Understanding $X(3862)$, $X(3872)$, and $X(3930)$ in a Friedrichs-model-like scheme

Zhi-Yong Zhou*

School of Physics, Southeast University, Nanjing 211189, People's Republic of China

Zhiguang Xiao†

Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China

(Received 21 April 2017; published 26 September 2017)

We developed a Friedrichs-model-like scheme in studying the hadron resonance phenomenology and present that the hadron resonances might be regarded as the Gamow states produced by a Hamiltonian in which the bare discrete state is described by the result of the usual quark potential model and the interaction part is described by the quark pair creation model. In an almost parameter-free calculation, the $X(3862)$, $X(3872)$, and $X(3930)$ state could be simultaneously produced with a quite good accuracy by coupling the three P-wave states, $\chi_{c2}(2P)$, $\chi_{c1}(2P)$, $\chi_{c0}(2P)$ predicted in the Godfrey-Isgur model to the $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$ continuum states. At the same time, we predict that the $h_c(2P)$ state is at about 3890 MeV with a pole width of about 44 MeV. In this calculation, the $X(3872)$ state has a large compositeness. This scheme may shed more light on the long-standing problem about the general discrepancy between the prediction of the quark model and the observed values, and it may also provide reference for future search for the hadron resonance state.

DOI: 10.1103/PhysRevD.96.054031

As regards the charmonium spectrum above the open-flavor thresholds, general discrepancies between the predicted masses in the quark potential model and the observed values have been highlighted for several years. Typically, among the P-wave $n^{2s+1}L_J = 2^3P_2$, 2^3P_1, 2^3P_0, and 2^1P_1 states, the $X(3930)$, discovered by the Belle collaboration [1], is now assigned to the $\chi_{c2}(2P)$ charmonium state though its mass is about 50 MeV lower than the prediction in the quark potential model [2–4]. The properties of the other P-wave states have not been firmly determined yet. The $X(3872)$ was first observed in the $B^\pm \to K^\pm J/\psi\pi^+\pi^-$ by the Belle collaboration in 2003 [5]. Although its quantum number is 1^{++}, the same as the $\chi_{c1}(2P)$, the pure charmonium interpretation was soon given up for the difficulties in explaining its decays. The pure molecular state explanation of $X(3872)$ also encounters difficulties in understanding its radiative decays. So its nature remains to be obscure up to now. As for the $\chi_{c0}(2P)$ state, the $X(3915)$ was assigned to it several years ago, but this assignment is questioned for the mass splitting between $\chi_{c2}(2P)$ and $\chi_{c0}(2P)$, and its dominant decay mode [6,7]. In Ref. [8], analyses of the angular distribution of $X(3915)$ to the final leptonic and pionic states also support the possibility of being a 2^{++} state, which means that it might be the same tensor state as the $X(3930)$. Very recently, the Belle collaboration announced a new result about the signal of $X(3862)$ which could be a candidate for the $\chi_{c0}(2P)$ [9]. The 2^1P_1 state has not been discovered yet. These puzzles have been discussed exhaustively in the literature (see Refs. [10–12] for example), but a consistent description is still missing.

In this paper, we adopt the idea of Gamow states and the solvable extended Friedrichs model developed recently [13–15], usually discussed in the pure mathematical physics literature, to study the resonance phenomena in the hadron physics, in particular the charmonium spectrum. Using the eigenvalues and wave functions for mesons in the Godfrey-Isgur (GI) model [3] as input and modeling the interaction by the quark pair production (QPC) model, we found that the first excited 2^{++}, 1^{++}, and 0^{++} charmonium states could be reproduced with good accuracy in an almost parameter-free calculation, and the mass and width of the 1^{+-} state are also obtained as a prediction. These results are helpful in resolving the long-standing puzzle of identifying the observed P-wave state, and also shed more light on the interpretation of the enigmatic $X(3872)$ state. Furthermore, this method can also provide the explicit wave functions of resonances, "compositeness" and "elementariness" parameters for bound states, and scattering S-matrix involving these resonances [13–15], which are rigorously obtained in the Friedrichs model and have important applications in further studies of the resonance properties. This scheme provides a general framework to incorporate the hadron interaction corrections to the spectra predicted by the quark model, and can be used in evaluating the other mass spectra above the open-flavor threshold to reconcile the gaps

*zhouzhy@seu.edu.cn
†Corresponding author.
xiaozg@ustc.edu.cn

between the quark potential model predictions and the experimental results.

To introduce our theoretical framework, we begin by recalling some basic facts about the Friedrichs model. A resonance exhibiting a peak structure in the invariant mass spectrum of the final states could be understood as a Gamow state in the famous Friedrichs model in mathematical physics [16]. In the simplest version of the Friedrichs model, the full Hamiltonian H is separated into the free part and the interaction part as

$$H = H_0 + V, \tag{1}$$

and the free Hamiltonian

$$H_0 = \omega_0|0\rangle\langle 0| + \int_{\omega_{\mathrm{th}}}^{\infty} \omega|\omega\rangle\langle\omega| d\omega \tag{2}$$

has a discrete eigenstate $|0\rangle$ with eigenvalue $\omega_0 > \omega_{\mathrm{th}}$, and continuum eigenstates $|\omega\rangle$ with eigenvalues $\omega \in [\omega_{\mathrm{th}}, \infty)$, ω_{th} being the threshold for the continuum states, and they are normalized as

$$\langle 0|0\rangle = 1, \quad \langle\omega|\omega'\rangle = \delta(\omega - \omega'), \quad \langle 0|\omega\rangle = \langle\omega|0\rangle = 0. \tag{3}$$

The interaction part serves to couple the discrete state and the continuous state as

$$V = \lambda \int_{\omega_{\mathrm{th}}}^{\infty} [f(\omega)|\omega\rangle\langle 0| + f^*(\omega)|0\rangle\langle\omega|]\mathrm{d}\omega, \tag{4}$$

where the $f(\omega)$ function denotes the coupling form factor between the discrete state and the continuum state and λ denotes the coupling strength. This eigenvalue problem for the Hamiltonian can be exactly solved. In the rigged-Hilbert-space formulation of the quantum mechanics developed by Bohm and Gadella, the discrete state becomes a generalized eigenstate with a complex eigenvalue, which corresponds to the resonance state called Gamow state [17,18]. The relation of the Gamow state and the pole in the scattering amplitude in the S-matrix theory is also straightforward [18]. By summing the perturbation series, Prigogine and his collaborators also obtained a similar mathematical structure [19]. Properties of Gamow states could be represented by the zero point of the $\eta(x)$ function on the unphysical sheet of the complex energy plane, where

$$\eta^{\pm}(x) = x - \omega_0 - \lambda^2 \int_{\omega_{\mathrm{th}}}^{\infty} \frac{|f(\omega)|^2}{x - \omega \pm i\epsilon} \mathrm{d}\omega. \tag{5}$$

In general, when λ increases from 0, the zero point moves away from the real axis to the second Riemann sheet. The wave function of the Gamow state is expressed as

$$|z_R\rangle = N_R\left(|0\rangle + \lambda \int_{\omega_{\mathrm{th}}}^{\infty} \mathrm{d}\omega \frac{f(\omega)}{[z_R - \omega]_+}|\omega\rangle\right), \tag{6}$$

and the conjugate for a pair of resonance poles on the second Riemann sheet, where the $[\cdots]_{\pm}$ means the analytical continuations of the integration [13]. There could also be bound-state and virtual-state solutions both for $\omega_0 > \omega_{\mathrm{th}}$ and $\omega_0 < \omega_{\mathrm{th}}$, with the wave function being

$$|z_{B,V}\rangle = N_{B,V}\left(|0\rangle + \lambda \int_{\omega_{\mathrm{th}}}^{\infty} \frac{f(\omega)}{z_{B,V} - \omega}|\omega\rangle \mathrm{d}\omega\right), \tag{7}$$

for a bound (virtual) state $|z_B\rangle$ ($|z_V\rangle$) at z_B (z_V) on the first (second) Riemann sheet below the threshold, where $N_{R,B,V}^{(*)}$ are the normalizations. For virtual states, the integral should be continued to z_V on the second sheet. A generalization of the Friedrichs model to include multiple discrete states and multiple continuum states is also worked out and readers are referred to Refs. [13–15] for more detailed discussions.

Inspired from QCD one-gluon exchange interaction and the confinement, the Godfrey-Isgur model [3], with partially relativized linear confinement, Coulomb-type, and color-hyperfine interactions, provides very successful predictions to the mass spectra of the conventional meson states composed of u, d, s, c, and b quarks, but its predictions with regard to the states above the open-flavor thresholds are not as good as those below. These discrepancies might arise from the neglecting of the coupling between these "bare" meson states and their decay channels (both open and closed) as they mentioned [3]. In our scheme, the GI's Hamiltonian which provides the discrete bare hadron eigenstates can be effectively viewed as the free Hamiltonian in the Friedrichs model, and the interactions between the bare states of H_0 and the continuum states are modeled by the QPC model [20] and will generate the corrections to the spectrum above the open-flavor thresholds. The stronger the coupling is, the larger the influence is. The wave functions in the QPC model are chosen to be the same as the eigenstate solutions in the GI model which is approximated by a combination of a set of harmonic oscillator basis. Since the Okubo-Zweig-Iizuka (OZI)-allowed channel will be more strongly coupled to the bare states than the OZI-suppressed channel, the pole shift is dominantly caused by these channels. So, we include only the OZI allowed channels in our analysis.

In the spirit of the Friedrichs model, suppose a discrete state $|0; JM\rangle$ with spin J, coupled to a continuum composed of two hadrons $|p; JM, LS\rangle$ with a total angular momentum quantum numbers J, M, orbital angular momentum quantum number L, total spin S, the center of mass (c.m.) momentum p for the two particles, and the reduced mass μ. In the nonrelativistic theory, the free Hamiltonian in the c.m. frame can be expressed as

$$H_0 = M_0 \sum_M |0; JM\rangle\langle 0; JM| + \sum_{L,S} \int p^2 \mathrm{d}p\omega |p; JM; LS\rangle\langle p; JM; LS|. \quad (8)$$

The interaction between the discrete states and the continuum states is rotationally invariant and we can confine ourselves to a fixed JM channel and omit the JM indices. The matrix elements of the interaction potentials can be expressed as [15]

$$H_{01} = \sum_{S,L} \int d\omega f_{SL}(\omega)|0\rangle\langle\omega, LS| + \mathrm{H.c.} \quad (9)$$

by absorbing a phase space factor $\sqrt{\mu p}$ in both $f_{SL}(\omega)$ and $|\omega, LS\rangle$.

The definition of the meson state is different from the one in Ref. [21] by omitting the factor $\sqrt{2E}$ to ensure the correct normalizations. Then, the meson coupling $A \to BC$ can be defined as the transition matrix element

$$\langle BC|T|A\rangle = \delta^3(\vec{P_f} - \vec{P_i}) M^{ABC}, \quad (10)$$

where the transition operator T is the one in the QPC model,

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d^3\vec{p_3} d^3\vec{p_4} \delta^3(\vec{p_3} + \vec{p_4}) \times \mathcal{Y}_1^m\left(\frac{\vec{p_3} - \vec{p_4}}{2}\right) \chi^{34}_{1-m} \phi_0^{34} \omega_0^{34} b_3^\dagger(\vec{p_3}) d_4^\dagger(\vec{p_4}). \quad (11)$$

By the standard derivation one can obtain the amplitude M^{ABC} and the partial wave amplitude $M^{SL}(P(\omega))$ as in Ref. [20]. Then the form factor f_{SL} which describes the interaction between $|A\rangle$ and $|BC\rangle$ in the Friedrichs model can be obtained as

$$f_{SL}(\omega) = \sqrt{\mu P(\omega)} M^{SL}(P(\omega)), \quad (12)$$

where $P(\omega) = \sqrt{\frac{2M_B M_C(\omega - M_B - M_C)}{M_B + M_C}}$ is the c.m. momentum, M_B and M_C being the masses of meson B and C respectively. Now, after including more continuum states, the $\eta(z)$ function can be expressed as

$$\eta^\pm(z) = z - \omega_0 - \sum_n \int_{\omega_{\mathrm{th},n}}^\infty \frac{\sum_{S,L} |f^n_{SL}(\omega)|^2}{z - \omega \pm i\epsilon} \mathrm{d}\omega, \quad (13)$$

where $\omega_{\mathrm{th},n}$ denotes the energy of the nth threshold. Notice that this function can be continued to an analytic function $\eta(z)$ defined on a 2^n-sheet Riemann surface in the case of n thresholds. The poles of a scattering amplitude are just the zeros of the $\eta(z)$ function [15], and its real part and imaginary part represent the mass and half-width of the Gamow state. Only the Gamow states close to the physical region could significantly influence the observables such as cross section or invariant mass spectrum of the final states.

With the parameters in the GI model [3], we first reproduced the results of GI by approximating the wave function of the P-wave charmonium states and the charmed mesons with 30 harmonic oscillator wave function basis. Using these wave functions of the meson states in the QPC model, one could then obtain the coupling form factor in the Friedrichs model. The only parameter of the QPC model is γ, which represents the quark pair production strength from the vacuum. We choose it to be the typical value 6.9 [4,22]. So there is no free parameter in our calculation.

The coupled channels are chosen up to $D^*\bar{D}^*$ in four cases. The $\chi_{c2}(2P)$ state can couple to $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ in both S- and D-wave. For the $\chi_{c1}(2P)$ and $h_c(2P)$ states, the coupled channels are $D\bar{D}^*$, and $D^*\bar{D}^*$. In the case of the $\chi_{c0}(2P)$ state, the coupled channels are $D\bar{D}$, and $D^*\bar{D}^*$.

The poles of scattering amplitude [zero of the $\eta(z)$ function] could be extracted by analytically continuing $\eta(z)$ to the closest Riemann sheet. To make this scheme more friendly to the experimentists, one may approximate $\eta(z)$ by a Breit-Wigner parametrization as $\eta(z) \approx z - M_{\mathrm{BW}} + i\Gamma_{\mathrm{BW}}/2$, and the mass parameter is determined by solving

$$M_{\mathrm{BW}} - \omega_0 - \sum_n \mathcal{P} \int_{\omega_{\mathrm{th},n}}^\infty \frac{\sum_{SL} |f^n_{SL}(\omega)|^2}{M_{\mathrm{BW}} - \omega} \mathrm{d}\omega = 0, \quad (14)$$

on the real axis where $\mathcal{P}\int$ means principal value integration and the Breit-Wigner partial width of the nth open channel is expressed as

$$\Gamma^n_{\mathrm{BW}} = 2\pi \sum_{S,L} |f^n_{SL}(M_{\mathrm{BW}})|^2, \quad (15)$$

and the total width $\Gamma_{\mathrm{tot}} = \sum_n \Gamma^n_{\mathrm{BW}}$. It is worth mentioning that this approximation, Eqs. (14) and (15) together, is only valid when it is used to represent a narrow resonance far away from the thresholds.

The numerical results of the extracted pole position and related Breit-Wigner parameters are shown in Table I. If there is only one open channel, usually one Gamow state which originates from the bare state is expected, but sometimes there could also be an extra virtual state or bound state generated by the form factor $f(\omega)f(\omega)^*$ when the coupling is strong, which exhibits the molecular nature of this state. Here, the $X(3872)$ is just of this nature. In Ref. [13], we discussed the general condition for this kind of virtual or bound state poles.

For the 2^3P_2 channel, $D\bar{D}$ and $D\bar{D}^*$ thresholds are open for the $\chi_{c2}(2P)$. This pole is shifted from the GI's value down to about 17 MeV below the observed value. Its width is about 12 MeV, a little smaller than the observed one. The branching ratio between $D\bar{D}$ and $D\bar{D}^*$ is 7.7, which demonstrates that $D\bar{D}$ is its dominant decay channel.

TABLE I. Comparison of the experimental masses and the total widths (in MeV) [23] with our results.

$n^{2s+1}L_J$	M_{expt}	Γ_{expt}	M_{BW}	Γ_{BW}	Pole	GI
2^3P_2	3927.2 ± 2.6	24 ± 6	3910	12	3908-5i	3979
2^3P_1	3942 ± 9	37^{+27}_{-17}			3917-45i	3953
	3871.69 ± 0.17	< 1.2	3871	0	3871-0i	
2^3P_0	3862^{+66}_{-45}	201^{+242}_{-149}	3860	25	3861-11i	3917
2^1P_1			3890	26	3890-22i	3956

Its decay probability to $D\bar{D}^*$ is relatively small, but there is still some possibility that there is the contribution of $X(3930)$ in the observed $D\bar{D}^*$ mass distribution in experiments.

In the 2^3P_1 channel, one pole is shifted down from the bare state to about 3917 MeV with fairly large width, while another bound state pole emerges just below the threshold around 3872 MeV without any tuning of the parameter, which is consistent with the $X(3872)$ found in the experiment. If the coupling strength γ is tuned smaller, this bound-state pole will move across the $D\bar{D}^*$ threshold to the second sheet and becomes a virtual state pole. This pole is dynamically generated from the form factor which is an evidence of the molecular origin of the state. It is natural to assign this bound state pole to the $X(3872)$, and the higher state generated from GI's bare state might be related to the $X(3940)$ state.

In the 2^3P_0 channel, the $\chi_{c0}(2P)$ state is found to be a narrow resonance right at about 3860 MeV. We noticed that the mass newly observed $\chi_{c0}(2P)$ candidate is just at 3862 MeV with a width 201^{+242}_{-149} MeV [9], having a large uncertainty. Although the $D\bar{D}$ channel is OZI allowed for the $\chi_{c0}(2P)$ state, in this calculation we find that the coupling between the $D\bar{D}$ channel and the bare $\chi_{c0}(2P)$ is unexpectedly weak which causes the narrow width. This narrow width is roughly only twice the bin size of the data in [1,24] and smaller than the one in the more recent [9]. So, in the future experiments, we propose a further exploration with a higher resolution in this energy region to see whether there is a narrow signal missing in the present data. An interesting observation is that there seems to be a simultaneous small excess at the vicinity of about 3860 MeV in the $\gamma\gamma \to D\bar{D}$ experiment of both Belle [1] and *BABAR* collaborations [24]. Notice the data points in the region of 3850 MeV $< m(D\bar{D}) <$ 3875 MeV in Fig. 1.

The $h_c(2P)$ state is predicted at around 3890 MeV in this scheme. As we have mentioned, $\chi_{c2}(2P)$, $\chi_{c1}(2P)$, $h_c(2P)$ all couple to the $D\bar{D}^*$ channel, which has no definite C-parity. This means that the enhancement above the $D\bar{D}^*$ threshold contains all the contributions from these states. To detect the $h_c(2P)$ signal, one needs to look for it in a negative C-parity channel such as $\eta_c\gamma$ in this energy region.

Further remarks about the $X(3872)$ are in order. In our calculation, although without tuning of the standard parameter it is found to be a bound state, we cannot

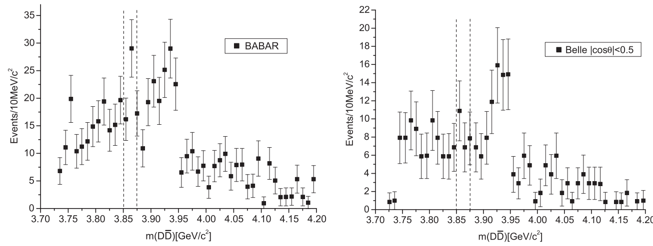

FIG. 1. The mass distribution of $\gamma\gamma \to D\bar{D}$ from *BABAR* [24] and Belle [1]. The data of Belle is the one for $|\cos\theta^*| < 0.5$. The two dashed lines are set at $m(D\bar{D}) = 3850$ MeV and 3875 MeV.

exclude the possibility of a virtual state nature [25,26], since only a small shift down of the γ parameter will move it to the second sheet. In [27], improving the approach adopted in [28], a dispersion relation method combined with the QPC model is also used in discussing the charmoniumlike states, where $X(3872)$ can also be produced. However, since the wave function for the $X(3872)$ cannot be obtained there and the wave function used in the QPC model there is inaccurate, further discussion on the nature of the $X(3872)$ may not be accurate. In the present scheme, the exactly solvable Friedrichs model provides a more solid theoretical setup and since the more accurate hadron wave function of the GI model is used in the QPC model, the result here would more accurately describe the nature of the $X(3872)$. Moreover, since the wave function for the $X(3872)$ can be rigorously solved in terms of the discrete state and the continuum states, one can also find out its compositeness and elementariness. The compositeness of a bound state, defined as the probability of finding the nth continuous states in the bound state, is expressed in the Friedrichs model as

$$X_n = \frac{1}{N^2}\int_{\omega_{\mathrm{th},n}}^{\infty} \mathrm{d}\omega \sum_{S,L} \frac{|f^n_{SL}(\omega)|^2}{(m_B-\omega)^2}, \tag{16}$$

where the normalization factor is

$$N = (1 + \sum_n \int_{\omega_{\mathrm{th},n}}^{\infty} \mathrm{d}\omega \sum_{S,L} \frac{|f^n_{SL}(\omega)|^2}{(m_B-\omega)^2})^{1/2}. \tag{17}$$

If the $X(3872)$ is a bound state, the relative ratio of finding $c\bar{c}$ and $D\bar{D}^*$ in the state is about 1∶2.7, showing the dominance of the continuum part in this state, which also demonstrates its molecular dominant nature [29–32]. In comparison, in [32], by analyzing the production rate of CMS [33] and CDF [34] data within the framework of NRQCD factorization, the $c\bar{c}$ component is estimated to be 22%–44%, which is consistent with our value. However, our result is different from [35], in which the $c\bar{c}$ component is about 6%. Nevertheless, both results favor a large $D\bar{D}^*$ component. Another QCD sum-rule analysis [36] predicts a larger $c\bar{c}$ component, about 97%, but the mass of the state is too low, at around 3.77 GeV, compared to the observed one of $X(3872)$.

It is worth emphasizing that for a resonance, the compositeness and elementariness parameters will become complex numbers [15], so they have no rigourous definitions, but some definitions proposed in the literature [37,38] might be able to approximately describe these quantities.

In this paper, using the exactly solvable Friedrichs model, we propose a general framework to include the hadron interaction corrections to the quark model spectrum predictions, in particular to the generally accepted GI's standard results. The explicit wave function for the resonances can be obtained and the compositeness and elementariness of the bound states can be calculated which are important for further study of the properties of the state. Using this scheme, we could reproduce the first excited P-wave charmoniumlike states. In particular, we find that the $X(3872)$ could be dynamically generated in a natural way by the coupling of the bare $\chi_{c1}(2P)$ state and continuum states, but its molecular components are larger. The $\chi_{c0}(2P)$ is found unexpectedly to be a narrow one. We also predict the appearance of the $h_c(2P)$ state to be at about 3890 MeV with a pole width of about 44 MeV. This scheme is promising in matching the predictions of the GI model with the observed states. The acceptable consistency of our results and experiments means that the hadron interactions really give large corrections to the GI's results for open flavor channels which can reconcile the discrepancy between the quark model prediction and the experiments.

ACKNOWLEDGMENTS

Helpful discussions with Dian-Yong Chen, Hai-Qing Zhou, Ce Meng, and Xiao-Hai Liu are appreciated. Z. X. is supported by China National Natural Science Foundation under Contracts No. 11105138, No. 11575177 and No. 11235010. Z. Z. is supported by the Natural Science Foundation of Jiangsu Province of China under Contract No. BK20171349.

[1] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 082003 (2006).
[2] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D **17**, 3090 (1978); **21**, 313(E) (1980).
[3] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[4] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[5] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[6] F.-K. Guo and U.-G. Meissner, Phys. Rev. D **86**, 091501 (2012).
[7] S. L. Olsen, Phys. Rev. D **91**, 057501 (2015).
[8] Z.-Y. Zhou, Z. Xiao, and H.-Q. Zhou, Phys. Rev. Lett. **115**, 022001 (2015).
[9] K. Chilikin *et al.* (Belle Collaboration), Phys. Rev. D **95**, 112003 (2017).
[10] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rep. **639**, 1 (2016).
[11] A. Esposito, A. Pilloni, and A. D. Polosa, Phys. Rep. **668**, 1 (2017).
[12] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Prog. Part. Nucl. Phys. **93**, 143 (2017).
[13] Z. Xiao and Z.-Y. Zhou, Phys. Rev. D **94**, 076006 (2016).
[14] Z. Xiao and Z.-Y. Zhou, J. Math. Phys. (N.Y.) **58**, 062110 (2017).
[15] Z. Xiao and Z.-Y. Zhou, J. Math. Phys. (N.Y.) **58**, 072102 (2017).
[16] K. O. Friedrichs, Commun. Pure Appl. Math. **1**, 361 (1948).
[17] A. Bohm and M. Gadella, *Dirac Kets, Gamow Vectors and Gel'fand Triplets*, edited by A. Bohm and J. D. Dollard, Lecture Notes in Physics Vol. 348 (Springer, Berlin, 1989).
[18] O. Civitarese and M. Gadella, Phys. Rep. **396**, 41 (2004).
[19] T. Petrosky, I. Prigogine, and S. Tasaki, Physica A (Amsterdam) **173A**, 175 (1991).
[20] H. G. Blundell and S. Godfrey, Phys. Rev. D **53**, 3700 (1996).
[21] C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).
[22] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[23] C. Patrignani *et al.*, Chin. Phys. C **40**, 100001 (2016).
[24] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **81**, 092003 (2010).
[25] C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, Phys. Rev. D **76**, 034007 (2007).
[26] X.-W. Kang and J. A. Oller, Eur. Phys. J. C **77**, 399 (2017).
[27] Z.-Y. Zhou and Z. Xiao, Eur. Phys. J. A **50**, 165 (2014).
[28] M. R. Pennington and D. J. Wilson, Phys. Rev. D **76**, 077502 (2007).
[29] N. A. Tornqvist, Z. Phys. C **61**, 525 (1994).
[30] D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, Phys. Rev. D **81**, 014029 (2010).
[31] F.-K. Guo, C. Hanhart, U.-G. Meiner, Q. Wang, and Q. Zhao, Phys. Lett. B **725**, 127 (2013).
[32] C. Meng, H. Han, and K.-T. Chao, arXiv:1304.6710.
[33] S. Chatrchyan *et al.* (CMS Collaboration), J. High Energy Phys. 04 (2013) 154.
[34] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
[35] M. Takizawa and S. Takeuchi, Prog. Theor. Exp. Phys. **2013**, 093D01 (2013).
[36] R. D. Matheus, F. S. Navarra, M. Nielsen, and C. M. Zanetti, Phys. Rev. D **80**, 056002 (2009).
[37] T. Sekihara, T. Hyodo, and D. Jido, Prog. Theor. Exp. Phys. **2015**, 063D04 (2015).
[38] Z.-H. Guo and J. A. Oller, Phys. Rev. D **93**, 096001 (2016).