Process-independent strong running coupling

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We unify two widely different approaches to understanding the infrared behavior of quantum chromodynamics (QCD), one essentially phenomenological, based on data, and the other computational, realized via quantum field equations in the continuum theory. Using the latter, we explain and calculate a process-independent running coupling for QCD, a new type of effective charge that is an analogue of the Gell-Mann–Low effective coupling in quantum electrodynamics. The result is almost identical to the process-dependent effective charge defined via the Bjorken sum rule, which provides one of the most basic constraints on our knowledge of nucleon spin structure. This reveals the Bjorken sum to be a near direct means by which to gain empirical insight into QCD's Gell-Mann–Low effective charge.

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I. INTRODUCTION

In quantum gauge field theories defined in four spacetime dimensions, the Lagrangian couplings and masses do not remain constant. Instead, owing to the need for ultraviolet (UV) renormalization, they come to depend on a mass scale, which can often be related to the energy or momentum at which a given process occurs. The archetype is quantum electrodynamics (QED), for which a sensible perturbation theory can be defined [1]. Within this framework, owing to the Ward identity [2], there is a single running coupling, measuring the strength of the photon–charged-fermion vertex, which can be obtained by summing the collection of virtual processes that change the bare photon into a dressed object, *viz.* by computing the photon vacuum polarization. QED's running-coupling is known to great accuracy [3] and the running has been observed directly [4,5].

A new coupling appears when electromagnetism is combined with weak interactions to produce the Standard Electroweak Model [6]. It may be characterized by $\sin^2\theta_W$, where θ_W is a scale-dependent angle which specifies the particular mixing between the model's defining neutral gauge bosons that produces the observed photon and Z^0-boson. A perturbation theory can also be defined for the electroweak theory [7] so that $\sin^2\theta_W$ can be computed and compared with precise experiments [3].

At first sight, the addition of quantum chromodynamics (QCD) [8] to the Standard Model does not qualitatively change anything, despite the presence of four possibly distinct strong-interaction vertices (gluon-ghost, three-gluon, four-gluon and gluon-quark) in the renormalized theory. An array of Slavnov-Taylor identities (STIs) [9,10], implementing BRST symmetry [11,12] (a generalization of non-Abelian gauge invariance for the quantized theory) ensures that a single running-coupling characterizes all four interactions on the domain within which perturbation theory is valid. The difference here is that whilst QCD is asymptotically free and extant evidence suggests that perturbation theory is valid at large momentum scales, all dynamics is nonperturbative at those scales typical of everyday strong-interaction phenomena, e.g. $\zeta \lesssim m_p$, where m_p is the proton's mass.

The questions that arise are how many distinct running-couplings exist in nonperturbative QCD, and how can they be computed? Given that there are four individual, apparently UV-divergent interaction vertices in the perturbative treatment of QCD, there could be as many as four distinct couplings at infrared (IR) momenta. (Of course, if nonperturbatively there are two or more couplings, they must all become equivalent on the perturbative domain.) We will argue herein that, nonperturbatively, too, QCD possesses a unique running coupling. The alternative admits the possibility of a different renormalization-group-invariant (RGI) intrinsic mass-scale for each coupling and no guarantee of a connection between them. In such circumstances, BRST symmetry would likely be irreparably broken by nonperturbative dynamics and one would be pressed to conclude that QCD was non-renormalizable owing to IR dynamics. There is no empirical evidence to support such a conclusion: QCD does seem to be a well-defined theory at all momentum scales, owing to the dynamical generation of gluon [13–18] and quark masses [19–21], which are large at IR momenta.

II. PROCESS-INDEPENDENT RUNNING COUPLING

Poincaré covariance is of enormous importance in modern physics, e.g. it places severe limitations on the nature and number of those independent amplitudes that are required to fully specify any one of a gauge theory's n-point Schwinger functions (Euclidean Green functions). Analyses and quantization procedures that violate Poincaré covariance lead to a rapid proliferation in the number of such functions. For example, the gluon 2-point function (propagator, $D_{\mu\nu}$) is completely specified by one scalar function in the class of linear covariant gauges; but, in the class of axial gauges, two unconnected functions are required and unphysical, kinematic singularities are present in the associated tensors [22]. Consequently, covariant gauges are typically preferred for concrete calculations in both continuum and lattice-regularized studies of QCD. In fact, a Landau gauge is the most common choice because, *inter alia*, it is a fixed point of the renormalization group and readily implemented in lattice-QCD [23]. Herein, therefore, we use a Landau gauge; and, moreover, employ a physical momentum-subtraction renormalization scheme, detailed elsewhere [24].

As noted in Sec. I, there is a particular simplicity to QED, *viz.* the unique running coupling, a process-independent effective charge, can be obtained simply by computing the photon vacuum polarisation. This is because ghost-fields decouple in Abelian theories; and, consequently, one has the Ward identity, which guarantees that the electric-charge renormalization constant is equivalent to that of the photon field. Stated physically, the impact of dressing the interaction vertices is absorbed into the vacuum polarization. This is not generally true in QCD because ghost-fields do not decouple.

There is one approach to analyzing QCD's Schwinger functions, however, that preserves some of QED's simplicity; namely, the combination of the pinch technique (PT) [25–30] and background field method (BFM) [31,32]. This framework can be seen as a means by which QCD can be made to "look" Abelian: one systematically rearranges classes of diagrams and their sums in order to obtain modified Schwinger functions that satisfy linear STIs. In the gauge sector, in Landau gauge, this produces a modified gluon dressing function from which one can compute the QCD running coupling, i.e. the polarization captures all required features of the renormalization group. Furthermore, the coupling is process independent: one obtains precisely the same result, independent of the scattering process considered, whether gluon + gluon → gluon + gluon, quark + quark → quark + quark, etc. This clean connection between the coupling and the gluon vacuum polarization relies on another particular feature of QCD, *viz.* in Landau gauge the renormalization constant of the gluon-ghost vertex is not only finite but unity [9], in consequence of which the effective charge obtained from the PT-BFM gluon vacuum polarization is directly connected with that deduced from the gluon-ghost vertex [24], sometimes called the "Taylor coupling," $\alpha_{\rm T}$ [33–35].

Writing these statements explicitly, with $T_{\mu\nu}(k) = \delta_{\mu\nu} - k_\mu k_\nu/k^2$, one has [36,37]

$$\alpha(\zeta^2) D^{\rm PB}_{\mu\nu}(k;\zeta) = \frac{\alpha(\zeta^2)\Delta_F(k^2;\zeta)}{[1+G(k^2;\zeta)]^2} T_{\mu\nu}(k) \tag{1a}$$

$$= \hat{d}(k^2) T_{\mu\nu}(k), \tag{1b}$$

$$\mathcal{I}(k^2) := k^2 \hat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}, \tag{1c}$$

where: $\alpha(\zeta^2) = g^2(\zeta^2)/[4\pi]$, ζ is the renormalization scale; $D^{\rm PB}_{\mu\nu}$ is the PT-BFM gluon two-point function; $D_{\mu\nu}(k) = \Delta_F(k^2)T_{\mu\nu}(k)$ is the canonical gluon two-point function; $\hat{d}(k^2)$ is the RGI running-interaction discussed in Ref. [24]; F is the dressing function for the ghost propagator; G is that piece of the gluon-ghost vacuum polarisation that can be isolated by transverse projection, and L is that longitudinal part which vanishes at $k^2 = 0$. In terms of these quantities, QCD's matter-sector gap equation can be written $(k = p - q)$

$$S^{-1}(p) = Z_2(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p), \tag{2a}$$

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \hat{d}(k^2) T_{\mu\nu}(k)\gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p), \tag{2b}$$

where the usual $Z_1\Gamma^a_\nu$ has become $Z_2\hat{\Gamma}^a_\nu$, with the latter being a PT-BFM gluon-quark vertex that satisfies an Abelian-like Ward-Green-Takahashi identity [30] and $Z_{1,2}$ are, respectively, the gluon-quark vertex and quark wave function renormalization constants.

The RGI interaction, $\hat{d}(k^2)$, in Eqs. (1) has been computed. The most up-to-date result is discussed in Refs. [36,37]. These analyses make explicit a remarkable feature of QCD; namely, the interaction saturates at infrared momenta:

$$\hat{d}(k^2 = 0) = \alpha(\zeta^2)/m_g^2(\zeta) = \alpha_0/m_0^2, \tag{3}$$

where $\alpha_0 := \alpha(0) \approx 0.9\pi$, $m_0 := m_g(0) \approx m_p/2$, i.e. the gluon sector of QCD is characterized by a nonperturbatively-generated infrared mass-scale [13–18]. With this in mind, we define a RGI function

$$\mathcal{D}(k^2) = \Delta_{\rm F}(k^2;\zeta)/[m_0^2 \Delta_{\rm F}(0;\zeta)], \tag{4}$$

employing for $\Delta_{\rm F}$ a parametrization of continuum- and/or lattice-QCD calculations of the canonical gluon two-point function built such that

$$\frac{1}{\mathcal{D}(k^2)} = \begin{cases} m_0^2 + {\rm O}(k^2 \ln k^2) & k^2 \ll m_0^2 \\ k^2 + {\rm O}(1) & k^2 \gg m_0^2 \end{cases}, \tag{5}$$

so that the nonperturbative IR behavior is preserved and the UV anomalous dimension remains in $\hat{d}(k^2)$. (Practical details are provided in Sec. III). Using Eq. (4),

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi\hat{\alpha}_{\mathrm{PI}}(k^2)\mathcal{D}_{\mu\nu}(k^2)\gamma_\mu S(q)\hat{\Gamma}^a_\nu(q,p), \quad (6)$$

where $\mathcal{D}_{\mu\nu} = \mathcal{D}T_{\mu\nu}$ and the dimensionless product

$$\hat{\alpha}_{\mathrm{PI}}(k^2) = \hat{d}(k^2)/\mathcal{D}(k^2) \quad (7)$$

is a RGI running coupling (effective charge): by construction, $\hat{\alpha}_{\mathrm{PI}}(k^2) = \mathcal{I}(k^2)$ on $k^2 \gg m_0^2$.

The product in Eq. (7) has many important qualities. For instance, it is process independent: as noted above, the same function appears irrespective of the initial and final parton systems. Moreover, it unifies a diverse and extensive array of hadron observables [36]; a property that is evident in the fact that the dressed-quark self-energy serves as a generating functional for the Bethe-Salpeter kernel in all meson channels and the product $\hat{\alpha}_{\mathrm{PI}}(k^2)$ is untouched by the generating procedure in all flavored systems [38–41]. Finally, although $\hat{\alpha}_{\mathrm{PI}}(k^2)$ is RGI and process-independent in any gauge, it is sufficient to know $\hat{\alpha}_{\mathrm{PI}}(k^2)$ in the Landau gauge (the choice for easiest computation) because: $\hat{\alpha}_{\mathrm{PI}}(k^2)$ is form-invariant under gauge transformations, since the identity established by Eqs. (1a), (1b) is the same in all linear covariant gauges [42]; and, crucially, gauge covariance ensures that such transformations are implemented by multiplying a simple factor into the configuration space transform of the gap equation's solution and may consequently be absorbed into the dressed-quark two-point function [43].

III. COMPUTING THE RUNNING COUPLING

The effective charge defined in Eq. (7) is a product of known quantities: both $\hat{d}(k^2)$ and the canonical gluon two-point function have been extensively studied and tightly constrained using continuum and lattice methods [36,37,44]. Indeed, the known forms of these functions provide a unified, quantitatively reliable explanation of numerous hadron physics observables [36,44]. It is therefore straightforward to combine existing results and compute $\hat{d}(k^2)$, a procedure [37] which yields the function depicted in Fig. 1. For this purpose we used a $[n, n+1]$, $n = 1$, Padé approximant to simultaneously interpolate the IR behavior of contemporary lattice results for $D_{\mu\nu}(k)$ and express the UV constraint on $\Delta_{\mathrm{F}}(k^2;\zeta)$ specified in Eq. (5). The result is given in line 2, Eq. (10) of Ref. [37] and yields $m_0 = 0.45$ GeV via Eqs. (4), (5). (Using $n \geq 2$ yields no noticeable fit improvement, but $n = 0$ is incapable of representing modern lattice data).

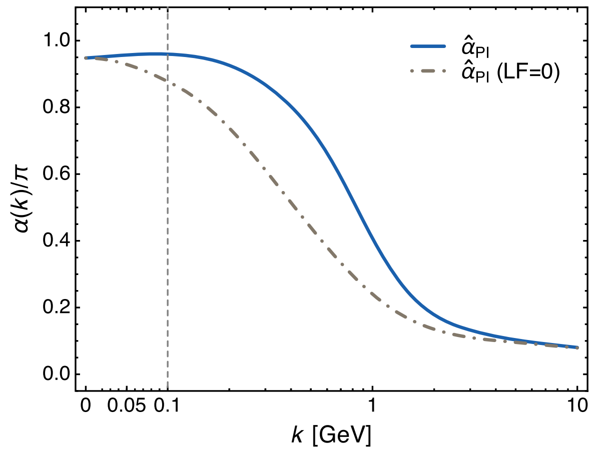

FIG. 1. Solid (blue) curve, complete effective charge in Eq. (7); and dot-dashed (black) curve, Taylor-scheme effective charge, i.e. computed in the absence of crucial pieces of the gluon-ghost vacuum polarisation [$LF \equiv 0$ in Eq. (1c)]. The k-axis scale is linear to the left of the vertical line and logarithmic otherwise, an artifice which enables us to show saturation of the effective charge.

It is worth highlighting some important features of the effective charge in Fig. 1. First, it is a parameter-free prediction: the curve is completely determined by results obtained for the gluon and ghost two-point functions using continuum and lattice-regularized QCD. Second, it is physical, in the sense that there is no Landau pole, and it saturates in the IR: $\hat{\alpha}_{\mathrm{PI}}(k^2 = 0) = \alpha_0 \approx 0.9\pi$, i.e. the coupling possesses an infrared fixed point [45]. Third, the prediction is equally concrete and sound at all spacelike momenta, connecting the IR and UV domains, and precisely reproducing the known behavior of the Taylor coupling at large k^2 [33–35], with no need for an *ad hoc* "matching procedure," such as that employed in models [46]. Finally, our result is essentially nonperturbative, obtained by combining self-consistent solutions of gauge-sector gap equations with lattice simulations, augmented only by a physical procedure for setting a single mass-scale [37]. There are indications [47–49] that the effective charge in Fig. 1 could prove useful in developing a modern dynamical perturbation theory [50].

It is evident in Fig. 1 that ghost-gluon interactions are critical. The RGI product LF in Eq. (1c) expresses effects of gluon-ghost scattering that are essential to ensuring $\hat{\alpha}_{\mathrm{PI}}$ is process-independent. It is also quantitatively important, introducing a roughly 60% enhancement of $\hat{\alpha}_{\mathrm{PI}}(k^2)$ for $k \simeq m_0$. It must also, therefore, be physically significant because the strength of the running-coupling at IR momenta determines the magnitude of dynamical chiral symmetry breaking (DCSB) [36,37,44]; and DCSB is a crucial emergent phenomenon in QCD, possibly inseparable from confinement in the unquenched theory [51], i.e. when dynamical light quarks are active.

IV. COMPARISON OF EFFECTIVE CHARGES

Another approach to determining an "effective charge" in QCD was introduced in Ref. [52]. This is a process-dependent procedure; namely, an effective running coupling is defined to be completely fixed by the leading-order term in the perturbative expansion of a given observable in terms of the canonical running coupling. An obvious difficulty, or perhaps drawback, of such a scheme is the process-dependence itself. Naturally, effective charges from different observables can in principle be algebraically connected to each other via an expansion of one coupling in terms of the other. However, any such expansion contains infinitely many terms [46]; and this connection does not imbue a given process-dependent charge with the ability to predict any other observable, since the expansion is only defined *a posteriori*, i.e. after both effective charges are independently constructed.

One such process-dependent effective charge is $\alpha_{g_1}(k^2)$, which is defined via the Bjorken sum rule [53,54]:

$$\int_0^1 dx[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}\left[1 - \frac{1}{\pi}\alpha_{g_1}(k^2)\right], \qquad (8)$$

where $g_1^{p,n}$ are the spin-dependent proton and neutron structure functions, whose extraction requires measurements using polarized targets, and g_A is the nucleon isovector axial-charge [55]. The merits of this definition are outlined in Ref. [46]. They include the existence of data for a wide range of k^2 [56–81]; tight sum-rules constraints on the behavior of the integral at the IR and UV extremes of k^2; and the isospin nonsinglet feature of the difference, which suppresses contributions from numerous processes that are hard to compute and hence might muddy interpretation of the integral in terms of an effective charge.

The world's data on the process-dependent effective charge $\alpha_{g_1}(k^2)$ are depicted in Fig. 2 and therein compared with our prediction for the process-independent RGI running coupling $\hat{\alpha}_{\mathrm{PI}}(k^2)$. Owing to asymptotic freedom, all reasonable definitions of a QCD effective charge must agree on $k^2 \gtrsim 1\ \mathrm{GeV}^2$ and our approach guarantees this connection. To be specific, in terms of the widely-used $\overline{\mathrm{MS}}$ running-coupling [3]:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)(1 + 1.14\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots), \qquad (9a)$$

$$\hat{\alpha}_{\mathrm{PI}}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)(1 + 1.09\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots), \qquad (9b)$$

where Eq. (9a) may be built from, e.g. Refs. [82,83].

Significantly, there is also near precise agreement with data on the IR domain, $k^2 \lesssim m_0^2$, and complete accord on $k^2 \geq m_0^2$. Figure 1 makes plain that any agreement on $k^2 \in [0.01, 1]\ \mathrm{GeV}^2$ is nontrivial because ghost-gluon interactions produce as much as 40% of $\hat{\alpha}_{\mathrm{PI}}(k^2)$ on this domain: if these effects were omitted from the gluon vacuum polarization, then α_{g_1} and $\hat{\alpha}_{\mathrm{PI}}$ would differ by roughly a factor of two on the critical domain of transition between strong and perturbative QCD.

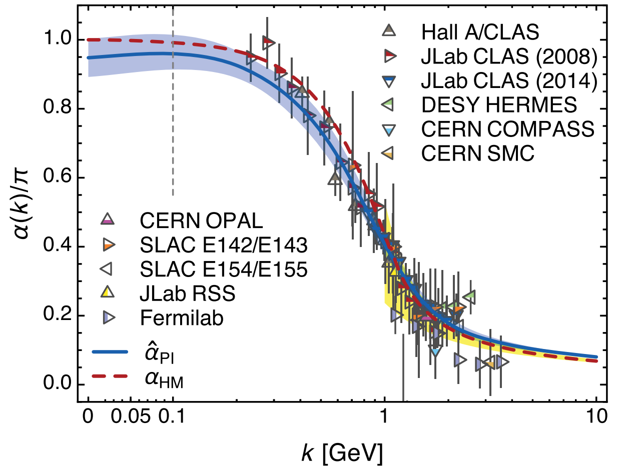

FIG. 2. Solid (blue) curve: predicted process-independent RGI running coupling $\hat{\alpha}_{\mathrm{PI}}(k^2)$, Eq. (7). The shaded (blue) band bracketing this curve combines a 95% confidence-level window based on existing lattice-QCD results for the gluon two-point function with an error of 10% in the continuum extraction of the RGI product LF in Eqs. (1). World data on α_{g_1} [56–81]. The shaded (yellow) band on $k > 1$ GeV represents α_{g_1} obtained from the Bjorken sum by using QCD evolution [84–86] to extrapolate high-k^2 data into the depicted region, following Refs. [56,57]; and, for additional context, the dashed (red) curve is the light-front holographic model of α_{g_1} canvassed in Ref. [46].

At this point we would like to mention that other studies have considered quantities which are related, in one way or another, to the effective charge, $\hat{\alpha}_{\mathrm{PI}}(k^2)$, depicted in Figs. 1 and 2. Pertinent examples are described in Refs. [87,88], which arrive at couplings with far-IR values of $(1.3 - 1.9)\pi$ and $(1.1 - 1.6)\pi$, respectively. Notably, the former employed quenched lattice results for the gluon two-point function, $\Delta_{\mathrm{F}}(k^2)$ in Eq. (4), and both used a range of estimates for the gluon mass-scale based on then-contemporary phenomenology. Those elements explain the differences between the IR saturation values in Refs. [87,88] and our final result: $\hat{\alpha}_{\mathrm{PI}}(0) = (0.9 \pm 0.1)\pi$, which is obtained using modern unquenched lattice results for the gluon.

Of equal or greater importance is the pointwise behavior of those charges, i.e. their *running*. Ref. [88] set $L \equiv 0$ in Eq. (1c) and so ignored material contributions from ghost-gluon dynamics, whose importance we have repeatedly emphasized. Furthermore, both Refs. [87,88] assumed that the effect of the gluon vacuum polarization is completely expressed by writing $\mathcal{D}(k^2) = 1/[k^2 + m^2(k^2)]$, with $m^2(k^2)$ monotonically decreasing from its maximum value at $k^2 = 0$; whereas, in reality, $\mathcal{D}(k^2) = 1/[J(k^2)k^2 + m^2(k^2)]$ on $k^2 \lesssim 2\ \mathrm{GeV}^2$, with $k^2 J(k^2)$ initially negative at far-IR momenta before turning to approach its perturbative form, which is reached on $k^2 \gtrsim 2\ \mathrm{GeV}^2$ [37]. The charges in Refs. [87,88] therefore omit effects which are crucial to obtaining a sound prediction for the running of the process-independent effective charge $\hat{\alpha}_{\mathrm{PI}}(k^2)$: in fact, they much overestimate the charge on $k^2 \lesssim 2\ \mathrm{GeV}^2$.

It is also worth highlighting that Refs. [33,34] focus solely on the Taylor coupling, which, as seen readily using Eqs. (1), (7), is only indirectly related to $\hat{\alpha}_{\mathrm{PI}}(k^2)$:

$$\hat{\alpha}_{\mathrm{PI}}(k^2) = \frac{1}{k^2\mathcal{D}(k^2)}\frac{\alpha_{\mathrm{T}}(k^2)}{[1-L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}. \quad (10)$$

Hence, a comparison is not meaningful.

V. CONCLUSIONS

We have defined and calculated a process-independent running coupling for QCD, $\hat{\alpha}_{\mathrm{PI}}(k^2)$ [Eq. (7), Fig. 1]. This is a new type of effective charge, which is an analogue of the Gell-Mann–Low effective coupling in QED, being completely determined by the gauge-boson two-point function. Our prediction for $\hat{\alpha}_{\mathrm{PI}}(k^2)$ is parameter-free, being obtained by combining the self-consistent solution of a set of Dyson-Schwinger equations with results from lattice-QCD; and it smoothly unifies the nonperturbative and perturbative domains of the strong-interaction theory. This process-independent running coupling is known to unify a vast array of observables, e.g. the pion mass and decay constant, and the light meson spectrum [89]; the parton distribution amplitudes of light- and heavy-mesons [90–92], associated elastic and transition form factors [93,94], etc.

Finally, and perhaps surprisingly at first sight, $\hat{\alpha}_{\mathrm{PI}}(k^2)$ is almost pointwise identical at infrared momenta to the process-dependent effective charge, α_{g_1}, defined via the Bjorken sum rule, one of the most basic constraints on our knowledge of nucleon spin structure, and in complete agreement on the domain of perturbative momenta [Fig. 2]. Equivalence on the perturbative domain is guaranteed for any two reasonable definitions of QCD's effective charge, but here the subleading terms differ by just 4% [Eqs. (9)]. An excellent match at infrared momenta, i.e. below the scale at which perturbation theory would locate the Landau pole, is nontrivial; and crucial to this agreement is the careful treatment and incorporation of a special class of gluon-ghost scattering effects. One is naturally compelled to ask how these two apparently unrelated definitions of a QCD effective charge can be so similar? We attribute this outcome to a physically useful feature of the Bjorken sum rule, *viz.* it is an isospin nonsinglet relation and hence contributions from many hard-to-compute processes are suppressed, and these same processes are omitted in our computation of $\hat{\alpha}_{\mathrm{PI}}(k^2)$.

The analysis herein unifies two vastly different approaches to understanding the infrared behavior of QCD, one essentially phenomenological and the other deliberately computational, embedded within QCD. There is no Landau pole in our predicted running-coupling. In fact, there is an inflection point at $\sqrt{k^2} = 0.7$ GeV, marking a transition wall at which, as momenta decreasing from the ultraviolet promote growth in the coupling, that coupling turns away from the Landau pole, the growth slows, and finally the coupling saturates: $\hat{\alpha}_{\mathrm{PI}}(k^2 = 0) \approx 0.9\pi$ [Fig. 2]. This unification identifies the Bjorken sum rule as a near direct means by which to gain empirical insight into a QCD analogue of the Gell-Mann–Low effective charge.

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