Pauli form factor of quark and nontrivial topological structure of the QCD

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We calculate the electromagnetic Pauli form factor of quark induced by the nontrivial topological fluctuations of QCD vacuum called instantons. It is shown that such a contribution is significant. We discuss the possible implications of our result in the photon-hadron reactions and in the dynamics of quark-photon interactions in the dense/hot quark matter.

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I. INTRODUCTION

Nowadays the study of electromagnetic structure of the elementary particles is one of the hottest topics in the Standard Model (SM). One well-known puzzle is the experimental value of the muon anomalous magnetic moment, which shows the significant deviation from the SM prediction (see recent reviews [\[1,2\]\)](#page-4-0). Electromagnetic probes of the hadrons give very important information about the structure of the strong interaction [\[3\]](#page-4-1). Various models have been developed which enable us to study electromagnetic properties of hadrons in terms of form factors [4–[13\].](#page-4-2) In the past decades, the significant progress has been made, especially regarding the study of the relation between generalized parton distribution functions (GPD) and electromagnetic form factors of hadrons [\[14,15\]](#page-4-3). The quark form factors carry the information about internal structure of the constituent quark and provide the bridge between partonic picture of the hadrons and their constituent structure [\[6,8,16](#page-4-4)–21].

In this paper, we consider a new nonperturbative contribution to electromagnetic Pauli form factor (EPFF) of a quark arisen from an instanton induced quark-gluon vertex. The instanton is the well-known solution of the QCD equation of motion in the Euclidean space-time, which has a nonzero topological charge. It was shown that instantons play a very important role in hadron physics (see the reviews [22–[24\]](#page-5-0)). In particular, the instantons lead to the spontaneous chiral symmetry breaking (SCSB) in the strong interaction, which is not only one of the main sources for the observed hadron masses but also leads to the various anomalies observed in the spin-dependent cross sections [\[6,16,25](#page-4-4)–29]. One of the cornerstones of the instanton-based theory of the spin effects in the strong interaction is the instanton-induced anomalous chromomagnetic quark-gluon interaction introduced in [\[16\].](#page-4-5) The strength of this interaction is determined by the dynamical mass of the quark in the instanton vacuum [\[23,30\]](#page-5-1), which is directly related to the phenomenon of the SCSB. The first attempt to estimate the effect of instantons to EPFF was made in [\[17\]](#page-4-6), where the so-called instanton's perturbative theory was used. This approach was developed in the papers [\[31](#page-5-2)–33] to obtain the effect of the small size of the instantons to the deepinelastic scattering (DIS) at a large transfer momentum $Q^2 = -q^2$. However, the final result for their contribution to DIS at the large Q^2 was found to be very small. The same conclusion is also valid for the contribution of the small instantons to the large Q^2 asymptotic of the EPFF of quark obtained in [\[17\].](#page-4-6) Here, we will use another way to calculate the instanton contribution to EPFF. This approach is based on the effective quark-gluon vertex induced by instantons and allows us to obtain the prediction for EPFF in the wide interval of the Q^2 including even the very important case of the real photon, $Q^2 = 0$.

II. THE CONTRIBUTION OF ANOMALOUS QUARK-GLUON INTERACTION TO THE QUARK ELECTROMAGNETIC FORM FACTOR

The general vertex for a photon-quark interaction for on shell quark is

$$
\Gamma^{\mu} = \gamma^{\mu} F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2M_q} F_2(Q^2),\tag{1}
$$

where F_1 , F_2 are electromagnetic Dirac and Pauli form factors, respectively, M_q is the dynamical mass of the quark, and $\sigma_{\mu\nu} = i(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})/2$. The anomalous quarkgluon chromomagnetic (AQGC) vertex induced by the instantons can be written in the form [\[16,23,30\]](#page-4-5)

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$$
V_{\mu}^{a}(k_1^2, k_2^2, t^2) = \frac{ig_s \sigma^{\mu\nu} q_{\nu}}{2M_q} F_2(k_1^2, k_2^2, t^2) t^a,
$$
 (2)

where k_1^2 and k_2^2 are the virtuality of the initial and final quarks, respectively, $t = k_1 - k_2$, and the general case for nonzero virtualities of quarks and a gluon is considered. The form factor $F_2(k_1^2, k_2^2, t^2)$ suppresses the AQGC vertex
at short distances when the respective virtualities are large at short distances when the respective virtualities are large. Within the instanton model, it is explicitly related to the Fourier-transformed of both the quark zero-mode and instanton field, which take the forms

$$
F_2(k_1^2, k_2^2, t^2) = \mu_a F_q(|k_1|\rho/2) F_q(|k_2|\rho/2) F_g(|t|\rho), \quad (3)
$$

where

$$
F_q(z) = -z \frac{d}{dz} (I_0(z) K_0(z) - I_1(z) K_1(z)),
$$

\n
$$
F_g(z) = \frac{4}{z^2} - 2K_2(z),
$$
\n(4)

 $I_{\nu}(z)$, $K_{\nu}(z)$ are the modified Bessel functions, ρ is the instanton size, and $\mu_a = F_2(0, 0, 0)$ is the anomalous quark chromomagnetic moment (AQCM). Within the instanton liquid model [\[22,23\]](#page-5-0), where all instantons have the same size $\rho_c \approx 1/3$ fm, AQCM is [\[23,30\]](#page-5-1)

$$
\mu_a = -\frac{3\pi (M_q \rho_c)^2}{4\alpha_s(\rho_c)}.\tag{5}
$$

The first feature is that the strong coupling constant enters into the denominator showing a clear nonperturbative origin of AQCM. The second feature is the negative sign of AQCM. As we will see below, this sign of AQCM leads to the positive sign of the anomalous quark magnetic moment (AQMM). The value of AQCM strongly depends on the dynamical quark mass, which is $M_q = 170$ MeV in the mean field approximation (MFA) [\[22\]](#page-5-0) and M_q = 350 MeV in the Diakonov-Petrov model (DP) [\[23\]](#page-5-1). Therefore, for the value of the strong coupling constant in the instanton model, $\alpha_s(\rho_c) \approx 0.5$ and average size of instantons $\rho_c = 1/600 \text{ MeV}^{-1}$ [\[23\]](#page-5-1), we get

$$
\mu_a^{\text{MFA}} = -0.4 \qquad \mu_a^{\text{DP}} = -1.6. \tag{6}
$$

The contribution to the electromagnetic Pauli form factor coming from the AQGC vertex is obtained by the consideration of the diagrams presented in Fig. [1](#page-1-0).

To perform analytical calculations the gaussian approximation for the form factors in Eq. [\(4\)](#page-1-1)

$$
F_g(k_E^2) \approx F_q(k_E^2) \approx e^{-k_E^2/\Lambda^2} \tag{7}
$$

is used with $\Lambda = 2/\rho_c$. At low virtuality, Eq. [\(7\)](#page-1-2) agrees with Eq. [\(4\)](#page-1-1) well numerically; hence, we can substitute the former for the latter without losing much accuracy. Such approximation was also adopted in paper [\[34\]](#page-5-3), see Eq. (42). Furthermore, the Gaussian approximation presented in Eq. [\(7\)](#page-1-2) enables us to obtain an analytical result for

FIG. 1. The diagrams with anomalous quark-gluon chromomagnetic vertex induced by instantons which contribute to EMFF of the quark. The vertex is denoted by a solid blob. p and p' are the momenta of the external legs, t is the momentum of the exchanged gluon.

EPFF, which might be very important for future calculations of various hadron properties.

The contribution from Fig. $1(b)$ is the same as that from Fig. [1\(a\);](#page-1-0) hence, the final result should be doubled. Therefore, the total matrix element is¹

$$
i\mathcal{M} \equiv -e_q C_F g_s^2 \frac{\mu_a}{M_q} \int \frac{d^4 t}{(2\pi)^4} \frac{F_g(t^2) F_q(k'^2) N}{(k'^2 - M_q^2)(k^2 - M_q^2) t^2}
$$

=
$$
-ie_q \bar{u}(p') \Gamma^\mu(p, p') u(p), \qquad (8)
$$

where $C_F = \text{tr}(T^a T^a) = \frac{4}{3}$ is the color factor, e_q is the electric charge of the quark and electric charge of the quark, and

$$
N \equiv -i\bar{u}(p')\sigma^{\alpha\rho}(\not\!k' + M_q)\gamma^{\mu}(\not\!k + M_q)\gamma_{\rho}t_{\alpha}u(p). \quad (9)
$$

One way to extract Pauli form factor $F_2(Q^2)$ from iM is to rearrange the gamma matrices in Eq. [\(9\)](#page-1-3) and find the term proportional to $i\sigma^{\mu\nu}/2M_a$. However, a simpler way is to use projector operator method [\[35,36\]](#page-5-4), by making use of the identity

$$
F_2(q^2) = \text{tr}\{(\not p + M_q)\Lambda_\rho^{(2)}(p', p)(\not p + M_q)\Gamma^\rho(p', p)\},\tag{10}
$$

where $q^2 = (p' - p)^2 \equiv -Q^2$ and

$$
\Lambda_{\rho}^{(2)}(p',p) \equiv \frac{M_q^2}{k^2 (4M_q^2 - k^2)} \left[\gamma_{\rho} + \frac{k^2 + 2M_q^2}{M_q (k^2 - 4M_q^2)} (p' + p)_{\rho} \right].
$$

¹For a careful calculation of the hadron properties within the models with an anomalous quark chromomagnetic (AQGC) vertex, one needs additionally to consider the confinement effects which results in an nonzero virtuality $k_{conf} \approx \Lambda_{QCD} \approx 200 \text{ MeV}$ of the initial and final quark legs in Fig. [1.](#page-1-0) In principle, this effect can be taken into account by the Fourier-transformed quark zero modes presented in Eq. [\(4\)](#page-1-1). But one can not expect a large contribution because according to Eq. [\(7\)](#page-1-2), the suppression factor is ~exp $\left(-(\Lambda_{\text{QCD}}\rho_c)^2/4\right)$, where $1/\rho_c = 600$ MeV.

By working in the Euclidean space-time,with the help of Feynman parametrization and identity

$$
\frac{1}{k^n} = \int_0^\infty d\alpha \frac{\alpha^{n-1}}{(n-1)!} e^{-\alpha k},\tag{11}
$$

we obtain

$$
F_2(Q^2) = \frac{\mu_a e_q g_s^2}{12\pi^2} \int d^3x \int_0^\infty d\alpha \frac{\alpha^2}{\Delta^2}
$$

× $\exp\left\{-M_q^2 \left[\Delta(v_1 + v_2)^2 - \frac{1}{\Lambda^2}\right] - Q^2 \Delta v_1 v_2\right\}$
× $\left\{\frac{3v_1 + 6v_2 - 7}{\Delta} - Q^2 v_1 v_2 (v_1 + 2v_2 - 3) - M_q^2 (v_1 + v_2) ((v_1 + v_2)(v_1 + 2v_2) - 2v_2)\right\},$ (12)

where $\int d^3x \equiv 2 \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$ and

$$
\Delta \equiv \alpha + \frac{2}{\Lambda^2},\tag{13}
$$

$$
v_1 \equiv x_2 \frac{\alpha}{\Delta},\tag{14}
$$

$$
v_2 \equiv x_1 \frac{\alpha}{\Delta} + \frac{1}{\Lambda^2 \Delta}.
$$
 (15)

III. NUMERICAL RESULTS

In our model, the form factor $F_2(Q^2)$ is proportional to the quark charge. Therefore, there is the relation between u- and d-quark form factors

$$
F_2^d(Q^2) = -\frac{1}{2}F_2^u(Q^2). \tag{16}
$$

For simplicity, below only the result for the u-quark case is presented in the figures. In Fig. [2,](#page-2-0) the result of the calculation of the electromagnetic form factor as the function of Q^2 is presented for two different masses of the u quark. Our numerical result can be fitted very well by the formula

$$
F_2(Q^2, M_q) = \frac{F_2(0, M_q)}{1 + \rho_c Q^2 / (4.7 M_q)},
$$
(17)

which can be useful for the applications. We would like to emphasize that a positive sign of the F_2 form factor for a u quark (see Fig. [2](#page-2-0)) is fixed by the negative sign of the AQCM, Eq. [\(5\)](#page-1-4). In the Fig. [3](#page-2-1), the dependency of the value of the magnetic moment of the u quark on the value of its dynamical mass is shown. Its behavior as a function of

FIG. 2. The F_2 form factor as a function of Q^2 for the different dynamical quark mass M_q in the comparison with the fit given by Eq. [\(17\).](#page-2-2)

quark mass in the range between 80 and 500 MeV can be fitted very well with the linear function

$$
\mu_a^u \approx \frac{2}{3}(-0.065 + 0.97(M_q \rho_c)).
$$
 (18)

The results for $\mu_a^{e,u} = F^u_2(Q^2 = 0)$ at the two different
lues of the dynamical quark masses obtained in the mean values of the dynamical quark masses obtained in the mean field approximation [\[22\]](#page-5-0) and within the Diakonov-Petrov model [\[23\]](#page-5-1) are

$$
\mu_a^{e,u} = 0.33
$$
 for $M_q = 350$ MeV,
\n $\mu_a^{e,u} = 0.14$ for $M_q = 170$ MeV. (19)

Our value for the quark magnetic moment at M_q = 350 MeV is in the qualitative agreement with the result of a calculation within a different approach based on the Dyson-Schwinger equation [\[37\]](#page-5-5). However, we would like to emphasize that in this paper, the Q^2 dependency of the EPFF is not considered. This Q^2 dependency in our model

FIG. 3. Behavior of μ_a^u versus quark mass: exact expression (black solid line), linear fit (blue dashed-dotted), expansion Eq. [\(24\)](#page-3-0) (red dotted).

is presented in the Fig. [2.](#page-2-0) One can mention its rather strong dependency on the virtuality of photon. In the model, it is coming from the quark and gluon form factors presented in the Eq. [\(7\)](#page-1-2).

IV. THE LARGE O^2 BEHAVIOR OF EPFF

The formula for expansion of the form factor at large $Q^2 \gg M_q^2$ is

$$
F_2(Q^2) \approx -\frac{1}{Q^2} \frac{e_q C_F g_s^2 \mu_a}{4\pi^2} \sum_{n=0}^{\infty} \int_0^{\infty} dk^2 \frac{k^{2(n+2)}}{n!(n+2)!} M_q^{2n}
$$

$$
\times [F_g D_g]_{n+1} \left(2[F_q D_q]_{n-1} - M_q^2 \frac{k^2}{(n+3)} [F_q D_q]_{n+1} \right),
$$

where

$$
[F_q D_q]_{-1} = -\int_{k^2}^{\infty} dl^2 F_q(l^2) D_q(l^2)
$$

\n
$$
[F_q D_q]_0 = F_q(k^2) D_q(k^2)
$$

\n
$$
[F_q D_q]_{+1} = -\left(\frac{d}{dk^2}\right) (F_q(k^2) D_q(k^2))
$$

\n...
\n
$$
[F_q D_q]_{+n} = \left(-\frac{d}{dk^2}\right)^n (F_q(k^2) D_q(k^2)), \qquad (20)
$$

and the same formula for $[F_gD_g]_i$ with the corresponding
changing of index $g \to g$. Using the expressions for the changing of index $q \rightarrow g$. Using the expressions for the form factors in the gluon and quark sector in Eq. [\(7\)](#page-1-2) and $D_g(k^2) = 1/k^2$, $D_g(k^2) = 1/(k^2 + M_g^2)$, one can rewrite it in the form

$$
F_2(Q^2) \approx \frac{1}{Q^2} \frac{e_q C_F g_s^2 \mu_a}{4\pi^2} \left\{ \int_0^\infty dk^2 \frac{e^{-k^2/\Lambda^2}}{k^2 + M_q^2} \right.\n\times \left[2\Lambda^2 - e^{-k^2/\Lambda^2} (k^2 + 2\Lambda^2) \right]\n+ \sum_{n=1}^\infty \int_0^\infty dk^2 \frac{k^{2(n+2)} M_q^{2n}}{(n-1)!(n+2)!} \n\times \left(\frac{2}{n} (F_g D_g)_{n+1} (F_q D_q)_{n-1} - (F_g D_g)_n (F_q D_q)_n \right) \right\}
$$
(21)

and show that the main contribution to EPFF is coming from the first term in brackets. Moreover, one can perform an additional expansion over M_q^2/Λ^2 . In this approximation and ignoring the second term in Eq. [\(21\),](#page-3-1) one can write the leading orders in M_q^2/Λ^2 expansion in the form

$$
F_2(Q^2) \approx 4e_q \frac{M_q^2}{Q^2} \left(2\ln(2) - \frac{1}{2} + \frac{M_q^2}{\Lambda^2} \left[\ln \left(8 \frac{M_q^2}{\Lambda^2} \right) - 2 + \gamma_E \right] \right), \quad (22)
$$

where γ_E is the Euler's constant. By using the relation $\Lambda \approx 2/\rho_c$, it can be rewritten as

$$
F_2(Q^2) \approx 4e_q \frac{M_q^2}{Q^2} \left(2\ln(2) - \frac{1}{2} + \frac{(M_q \rho_c)^2}{4} \left[\ln\left(2(M_q \rho_c)^2\right) - 2 + \gamma_E \right] \right). \tag{23}
$$

Therefore, at large Q^2 , the form factor behaves as $F_2(Q^2) \sim 1/Q^2$.

Instanton corresponds to the subbarrier transition between vacua with different topological charges, the height of the potential barrier between these vacua is given by the energy of the so-called sphaleron $E_{sph} =$ $3\pi/(4\alpha_s(\rho)\rho)$ [\[23\].](#page-5-1) For $\rho = \rho_c \approx 0.3$ fm and $\alpha(\rho_c) \approx 0.5$, we obtain $E_{sph} \approx 3$ GeV, which is rather a large value; therefore, the zero-mode approximation should be valid as far as the external energy scale satisfies $Q \leq 3$ GeV. For $Q \gg E_{sph}$, the nonzero modes effects became important. Furthermore, instead of using a fixed instanton size, one should integrate over ρ in the spirit of the paper [\[31\].](#page-5-2)

V. THE LOW Q^2 BEHAVIOR OF EPFF

It can be shown that in the limit $Q^2 \rightarrow 0$, the form factor is

$$
F_2(0) \approx e_q (M_q \rho_c)^2 \left(y + \frac{(192y + 211)}{288} (M_q \rho_c)^2 + \frac{(1536y + 3089)}{9216} (M_q \rho_c)^4 + \cdots \right),
$$
 (24)

where

$$
y = \ln\left(\frac{2}{(M_q \rho_c)^2}\right) - \gamma_E - \frac{1}{4}.
$$

The expansion given by Eq. [\(24\)](#page-3-0) describes the exact result very well in the region of small $M_q\rho_c < 1$, Fig. [3](#page-2-1). One can see that $F_2(0)$ vanishes in the limit $\overline{M}_q \to 0$. It means that this contribution to the form factor is directly related to the phenomenon of SCSB.

VI. CONCLUSION

In this paper, we calculate the quark electromagnetic form factor within the nonperturbative approach based on the instanton picture for the QCD vacuum. It is shown that the anomalous quark-gluon chromomagnetic interaction induced by instantons leads to a large magnetic moment of u and d quarks. Possible applications of our results are as follows. One of the tasks is to consider the influence of EPFF on the hadron electromagnetic form factors.

Recently, the electromagnetic form factor for a pion was calculated in the framework of the Dyson-Schwinger equation with a nonlocal photon-quark vertex in the paper [\[38\]](#page-5-6) (see also [\[39\]](#page-5-7)). The normalization requires $F_\pi(Q^2 = 0) = 1$ for the pion eletromagnetic form factor. It will be one of our future tasks to study whether this condition is satisfied by instanton induced quark-gluon and quark-photon interactions, taking into account their nonlocal effects.

We would like to mention that instanton contribution to the electromagnetic form factors of proton, neutron, and pion were calculated using different versions of the instanton model in [10–[13,40\]](#page-4-7) in semiclassical approaches to the corresponding correlators. However, it would be interesting to study the electromagnetic properties of hadrons based on the constituent quark model with an effective quark-photon and quark-gluon vertices induced by instantons. In this way, one can take into consideration the confinement effects as well in spirit of the calculation of nucleon electromagnetic form factors carried out in [\[8\]](#page-4-8) for constituent quarks with inner structure. In our model, it is evident that due to the existence of an additional scale related to the instanton size $\rho_c \approx 1/3$ fm, one can expect the deviation of the Q^2 dependency of the hadron form factors from the quark-counting rule prediction [\[41](#page-5-8)–43].

It is well-known that models with an nonperturbative nonlocal interaction and conserved external currents could be considered in the framework of gauged effective Lagrangians [\[2,39,44,45\]](#page-4-9) or with the help of schemes based on Dyson-Schwinger and Bethe-Salpeter equations [\[9,20,21,37\]](#page-4-10). In our work, we consider the quarks with a constant constituent mass due to the SCSB and the effective AQGC vertex induced by instantons, as shown in Eq. [\(2\)](#page-0-5). This model suffices to mimic the main properties of the anomalous gluon contribution to F_2 , while the full treatment, in the framework of effective Lagrangian, will only slightly change the numerical values.

On the other hand, one can expect that corrections from mesonic fluctuations should be of order $1/N_c$, which could serve as a naive estimation for the precision of our model.

We should stress that EPFF leads to quark spin flip. Therefore, it should make a contribution to various spindependent photon-hadron cross sections, including polarized semi-inclusive DIS. Another possible application, in the line of [\[46\],](#page-5-9) is the study of the influence of the nonzero value of the anomalous quark magnetic moment on the dynamics of the quark-gluon plasma (QGP) in the strong magnetic field. We would like to emphasize that our new type of photon-quark interaction is very sensitive to the topological structure of the QCD vacuum, which might be drastically changed during the deconfinement transition [\[47](#page-5-10)–50]. This phenomenon can lead to, for example, the suppression of a direct photon production induced by our anomalous quark-photon vertex in the QGP.

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- [1] M. Lindner, M. Platscher, and F.S. Queiroz, [arXiv:](http://arXiv.org/abs/1610.06587) [1610.06587.](http://arXiv.org/abs/1610.06587)
- [2] A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhevlakov, [Eur. Phys. J. Web Conf.](https://doi.org/10.1051/epjconf/201612502007) 125, 02007 (2016).
- [3] V. Punjabi, C. F. Perdrisat, M. K. Jones, E. J. Brash, and C. E. Carlson, [Eur. Phys. J. A](https://doi.org/10.1140/epja/i2015-15079-x) 51, 79 (2015).
- [4] G. Altarelli, S. Petrarca, and F. Rapuano, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(96)00131-1) 373, [200 \(1996\)](https://doi.org/10.1016/0370-2693(96)00131-1).
- [5] S. Scopetta and V. Vento, Phys. Rev. D 69[, 094004 \(2004\).](https://doi.org/10.1103/PhysRevD.69.094004)
- [6] N. Kochelev, H. J. Lee, B. Zhang, and P. Zhang, [Phys. Lett.](https://doi.org/10.1016/j.physletb.2016.04.027) B 757[, 420 \(2016\)](https://doi.org/10.1016/j.physletb.2016.04.027).
- [7] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, [J.](https://doi.org/10.1088/0954-3899/42/9/095005) Phys. G 42[, 095005 \(2015\)](https://doi.org/10.1088/0954-3899/42/9/095005).
- [8] R. Petronzio, S. Simula, and G. Ricco, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.67.094004) 67, [094004 \(2003\);](https://doi.org/10.1103/PhysRevD.67.094004) 68[, 099901\(E\) \(2003\).](https://doi.org/10.1103/PhysRevD.68.099901)
- [9] J. Arrington, C. D. Roberts, and J. M. Zanotti, [J. Phys. G](https://doi.org/10.1088/0954-3899/34/7/S03) 34, [S23 \(2007\).](https://doi.org/10.1088/0954-3899/34/7/S03)
- [10] P. Faccioli, A. Schwenk, and E. V. Shuryak, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.67.113009) 67[, 113009 \(2003\).](https://doi.org/10.1103/PhysRevD.67.113009)
- [11] P. Faccioli, A. Schwenk, and E. V. Shuryak, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(02)02886-1) 549[, 93 \(2002\).](https://doi.org/10.1016/S0370-2693(02)02886-1)
- [12] P. Faccioli and E. V. Shuryak, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.65.076002) 65, 076002 [\(2002\).](https://doi.org/10.1103/PhysRevD.65.076002)
- [13] A. Blotz and E. V. Shuryak, Phys. Rev. D 55[, 4055 \(1997\).](https://doi.org/10.1103/PhysRevD.55.4055)
- [14] X.-D. Ji, Phys. Rev. D 55[, 7114 \(1997\).](https://doi.org/10.1103/PhysRevD.55.7114)
- [15] A. V. Radyushkin, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(96)00528-X) 380, 417 (1996).
- [16] N. I. Kochelev, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(98)00262-7) 426, 149 (1998).
- [17] N. I. Kochelev, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(03)00727-5) **565**, 131 (2003).
- [18] A. E. Dorokhov and I. O. Cherednikov, [Ann. Phys.](https://doi.org/10.1016/j.aop.2004.05.003) [\(Amsterdam\)](https://doi.org/10.1016/j.aop.2004.05.003) 314, 321 (2004).

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- [19] S. Simula, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2003.09.032) **574**, 189 (2003).
- [20] C. D. Roberts, [Prog. Part. Nucl. Phys.](https://doi.org/10.1016/j.ppnp.2007.12.034) 61, 50 (2008).
- [21] P. Maris and C. D. Roberts, [Int. J. Mod. Phys. E](https://doi.org/10.1142/S0218301303001326) 12, 297 [\(2003\).](https://doi.org/10.1142/S0218301303001326)
- [22] T. Schäfer and E. V. Shuryak, [Rev. Mod. Phys.](https://doi.org/10.1103/RevModPhys.70.323) 70, 323 [\(1998\).](https://doi.org/10.1103/RevModPhys.70.323)
- [23] D. Diakonov, [Prog. Part. Nucl. Phys.](https://doi.org/10.1016/S0146-6410(03)90014-7) **51**, 173 (2003).
- [24] N. I. Kochelev, Fiz. Elem. Chastits At. Yadra 36, 1157 (2005) Phys. Part. Nucl. 36, 608 (2005).
- [25] D. Ostrovsky and E. Shuryak, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.71.014037) **71**, 014037 [\(2005\).](https://doi.org/10.1103/PhysRevD.71.014037)
- [26] I. O. Cherednikov, U. D'Alesio, N. I. Kochelev, and F. Murgia, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2006.09.019) 642, 39 (2006).
- [27] N. Kochelev and N. Korchagin, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2014.01.003) 729, 117 [\(2014\).](https://doi.org/10.1016/j.physletb.2014.01.003)
- [28] P. Hoyer and M. Jarvinen, [J. High Energy Phys. 10 \(2005\)](https://doi.org/10.1088/1126-6708/2005/10/080) [080.](https://doi.org/10.1088/1126-6708/2005/10/080)
- [29] Y. Qian and I. Zahed, [Ann. Phys. \(Amsterdam\)](https://doi.org/10.1016/j.aop.2016.09.002) 374, 314 [\(2016\).](https://doi.org/10.1016/j.aop.2016.09.002)
- [30] N. Kochelev, [Phys. Part. Nucl. Lett.](https://doi.org/10.1134/S1547477110050043) 7, 326 (2010).
- [31] S. Moch, A. Ringwald, and F. Schrempp, [Nucl. Phys.](https://doi.org/10.1016/S0550-3213(97)00592-0) **B507**, [134 \(1997\)](https://doi.org/10.1016/S0550-3213(97)00592-0).
- [32] A. Ringwald and F. Schrempp, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(01)00216-7) 503, 331 [\(2001\).](https://doi.org/10.1016/S0370-2693(01)00216-7)
- [33] A. Ringwald and F. Schrempp, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(99)00682-6) 459, 249 [\(1999\).](https://doi.org/10.1016/S0370-2693(99)00682-6)
- [34] D. E. Kharzeev, Y. V. Kovchegov, and E. Levin, [Nucl. Phys.](https://doi.org/10.1016/S0375-9474(01)00352-9) A690[, 621 \(2001\)](https://doi.org/10.1016/S0375-9474(01)00352-9).
- [35] S.J. Brodsky and J.D. Sullivan, [Phys. Rev.](https://doi.org/10.1103/PhysRev.156.1644) 156, 1644 [\(1967\).](https://doi.org/10.1103/PhysRev.156.1644)
- [36] M. Knecht and A. Nyffeler, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.65.073034) 65, 073034 [\(2002\).](https://doi.org/10.1103/PhysRevD.65.073034)
- [37] L. Chang, Y. X. Liu, and C. D. Roberts, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.106.072001) 106[, 072001 \(2011\).](https://doi.org/10.1103/PhysRevLett.106.072001)
- [38] L. Chang, I. C. Cloet, C. D. Roberts, S. M. Schmidt, and P. C. Tandy, Phys. Rev. Lett. 111[, 141802 \(2013\)](https://doi.org/10.1103/PhysRevLett.111.141802).
- [39] A. E. Dorokhov, A. E. Radzhabov, and M. K. Volkov, [Eur.](https://doi.org/10.1140/epja/i2003-10177-0) Phys. J. A 21[, 155 \(2004\)](https://doi.org/10.1140/epja/i2003-10177-0).
- [40] H. Forkel and M. Nielsen, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(94)01596-5) 345, 55 (1995).
- [41] V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, [Lett. Nuovo Cimento Soc. Ital. Fis.](https://doi.org/10.1007/BF02777988) 5, 907 (1972); 5[, 907](https://doi.org/10.1007/BF02777988) [\(1972\).](https://doi.org/10.1007/BF02777988)
- [42] S. J. Brodsky and G. R. Farrar, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.31.1153) 31, 1153 [\(1973\).](https://doi.org/10.1103/PhysRevLett.31.1153)
- [43] S. J. Brodsky and G. R. Farrar, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.11.1309) 11, 1309 [\(1975\).](https://doi.org/10.1103/PhysRevD.11.1309)
- [44] A. E. Dorokhov and W. Broniowski, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s2003-01386-x) 32, 79 [\(2003\).](https://doi.org/10.1140/epjc/s2003-01386-x)
- [45] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.81.034010) 81, 034010 [\(2010\).](https://doi.org/10.1103/PhysRevD.81.034010)
- [46] S. Fayazbakhsh and N. Sadooghi, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.90.105030) 90, 105030 [\(2014\).](https://doi.org/10.1103/PhysRevD.90.105030)
- [47] E. M. Ilgenfritz and E. V. Shuryak, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(94)90007-8) 325, 263 [\(1994\).](https://doi.org/10.1016/0370-2693(94)90007-8)
- [48] E. M. Ilgenfritz and E. V. Shuryak, [Nucl. Phys.](https://doi.org/10.1016/0550-3213(89)90617-2) B319, 511 [\(1989\).](https://doi.org/10.1016/0550-3213(89)90617-2)
- [49] T. Schäfer, E. V. Shuryak, and J. J. M. Verbaarschot, [Phys.](https://doi.org/10.1103/PhysRevD.51.1267) Rev. D 51[, 1267 \(1995\).](https://doi.org/10.1103/PhysRevD.51.1267)
- [50] T. Schäfer and E. V. Shuryak, Phys. Rev. D 53[, 6522 \(1996\).](https://doi.org/10.1103/PhysRevD.53.6522)