\} = \{\rho(1700), K^*(1680), \omega(1650), \phi(1930)\}$.

The Lagrangian terms of our model $\mathcal{L}_{iVP}$ with $i = E$, D presented in Sec. II, Eq. (8), are given by

$$\begin{aligned}
\mathcal{L}_{iVP} &= g_{iVP}\mathrm{Tr}(\tilde{V}_i^{\mu\nu}\{V_{\mu\nu}, P\}) = 2g_{iVP}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}((\partial_\alpha V_{i,\beta})\{(\partial_\mu V_\nu), P\}) \\
&= \frac{g_{iVP}}{2}\epsilon^{\mu\nu\alpha\beta}\{(\partial_\alpha\rho_{i,\beta}^0)(2\pi^0(\partial_\mu\omega_\nu) + 2\eta_N(\partial_\mu\rho_\nu^0) - \bar{K}^0(\partial_\mu K_\nu^{*0}) - K^0(\partial_\mu\bar{K}_\nu^{*0}) + K^+(\partial_\mu K_\nu^{*-}) + K^-(\partial_\mu K_\nu^{*+})) \\
&\quad + \sqrt{2}(\partial_\alpha\rho_{i,\beta}^-)(\sqrt{2}\pi^+(\partial_\mu\omega_\nu) + \sqrt{2}\eta_N(\partial_\mu\rho_\nu^+) + K^+(\partial_\mu\bar{K}_\nu^{*0}) + \bar{K}^0(\partial_\mu K_\nu^{*+})) \\
&\quad + \sqrt{2}(\partial_\alpha\rho_{i,\beta}^+)(\sqrt{2}\pi^-(\partial_\mu\omega_\nu) + \sqrt{2}\eta_N(\partial_\mu\rho_\nu^-) + K^-(\partial_\mu K_\nu^{*0}) + K^0(\partial_\mu K_\nu^{*-})) \\
&\quad + \sqrt{2}(\partial_\alpha\phi_{i,\beta})(2\eta_S(\partial_\mu\phi_\nu) + K^0(\partial_\mu\bar{K}_\nu^{*0}) + \bar{K}^0(\partial_\mu K_\nu^{*0}) + K^+(\partial_\mu K_\nu^{*-}) + K^-(\partial_\mu K_\nu^{*+})) \\
&\quad + (\partial_\alpha\omega_{i,\beta})(2\pi^0(\partial_\mu\rho_\nu^0) + 2\pi^+(\partial_\mu\rho_\nu^-) + 2\pi^-(\partial_\mu\rho_\nu^+) + 2\eta_N(\partial_\mu\omega_\nu) \\
&\quad + K^0(\partial_\mu\bar{K}_\nu^{*0}) + \bar{K}^0(\partial_\mu K_\nu^{*0}) + K^+(\partial_\mu K_\nu^{*-}) + K^-(\partial_\mu K_\nu^{*+})) \\
&\quad + (\partial_\alpha K_{i,\beta}^{*0})(\bar{K}^0(\partial_\mu\omega_\nu) - \pi^0(\partial_\mu\bar{K}_\nu^{*0}) + \sqrt{2}\pi^+(\partial_\mu K_\nu^{*-}) - \bar{K}^0(\partial_\mu\rho_\nu^0) + \sqrt{2}K^-(\partial_\mu\rho_\nu^+) \\
&\quad + \eta_N(\partial_\mu\bar{K}_\nu^{*0}) + \sqrt{2}\eta_S(\partial_\mu\bar{K}_\nu^{*0}) + \sqrt{2}\bar{K}^0(\partial_\mu\phi_\nu)) \\
&\quad + (\partial_\alpha\bar{K}_{i,\beta}^{*0})(K^0(\partial_\mu\omega_\nu) - \pi^0(\partial_\mu K_\nu^{*0}) + \sqrt{2}\pi^-(\partial_\mu K_\nu^{*+}) - K^0(\partial_\mu\rho_\nu^0) + \sqrt{2}K^+(\partial_\mu\rho_\nu^-) \\
&\quad + \eta_N(\partial_\mu K_\nu^{*0}) + \sqrt{2}\eta_S(\partial_\mu K_\nu^{*0}) + \sqrt{2}K^0(\partial_\mu\phi_\nu)) \\
&\quad + (\partial_\alpha K_{i,\beta}^{*-})(K^+(\partial_\mu\omega_\nu) + \pi^0(\partial_\mu K_\nu^{*+}) + \sqrt{2}\pi^+(\partial_\mu K_\nu^{*0}) + K^+(\partial_\mu\rho_\nu^0) + \sqrt{2}K^0(\partial_\mu\rho_\nu^+) \\
&\quad + \eta_N(\partial_\mu K_\nu^{*+}) + \sqrt{2}\eta_S(\partial_\mu K_\nu^{*+}) + \sqrt{2}K^+(\partial_\mu\phi_\nu)) \\
&\quad + (\partial_\alpha K_{i,\beta}^{*+})(K^-(\partial_\mu\omega_\nu) + \pi^0(\partial_\mu K_\nu^{*-}) + \sqrt{2}\pi^-(\partial_\mu\bar{K}_\nu^{*0}) + K^-(\partial_\mu\rho_\nu^0) + \sqrt{2}\bar{K}^0(\partial_\mu\rho_\nu^-) \\
&\quad + \eta_N(\partial_\mu K_\nu^{*-}) + \sqrt{2}\eta_S(\partial_\mu K_\nu^{*-}) + \sqrt{2}K^-(\partial_\mu\phi_\nu))\}. \qquad \text{(A2)}
\end{aligned}$$

We recall that for $i=E$ the states correspond to $\{\rho, K^*, \phi, \omega\} = \{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ and for $i=D$ to $\{\rho, K^*, \phi, \omega\} = \{\rho(1700), K^*(1680), \omega(1650), \phi(1930)\}$.

APPENDIX B: COUPLING TO THE PHOTON VIA VMD

Let us start from a single neutral ρ^0 meson. Its coupling to an electron-positron pair can be written down as

$$\mathcal{L}_{\rho e^+e^-} = g_{\rho e^+e^-}\rho^0_\mu \bar{\psi}_e \gamma^\mu \psi_e. \tag{B1}$$

Then, the decay into e^+e^- reads

$$\Gamma_{\rho\to e^+e^-} = \frac{\sqrt{\frac{m_\rho^2}{4} - m_e^2}}{6\pi m_\rho^2}(m_\rho^2 + 2m_e^2) g^2_{\rho e^+e^-}. \tag{B2}$$

The interaction (B1) can be obtained in the framework of vector meson dominance (according to the so-called VMD-1 of Ref. [37]) by considering the Lagrangian

$$\mathcal{L}_{VMD,\rho} = e_0 A_\mu \bar{\psi}_e \gamma^\mu \psi_e - \frac{e_0}{2g_\rho}\rho^0_{\mu\nu}F^{\mu\nu}, \tag{B3}$$

where the coupling g_ρ appears also in the decay amplitude of the process $\rho^0 \to \pi^+\pi^-$. The ρ^0 meson first transforms to a photon, which then generates a lepton pair. As a consequence, the following relation holds:

$$g_{\rho e^+e^-} = \frac{e_0}{g_\rho}. \tag{B4}$$

In fact $\rho^0_{\mu\nu}F^{\mu\nu} \to 2q^2\rho^0_\mu A^\mu$; hence one gets in the corresponding amplitude (upon using the Feynman rules)

$$\frac{e_0}{2g_\rho}2q^2\frac{1}{q^2} = \frac{e_0}{g_\rho}. \tag{B5}$$

Finally, VMD-1 implies that

$$\Gamma_{\rho\to e^+e^-} = \frac{\sqrt{\frac{m_\rho^2}{4} - m_e^2}}{6\pi m_\rho^2}(m_\rho^2 + 2m_e^2)\left(\frac{e}{g_\rho}\right)^2. \tag{B6}$$

The very same formula can be used for the decay into a muon pair:

$$\Gamma_{\rho\to \mu^+\mu^-} = \frac{\sqrt{\frac{m_\rho^2}{4} - m_\mu^2}}{6\pi m_\rho^2}(m_\rho^2 + 2m_\mu^2)\left(\frac{e}{g_\rho}\right)^2. \tag{B7}$$

Moreover, also the decay of the other neutral scalar states can be obtained as (straightforward changes due to different charges of quarks must be taken into account)

$$\Gamma_{\omega\to e^+e^-} = \frac{\sqrt{\frac{m_\omega^2}{4} - m_e^2}}{6\pi m_\omega^2}(m_\omega^2 + 2m_e^2)\left(\frac{e}{3g_\rho}\right)^2, \tag{B8}$$

$$\Gamma_{\phi\to e^+e^-} = \frac{\sqrt{\frac{m_\phi^2}{4} - m_e^2}}{6\pi m_\phi^2}(m_\phi^2 + 2m_e^2)\left(-\frac{\sqrt{2}}{3}\frac{e}{g_\rho}\right)^2. \tag{B9}$$

The extension of the latter two to the decays into a muon pair is straightforward.

The extension to the full nonet is then obtained by using the matrix for vector mesons introduced in Sec. II, which we rewrite here for convenience:

$$V_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega}{\sqrt{2}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K(892)^{*+} \\ \rho^- & \frac{\omega}{\sqrt{2}} - \frac{\rho^0}{\sqrt{2}} & K(892)^{*0} \\ K(892)^{*-} & \bar{K}(892)^{*0} & \phi \end{pmatrix}. \tag{B10}$$

The VMD approach reads

$$\mathcal{L}_{VMD,\mathrm{full}} = e_0 A_\mu \bar{\psi}_e \gamma^\mu \psi_e - \frac{e}{g_\rho}F_{\mu\nu}\mathrm{Tr}[V^{\mu\nu}Q] \tag{B11}$$

with

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}. \tag{B12}$$

Finally, the photon-meson mixing can be taken into account by performing the shift

$$V_{\mu\nu} \to V_{\mu\nu} + \frac{e_0}{g_\rho}F_{\mu\nu}Q. \tag{B13}$$

This is the shift that we have applied in order to determine the decay of the type $R \to \gamma P$ studied in this work. Intuitively, one has a decay chain of the type $R \to VP \to \gamma P$, where in the second step the transition $V \to \gamma$ has taken place according to VMD.

APPENDIX C: ERRORS OF THE COUPLING CONSTANTS AND THEIR PROPAGATION

Let us consider the χ^2 function $F \equiv F(x_k)$, where x_k are the parameters of the theory with $k = 1, \ldots, N$ [in our cases, F corresponds to Eq. (24) and to Eq. (29) respectively, and the parameters x_1 and x_2 are the coupling constants]. We look for the minimum of F by solving $\partial_q F(x_k) = 0 \to x_k = x_k^{\min}$. The Taylor expansion reads

$$F(x_k) = F(x_k^{\min}) + (x_k - x_k^{\min}) H_{kq} (x_q - x_q^{\min}) + \cdots,$$
$$H_{kq} = \frac{1}{2} \frac{\partial^2 F}{\partial x_k \partial x_q}\bigg|_{x_k = x_k^{\min}}. \tag{C1}$$

The matrix H, with elements H_{kq}, is the Hesse matrix of the function F evaluated at the minimum. We introduce the matrix B such that $BHB^t = D = \mathrm{diag}\{\lambda_1, \ldots, \lambda_N\}$ and the new variables $z_k = B_{kq}(x_q - x_q^{\min})$ (note that $B_{kq} = \frac{\partial z_k}{\partial x_q}$). As function of z_k, we get

$$F \equiv F(z_k) = F(x_k^{\min}) + z_k^2 \lambda_k + \cdots \tag{C2}$$

Therefore the error of z_k is given by $\delta z_k = 1/\sqrt{\lambda_k}$ (increments of 1 of the χ^2). Next, let us consider an arbitrary function of the parameters $G \equiv G(x_k)$, which represents some physical quantity of interest (in our examples, it can be a decay width or a ratio of decay widths). The physical value of G is clearly given by $G(x_k^{\min})$. Its error is evaluated with respect to the (mutually independent) parameters z_k:

$$\delta G = \sqrt{\left(\frac{\partial G}{\partial z_k}\bigg|_{z_k = 0} \delta z_k \right)^2}, \tag{C3}$$

where

$$\frac{\partial G}{\partial z_k}\bigg|_{z_k=0} = \frac{\partial G}{\partial x_q}\bigg|_{x_k = x_k^{\min}} B^t_{qk} = \frac{\partial G}{\partial x_q}\bigg|_{x_k = x_k^{\min}} B_{kq}. \tag{C4}$$

The errors of the parameters x_r are calculated by setting $G = x_r$, out of which (upon using $\frac{\partial x_r}{\partial z_k} = B^t_{rk} = B_{kr}$)

$$\delta x_r = \sqrt{(B_{kr} \delta z_k)^2} = \sqrt{H^{-1}_{rr}}. \tag{C5}$$

(For the last equality we used $H^{-1} = B^t D^{-1} B \to H^{-1}_{rr} = B^t_{rq} D^{-1}_{qk} B_{kr} = B^t_{rk} \delta z_k^2 B_{kr} = B^2_{kr} \delta z_k^2$.) In this way we evaluated the parameter errors in Sec. III. A. It is also interesting to mention that the naive evaluation of the error of G as

$$\delta G^{\mathrm{naive}} = \sqrt{\left(\frac{\partial G}{\partial x_k}\bigg|_{x_k = x_k^{\min}} \delta x_k \right)^2} \tag{C6}$$

is in general not correct (typically, it is an overestimation of the error δG).

Finally, we turn to our concrete examples. For the nonet of radially excited vector states, we identify $x_1 = g_{EPP}$ and $x_2 = g_{EVP}$, and F is given by Eq. (24). Here, the Hesse matrix is from the very beginning diagonal. Then, in this particular case $\delta G^{\mathrm{naive}} = \delta G$; hence the error of a certain quantity $G(g_{EPP}, g_{EVP})$ reads

$$\delta G = \sqrt{\left(\frac{\partial G}{\partial g_{EPP}}\bigg|_{\min} \delta g_{EPP} \right)^2 + \left(\frac{\partial G}{\partial g_{EVP}}\bigg|_{\min} \delta g_{EVP} \right)^2} \tag{C7}$$

where "min" refers to the values of Eq. (25). In this way all the errors of the quantities of Sec. III. B were evaluated.

Concerning the nonet of orbitally excited vector states, we set $x_1 = g_{DPP}$ and $x_2 = g_{DVP}$ and use F from Eq. (29). While for the decays in Tables IX, X, and XI (which involve only one of the two coupling constants) the naive procedure would still be valid, this is not true in general and the diagonalization is necessary. For instance, for the ratios of coupling constants (entering into the various decay ratios studied in Sec. III. C), one sets $G = g^2_{DPP}/g^2_{DVP}$. The central value reads 5.4 and the corresponding error is $\delta G = 1.8$, whereas $\delta G^{\mathrm{naive}} = 2.7$ would be an overestimation.

In this work, we have limited our evaluation of the errors to the couplings discussed above because they represent the largest source of indeterminacy. Yet, as mentioned in the text, other error sources (not included here) exist, such as masses and flavor-breaking and large-N_c suppressed terms.

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