Hidden-charm decays of $Y(4390)$ in a hadronic molecular scenario

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In the present work, the hidden-charm decays of $Y(4390)$ are investigated in a $D^*\bar{D}_1 + \mathrm{H.c.}$ molecular scenario. We find in this frame the observation of the $Y(4390)$ in the $e^+e^- \to \pi^+\pi^- h_c$ process and the absence of this state in the $e^+e^- \to \pi^+\pi^- J/\psi$ process are very natural. The partial width of $Y(4390) \to \pi^+\pi^- h_c$ could reach up to 1.26 MeV, which is large enough to be observed. The result also indicates that the partial widths of $Y(4390) \to \eta J/\psi$ and $Y(4390) \to \eta h_c$ are of the same order of magnitude as the one of $Y(4390) \to \pi^+\pi^- h_c$, which could be tested by the precise measurements at BES III and Belle II.

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I. INTRODUCTION

The electron-positron annihilation process is an unique platform of observing vector charmonia and charmonium-like states. In this process, a number of charmonium-like states have been observed, such as $Y(4008)$ [1], $Y(4220)$ [2], $Y(4260)$ [1,3], $Y(4360)$ [4,5], $Y(4630)$ [6], and $Y(4660)$ [5]. This is not the end of the ever-lengthening particle list. Recently, the BESIII Collaboration reported their precise measurements of the cross-sections for the $e^+e^- \to \pi^+\pi^- J/\psi$ [7] and $e^+e^- \to \pi^+\pi^- h_c$ [8] with the center-of-mass energy up to 4.6 GeV, where three charmonium-like state, $Y(4220)$, $Y(4320)$ and $Y(4390)$, were observed. $Y(4220)$ were reported in both $e^+e^- \to \pi^+\pi^- J/\psi$ and $e^+e^- \to \pi^+\pi^- h_c$ processes [7,8], and its resonance parameters are also consistent with those of the one observed in the $e^+e^- \to \omega\chi_{c0}$ [2]. The charmonium-like states $Y(4320)$ and $Y(4390)$ were observed for the first time, and the former one was reported in the cross-sections for the $e^+e^- \to \pi^+\pi^- J/\psi$ [8], while the later one was only observed in the $e^+e^- \to \pi^+\pi^- h_c$ process [8].

The observations of the vector charmonium-like states stimulate theorist's great interest in their intrinsic structures. Taking the long-standing $Y(4260)$ as an example, the mass of this state is far away from the relativistic quark model [9], and more importantly, this state is only observed in the cross-sections for the $e^+e^- \to \pi^+\pi^- J/\psi$ [1,3], while in the open charm channels [10–13] and R-value scan [14–19] there is no signal of this states. Due to the particular property of $Y(4260)$, some exotic interpretations are proposed. In Ref. [20], the author found that the charmonium and molecular interpretations were not supported by their calculation, and a hybrid charmonium interpretation could explain the decay property of the $Y(4260)$ [21]. In Ref. [22], $Y(4260)$ was proposed to be the first orbital excitation of the diquark-antidiquark $[cs][\bar{c}\,\bar{s}]$ state. While the relativistic quark model estimation indicated that the P wave diquark-antidiquark $[cq][\bar{c}\,\bar{q}]$ state could be a more natural explanation of $Y(4260)$ [23] and in such a kind of picture, the radiative transition between $Y(4260)$ and $X(3872)$ was investigated [24]. The nonresonance interpretations in Refs. [25–27] indicated that the line shape of the cross-sections for $e^+e^- \to \pi^+\pi^- J/\psi$ could be reproduced by the interferences of the nearby charmonia. However, the recent measurement of $Y(4260)$ in the cross-sections for $e^+e^- \to \pi^+\pi^- h_c$ [8] challenged such kind of interference picture.

In addition, it should be noticed that the mass of the $Y(4260)$ is 4251 ± 9 MeV, which is about 40 MeV below the $D_1\bar{D}$ threshold, which indicates that the $Y(4260)$ could be a molecular state composed of $D_1\bar{D} + \mathrm{H.c.}$ The meson exchange model calculations in Ref. [28] found that $Y(4260)$ could be accommodated as a $D_1\bar{D} + \mathrm{H.c.}$ molecule. The estimation of the chiral quark model also supported the $D_1\bar{D} + \mathrm{H.c.}$ molecular interpretation of $Y(4260)$ [29]. The investigations of the decays and productions of the $Y(4260)$ in the molecular scenario are also in line with the corresponding experimental data [30–32].

As for the newly observed $Y(4390)$, it is observed in the $e^+e^- \to \pi^+\pi^- h_c$ process [8]. The resonance parameters of $Y(4390)$ are presented in Table I and those for $Y(4260)$ are also listed for a comparison. The widths of $Y(4260)$ and $Y(4390)$ are very similar and their masses satisfy,

$$m_{Y(4390)} - m_{Y(4260)} \simeq m_{D^*} - m_D \simeq 140 \text{ MeV}.$$

Such kinds of similarities indicate that $Y(4390)$ could be a $D^*\bar{D}_1 + \mathrm{H.c.}$ molecule, which is a counterpart of the

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TABLE I. A comparison of the resonance parameters of $Y(4260)$ and $Y(4360)$.

State	Mass (MeV)	Width (MeV)
$Y(4260)$ [34]	4251 ± 9	120 ± 12
$Y(4390)$ [8]	$4391.6^{+6.3}_{-6.8} \pm 1.0$	$139.5^{+16.2}_{-20.6} \pm 0.6$

$Y(4260)$. In Ref. [33], we calculate the potential of the $D_1\bar{D}^*$ system in a one-boson-exchange model. By solving corresponding quasipotential Bethe-Salpeter equation, we find a bound state solution, which well corresponds to the $Y(4390)$.

To further test the molecular interpretations of the $Y(4390)$, we should answer the question why this state is only observed in $e^+e^- \to \pi^+\pi^- h_c$ process, which should be suppressed due to the spin flip of the heavy quark, while in the spin conserved process, $e^+e^- \to \pi^+\pi^- J/\psi$, this state is absent. In the present work, we further investigate the hidden-charm decays of the $Y(4390)$ in a $D_1\bar{D}^*$ molecular scenario, especially the decay process $Y(4390) \to h_c\pi^+\pi^-$, which could be a crucial test of the molecular assignment.

This work is organized as follows. The hidden-charm decays are presented in the following section. The numerical results and discussions are given in Sec. III and Sec. IV is devoted to a summary.

II. HIDDEN-CHARM DECAYS OF $Y(4390)$

Before the discussion of $Y(4390)$ decay, we recall another two charmonium-like but charged states $Z_c(3900)$ and $Z_c(4020)$ (thereafter we denote these two states as Z_c and Z_c'), which would help us to understand the observation of $Y(4390)$. Z_c was first reported in the $J/\psi\pi$ invariant mass spectrum of the $e^+e^- \to \pi^+\pi^- J/\psi$ [35,36], and then in the $D\bar{D}^*$ invariant mass spectrum of the $e^+e^- \to \pi D^* D$ process [37]. As a counterpart of the Z_c, Z_c' was successively observed in the $h_c\pi$ invariant mass spectrum of $e^+e^- \to \pi^+\pi^- h_c$ process and $D^*\bar{D}^*$ invariant mass spectrum of $e^+e^- \to \pi D^*\bar{D}^*$ [38,39]. The observed mass of the Z_c and Z_c' are very close to the thresholds of $D\bar{D}^*$ and $D^*\bar{D}^*$, respectively. The resonance parameters and decay behaviors indicated that Z_c and Z_c' could be the molecular state composed of $D\bar{D}^* + \mathrm{H.c.}$ and $D^*\bar{D}^*$, respectively [40–43]. In the molecular scenario, the flavor wave functions of the Z_c and Z_c' are,

$$|Z_c\rangle = \frac{1}{\sqrt{2}}[D\bar{D}^* + \bar{D}D^*], |Z_c'\rangle = D^*\bar{D}^*, \tag{1}$$

respectively.

As for $Y(4390)$, it was reported in the $e^+e^- \to \pi^+\pi^- h_c$, which is a spin flipped process. However, this structure is absent in the spin conserved process, $e^+e^- \to \pi^+\pi^- J/\psi$, This particular phenomena could be understood qualitatively in the $D_1\bar{D}^* + \mathrm{H.c.}$ molecular scenario. The molecular state decays could occur via quark rearrangement, and in the hadronic level, the molecular components could couple to the final state via exchanging a proper meson [44–46]. As for $Y(4390)$, it could transit into a charmonium by exchanging charmed mesons. As shown in Fig. 1(a) and (b), the molecular state $Y(4390)$ couples to its components $D_1\bar{D}^* + \mathrm{H.c.}$ via S wave, and the $D_1/\bar{D}_1$ transits into $D^*/\bar{D}^*$ via the emission of pion and the $D^*/\bar{D}^*$ and $\bar{D}^*/D^*$ in the molecule could only couple to Z_c' as indicated by the wave functions of the Z_c and Z_c' in Eq. (1). All these three vertexes in the triangle diagrams are $S-$wave coupling, and furthermore, Z_c' has large branching ratio to $h_c\pi$, but can not decay into $J/\psi\pi$. Thus, Z_c' plays principle role in understanding both the observed decay mode $\pi^+\pi^- h_c$ and the unobserved mode $\pi^+\pi^- J/\psi$. Besides the observed channel, we also notice the measurements of the cross-sections for $e^+e^- \to \eta J/\psi$ and $e^+e^- \to \eta h_c$ in the vicinity of 4.39 GeV [47,48]. Thus, searching for the signal of $Y(4390)$ in these final states is also interesting. The corresponding diagrams contribute to $Y(4390) \to \eta(\eta')J/\psi$ and ηh_c are presented in Fig. 1(c) and (d), respectively.

In the $D_1\bar{D}^* + \mathrm{H.c.}$ molecular scenario, $Y(4390)$ should couple to its components dominantly via S wave, and the corresponding effective Lagrangian is in the form,

$$\mathcal{L} = \frac{g_Y}{\sqrt{2}}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu Y^\nu (D_1^\alpha \bar{D}^{*\beta} + \bar{D}_1^\alpha D^{*\beta}), \tag{2}$$

where Y and D_1, D^* indicate $Y(4390)$, $D_1(2420)$, and $D^*(2010)$, respectively.

Besides the above effective Lagrangian, the following effective coupling between the charmonium/light mesons and charmed mesons are also involved in our present estimations,

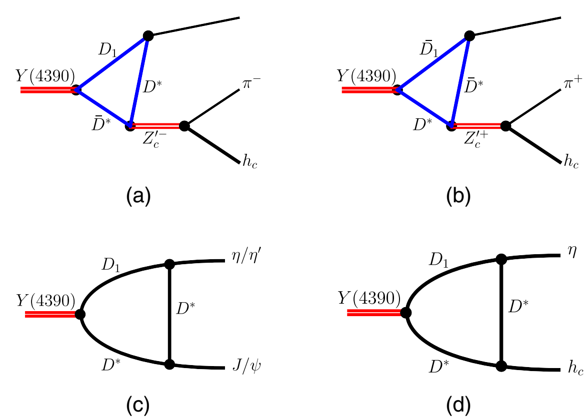

FIG. 1. Diagrams contribute to the considered hidden-charm decays. Diagrams $(a)-(b)$ indicate $Y(4390) \to \pi^+\pi^- h_c$ process via Z_c'. Diagrams (c) and (d) are correspond to the η/η' transitions from $Y(4390)$ to J/ψ and h_c, respectively.

$$\mathcal{L}_{J/\psi\mathcal{D}^*\mathcal{D}^*} = ig_{J/\psi\mathcal{D}^*\mathcal{D}^*}\psi^\mu(\mathcal{D}^*_\nu\partial^\nu\mathcal{D}^{*\dagger}_\mu - \partial^\nu\mathcal{D}^*_\mu\mathcal{D}^{*\dagger}_\nu - \mathcal{D}^*_\nu\overset{\leftrightarrow}{\partial}_\mu\mathcal{D}^{*\nu\dagger}),$$
$$\mathcal{L}_{h_c\mathcal{D}^*\mathcal{D}^*} = ig_{h_c\mathcal{D}^*\mathcal{D}^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu h_{c\nu}\mathcal{D}^*_\alpha\mathcal{D}^{*\dagger}_\beta,$$
$$\mathcal{L}_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}} = g_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}}(3\mathcal{D}^\mu_1(\partial^\mu\partial^\nu\mathcal{P})\mathcal{D}^{*\nu\dagger} - \mathcal{D}^\mu_1(\partial^\nu\partial_\nu\mathcal{P})\mathcal{D}^{*\dagger}_\mu), \tag{3}$$

which can be constructed by the heavy quark limit and chiral symmetry [49–51]. $\mathcal{P}$ is the matrix form of the pseudoscalar mesons, which is,

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & K^0 \\ K^- & \bar{K}^0 & \gamma\eta + \delta\eta' \end{pmatrix}, \tag{4}$$

where $\alpha = (\cos\theta - \sqrt{2}\sin\theta)/\sqrt{6}$, $\beta = (\sin\theta + \sqrt{2}\cos\theta)/\sqrt{6}$, $\gamma = (-2\cos\theta - \sqrt{2}\sin\theta)/\sqrt{6}$, $\delta = (-2\sin\theta + \sqrt{2}\cos\theta)/\sqrt{6}$. According to the analyses in Refs. [52,53], we take the mixing angle $\theta = -19.1°$.

The effective Lagrangians related to Z'_c should dependent on its nature, which has not yet been ascertained. Here, we adopt the coupling of $Z'_cD^*D^*$ to be the same as the one of $h_cD^*D^*$ since the J^P quantum numbers of the Z'_c are the same as those of h_c. In addition, Z'_c could couple to $h_c\pi$ via a P wave. Thus the Z'_c related effective Lagrangians are,

$$\mathcal{L}_{Z'_cD^*D^*} = ig_{h_c\mathcal{D}^*\mathcal{D}^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu h_{c\nu}\mathcal{D}^*_\alpha\mathcal{D}^{*\dagger}_\beta,$$
$$\mathcal{L}_{Z'_ch_c\pi} = g_{Z'_ch_c\pi}\varepsilon_{\mu\nu\alpha\beta}Z'^\mu_c\partial^\nu h^\alpha_c\partial^\beta\pi. \tag{5}$$

The coupling constants in the Lagrangians as shown in Eqs. (2), (3), and (5) will be discussed in the following section.

With above effective Lagrangians, we can obtain the amplitudes of $Y(4390)(p_0) \rightarrowtail [D_1(p_1)\bar{D}^*(p_2)]D^*(q) \rightarrowtail \pi^+(p_3)\pi^-(p_4)h_c(p_5)$ corresponding to Fig. 1(a), which is,

$$\begin{aligned}\mathcal{M}_a =& \int\frac{d^4q}{(2\pi)^4}\left[\frac{g_Y}{\sqrt{2}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu_Y(-ip^\mu_0)\right][g_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}}(3(ip_3\rho)(ip_{3\lambda})\\ &-(ip_3)^2g_{\rho\lambda})][ig_{Z'_cD^*D^*}\varepsilon_{\theta\phi\delta\tau}(ip^\theta_4 + ip^\theta_5)]\\ &\times[g_{Z'_ch_c\pi}\varepsilon_{\gamma\eta\kappa\xi}(ip^\eta_5)(ip^\xi_4)\epsilon^\kappa_{h_c}]\frac{-g^{\alpha\rho} + p^\alpha_1p^\rho_1/m^2_{D_1}}{p^2_1 - m^2_{D_1}}\\ &\times\frac{-g^{\beta\tau} + p^\beta_2p^\tau_2/m^2_{D^*}}{p^2_2 - m^2_{D^*}}\frac{-g^{\delta\lambda} + q^\delta q^\lambda/m^2_{D^*}}{q^2 - m^2_{D^*}}\\ &\times\frac{-g^{\phi\gamma} + (p^\gamma_4 + p^\gamma_5)(p^\phi_4 + p^\phi_5)/m^2_{Z'_c}}{(p4 + p5)^2 - m^2_{Z'_c}}\mathcal{F}^2(q^2, m^2_{D^*}).\end{aligned} \tag{6}$$

As the reflection of Fig. 1(a), the amplitude corresponding to Fig. 1(b) could be obtained from $\mathcal{M}_a$ by performing the following replacement,

$$\mathcal{M}_b = \mathcal{M}_a|_{p_3\leftrightarrow p_4}, \tag{7}$$

Besides the observed channel $\pi^+\pi^-h_c$, $Y(4390)$ could decay into $\eta(\eta')J/\psi$ and ηh_c, and the amplitudes corresponding to Fig. 1(c) and (d) are,

$$\begin{aligned}\mathcal{M}_c =& \int\frac{d^4q}{(2\pi)^4}\left[\frac{g_Y}{\sqrt{2}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu_Y(-ip^\mu_0)\right][g_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}}(3(ip_{3\rho})(ip_{3\lambda})\\ &\times-(ip_3)^2g_{\rho\lambda})][ig_{J/\psi D^*D^*}\epsilon^\theta_{J/\psi}(-ip_{2\delta}g_{\theta\tau} + iq_\tau g_{\theta\delta}\\ &+(ip_{2\theta} - iq_\theta)g_{\delta\tau})]\frac{-g^{\alpha\rho} + p^\alpha_1p^\rho_1/m^2_{D_1}}{p^2_1 - m^2_{D_1}}\\ &\times\frac{-g^{\beta\tau} + p^\beta_2p^\tau_2/m^2_{D^*}}{p^2_2 - m^2_{D^*}}\frac{-g^{\delta\lambda} + q^\delta q^\lambda/m^2_{D^*}}{q^2 - m^2_{D^*}}\mathcal{F}^2(q^2, m^2_{D^*}),\\ \mathcal{M}_d =& \int\frac{d^4q}{(2\pi)^4}\left[\frac{g_Y}{\sqrt{2}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu_Y(-ip^\mu_0)\right][g_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}}(3(ip_{3\rho})(ip_{3\lambda})\\ &\times-(ip_3)^2g_{\rho\lambda})][ig_{h_cD^*D^*}\varepsilon_{\theta\phi\delta\tau}\epsilon^\phi_{h_c}(ip^\theta_4 + ip^\theta_5)]\\ &\times\frac{-g^{\alpha\rho} + p^\alpha_1p^\rho_1/m^2_{D_1}}{p^2_1 - m^2_{D_1}}\frac{-g^{\beta\tau} + p^\beta_2p^\tau_2/m^2_{D^*}}{p^2_2 - m^2_{D^*}}\\ &\times\frac{-g^{\delta\lambda} + q^\delta q^\lambda/m^2_{D^*}}{q^2 - m^2_{D^*}}\mathcal{F}^2(q^2, m^2_{D^*}),\end{aligned} \tag{8}$$

where $\mathcal{F}(q^2, m^2_{D^*})$ is a form factor introduced to depict the structure effect of the interaction vertexes and offshellness of the exchanged D^* meson. In the present estimation, we take,

$$\mathcal{F}(q^2, m^2_{D^*}) = \frac{m^2_{D^*} - \Lambda^2}{q^2 - \Lambda^2}, \tag{9}$$

where the parameter Λ can be further reparameterized as $\Lambda = \alpha\Lambda_{\mathrm{QCD}} + m_{D^*}$ with $\Lambda_{\mathrm{QCD}} = 220$ MeV. The dimensionless parameter α should be of order one, since Λ should not be far way from the mass of the exchanged meson [54].

The total amplitudes of $Y(4390) \to \pi^+\pi^-h_c$ is,

$$\mathcal{A}^{\mathrm{Tot}}_{\pi^+\pi^-h_c} = \mathcal{M}_a + \mathcal{M}_b, \tag{10}$$

and the differential partial width of $Y(4390) \to \pi^+\pi^-h_c$ reads

$$d\Gamma_{\pi^+\pi^-h_c} = \frac{1}{(2\pi)^3}\frac{1}{32m^3_Y}\overline{|A^{\mathrm{Tot}}_{\pi^+\pi^-h_c}|^2}dm^2_{h_c\pi}dm^2_{\pi\pi}, \tag{11}$$

where the overline above indicates the sum over the spin of the final states and the average of spin of $Y(4390)$.

As for $Y(4390) \to \eta(\eta')J/\psi$ and $Y(4390) \to \eta h_c$, the differential decay width is,

$$d\Gamma = \frac{1}{32\pi^2}\overline{|\mathcal{A}_{\mathrm{Tot}}|^2}\frac{|\vec{p_f}|}{m^2_Y}d\Omega, \tag{12}$$

where $\mathcal{A}_{\mathrm{Tot}} = 2\mathcal{M}_c$ and $\mathcal{A}_{\mathrm{Tot}} = 2\mathcal{M}_d$ for $\eta(\eta')J/\psi$ mode and ηh_c mode, respectively. $\vec{p_f}$ is the momentum of the final states in the initial rest frame.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Coupling constants

Before we estimate the partial width of the considered processes, the involved coupling constants should be clarified. In the nonrelativistic limit, the coupling of $Y(4390)$ and its components in Eq. (2) should be

$$\mathcal{L} = \frac{g_Y}{\sqrt{2}} m_Y \epsilon_{ijk} Y^i (D_1^j \bar{D}^{*k} + \bar{D}_1^j D^{*k}), \tag{13}$$

and the coupling constants g_Y could be related to the probability of $D_1\bar{D}^* + \mathrm{H.c.}$ component in $Y(4390)$ denoted by c and the binding energy E_b by [55–57]

$$g_Y^2 \simeq 16\pi \frac{(m_{D^*} + m_{D_1})^2}{m_Y^2} c^2 \sqrt{\frac{2E_b}{\mu}}, \tag{14}$$

with $\mu = m_{D^*} m_{D_1}/(m_{D_1} + m_{D^*})$ is the reduced mass. In the molecular scenario, $Y(4390)$ is a pure $D_1\bar{D}^* + \mathrm{H.c.}$ molecular state, thus $c = 1$. With above relationship, the coupling constants g_Y could be estimated to be 3.51–3.84, where the uncertainty comes from the measurement error of the mass of $Y(4390)$ [8].

As a cross-check of the reliability of the nonrelativistic limit, we also estimated the coupling constants by using the compositeness condition with a local interaction, which has been widely used to discuss the molecular decays [44,45,58,59]. In the approach, the local interaction is described by a correlation function with a model parameter Λ, which should be of 1 GeV. When taking $\Lambda = 1$ GeV, the coupling constant g_Y is estimated to be 3.54–4.13, which is consistent with the one evaluated in nonrelativistic limit.

In the heavy-quark limit and chiral symmetry, the coupling constants relevant to the effective Lagrangians in Eq. (3) could related to some gauge coupling by [49–51],

$$\begin{aligned} g_{J/\psi \mathcal{D}^* \mathcal{D}^*} &= \frac{m_D}{m_{\mathcal{D}^*}} \frac{m_{J/\psi}}{f_{J/\psi}}, \\ g_{h_c \mathcal{D}^* \mathcal{D}^*} &= 2 g_1 \frac{m_{\mathcal{D}^*}}{\sqrt{m_{h_c}}}, \\ g_{\mathcal{D}_1 \mathcal{D}^* \mathcal{P}} &= -\frac{\sqrt{6}}{3} \frac{h_1 + h_2}{\Lambda_\chi f_\pi} \sqrt{m_{\mathcal{D}_1} m_{\mathcal{D}^*}}, \end{aligned} \tag{15}$$

where $f_{J/\psi} = 416$ MeV is the decay constants of the J/ψ, which could be estimated from the leptonic width of J/ψ. The gauge coupling g_1 can relate to the decay constant of χ_{c0} via $g_1 = -\sqrt{m_{\chi_c 0}/3}/f_{\chi_{c0}}$ with $f_{\chi_{c0}} = 510$ MeV [60]. $\Lambda_\chi \simeq 1$ GeV and $f_\pi = 132$ MeV are the chiral symmetry breaking scale and the decay constants of pion. The coupling constant $h' = h_1 + h_2$ is estimated to be 0.65 in a constituent quark-meson model [61].

To date, the nature of the Z'_c keeps unknown. Here we could estimate the effective coupling constants in Eq. (5) based on the limited experimental measurements. The cross section for $e^+e^- \to \pi^\pm Z_c'^\mp \to \pi^+\pi^- h_c$ is reported to be $(7.4 \pm 1.7 \pm 2.1 \pm 1.2)$ pb at 4.26 GeV [38]. In the same center-of-mass energy, the cross-section for $e^+e^- \to \pi^\pm (D^*\bar{D}^*)^\mp$ is measured to be $(137 \pm 9 \pm 15)$ pb and the ratio $\sigma(e^+e^- \to \pi^\pm Z_c'^\mp \to \pi^\pm (D^*\bar{D}^*)^\mp)/\sigma(e^+e^- \pi^\pm (D^*\bar{D}^*)^\mp)$ is fitted to be $0.65 \pm 0.09 \pm 0.06$ [39]. Thus, the cross-section for $e^+e^- \to \pi^\pm Z_c'^\mp \to \pi^\pm (D^*\bar{D}^*)^\mp$ is estimated to be (89 ± 14) pb, where only the statistical uncertainties are considered. Thus, ratio of the partial widths $\Gamma(Z'_c \to D^*\bar{D}^*)$ and $\Gamma(Z'_c \to h_c\pi)$ could be,

$$\begin{aligned} \frac{\Gamma(Z'_c \to D^*\bar{D}^*)}{\Gamma(Z'_c \to h_c\pi)} &= \frac{\sigma(e^+e^- \to \pi^\pm Z_c'^\mp \to \pi^\pm (D^*\bar{D}^*)^\mp)}{\sigma(e^+e^- \to \pi^\pm Z_c'^\mp \to \pi^+\pi^- h_c)} \\ &= 12.0 \pm 3.3. \end{aligned} \tag{16}$$

Under the approximation that Z'_c dominantly decays into $D^*\bar{D}^*$ and $h_c\pi$, the branching ratio of $Z'_c \to \pi h_c$ could be,

$$\mathcal{B}(Z'_c \to \pi h_c) \simeq (7.52 \pm 1.87)\% \tag{17}$$

The width of Z'_c is 13 ± 5 MeV [34], thus, with the center values of the branching ratio and total width, one can roughly estimate the partial widths of $Z'_c \to D^*\bar{D}^*$ and $Z'_c \to h_c\pi$ are about 12 MeV and 1 MeV, respectively. With the partial widths and the effective Lagrangains in Eq. (5), one can obtain $g_{Z'_c D^* D^*} = 1.08$ and $g_{Z'_c h_c \pi} = 0.65\ \mathrm{GeV}^{-1}$.

B. Decay width

With the above preparations, we could calculate the hidden-charm decay width of $Y(4390)$ in a $D_1\bar{D}^* + H.c$ molecular scenario. In the present calculation, one model parameter α is introduced, which should be of order one. In Fig. 2, the partial width of $Y(4390) \to \pi^+\pi^- h_c$ depending on parameter α is presented. The red solid curve is the estimated results with the center values of the mass of $Y(4390)$. The grey band indicates the uncertainty caused by the measurement error of the mass of $Y(4390)$. In the narrow width approximation, the partial width of the $Y(4390) \to \pi^+\pi^- h_c$ could be proportional to $\Gamma_{Y(4390)\to Z'_c\pi} \mathcal{B}(Z'_c \to h_c\pi)$, which is independent on the width of Z'_c. In the present calculation, we take the uncertainty of $\mathcal{B}(Z'_c \to h_c\pi)$ into consideration, which is indicated by the light grey brands in Fig. 2. Here, we vary α from 2 to 3, and in this parameter range, we find the partial width of $Y(4390) \to \pi^+\pi^- h_c$ very weakly depends on the model parameter. In particular, the center value of the partial width increases from 0.74 to 0.85 MeV, and when considering the mass uncertainties of $Y(4390)$ and the uncertainty of the branching ratio of $Z'_c \to \pi h_c$, this partial width could reach up to 1.26 MeV and the corresponding branching ratio is of order one percent. Such a large branching ratio could answer why this state could be observed in the $\pi^+\pi^- h_c$ mode.

Besides the observed channel, $Y(4390)$ could also transit into J/ψ or h_c by emitting a η/η'. The α dependent partial widths of these hidden-charm decays are presented in

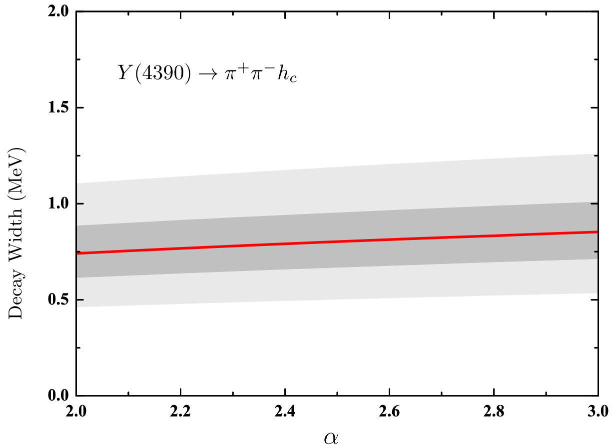

FIG. 2. The α dependences of the partial width of $Y(4390) \to \pi^+\pi^- h_c$.

Fig. 3. The red curves are the results obtained with center value of the mass of $Y(4390)$, while the bands are the uncertainties resulted from the measurement error of the mass of $Y(4390)$. Different from the $\pi^+\pi^- h_c$ mode, the partial width of these two-body hidden-charm decays slightly depend on the model parameter α and very weakly depend on the mass of $Y(4390)$.

As for $Y(4390) \to J/\psi\eta$, the partial width increase from 0.57 MeV to 1.82 MeV when α increasing from 2 to 3. Taking the evaluated center values of $\Gamma(Y(4390) \to \pi^+\pi^- h_c)$, we can obtain the ratio of $\Gamma(Y(4390) \to J/\psi\eta)$ and $\Gamma(Y(4390) \to \pi^+\pi^- h_c)$ to be,

$$\frac{\Gamma(Y(4390) \to J/\psi\eta)}{\Gamma(Y(4390) \to \pi^+\pi^- h_c)} = 0.52\text{–}2.29. \tag{18}$$

The cross section for $e^+e^- \to \pi^+\pi^- h_c$ is reported to be $45.5^{+12.9}_{-9.0}$ pb at the center-of-mass energy of 4.3874 GeV [8] and the cross section for $e^+e^- \to \eta J/\psi$ is measured to be $11.7^{+8.5}_{-5.4} \pm 0.6$ pb at the center-of-mass energy of 4.390 GeV. With the assumption that all the signals at the center-of-mass energy of 4.390 GeV come from

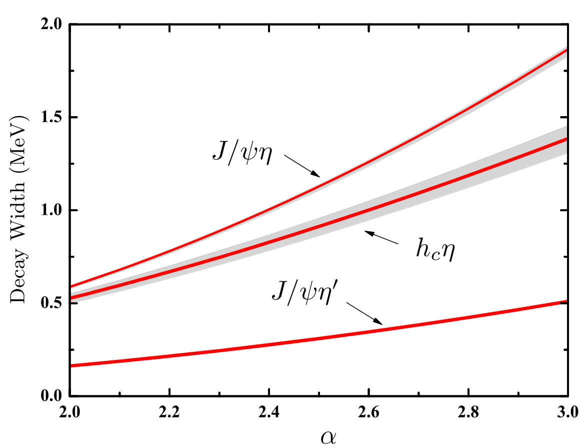

FIG. 3. The partial widths of the hidden-charm decay of $Y(4390)$ depending on the model parameter α.

the resonance contribution of $Y(4390)$, one can find the ratio of the cross sections of $e^+e^- \to \eta J/\psi$ and $e^+e^- \to \pi^+\pi^- h_c$ should be equal to $\Gamma(Y(4390) \to J/\psi\eta)/\Gamma(Y(4390) \to \pi^+\pi^- h_c)$. With the measured cross-sections, we can approximately estimate

$$\frac{\sigma(e^+e^- \to \eta J/\psi)}{\sigma(e^+e^- \to \pi^+\pi^- h_c)} = 0.11\text{–}0.55, \tag{19}$$

which could overlap with the ratio in Eq. (18) estimated in the molecular scenario. As for $Y(4390) \to \eta' J/\psi$, this process is suppressed due to the $\eta - \eta'$ mixing as well as the phase space. The estimated partial width varies from 0.16 MeV to 0.51 MeV, thus the ratio of $\Gamma(Y(4390) \to \eta' J/\psi)$ and $\Gamma(Y(4390) \to \eta J/\psi)$ is estimated to be,

$$\frac{\Gamma(Y(4390) \to \eta' J/\psi)}{\Gamma(Y(4390) \to \eta J/\psi)} = 0.19\text{–}0.28, \tag{20}$$

which could be test by further experimental measurements in the BES III and Belle II.

The partial width of $Y(4390) \to \eta h_c$ is estimated to be 0.50–1.45 MeV in the considered parameter range when taking the measurement error of the mass of $Y(4390)$ into consideration. We can also obtain the following ratios,

$$\frac{\Gamma(Y(4390) \to \eta h_c)}{\Gamma(Y(4390) \to \pi^+\pi^- h_c)} = 0.45\text{–}2.63,$$
$$\frac{\Gamma(Y(4390) \to \eta h_c)}{\Gamma(Y(4390) \to \eta J/\psi)} = 0.80\text{–}0.88. \tag{21}$$

In the cross-sections for $e^+e^- \to \eta h_c$, no significant signal is observed at center-of-mass energy of 4.3874 MeV under the present measurement precision and the upper limit of the cross-section is 26.2 pb. Thus, we can conclude,

$$\frac{\sigma(e^+e^- \to \eta h_c)}{\sigma(e^+e^- \to \pi^+\pi^- h_c)} < 0.71, \frac{\sigma(e^+e^- \to \eta h_c)}{\sigma(e^+e^- \to \pi^+\pi^- h_c)} < 4.15, \tag{22}$$

which are also consistent with our estimated ratios in Eq. (21).

IV. SUMMARY

Recently, the BES III Collaboration reported three charmonium-like states in the cross-sections for $e^+e^- \to \pi^+\pi^- J/\psi$ and $e^+e^- \to \pi^+\pi^- h_c$, which include $Y(4220)$, $Y(4320)$, and $Y(4390)$. These observations make the vector charmonium-like states abundant and stimulate theorists' great interests in the nature of those states. We notice that the mass splitting of the $Y(4260)$ and $Y(4390)$ are very close to the one of D and D^*, which indicates that $Y(4390)$ could be a counterpart of the $Y(4260)$ in the molecular scenario, i.e., they are the molecular states of $D^*\bar{D}_1 +$ H.c. and $D\bar{D}_1 +$ H.c., respectively. The potential model in the Bethe-Salpeter approach also supports such interpretation [33].

To further test the molecular possibility of $Y(4390)$, we investigated its hidden-charm decays in the molecular scenario. We find it is very natural to understand the observations of $Y(4390)$ in the $e^+e^- \to \pi^+\pi^- h_c$ as well as the absence in the $\pi^+\pi^- J/\psi$ mode if we assign $Y(4390)$ as a $D^*\bar{D}_1 + \mathrm{H.c.}$ molecule. The numerical calculations also indicate the branching ratio of $Y(4390) \to \pi^+\pi^- h_c$ should be of order of one percent, which is large enough to be observed.

Some other unobserved hidden-charm decay processes are also considered in the present work, The partial widths of the $Y(4390) \to \eta(\eta')J/\psi$ and $Y(4390) \to \eta h_c$ slightly depend on the model parameter. The estimated $\Gamma(Y(4390) \to \eta J/\psi)$ and $\Gamma(Y(4390) \to \eta h_c)$ are of the same order of magnitude as $\Gamma(Y(4390) \to \pi^+\pi^- h_c)$. We find the ratios of the considered partial widths are consistent with the present experimental measurements, which indicates that the assignment of $Y(4390)$ as a $D^*D_1 + \mathrm{H.c.}$ is possible. The present calculations indicate that the signals of the $Y(4390)$ in the cross-sections for $e^+e^- \to \eta J/\psi$ and $e^+e^- \to \eta h_c$ are significant, which indicates that searching $Y(4390)$ in these processes could be accessible by BES III and Belle II.

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