Twist-4 contributions to semi-inclusive e^+e^- annihilation process

Wei-hua Yang, Kai-bao Chen, and Zuo-tang Liang

School of Physics & Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan, Shandong 250100, China

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We present the complete twist-4 results for the semi-inclusive annihilation process $e^+ + e^- \to h + \bar{q} + X$ at the tree level of perturbative quantum chromodynamics. The calculations are carried out by using the formalism obtained by applying the collinear expansion to this process, where the multiple gluon scattering is taken into account and gauge links are obtained systematically and automatically. We present the results for structure functions in terms of gauge invariant fragmentation functions up to twist-4 and the corresponding results for the azimuthal asymmetries and polarizations of hadrons produced. The results obtained show in particular that, similar to that for semi-inclusive deeply inelastic scattering, for structure functions associated with the sine or cosine of an odd number of azimuthal angle(s), there are only twist-3 contributions, while for those of an even number of azimuthal angle(s), there are leading twist and twist-4 contributions. For all of those structure functions that have leading twist contributions, there are twist-4 addenda to them. Hence, twist-4 contributions may even have large influences on extracting leading twist fragmentation functions from the data. We also suggest a method for a rough estimation of twist-4 contributions based on the leading twist fragmentation functions.

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I. INTRODUCTION

Parton distribution functions (PDFs) and fragmentation functions (FFs) are two important quantities in describing high-energy reactions. When three dimensional, i.e., the transverse momentum dependent (TMD) PDFs and/or FFs are considered, the sensitive quantities studied in experiments are often different azimuthal asymmetries [1–26]. In such cases, higher twist contributions can be very significant and play a very important role in studying these TMD PDFs and/or FFs. Twist-3 contributions to semi-inclusive deeply inelastic scattering (SIDIS) [27–34] and e^+e^--annihilations [35–40] have been extensively calculated in recent years. Results have been given for cross section and different azimuthal asymmetries in terms of gauge invariant PDFs and/or FFs.

In a recent paper, we have carried out the calculations of twist-4 contributions to the SIDIS process $e^-N \to e^-qX$ [41]. The results obtained show a very distinct feature, i.e., while all twist-3 contributions lead to azimuthal asymmetries absent at leading twist, the twist-4 contributions are just addenda to the leading twist asymmetries. We have twist-4 contributions to all of the eight leading twist structure functions corresponding to the eight leading twist TMD PDFs. This implies that studying twist-4 contributions is important not only for itself but also in determining leading twist TMD PDFs. It may leads to significant modifications in extracting leading twist PDFs from experimental data [1–19].

While three-dimensional PDFs are best studied in SIDIS, three-dimensional FFs are best studied in the semi-inclusive process $e^+e^- \to h\bar{q}X$. Moreover, we can study not only the vector polarization dependent FFs but also tensor polarization dependent FFs. In view of the conclusions presented in [38,39], it is natural and important to extend the twist-4 calculations to $e^+e^- \to h\bar{q}X$.

In this paper, we present the twist-4 studies to the semi-inclusive annihilation process $e^+e^- \to h\bar{q}X$. We present the complete calculations at tree level in perturbative quantum chromodynamics (pQCD) and show the results of the structure functions in terms of gauge invariant FFs. We present the results for the unpolarized, the vector polarization dependent, and the tensor polarization dependent parts, respectively. We also present the azimuthal asymmetries and hadron polarizations in terms of the gauge invariant FFs.

The higher twist calculations presented in, e.g., [31–33,37–39,41] benefited very much from the collinear expansion. We found out that in dealing with higher twist effects in a quantum chromodynamics (QCD) parton model for high energy reactions, collinear expansion is indeed extremely important and powerful. It provides not only the correct formalism where the differential cross section or the hadronic tensor is given in terms of gauge invariant PDFs and/or FFs, but also very simplified expressions so that even twist-4 contributions can be calculated. The collinear expansion has been first introduced in 1980 to 1990s and has been applied to inclusive processes [42–45]. It has been shown that it can also be applied to the semi-inclusive DIS process $e^-N \to e^-qX$ [31], and also recently to the inclusive $e^+e^- \to hX$ [37] and semi-inclusive process $e^+e^- \to h\bar{q}X$ [38]. As has been emphasized in [31–33,37–39,41], the collinear expansion is a necessary procedure for obtaining a hadronic tensor in terms of gauge-invariant PDFs and/or

FFs. Moreover, the hard parts after the collinear expansion are not only calculable but also reduced to a form independent of the parton momenta besides some delta-function. Correspondingly, the involved PDFs and/or FFs are not only gauge invariant but are also all defined via a quark-quark or quark-j-gluon-quark correlator with one independent parton momentum. Hence, the Lorentz decomposition of such quark-quark or quark-j-gluon-quark correlator is feasible, and higher twist calculations can be carried out.

The rest of the paper is organized as follows. In Sec. II, we present the formalism of $e^+e^- \to h\bar{q}X$, where we show both the results of the general kinematic analysis and those for the hadronic tensor in QCD parton model after collinear expansion. In Sec. III, we present the results for the hadronic tensor, the structure functions, the azimuthal asymmetries, and the hadron polarizations at the tree level up to twist-4. A short summary and discussion is given in Sec. IV.

II. THE FORMALISM

To be explicit, we consider the semi-inclusive process $e^+e^- \to Z^0 \to h\bar{q}X$ where $\bar{q}$ denotes an antiquark that corresponds to a jet of hadrons in experiments and h denotes the outgoing hadron. The cross section is given by

$$\frac{2E_pE_{k'}d\sigma}{d^3pd^3k'} = \frac{\alpha^2}{8\pi^3 sQ^4}\chi L^{\mu\nu}(l_1,l_2)W^{(si)}_{\mu\nu}(q,p,S,k'). \quad (2.1)$$

Here we use the notations as illustrated in Fig. 1; $\alpha = e^2/4\pi$, $\chi = Q^4/[(Q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2]\sin^4 2\theta_W$, $Q^2=s=q^2$, θ_W is the Weinberg angle, M_Z is the Z-boson mass and Γ_Z is the decay width. The leptonic tensor is given by

$$L_{\mu\nu}(l_1,l_2) = c_1^e(l_{1\mu}l_{2\nu}+l_{1\nu}l_{2\mu}-g_{\mu\nu}l_1\cdot l_2)+ic_3^e\varepsilon_{\mu\nu l_1 l_2}, \quad (2.2)$$

where $c_1^e=(c_V^e)^2+(c_A^e)^2$ and $c_3^e = 2c_V^ec_A^e$; c_V^e and c_A^e are defined in the weak interaction current $J_\mu(x) = \bar{\psi}(x)\Gamma_\mu\psi(x)$ and $\Gamma_\mu = \gamma_\mu(c_V^e - c_A^e\gamma^5)$. Similar notations are also used for quarks where we use a superscript q to replace e. We use also the shorthand notations such as $\varepsilon_{\mu\nu AB}\equiv\varepsilon_{\mu\nu\alpha\beta}A^\alpha B^\beta$. The hadronic tensor is defined as

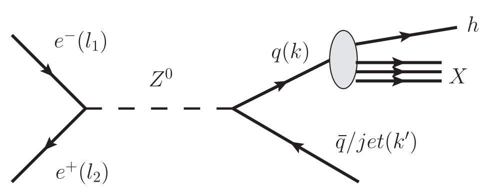

FIG. 1. Illustrating diagram for $e^+ + e^- \to h + \bar{q} + X$.

$$W^{(si)}_{\mu\nu}(q,p,S,k') = \frac{1}{2\pi}\sum_X(2\pi)^4\delta^4(q-p-k'-p_X) \times\langle 0|J_\nu(0)|p,S,k';X\rangle\langle p,S,k';X|J_\mu(0)|0\rangle, \quad (2.3)$$

where S denotes the polarization of the hadron and $J_\mu(x)$ is the quark electroweak current. It is related to the inclusive hadronic tensor $W^{(\mathrm{in})}_{\mu\nu}(q,p,S)$ by

$$W^{(\mathrm{in})}_{\mu\nu}(q,p,S) = \int\frac{d^3k'}{(2\pi)^3 2E_{k'}}W^{(si)}_{\mu\nu}(q,p,S,k'). \quad (2.4)$$

If we consider only the transverse momentum $k'_\perp$ dependence, we have

$$\frac{E_pd\sigma}{d^3pd^2k'_\perp} = \frac{\alpha^2\chi}{4\pi^2 sQ^4}L^{\mu\nu}(l_1,l_2)W_{\mu\nu}(q,p,S,k'_\perp), \quad (2.5)$$

where $W_{\mu\nu}(q,p,S,k'_\perp)$ is the TMD semi-inclusive hadronic tensor given by

$$W_{\mu\nu}(q,p,S,k'_\perp) = \int\frac{dk'_z}{(2\pi)2E_{k'}}W^{(si)}_{\mu\nu}(q,p,S,k'). \quad (2.6)$$

A. The general form of the cross section in terms of structure functions

Formally, the general form of the cross section for $e^+e^- \to h\bar{q}X$ is exactly the same as that for $e^+e^- \to V\pi X$ discussed in detail in [39]. We summarize the results here.

The hadronic tensor is divided into a symmetric and an antisymmetric part, $W_{\mu\nu} = W^S_{\mu\nu} + iW^A_{\mu\nu}$, each of them is given by a linear combination of a set of basic Lorentz tensors (BLTs), i.e.,

$$W^{S\mu\nu} = \sum_{\sigma,j}W^S_{\sigma j}h^{S\mu\nu}_{\sigma j} + \sum_{\sigma,j}\tilde{W}^S_{\sigma j}\tilde{h}^{S\mu\nu}_{\sigma j}, \quad (2.7)$$

$$W^{A\mu\nu} = \sum_{\sigma,j}W^A_{\sigma j}h^{A\mu\nu}_{\sigma j} + \sum_{\sigma,j}\tilde{W}^A_{\sigma j}\tilde{h}^{A\mu\nu}_{\sigma j}, \quad (2.8)$$

where $h^{\mu\nu}$ and $\tilde{h}^{\mu\nu}$ represent the space reflection even and space reflection odd BLTs, respectively. The subscript σ specifies the polarization.

As has been found out in [39], a distinct feature for BLTs in semi-inclusive reactions such as $e^+e^- \to h\bar{q}X$ is that the polarization dependent BLTs can be taken as a product of the unpolarized BLTs and polarization dependent Lorentz scalar(s) or pseudo-scalar(s). There are nine unpolarized BLTs given by

$$h^{S\mu\nu}_{Ui} = \left\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_q^\mu p_q^\nu, k_q'^\mu k_q'^\nu, p_q^{\{\mu}k_q'^{\nu\}}\right\}, \quad (2.9)$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \{\varepsilon^{\{\mu q p k'} p_q^{\nu\}}, \varepsilon^{\{\mu q p k'} k_q'^{\nu\}}\} \tag{2.10}$$

$$h_U^{A\mu\nu} = \{p_q^{[\mu} k_q'^{\nu]}\}, \tag{2.11}$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \{\varepsilon^{\mu\nu q p}, \varepsilon^{\mu\nu q k'}\}. \tag{2.12}$$

The subscript U denotes the unpolarized part. Here $p_q \equiv p - q(p\cdot q)/q^2$ satisfying $p_q \cdot q = 0$, $A^{\{\mu}B^{\nu\}} \equiv A^\mu B^\nu + A^\nu B^\mu$, and $A^{[\mu}B^{\nu]} \equiv A^\mu B^\nu - A^\nu B^\mu$.

The vector polarization dependent BLTs are given by

$$h_{Vi}^{S\mu\nu} = \{[\lambda_h, (k'_\perp \cdot S_T)]\tilde{h}_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S} h_{Uj}^{S\mu\nu}\}, \tag{2.13}$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \{[\lambda_h, (k'_\perp \cdot S_T)]h_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S} \tilde{h}_{Uj}^{S\mu\nu}\}, \tag{2.14}$$

$$h_{Vi}^{A\mu\nu} = \{[\lambda_h, (k'_\perp \cdot S_T)]\tilde{h}_{Ui}^{A\mu\nu}, \varepsilon_\perp^{k'S} h_U^{A\mu\nu}\}, \tag{2.15}$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \{[\lambda_h, (k'_\perp \cdot S_T)]h_U^{A\mu\nu}, \varepsilon_\perp^{k'S} \tilde{h}_{Uj}^{A\mu\nu}\}, \tag{2.16}$$

where $\varepsilon_\perp^{k'S} = \varepsilon_\perp^{\alpha\beta} k'_\alpha S_\beta$, $\varepsilon_\perp^{\alpha\beta} = \varepsilon^{\mu\nu\alpha\beta}\bar{n}_\mu n_\nu$; λ_h is the helicity of the outgoing hadron and S_T denotes the transverse polarization components. There are 27 such vector polarized BLTs in total.

The tensor polarized part is composed of S_{LL}-, S_{LT}- and S_{TT}-dependent parts. There are nine S_{LL}-dependent BLTs, they are given by

$$h_{LLi}^{S\mu\nu} = S_{LL} h_{Ui}^{S\mu\nu}, \qquad \tilde{h}_{LLi}^{S\mu\nu} = S_{LL}\tilde{h}_{Ui}^{S\mu\nu}, \tag{2.17}$$

$$h_{LL}^{A\mu\nu} = S_{LL} h_U^{A\mu\nu}, \qquad \tilde{h}_{LLi}^{A\mu\nu} = S_{LL}\tilde{h}_{Ui}^{A\mu\nu}. \tag{2.18}$$

There are 18 S_{LT}-dependent ones, they are

$$h_{LTi}^{S\mu\nu} = \{(k'_\perp \cdot S_{LT}) h_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S_{LT}} \tilde{h}_{Uj}^{S\mu\nu}\}, \tag{2.19}$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \{(k'_\perp \cdot S_{LT}) \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S_{LT}} h_{Uj}^{S\mu\nu}\}, \tag{2.20}$$

$$h_{LTi}^{A\mu\nu} = \{(k'_\perp \cdot S_{LT}) h_U^{A\mu\nu}, \varepsilon_\perp^{k'S_{LT}} \tilde{h}_{Uj}^{A\mu\nu}\}, \tag{2.21}$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \{(k'_\perp \cdot S_{LT}) \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon_\perp^{k'S_{LT}} h_U^{A\mu\nu}\}. \tag{2.22}$$

There are also 18 S_{TT}-dependent ones, they are

$$h_{TTi}^{S\mu\nu} = \{S_{TT}^{k'k'} h_{Ui}^{S\mu\nu}, S_{TT}^{\tilde{k}'k'} \tilde{h}_{Uj}^{S\mu\nu}\}, \tag{2.23}$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \{S_{TT}^{k'k'} \tilde{h}_{Ui}^{S\mu\nu}, S_{TT}^{\tilde{k}'k'} h_{Uj}^{S\mu\nu}\}, \tag{2.24}$$

$$h_{TTi}^{A\mu\nu} = \{S_{TT}^{k'k'} h_U^{A\mu\nu}, S_{TT}^{\tilde{k}'k'} \tilde{h}_{Uj}^{A\mu\nu}\}, \tag{2.25}$$

$$\tilde{h}_{TTi}^{A,\mu\nu} = \{S_{TT}^{k'k'} \tilde{h}_{Ui}^{A\mu\nu}, S_{TT}^{\tilde{k}'k'} h_U^{A\mu\nu}\}, \tag{2.26}$$

where $S_{TT}^{k'k'} = k'_{\perp\alpha} S_{TT}^{\alpha\beta} k'_{\perp\beta}$ and $\tilde{k}'_{\perp\alpha} = \varepsilon_{\perp\alpha k'}$. There are in total 81 such BLTs. Correspondingly there should be 81 structure functions for $e^+e^- \to h\bar{q}X$.

The cross section is given in the helicity Gottfried-Jackson frame [39] where we choose $p = (E_p, 0, 0, p_z)$, and $l_1 = Q(1, \sin\theta, 0, \cos\theta)/2$, $k' = (E_{k'}, |\vec{k}'_\perp|\cos\varphi, |\vec{k}'_\perp|\sin\varphi, k'_z)$ and

$$S = \left(\lambda_h \frac{p_z}{M}, |S_T|\cos\varphi_S, |S_T|\sin\varphi_S, \lambda_h \frac{E_P}{M}\right), \tag{2.27}$$

$$S_{LT} = (0, |S_{LT}|\cos\varphi_{LT}, |S_{LT}|\sin\varphi_{LT}, 0), \tag{2.28}$$

$$S_{TT}^{x\mu} = (0, |S_{TT}|\cos 2\varphi_{TT}, |S_{TT}|\sin 2\varphi_{TT}, 0). \tag{2.29}$$

In this frame, the cross section is given by

$$\begin{aligned}\frac{E_p d\sigma}{d^3p d^2k'_\perp} = \frac{\alpha^2\chi}{4\pi^2 s^2}[&(\mathcal{W}_U + \tilde{\mathcal{W}}_U) + \lambda_h(\mathcal{W}_L + \tilde{\mathcal{W}}_L) \\ &+ |S_T|(\mathcal{W}_T + \tilde{\mathcal{W}}_T) + S_{LL}(\mathcal{W}_{LL} + \tilde{\mathcal{W}}_{LL}) \\ &+ |S_{LT}|(\mathcal{W}_{LT} + \tilde{\mathcal{W}}_{LT}) + |S_{TT}|(\mathcal{W}_{TT} + \tilde{\mathcal{W}}_{TT})],\end{aligned} \tag{2.30}$$

where we use $\mathcal{W}_\sigma$ and $\tilde{\mathcal{W}}_\sigma$ to denote the parity conserved and parity violated parts, respectively. For the unpolarized part, they are given by

$$\begin{aligned}\mathcal{W}_U = &(1+\cos^2\theta)W_{U1} + \sin^2\theta W_{U2} + \cos\theta W_{U3} \\ &+ \cos\varphi[\sin\theta W_{U1}^{\cos\varphi} + \sin 2\theta W_{U2}^{\cos\varphi}] \\ &+ \cos 2\varphi \sin^2\theta W_U^{\cos 2\varphi},\end{aligned} \tag{2.31}$$

$$\begin{aligned}\tilde{\mathcal{W}}_U = &\sin\varphi[\sin\theta \tilde{W}_{U1}^{\sin\varphi} + \sin 2\theta \tilde{W}_{U2}^{\sin\varphi}] \\ &+ \sin 2\varphi \sin^2\theta \tilde{W}_U^{\sin 2\varphi}.\end{aligned} \tag{2.32}$$

We note in particular that all the θ and φ dependences are given explicitly. The W_{Uj} and $\tilde{W}_{Uj}$ are scalar functions depending on s, $\xi = 2q\cdot p/q^2$ and $k'^2_\perp$ and are called "structure functions". The subscript j specifies different $\theta-$ dependence modes for the same $\varphi-$ dependence. We have 6 structure functions corresponding to parity conserved terms and 3 corresponding to parity violated terms in the unpolarized case.

The longitudinal polarization (helicity λ_h and spin alignment S_{LL}) dependent parts take exactly the same form as the unpolarized part. More precisely, $\tilde{\mathcal{W}}_L$ and $\mathcal{W}_{LL}$ take the same form as $\mathcal{W}_U$; $\mathcal{W}_L$ and $\tilde{\mathcal{W}}_{LL}$ take the same form as $\tilde{\mathcal{W}}_U$, i.e.,

$$\mathcal{W}_L = \sin\varphi[\sin\theta W_{L1}^{\sin\varphi} + \sin 2\theta W_{L2}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta W_L^{\sin 2\varphi}, \tag{2.33}$$

$$\tilde{\mathcal{W}}_L = (1+\cos^2\theta)\tilde{W}_{L1} + \sin^2\theta\tilde{W}_{L2} + \cos\theta\tilde{W}_{L3} + \cos\varphi[\sin\theta\tilde{W}_{L1}^{\cos\varphi} + \sin 2\theta\tilde{W}_{L2}^{\cos\varphi}] + \cos 2\varphi\sin^2\theta\tilde{W}_L^{\cos 2\varphi}; \quad (2.34)$$

$$\mathcal{W}_{LL} = (1+\cos^2\theta)W_{LL1} + \sin^2\theta W_{LL2} + \cos\theta W_{LL3} + \cos\varphi[\sin\theta W_{LL1}^{\cos\varphi} + \sin 2\theta W_{LL2}^{\cos\varphi}] + \cos 2\varphi\sin^2\theta W_{LL}^{\cos 2\varphi}, \quad (2.35)$$

$$\tilde{\mathcal{W}}_{LL} = \sin\varphi[\sin\theta\tilde{W}_{LL1}^{\sin\varphi} + \sin 2\theta\tilde{W}_{LL2}^{\sin\varphi}] + \sin 2\varphi\sin^2\theta\tilde{W}_{LL}^{\sin 2\varphi}. \quad (2.36)$$

For the transverse polarization dependent parts, there are three azimuthal angles, φ_S, φ_{LT} and φ_{TT} involved. The corresponding expressions are

$$\begin{aligned}\mathcal{W}_T = {}& \sin\varphi_S[\sin\theta W_{T1}^{\sin\varphi_S} + \sin 2\theta W_{T2}^{\sin\varphi_S}] + \sin(\varphi+\varphi_S)\sin^2\theta W_T^{\sin(\varphi+\varphi_S)} + \sin(\varphi-\varphi_S)[(1+\cos^2\theta)W_{T1}^{\sin(\varphi-\varphi_S)} \\ & + \sin^2\theta W_{T2}^{\sin(\varphi-\varphi_S)} + \cos\theta W_{T3}^{\sin(\varphi-\varphi_S)}] + \sin(2\varphi-\varphi_S)[\sin\theta W_{T1}^{\sin(2\varphi-\varphi_S)} + \sin 2\theta W_{T2}^{\sin(2\varphi-\varphi_S)}] \\ & + \sin(3\varphi-\varphi_S)\sin^2\theta W_T^{\sin(3\varphi-\varphi_S)},\end{aligned} \quad (2.37)$$

$$\begin{aligned}\tilde{\mathcal{W}}_T = {}& \cos\varphi_S[\sin\theta\tilde{W}_{T1}^{\cos\varphi_S} + \sin 2\theta\tilde{W}_{T2}^{\cos\varphi_S}] + \cos(\varphi+\varphi_S)\sin^2\theta\tilde{W}_T^{\cos(\varphi+\varphi_S)} + \cos(\varphi-\varphi_S)[(1+\cos^2\theta)\tilde{W}_{T1}^{\cos(\varphi-\varphi_S)} \\ & + \sin^2\theta\tilde{W}_{T2}^{\cos(\varphi-\varphi_S)} + \cos\theta\tilde{W}_{T3}^{\cos(\varphi-\varphi_S)}] + \cos(2\varphi-\varphi_S)[\sin\theta\tilde{W}_{T1}^{\cos(2\varphi-\varphi_S)} + \sin 2\theta\tilde{W}_{T2}^{\cos(2\varphi-\varphi_S)}] \\ & + \cos(3\varphi-\varphi_S)\sin^2\theta\tilde{W}_T^{\cos(3\varphi-\varphi_S)};\end{aligned} \quad (2.38)$$

$$\begin{aligned}\tilde{\mathcal{W}}_{LT} = {}& \sin\varphi_{LT}[\sin\theta\tilde{W}_{LT1}^{\sin\varphi_{LT}} + \sin 2\theta\tilde{W}_{LT2}^{\sin\varphi_{LT}}] + \sin(\varphi+\varphi_{LT})\sin^2\theta\tilde{W}_{LT}^{\sin(\varphi+\varphi_{LT})} \\ & + \sin(\varphi-\varphi_{LT})[(1+\cos^2\theta)\tilde{W}_{LT1}^{\sin(\varphi-\varphi_{LT})} + \sin^2\theta\tilde{W}_{LT2}^{\sin(\varphi-\varphi_{LT})} + \cos\theta\tilde{W}_{LT3}^{\sin(\varphi-\varphi_{LT})}] \\ & + \sin(2\varphi-\varphi_{LT})[\sin\theta\tilde{W}_{LT1}^{\sin(2\varphi-\varphi_{LT})} + \sin 2\theta\tilde{W}_{LT2}^{\sin(2\varphi-\varphi_{LT})}] + \sin(3\varphi-\varphi_{LT})\sin^2\theta\tilde{W}_{LT}^{\sin(3\varphi-\varphi_{LT})},\end{aligned} \quad (2.39)$$

$$\begin{aligned}\mathcal{W}_{LT} = {}& \cos\varphi_{LT}[\sin\theta W_{LT1}^{\cos\varphi_{LT}} + \sin 2\theta W_{LT2}^{\cos\varphi_{LT}}] + \cos(\varphi+\varphi_{LT})\sin^2\theta W_{LT}^{\cos(\varphi+\varphi_{LT})} \\ & + \cos(\varphi-\varphi_{LT})[(1+\cos^2\theta)W_{LT1}^{\cos(\varphi-\varphi_{LT})} + \sin^2\theta W_{LT2}^{\cos(\varphi-\varphi_{LT})} + \cos\theta W_{LT3}^{\cos(\varphi-\varphi_{LT})}] \\ & + \cos(2\varphi-\varphi_{LT})[\sin\theta W_{LT1}^{\cos(2\varphi-\varphi_{LT})} + \sin 2\theta W_{LT2}^{\cos(2\varphi-\varphi_{LT})}] + \cos(3\varphi-\varphi_{LT})\sin^2\theta W_{LT}^{\cos(3\varphi-\varphi_{LT})};\end{aligned} \quad (2.40)$$

$$\begin{aligned}\tilde{\mathcal{W}}_{TT} = {}& \sin(\varphi-2\varphi_{TT})[\sin\theta\tilde{W}_{TT1}^{\sin(\varphi-2\varphi_{TT})} + \sin 2\theta\tilde{W}_{TT2}^{\sin(\varphi-2\varphi_{TT})}] + \sin 2\varphi_{TT}\sin^2\theta\tilde{W}_{TT}^{\sin 2\varphi_{TT}} \\ & + \sin(2\varphi-2\varphi_{TT})[(1+\cos^2\theta)\tilde{W}_{TT1}^{\sin(2\varphi-2\varphi_{TT})} + \sin^2\theta\tilde{W}_{TT2}^{\sin(2\varphi-2\varphi_{TT})} + \cos\theta\tilde{W}_{TT3}^{\sin(2\varphi-2\varphi_{TT})}] \\ & + \sin(3\varphi-2\varphi_{TT})[\sin\theta\tilde{W}_{TT1}^{\sin(3\varphi-2\varphi_{TT})} + \sin 2\theta\tilde{W}_{TT2}^{\sin(3\varphi-2\varphi_{TT})}] + \sin(4\varphi-2\varphi_{TT})\sin^2\theta\tilde{W}_{TT}^{\sin(4\varphi-2\varphi_{TT})},\end{aligned} \quad (2.41)$$

$$\begin{aligned}\mathcal{W}_{TT} = {}& \cos(\varphi-2\varphi_{TT})[\cos\theta W_{TT1}^{\cos(\varphi-2\varphi_{TT})} + \sin 2\theta W_{TT2}^{\cos(\varphi-2\varphi_{TT})}] + \cos 2\varphi_{TT}\sin^2\theta W_{TT}^{\cos 2\varphi_{TT}} \\ & + \cos(2\varphi-2\varphi_{TT})[(1+\cos^2\theta)W_{TT1}^{\cos(2\varphi-2\varphi_{TT})} + \sin^2\theta W_{TT2}^{\cos(2\varphi-2\varphi_{TT})} + \cos\theta W_{TT3}^{\cos(2\varphi-2\varphi_{TT})}] \\ & + \cos(3\varphi-2\varphi_{TT})[\sin\theta W_{TT1}^{\cos(3\varphi-2\varphi_{TT})} + \sin 2\theta W_{TT2}^{\cos(3\varphi-2\varphi_{TT})}] + \cos(4\varphi-2\varphi_{TT})\sin^2\theta W_{TT}^{\cos(4\varphi-2\varphi_{TT})}.\end{aligned} \quad (2.42)$$

We note in particular the following: Since S is an axial vector and S_{LT} is a vector, we have one to one correspondence between $\mathcal{W}_T \leftrightarrow \tilde{\mathcal{W}}_{LT}$ and $\tilde{\mathcal{W}}_T \leftrightarrow \mathcal{W}_{LT}$. Also S_{LT}^μ corresponds to $S_{TT}^{k'\mu}$ in the BLTs given by Eqs. (2.19)–(2.26), hence φ_{LT} corresponds to $2\varphi_{TT} - \varphi$.

Having the general form of the differential cross section, we can express all other measurable quantities such as azimuthal asymmetries and different components of hadron polarizations in terms of structure functions. The longitudinal components are unique and defined with respect to the direction of the hadron momentum, i.e., in the helicity basis. The transverse directions can be chosen as the normal of the lepton-hadron plane (defined by the momenta of the hadron h and the electron e^-), i.e., the y-direction, or that of the hadron-jet plane (defined by the momenta of the hadron h and the $\bar{q}$). The expressions of these different components of polarizations in terms of the structure functions can easily be derived and take exactly the same form as those for $e^+e^- \to V\pi X$ given in [39]. We do not repeat them here.

If we integrate over $d^2k'_\perp$, we obtain the result for the inclusive process $e^+e^- \to hX$, i.e.,

$$\frac{E_p d\sigma^{(\mathrm{in})}}{d^3 p} = \frac{\alpha^2\chi}{s^2}[\mathcal{F}_U^{(\mathrm{in})} + \lambda_h\tilde{\mathcal{F}}_L^{(\mathrm{in})} + |S_T|(\mathcal{F}_T^{(\mathrm{in})} + \tilde{\mathcal{F}}_T^{(\mathrm{in})}) + S_{LL}\mathcal{F}_{LL}^{(\mathrm{in})} + |S_{LT}|(\mathcal{F}_{LT}^{(\mathrm{in})} + \tilde{\mathcal{F}}_{LT}^{(\mathrm{in})}) + |S_{TT}|(\mathcal{F}_{TT}^{(\mathrm{in})} + \tilde{\mathcal{F}}_{TT}^{(\mathrm{in})})], \tag{2.43}$$

$$\mathcal{F}_U^{(\mathrm{in})} = (1+\cos^2\theta)F_{U1}^{(\mathrm{in})} + \sin^2\theta F_{U2}^{(\mathrm{in})} + \cos\theta F_{U3}^{(\mathrm{in})}, \tag{2.44}$$

$$\tilde{\mathcal{F}}_L^{(\mathrm{in})} = (1+\cos^2\theta)\tilde{F}_{L1}^{(\mathrm{in})} + \sin^2\theta\tilde{F}_{L2}^{(\mathrm{in})} + \cos\theta\tilde{F}_{L3}^{(\mathrm{in})}, \tag{2.45}$$

$$\mathcal{F}_{LL}^{(\mathrm{in})} = (1+\cos^2\theta)F_{LL1}^{(\mathrm{in})} + \sin^2\theta F_{LL2}^{(\mathrm{in})} + \cos\theta F_{LL3}^{(\mathrm{in})}, \tag{2.46}$$

$$\mathcal{F}_T^{(\mathrm{in})} = \sin\varphi_S(\sin\theta F_{T1}^{(\mathrm{in})\sin\varphi_S} + \sin 2\theta F_{T2}^{(\mathrm{in})\sin\varphi_S}), \tag{2.47}$$

$$\tilde{\mathcal{F}}_T^{(\mathrm{in})} = \cos\varphi_S(\sin\theta\tilde{F}_{T1}^{(\mathrm{in})\cos\varphi_S} + \sin 2\theta\tilde{F}_{T2}^{(\mathrm{in})\cos\varphi_S}), \tag{2.48}$$

$$\mathcal{F}_{LT}^{(\mathrm{in})} = \cos\varphi_{LT}(\sin\theta F_{LT1}^{(\mathrm{in})\cos\varphi_{LT}} + \sin 2\theta F_{LT2}^{(\mathrm{in})\cos\varphi_{LT}}), \tag{2.49}$$

$$\tilde{\mathcal{F}}_{LT}^{(\mathrm{in})} = \sin\varphi_{LT}(\sin\theta\tilde{F}_{LT1}^{(\mathrm{in})\sin\varphi_{LT}} + \sin 2\theta\tilde{F}_{LT2}^{(\mathrm{in})\sin\varphi_{LT}}), \tag{2.50}$$

$$\mathcal{F}_{TT}^{(\mathrm{in})} = \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{(\mathrm{in})\cos 2\varphi_{TT}}, \tag{2.51}$$

$$\tilde{\mathcal{F}}_{TT}^{(\mathrm{in})} = \sin 2\varphi_{TT}\sin^2\theta\tilde{F}_{TT}^{(\mathrm{in})\sin 2\varphi_{TT}}. \tag{2.52}$$

We have in total 19 inclusive structure functions and they are just equal to the semi-inclusive counterparts integrated over $d^2k'_\perp/(2\pi)^2$.

B. Hadronic tensor in the QCD parton model

In the QCD parton model, at the tree level of pQCD, we need to consider the series of diagrams illustrated in Fig. 2 where diagrams with exchange of j gluon(s) ($j = 0, 1, 2, \cdots$) are included. After the collinear expansion, the TMD semi-inclusive hadronic tensor is obtained as

$$W_{\mu\nu}(q,p,S,k'_\perp) = \sum_{j,c}\tilde{W}_{\mu\nu}^{(j,c)}(q,p,S,k'_\perp), \tag{2.53}$$

where c denotes different cuts. The $\tilde{W}_{\mu\nu}^{(j,c)}$ is a trace of the collinear-expanded hard part and gauge invariant quark-j-gluon-quark correlator and can be simplified to [38]

$$\tilde{W}_{\mu\nu}^{(0)} = \frac{1}{2}\mathrm{Tr}[\hat{h}_{\mu\nu}^{(0)}\hat{\Xi}^{(0)}], \tag{2.54}$$

$$\tilde{W}_{\mu\nu}^{(1,L)} = -\frac{1}{4(p\cdot q)}\mathrm{Tr}[\hat{h}_{\mu\nu}^{(1)\rho}\hat{\Xi}_\rho^{(1)}], \tag{2.55}$$

$$\tilde{W}_{\mu\nu}^{(2,L)} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{N}_{\mu\nu}^{(2)\rho\sigma}\hat{\Xi}_{\rho\sigma}^{(2)} + \hat{h}_{\mu\nu}^{(1)\rho}\hat{\Xi}_\rho^{(2\prime)}], \tag{2.56}$$

$$\tilde{W}_{\mu\nu}^{(2,M)} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{h}_{\mu\nu}^{(2)\rho\sigma}\hat{\Xi}_{\rho\sigma}^{(2,M)}], \tag{2.57}$$

where the hard parts are given by

$$\hat{h}_{\mu\nu}^{(0)} = \frac{1}{p^+}\Gamma_\mu^q \not{n}\Gamma_\nu^q, \tag{2.58}$$

$$\hat{h}_{\mu\nu}^{(1)\rho} = \Gamma_\mu^q \not{n}\gamma_\perp^\rho \bar{\not{n}}\,\Gamma_\nu^q, \tag{2.59}$$

$$\hat{h}_{\mu\nu}^{(2)\rho\sigma} = \frac{p^+}{2}\Gamma_\mu^q \bar{\not{n}}\gamma_\perp^\rho \not{n}\gamma_\perp^\sigma \bar{\not{n}}\,\Gamma_\nu^q, \tag{2.60}$$

$$\hat{N}_{\mu\nu}^{(2)\rho\sigma} = q^-\Gamma_\mu^q\gamma_\perp^\rho \not{n}\gamma_\perp^\sigma\Gamma_\nu^q. \tag{2.61}$$

All the quark-quark and quark-j-gluon-quark correlators involved are functions of one parton momentum and the hadron momentum and spin, i.e., (z, $k_\perp$, p, S), and are given by

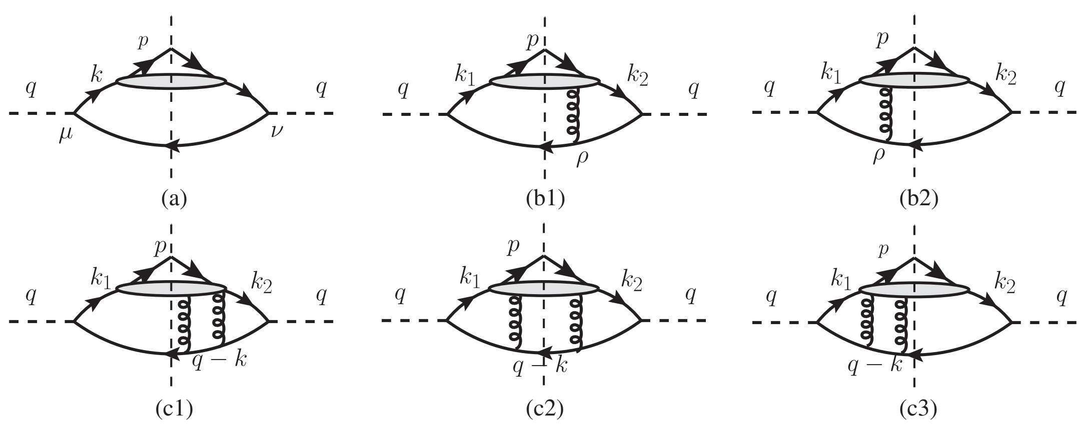

FIG. 2. The first few diagrams as examples of the considered diagram series with exchange of j-gluon(s) and different cuts. We see (a) $j = 0$, (b1) $j = 1$ and left cut, (b2) $j = 1$ and right cut, (c1) $j = 2$ and left cut, (c2) $j = 2$ and middle cut, and (c3) $j = 2$ and right cut, respectively.

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ip^+\xi^-/z + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty) \times \psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle, \tag{2.62}$$

$$\hat{\Xi}^{(1)}_\rho = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ip^+\xi^-/z + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty) \times D_{\perp\rho}(0)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle, \tag{2.63}$$

$$\hat{\Xi}^{(2)}_{\rho\sigma} = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} \int_0^\infty ip^+ d\eta^- e^{-ip^+\xi^-/z + ik_\perp\cdot\xi_\perp} \times \langle 0|\mathcal{L}^\dagger(\eta,\infty)D_{\perp\rho}(\eta)D_{\perp\sigma}(\eta)\mathcal{L}^\dagger(0,\eta) \times \psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle, \tag{2.64}$$

$$\hat{\Xi}^{(2\prime)}_\rho = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ip^+\xi^-/z + ik_\perp\cdot\xi_\perp} p^\sigma \langle 0|\mathcal{L}^\dagger(0,\infty) \times D_{\perp\rho}(0)D_\sigma(0)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle, \tag{2.65}$$

$$\hat{\Xi}^{(2,M)}_{\rho\sigma} = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ip^+\xi^-/z + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty) \times D_{\perp\rho}\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)D_{\perp\sigma}(\xi)\mathcal{L}(\xi,\infty)|0\rangle, \tag{2.66}$$

where $D_\rho = -i\partial_\rho + gA_\rho$, and $\mathcal{L}(0,y)$ is the gauge link. As a convention, the argument ξ in the quark filed operator ψ and gauge link represents $(0,\xi^-,\vec{\xi}_\perp)$. We note that the leading power contribution of $\tilde{W}^{(j)}_{\mu\nu}$ is twist-$(j+2)$. However, because of the factor p^σ in the definition of $\hat{\Xi}^{(2\prime)}_\rho$ given by Eq. (2.65), the second term in Eq. (2.56) has no contribution up to twist-4. The leading power contribution of this term is twist-5.

C. Decompositions of the quark-j-gluon-quark correlator

In $e^+e^- \to h\bar{q}X$, only the chiral even FFs are involved. We only need to consider the γ^α- and the $\gamma^5\gamma^\alpha$-term in the decomposition of the correlators in terms of the Γ-matrices such as $\hat{\Xi}^{(0)} = \Xi^{(0)}_\alpha\gamma^\alpha + \tilde{\Xi}^{(0)}_\alpha\gamma^5\gamma^\alpha + \cdots$. We write down all of the twist-4 terms in the decomposition of these correlators in the following. For $\hat{\Xi}^{(0)}$, they are given by

$$z\Xi^{(0)}_\alpha = \frac{M^2}{p^+} n_\alpha \left(D_3 - \frac{\varepsilon^{kS}_\perp}{M} D^\perp_{3T} + S_{LL}D_{3LL} + \frac{k_\perp\cdot S_{LT}}{M} D^\perp_{3LT} + \frac{S^{kk}_{TT}}{M^2} D^\perp_{3TT} \right), \tag{2.67}$$

$$z\tilde{\Xi}^{(0)}_\alpha = -\frac{M^2}{p^+} n_\alpha \left(\lambda_h G_{3L} - \frac{k_\perp\cdot S_T}{M} G^\perp_{3T} + \frac{\varepsilon^{kS_{LT}}_\perp}{M} G^\perp_{3LT} + \frac{S^{\tilde{k}k}_{TT}}{M^2} G^\perp_{3TT} \right). \tag{2.68}$$

Here, as in [39], D's and G's represent the γ^α- and $\gamma^5\gamma^\alpha$-type FFs, respectively. The digit j in the subscript denotes twist-$(j+1)$; the capital letter such as T, L, or LL denotes the hadron polarization. There are in total nine twist-4 chiral even FFs defined via $\hat{\Xi}^{(0)}$.

For $\hat{\Xi}^{(1)}_\rho$, the chiral even parts are

$$\begin{aligned}
z\Xi^{(1)}_{\rho\alpha} = {} & M^2 g_{\perp\rho\alpha}\left(D_{3d} - \frac{\varepsilon^{kS}_\perp}{M} D^\perp_{3dT} + S_{LL}D_{3dLL} + \frac{k_\perp\cdot S_{LT}}{M} D^\perp_{3dLT} + \frac{S^{kk}_{TT}}{M^2} D^\perp_{3dTT} \right) \\
& + k_{\perp\langle\rho}k_{\perp\alpha\rangle}\left(D^\perp_{3d} + \frac{\varepsilon^{kS}_\perp}{M} D^{\perp 2}_{3dT} + S_{LL}D^\perp_{3dLL} + \frac{k_\perp\cdot S_{LT}}{M} D^{\perp 2}_{3dLT} + \frac{S^{kk}_{TT}}{M^2} D^{\perp 2}_{3dTT} \right) \\
& + iM^2\varepsilon_{\perp\rho\alpha}\left(\lambda_h D_{3dL} - \frac{k_\perp\cdot S_T}{M} D^{\perp 3}_{3dT} + \frac{\varepsilon^{kS_{LT}}_\perp}{M} D^{\perp 3}_{3dLT} + \frac{S^{\tilde{k}k}_{TT}}{M^2} D^{\perp 3}_{3dTT} \right) \\
& + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}\left(\lambda_h D^\perp_{3dL} + \frac{k_\perp\cdot S_T}{M} D^{\perp 4}_{3dT} + \frac{\varepsilon^{kS_{LT}}_\perp}{M} D^{\perp 4}_{3dLT} + \frac{S^{\tilde{k}k}_{TT}}{M^2} D^{\perp 4}_{3dTT} \right),
\end{aligned} \tag{2.69}$$

$$\begin{aligned}
z\tilde{\Xi}^{(1)}_{\rho\alpha} = {} & iM^2\varepsilon_{\perp\rho\alpha}\left(G_{3d} - \frac{\varepsilon^{kS}_\perp}{M} G^\perp_{3dT} + S_{LL}G_{3dLL} + \frac{k_\perp\cdot S_{LT}}{M} G^\perp_{3dLT} + \frac{S^{kk}_{TT}}{M^2} G^\perp_{3dTT} \right) \\
& + \frac{i}{2}k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}\left(G^\perp_{3d} + \frac{\varepsilon^{kS}_\perp}{M} G^{\perp 2}_{3dT} + S_{LL}G^\perp_{3dLL} + \frac{k_\perp\cdot S_{LT}}{M} G^{\perp 2}_{3dLT} + \frac{S^{kk}_{TT}}{M^2} G^{\perp 2}_{3dTT} \right) \\
& + M^2 g_{\perp\rho\alpha}\left(\lambda_h G_{3dL} - \frac{k_\perp\cdot S_T}{M} G^{\perp 3}_{3dT} + \frac{\varepsilon^{kS_{LT}}_\perp}{M} G^{\perp 3}_{3dLT} + \frac{S^{\tilde{k}k}_{TT}}{M^2} G^{\perp 3}_{3dTT} \right) \\
& + ik_{\perp\langle\rho}k_{\perp\alpha\rangle}\left(\lambda_h G^\perp_{3dL} + \frac{k_\perp\cdot S_T}{M} G^{\perp 4}_{3dT} + \frac{\varepsilon^{kS_{LT}}_\perp}{M} G^{\perp 4}_{3dLT} + \frac{S^{\tilde{k}k}_{TT}}{M^2} G^{\perp 4}_{3dTT} \right),
\end{aligned} \tag{2.70}$$

where $k_{\perp\langle\rho}k_{\perp\alpha\rangle} \equiv k_{\perp\rho}k_{\perp\alpha} - k_\perp^2 g_{\perp\rho\alpha}/2$ and $g_{\perp\rho\alpha}$ is defined as $g_{\perp\rho\alpha} = g_{\rho\alpha} - \bar{n}_\rho n_\alpha - \bar{n}_\alpha n_\rho$. Here we add a subscript d to denote FFs defined via $\hat{\Xi}_\rho^{(1)}$.

Up to twist-4, we only need the leading power contributions from $\hat{\Xi}_{\rho\sigma}^{(2)}$ and $\hat{\Xi}_{\rho\sigma}^{(2,M)}$. For the chiral even part, we need only the $\bar{n}_\alpha$-terms. They are given by

$$\begin{aligned} z\Xi_{\rho\sigma\alpha}^{(2)} = p^+\bar{n}_\alpha\Big[& M^2 g_{\perp\rho\sigma}\Big(D_{3dd} - \frac{\varepsilon_\perp^{kS}}{M}D_{3ddT}^\perp + S_{LL}D_{3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D_{3ddLT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{3ddTT}^\perp\Big) \\ & + k_{\perp\langle\rho}k_{\perp\sigma\rangle}\Big(D_{3dd}^\perp + \frac{\varepsilon_\perp^{kS}}{M}D_{3ddT}^{\perp 2} + S_{LL}D_{3ddLL}^\perp - \frac{k_\perp\cdot S_{LT}}{M}D_{3ddLT}^{\perp 2} - \frac{S_{TT}^{kk}}{M^2}D_{3ddTT}^{\perp 2}\Big) \\ & + iM^2\varepsilon_{\perp\rho\sigma}\Big(\lambda_h D_{3ddL} - \frac{k_\perp\cdot S_T}{M}D_{3ddT}^{\perp 3} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{3ddLT}^{\perp 3} - \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{3ddTT}^{\perp 3}\Big) \\ & + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\sigma\}}\Big(\lambda_h D_{3ddL}^\perp + \frac{k_\perp\cdot S_T}{M}D_{3ddT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{3ddLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{3ddTT}^{\perp 4}\Big)\Big], \end{aligned} \tag{2.71}$$

$$\begin{aligned} z\tilde{\Xi}_{\rho\sigma\alpha}^{(2)} = p^+\bar{n}_\alpha\Big[& iM^2\varepsilon_{\perp\rho\sigma}\Big(G_{3dd} - \frac{\varepsilon_\perp^{kS}}{M}G_{3ddT}^\perp + S_{LL}G_{3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}G_{3ddLT}^\perp + \frac{S_{TT}^{kk}}{M^2}G_{3ddTT}^\perp\Big) \\ & + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\sigma\}}\Big(G_{3dd}^\perp + \frac{\varepsilon_\perp^{kS}}{M}G_{3ddT}^{\perp 2} + S_{LL}G_{3ddLL}^\perp - \frac{k_\perp\cdot S_{LT}}{M}G_{3ddLT}^{\perp 2} - \frac{S_{TT}^{kk}}{M^2}G_{3ddTT}^{\perp 2}\Big) \\ & + M^2 g_{\perp\rho\sigma}\Big(\lambda_h G_{3ddL} - \frac{k_\perp\cdot S_T}{M}G_{3ddT}^{\perp 3} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}G_{3ddLT}^{\perp 3} - \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{3ddTT}^{\perp 3}\Big) \\ & + k_{\perp\langle\rho}k_{\perp\sigma\rangle}\Big(\lambda_h G_{3ddL}^\perp + \frac{k_\perp\cdot S_T}{M}G_{3ddT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G_{3ddLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{3ddTT}^{\perp 4}\Big)\Big], \end{aligned} \tag{2.72}$$

where we use dd in the subscript to denote FFs defined via $\hat{\Xi}_{\rho\sigma}^{(2)}$. The decomposition of $\hat{\Xi}_{\rho\sigma}^{(2,M)}$ takes exactly the same form as that of $\hat{\Xi}_{\rho\sigma}^{(2)}$. We just add an additional superscript M to distinguish them from each other and omit the equations here.

From Eqs. (2.67)–(2.72), we see that for the twist-4 parts, the decomposition of Ξ and that of $\tilde{\Xi}$ have exact one to one correspondence. For each D_3, there is a G_3 corresponding to it. They always appear in pairs. Because of the Hermiticity of $\hat{\Xi}^{(0)}$ and $\hat{\Xi}_{\rho\sigma}^{(2,M)}$, the FFs defined via them are real. For those defined via $\hat{\Xi}_\rho^{(1)}$ and $\hat{\Xi}_{\rho\sigma}^{(2)}$, there is no such constraint so that they can be complex.

D. Relationships derived from the QCD equation of motion

From the QCD equation of motion, $\gamma\cdot D\psi = 0$, we relate the quark-j-gluon-quark correlators to the quark-quark correlator. For the two transverse components $\Xi_\perp^{(0)\rho}$ and $\tilde{\Xi}_\perp^{(0)\rho}$, we have

$$k^+\Xi_\perp^{(0)\rho} = -g_\perp^{\rho\sigma}\mathrm{Re}\Xi_{\sigma+}^{(1)} - \varepsilon_\perp^{\rho\sigma}\mathrm{Im}\tilde{\Xi}_{\sigma+}^{(1)}, \tag{2.73}$$

$$k^+\tilde{\Xi}_\perp^{(0)\rho} = -g_\perp^{\rho\sigma}\mathrm{Re}\tilde{\Xi}_{\sigma+}^{(1)} - \varepsilon_\perp^{\rho\sigma}\mathrm{Im}\Xi_{\sigma+}^{(1)}. \tag{2.74}$$

Equations (2.73) and (2.74) lead to a set of relationships between twist-3 FFs given in the unified form [39]

$$D_S^K - iG_S^K = -z(D_{dS}^K - G_{dS}^K), \tag{2.75}$$

where $S = $ null, L, T, LL, LT or TT and $K = $ null, $\perp$ or $\prime\perp$ whenever applicable [46]. Similarly, for the minus components of $\Xi_\alpha^{(0)}$ and $\tilde{\Xi}_\alpha^{(0)}$, we have

$$\begin{aligned} 2k^{+2}\Xi_-^{(0)} &= k^+(g_\perp^{\rho\sigma}\Xi_{\rho\sigma}^{(1)} + i\varepsilon_\perp^{\rho\sigma}\tilde{\Xi}_{\rho\sigma}^{(1)}) \\ &= -g_\perp^{\rho\sigma}\Xi_{\rho\sigma+}^{(2,M)} + i\varepsilon_\perp^{\rho\sigma}\tilde{\Xi}_{\rho\sigma+}^{(2,M)}, \end{aligned} \tag{2.76}$$

$$\begin{aligned} 2k^{+2}\tilde{\Xi}_-^{(0)} &= k^+(g_\perp^{\rho\sigma}\tilde{\Xi}_{\rho\sigma}^{(1)} + i\varepsilon_\perp^{\rho\sigma}\Xi_{\rho\sigma}^{(1)}) \\ &= -g_\perp^{\rho\sigma}\tilde{\Xi}_{\rho\sigma+}^{(2,M)} + i\varepsilon_\perp^{\rho\sigma}\Xi_{\rho\sigma+}^{(2,M)}. \end{aligned} \tag{2.77}$$

From Eqs. (2.76) and (2.77), we obtain a set of relationships between twist-4 FFs defined via $\hat{\Xi}^{(0)}$, $\hat{\Xi}^{(1)}$ and $\hat{\Xi}^{(2,M)}$. For longitudinal components, we have

$$D_3 = zD_{-3d} = -z^2 D_{+3dd}^M, \tag{2.78}$$

$$D_{3LL} = zD_{-3dLL} = -z^2 D_{+3ddLL}^M, \tag{2.79}$$

$$G_{3L} = zD_{-3dL} = z^2 D_{+3ddL}^M, \tag{2.80}$$

where $D_\pm \equiv D \pm G$ such as $D_{-3d} \equiv D_{3d} - G_{3d}$ and so on. For the transverse components, we have

$$D_{3S}^{\perp} = zD_{-3dS}^{\perp} = -z^2 D_{+3ddS}^{M\perp}, \tag{2.81}$$

$$G_{3S}^{\perp} = zD_{-3dS}^{\perp 3} = -\eta_S z^2 D_{+3ddS}^{M\perp 3}, \tag{2.82}$$

where $S = T, LT$ or TT represents the transverse components; η_S represents a sign that takes -1 for $S = T$ and $+1$ for $S = LT$ or TT as well as in Eqs. (2.86) and (2.87).

We note in particular that Eqs. (2.78)–(2.82) represent 27 equations in total that can be used to eliminate those twist-4 TMD FFs that are not independent in parton model results for a cross section.

E. Relationships between twist-4 and leading twist FFs at $g=0$

The higher twist FFs defined in Sec. II C are new and much involved. Currently, there is not much data available. If we neglect the multiple gluon scattering, i.e., set $g = 0$, we obtain a set of equations relating them to the leading twist counterparts. These relationships could be helpful in understanding the properties of these higher twist FFs, in particular at the present stage when few data are available. We give these relationships in this subsection.

By putting $g = 0$ into Eqs. (2.62)–(2.66), we obtain relationships such as $\hat{\Xi}_\rho^{(1)}|_{g=0} = -k_{\perp\rho}\hat{\Xi}^{(0)}|_{g=0}$, $\hat{\Xi}_{\rho\sigma}^{(2,M)}|_{g=0} = k_{\perp\rho}k_{\perp\sigma}\hat{\Xi}^{(0)}|_{g=0}$, $(\hat{\Xi}_{\rho\sigma}^{(2)} + \gamma^0\hat{\Xi}_{\sigma\rho}^{(2)\dagger}\gamma^0)|_{g=0} = z^2 k_{\perp\rho}k_{\sigma\perp}(\partial\hat{\Xi}^{(0)}/\partial z)|_{g=0}$. Together with the QCD equation of motion, we obtain the relationships between the leading twist FFs and twist-4 FFs in the following. For the twist-4 FFs defined via $\hat{\Xi}_\rho^{(1)}$, we obtain that for the longitudinal components

$$D_{3d} = \frac{k_\perp^2}{2M^2} D_{3d}^{\perp} = \frac{1}{z} D_3 = -\frac{k_\perp^2}{2M^2} zD_1, \tag{2.83}$$

$$D_{3dLL} = \frac{k_\perp^2}{2M^2} D_{3dLL}^{\perp} = \frac{1}{z} D_{3LL} = -\frac{k_\perp^2}{2M^2} zD_{1LL}, \tag{2.84}$$

$$G_{3dL} = i\frac{k_\perp^2}{2M^2} G_{3dL}^{\perp} = \frac{1}{z} G_{3L} = -\frac{k_\perp^2}{2M^2} zG_{1L}, \tag{2.85}$$

and for the transverse components

$$D_{3dS}^{\perp} = \eta_S \frac{k_\perp^2}{2M^2} D_{3dS}^{\perp 2} = \frac{1}{z} D_{3S}^{\perp} = -\frac{k_\perp^2}{2M^2} zD_{1S}^{\perp}, \tag{2.86}$$

$$G_{3dS}^{\perp 3} = i\eta_S \frac{k_\perp^2}{2M^2} G_{3dS}^{\perp 4} = \frac{1}{z} G_{3S}^{\perp} = \eta_S \frac{k_\perp^2}{2M^2} zG_{1S}^{\perp}, \tag{2.87}$$

where $S = T, LT$ or TT.

For those defined via $\hat{\Xi}_{\rho\sigma}^{(2)}$, we have, for the longitudinal components,

$$\mathrm{Re}D_{3dd} = \frac{k_\perp^2}{2M^2}\mathrm{Re}D_{3dd}^{\perp} = z^2 \frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}D_1, \tag{2.88}$$

$$\mathrm{Re}G_{3ddL} = \frac{k_\perp^2}{2M^2}\mathrm{Re}G_{3ddL}^{\perp} = -z^2 \frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}G_{1L}, \tag{2.89}$$

$$\mathrm{Re}D_{3ddLL} = \frac{k_\perp^2}{2M^2}\mathrm{Re}D_{3ddLL}^{\perp} = z^2 \frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}D_{1LL}, \tag{2.90}$$

and for the transverse components

$$\mathrm{Re}D_{3ddS}^{\perp} = -\frac{k_\perp^2}{2M^2}\mathrm{Re}D_{3ddS}^{\perp 2} = z^2 \frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}D_{1S}^{\perp}, \tag{2.91}$$

$$\mathrm{Re}G_{3ddS}^{\perp 3} = -\frac{k_\perp^2}{2M^2}\mathrm{Re}G_{3ddS}^{\perp 4} = -z^2 \frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}G_{1S}^{\perp}, \tag{2.92}$$

where $S = T, LT$ or TT. All others twist-4 FFs vanish and also time-reversal invariance demands $D_{1T}^{\perp} = 0$ in this case.

III. THE COMPLETE TWIST-4 RESULTS

By substituting Eqs. (2.58)–(2.61) and (2.67)–(2.72) into Eqs. (2.54)–(2.57), carrying out the traces, we obtain the hadronic tensor results at twist-4. Making the Lorentz contraction of the hadronic tensor with the leptonic tensor, we obtain the cross section up to twist-4. We compare the results with the general form of the cross section given by Eqs. (2.30)–(2.42) and obtain the results for the structure functions in terms of gauge invariant FFs. We present the complete results up to twist-4 in this section. For comparison, we also show the corresponding results at leading twist and twist-3. They can be found, e.g., in [38]. There are also contributions from the four-quark correlators at twist-4. We present them in Sec. III E. We also show the results for the inclusive processes.

A. The hadronic tensor at twist-4

The hadronic tensor up to twist-3 obtained using the formalism in Sec. II has been presented in e.g., [38]. We show only the twist-4 part obtained by substituting Eqs. (2.58)–(2.61) and Eqs. (2.67)–(2.72) into Eqs. (2.54)–(2.57).

For the contributions from $\tilde{W}_{\mu\nu}^{(0)}$, we use

$$\mathrm{Tr}[\hat{h}_{\mu\nu}^{(0)}\gamma^\alpha] = -\frac{4}{p^+}[c_1^q(g_{\mu\nu}n^\alpha - g_{\{\mu}^{\alpha}n_{\nu\}}) + ic_3^q\varepsilon_{\mu\nu}{}^{\alpha n}], \tag{3.1}$$

$$\mathrm{Tr}[\hat{h}_{\mu\nu}^{(0)}\gamma^5\gamma^\alpha] = \frac{4}{p^+}[c_3^q(g_{\mu\nu}n^\alpha - g_{\{\mu}^{\alpha}n_{\nu\}}) + ic_1^q\varepsilon_{\mu\nu}{}^{\alpha n}], \tag{3.2}$$

and obtain the twist-4 part of $\tilde{W}_{\mu\nu}^{(0)}$ as given by

$$\tilde{W}^{(0)}_{t4\mu\nu} = \frac{4M^2 n_\mu n_\nu}{z(p^+)^2}\left[c_1^q\left(D_3 - \frac{\varepsilon_\perp^{kS}}{M}D_{3T}^\perp + S_{LL}D_{3LL} + \frac{k_\perp\cdot S_{LT}}{M}D_{3LT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{3TT}^\perp\right)\right.$$
$$\left. + c_3^q\left(\lambda_h G_{3L} - \frac{k_\perp\cdot S_T}{M}G_{3T}^\perp + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G_{3LT}^\perp + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{3TT}^\perp\right)\right], \tag{3.3}$$

where we use a subscript $t4$ to denote the twist-4 part only.

For $\tilde{W}^{(1)}_{\mu\nu}$, we have contributions from $\tilde{W}^{(1,L)}_{\mu\nu}$ and $\tilde{W}^{(1,R)}_{\mu\nu} = \tilde{W}^{(1,L)*}_{\nu\mu}$. We calculate

$$\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^\alpha] = 4c_1^q[2n_\mu\bar{n}_\nu g_\perp^{\rho\alpha} + g_{\perp\mu\nu}g_\perp^{\rho\alpha} - g_{\perp\mu}^{\{\rho}g_{\perp\nu}^{\alpha\}}] - 4ic_3^q[2n_\mu\bar{n}_\nu\varepsilon_\perp^{\rho\alpha} + g^\rho_{\perp\mu}\varepsilon_{\perp\nu}{}^\alpha + g^\alpha_{\perp\nu}\varepsilon_{\perp\mu}{}^\rho], \tag{3.4}$$

$$\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^5\gamma^\alpha] = 4ic_1^q[2n_\mu\bar{n}_\nu\varepsilon_\perp^{\rho\alpha} + g^\rho_{\perp\mu}\varepsilon_{\perp\nu}{}^\alpha + g^\alpha_{\perp\nu}\varepsilon_{\perp\mu}{}^\rho] - 4c_3^q[2n_\mu\bar{n}_\nu g_\perp^{\rho\alpha} + g_{\perp\mu\nu}g_\perp^{\rho\alpha} - g_{\perp\mu}^{\{\rho}g_{\perp\nu}^{\alpha\}}], \tag{3.5}$$

and obtain

$$\tilde{W}^{(1,L)}_{t4\mu\nu} = -\frac{4M^2}{z(p\cdot q)}n_\mu\bar{n}_\nu\left[c_1^q\left(D_{-3d} - \frac{\varepsilon_\perp^{kS}}{M}D_{-3dT}^\perp + S_{LL}D_{-3dLL} + \frac{k_\perp\cdot S_{LT}}{M}D_{-3dLT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{-3dTT}^\perp\right)\right.$$
$$\left. + c_3^q\left(\lambda_h D_{-3dL} - \frac{k_\perp\cdot S_T}{M}D_{-3dT}^{\perp 3} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{-3dLT}^{\perp 3} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{-3dTT}^{\perp 3}\right)\right]$$
$$+ \frac{2}{z(p\cdot q)}k_{\perp\langle\mu}k_{\perp\nu\rangle}\left[c_1^q\left(D_{-3d}^\perp + \frac{\varepsilon_\perp^{kS}}{M}D_{-3dT}^{\perp 2} + S_{LL}D_{-3dLL}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{-3dLT}^{\perp 2} + \frac{S_{TT}^{kk}}{M^2}D_{-3dTT}^{\perp 2}\right)\right.$$
$$\left. - ic_3^q\left(\lambda_h D_{+3dL}^\perp + \frac{k_\perp\cdot S_T}{M}D_{+3dT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{+3dLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{+3dTT}^{\perp 4}\right)\right]$$
$$+ \frac{1}{z(p\cdot q)}k_{\perp\{\mu}\tilde{k}_{\perp\nu\}}\left[c_1^q\left(\lambda_h D_{+3dL}^\perp + \frac{k_\perp\cdot S_T}{M}D_{+3dT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{+3dLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{+3dTT}^{\perp 4}\right)\right.$$
$$\left. + ic_3^q\left(D_{-3d}^\perp + \frac{\varepsilon_\perp^{kS}}{M}D_{-3dT}^{\perp 2} + S_{LL}D_{-3dLL}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{-3dLT}^{\perp 2} + \frac{S_{TT}^{kk}}{M^2}D_{-3dTT}^{\perp 2}\right)\right]. \tag{3.6}$$

For $\tilde{W}^{(2)}_{\mu\nu}$, we have contributions from $\tilde{W}^{(2,M)}_{\mu\nu}$, $\tilde{W}^{(2,L)}_{\mu\nu}$ and $\tilde{W}^{(2,R)}_{\mu\nu} = \tilde{W}^{(2,L)*}_{\nu\mu}$. For that from $\tilde{W}^{(2,M)}_{\mu\nu}$, we calculate

$$\mathrm{Tr}[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\bar{n}\!\!\!/]p^+ = -8c_1^q p_\mu p_\nu g_\perp^{\rho\sigma} - 8ic_3^q p_\mu p_\nu\varepsilon_\perp^{\rho\sigma}, \tag{3.7}$$

$$\mathrm{Tr}[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\gamma^5\bar{n}\!\!\!/]p^+ = 8c_3^q p_\mu p_\nu g_\perp^{\rho\sigma} + 8ic_1^q p_\mu p_\nu\varepsilon_\perp^{\rho\sigma}, \tag{3.8}$$

and the result is given by

$$\tilde{W}^{(2,M)}_{t4\mu\nu} = -\frac{4M^2}{z(p\cdot q)^2}p_\mu p_\nu\left[c_1^q\left(D^M_{+3dd} - \frac{\varepsilon_\perp^{kS}}{M}D^{M\perp}_{+3ddT} + S_{LL}D^M_{+3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D^{M\perp}_{+3ddLT} + \frac{S_{TT}^{kk}}{M^2}D^{M\perp}_{+3ddTT}\right)\right.$$
$$\left. - c_3^q\left(\lambda_h D^M_{+3ddL} - \frac{k_\perp\cdot S_T}{M}D^{M\perp 3}_{+3ddT} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{M\perp 3}_{+3ddLT} - \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{M\perp 3}_{+3ddTT}\right)\right]. \tag{3.9}$$

For $\tilde{W}^{(2,L)}_{\mu\nu}$, we have

$$\mathrm{Tr}[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\bar{n}\!\!\!/]p^+ = 4(p\cdot q)c_1^q[g_\perp^{\rho\sigma}g_{\perp\mu\nu} + g^\rho_{\perp[\mu}g^\sigma_{\perp\nu]}] - 4(p\cdot q)ic_3^q[g^\rho_{\perp\mu}\varepsilon_{\perp\nu}{}^\sigma - g^\sigma_{\perp\nu}\varepsilon_{\perp\mu}{}^\rho], \tag{3.10}$$

$$\mathrm{Tr}[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\gamma^5\bar{n}\!\!\!/]p^+ = -4(p\cdot q)c_3^q[g_\perp^{\rho\sigma}g_{\perp\mu\nu} + g^\rho_{\perp[\mu}g^\sigma_{\perp\nu]}] + 4(p\cdot q)ic_1^q[g^\rho_{\perp\mu}\varepsilon_{\perp\nu}{}^\sigma - g^\sigma_{\perp\nu}\varepsilon_{\perp\mu}{}^\rho], \tag{3.11}$$

and the result is

$$\tilde{W}^{(2,L)}_{t4\mu\nu} = \frac{2M^2}{z(p\cdot q)}\Big\{g_{\perp\mu\nu}\Big[c_1^q\Big(D_{-3dd} - \frac{\varepsilon_\perp^{kS}}{M}D^\perp_{-3ddT} + S_{LL}D_{-3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D^\perp_{-3ddLT} + \frac{S_{TT}^{kk}}{M^2}D^\perp_{-3ddTT}\Big)$$
$$+ c_3^q\Big(\lambda_h D_{-3ddL} - \frac{k_\perp\cdot S_T}{M}D^{\perp 3}_{-3ddT} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{\perp 3}_{-3ddLT} - \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{\perp 3}_{-3ddTT}\Big)\Big]$$
$$+ i\varepsilon_{\perp\mu\nu}\Big[c_1^q\Big(\lambda_h D_{-3ddL} - \frac{k_\perp\cdot S_T}{M}D^{\perp 3}_{-3ddT} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{\perp 3}_{-3ddLT} - \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{\perp 3}_{-3ddTT}\Big)$$
$$+ c_3^q\Big(D_{-3dd} - \frac{\varepsilon_\perp^{kS}}{M}D^\perp_{-3ddT} + S_{LL}D_{-3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D^\perp_{-3ddLT} + \frac{S_{TT}^{kk}}{M^2}D^\perp_{-3ddTT}\Big)\Big]\Big\}. \quad (3.12)$$

We add all of the contributions from $\tilde{W}^{(0)}_{\mu\nu}$, $\tilde{W}^{(1)}_{\mu\nu}$, and $\tilde{W}^{(2)}_{\mu\nu}$ together and use the relationships given by Eqs. (2.78)–(2.82) to eliminate the not independent FFs. We obtain the twist-4 contributions to the hadronic tensor as given by

$$W_{t4\mu\nu} = \frac{4M^2}{z(p\cdot q)}\Big\{\frac{(zq-2p)_\mu(zq-2p)_\nu}{z^2(p\cdot q)}\Big[c_1^q\Big(D_3 - \frac{\varepsilon_\perp^{kS}}{M}D^\perp_{3T} + S_{LL}D_{3LL} + \frac{k_\perp\cdot S_{LT}}{M}D^\perp_{3LT} + \frac{S_{TT}^{kk}}{M^2}D^\perp_{3TT}\Big)$$
$$- c_3^q\Big(\lambda_h G_{3L} - \frac{k_\perp\cdot S_T}{M}G^\perp_{3T} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G^\perp_{3LT} + \frac{S_{TT}^{\tilde{k}k}}{M^2}G^\perp_{3TT}\Big)\Big]$$
$$+ \frac{k_{\perp\langle\mu}k_{\perp\nu\rangle}}{M^2}\Big[c_1^q\mathrm{Re}\Big(D^\perp_{-3d} + \frac{\varepsilon_\perp^{kS}}{M}D^{\perp 2}_{-3dT} + S_{LL}D^\perp_{-3dLL} + \frac{k_\perp\cdot S_{LT}}{M}D^{\perp 2}_{-3dLT} + \frac{S_{TT}^{kk}}{M^2}D^{\perp 2}_{-3dTT}\Big)$$
$$+ c_3^q\mathrm{Im}\Big(\lambda_h D^\perp_{+3dL} + \frac{k_\perp\cdot S_T}{M}D^{\perp 4}_{+3dT} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{\perp 4}_{+3dLT} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{\perp 4}_{+3dTT}\Big)\Big]$$
$$+ \frac{k_{\perp\{\mu}\tilde{k}_{\perp\nu\}}}{2M^2}\Big[c_1^q\mathrm{Re}\Big(\lambda_h D^\perp_{+3dL} + \frac{k_\perp\cdot S_T}{M}D^{\perp 4}_{+3dT} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{\perp 4}_{+3dLT} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{\perp 4}_{+3dTT}\Big)$$
$$- c_3^q\mathrm{Im}\Big(D^\perp_{-3d} + \frac{\varepsilon_\perp^{kS}}{M}D^{\perp 2}_{-3dT} + S_{LL}D^\perp_{-3dLL} + \frac{k_\perp\cdot S_{LT}}{M}D^{\perp 2}_{-3dLT} + \frac{S_{TT}^{kk}}{M^2}D^{\perp 2}_{-3dTT}\Big)\Big]$$
$$+ (c_1^q g_{\perp\mu\nu} + ic_3^q\varepsilon_{\perp\mu\nu})\mathrm{Re}\Big(D_{-3dd} - \frac{\varepsilon_\perp^{kS}}{M}D^\perp_{-3ddT} + S_{LL}D_{-3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D^\perp_{-3ddLT} + \frac{S_{TT}^{kk}}{M^2}D^\perp_{-3ddTT}\Big)$$
$$+ (c_3^q g_{\perp\mu\nu} + ic_1^q\varepsilon_{\perp\mu\nu})\mathrm{Re}\Big(\lambda_h D_{-3ddL} - \frac{k_\perp\cdot S_T}{M}D^{\perp 3}_{-3ddT} - \frac{\varepsilon_\perp^{kS_{LT}}}{M}D^{\perp 3}_{-3ddLT} - \frac{S_{TT}^{\tilde{k}k}}{M^2}D^{\perp 3}_{-3ddTT}\Big)\Big\}. \quad (3.13)$$

Here a summation over quark flavor is implicit. We can easily check that current conservation $q^\mu W_{t4\mu\nu} = q^\nu W_{t4\mu\nu} = 0$ is valid.

B. The structure functions

By making Lorentz contraction of the hadronic tensor given by Eq. (3.13) with the leptonic tensor, we obtain the differential cross section. By comparing the results obtained with the general form given by Eqs. (2.30)–(2.42) in terms of the structure functions, we obtain the twist-4 results of the structure functions in the QCD parton model at leading order pQCD. We now present the results. For comparison, we include also the leading twist and twist-3 results here.

Up to twist-4, we have contributions to all the 81 structure functions. Among them, 18 have both leading twist and twist-4 contributions. They are given by

$$zW_{U1} = c_1^e c_1^q(D_1 - 4\kappa_M^2\mathrm{Re}D_{-3dd}/z), \quad (3.14)$$

$$zW_{U3} = 2c_3^e c_3^q(D_1 - 4\kappa_M^2\mathrm{Re}D_{-3dd}/z), \quad (3.15)$$

$$z\tilde{W}_{L1} = c_1^e c_3^q(G_{1L} - 4\kappa_M^2\mathrm{Re}D_{-3ddL}/z), \quad (3.16)$$

$$z\tilde{W}_{L3} = 2c_3^e c_1^q(G_{1L} - 4\kappa_M^2\mathrm{Re}D_{-3ddL}/z), \quad (3.17)$$

$$zW_{T1}^{\sin(\varphi-\varphi_S)} = k_{\perp M}c_1^e c_1^q(D^\perp_{1T} - 4\kappa_M^2\mathrm{Re}D^\perp_{-3ddT}/z), \quad (3.18)$$

$$zW_{T3}^{\sin(\varphi-\varphi_S)} = 2k_{\perp M}c_3^e c_3^q(D^\perp_{1T} - 4\kappa_M^2\mathrm{Re}D^\perp_{-3ddT}/z), \quad (3.19)$$

$$z\tilde{W}_{T1}^{\cos(\varphi-\varphi_S)} = k_{\perp M}c_1^e c_3^q(G^\perp_{1T} - 4\kappa_M^2\mathrm{Re}D^{\perp 3}_{-3ddT}/z), \quad (3.20)$$

$$z\tilde{W}_{T3}^{\cos(\varphi-\varphi_S)} = 2k_{\perp M}c_3^e c_1^q(G^\perp_{1T} - 4\kappa_M^2\mathrm{Re}D^{\perp 3}_{-3ddT}/z), \quad (3.21)$$

$$zW_{LL1} = c_1^e c_1^q (D_{1LL} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLL}/z), \tag{3.22}$$

$$zW_{LL3} = 2c_3^e c_3^q (D_{1LL} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLL}/z), \tag{3.23}$$

$$zW_{LT1}^{\cos(\varphi-\varphi_{LT})} = -k_{\perp M} c_1^e c_1^q (D_{1LT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLT}^{\perp}/z), \tag{3.24}$$

$$zW_{LT3}^{\cos(\varphi-\varphi_{LT})} = -2k_{\perp M} c_3^e c_3^q (D_{1LT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLT}^{\perp}/z), \tag{3.25}$$

$$z\tilde{W}_{LT1}^{\sin(\varphi-\varphi_{LT})} = k_{\perp M} c_1^e c_3^q (G_{1LT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLT}^{\perp 3}/z), \tag{3.26}$$

$$z\tilde{W}_{LT3}^{\sin(\varphi-\varphi_{LT})} = 2k_{\perp M} c_3^e c_1^q (G_{1LT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddLT}^{\perp 3}/z), \tag{3.27}$$

$$zW_{TT1}^{\cos(2\varphi-2\varphi_{TT})} = k_{\perp M}^2 c_1^e c_1^q (D_{1TT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddTT}^{\perp}/z), \tag{3.28}$$

$$\begin{aligned} zW_{TT3}^{\cos(2\varphi-2\varphi_{TT})} &= 2k_{\perp M}^2 c_3^e c_3^q (D_{1TT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddTT}^{\perp}/z), \\ z\tilde{W}_{TT1}^{\sin(2\varphi-2\varphi_{TT})} &= -k_{\perp M}^2 c_1^e c_3^q (G_{1TT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddTT}^{\perp 3}/z), \\ z\tilde{W}_{TT3}^{\sin(2\varphi-2\varphi_{TT})} &= -2k_{\perp M}^2 c_3^e c_1^q (G_{1TT}^{\perp} - 4\kappa_M^2 \mathrm{Re} D_{-3ddTT}^{\perp 3}/z). \end{aligned} \tag{3.29}$$

Here, as in [41], we use $\kappa_M \equiv M/Q$ to symbolize higher twist contributions, i.e., κ_M symbolizes twist-3 and κ_M^2 is twist-4. We also use $k_{\perp M} \equiv |\vec{k}_\perp|/M$ to make the equations look more concise. We may note that the results obtained take a quite unified form, i.e., a twist-2 FF minus the real part of the corresponding twist-4 FF. Another 27 have only twist-4 contributions

$$z^2 W_{U2} = 8\kappa_M^2 c_1^e c_1^q D_3/z, \tag{3.30}$$

$$z^2 W_U^{\cos 2\varphi} = -2k_{\perp M}^2 \kappa_M^2 c_1^e c_1^q \mathrm{Re} D_{-3d}^{\perp}, \tag{3.31}$$

$$z^2 \tilde{W}_U^{\sin 2\varphi} = -2k_{\perp M}^2 \kappa_M^2 c_1^e c_3^q \mathrm{Im} D_{-3d}^{\perp}, \tag{3.32}$$

$$z^2 \tilde{W}_{L2} = 8\kappa_M^2 c_1^e c_3^q G_{3L}/z, \tag{3.33}$$

$$z^2 \tilde{W}_L^{\cos 2\varphi} = -2k_{\perp M}^2 \kappa_M^2 c_1^e c_3^q \mathrm{Im} D_{+3dL}^{\perp}, \tag{3.34}$$

$$z^2 W_L^{\sin 2\varphi} = 2k_{\perp M}^2 \kappa_M^2 c_1^e c_1^q \mathrm{Re} D_{+3dL}^{\perp}, \tag{3.35}$$

$$z^2 W_{T2}^{\sin(\varphi-\varphi_S)} = 8k_{\perp M} \kappa_M^2 c_1^e c_1^q D_{3T}^{\perp}/z, \tag{3.36}$$

$$z^2 W_T^{\sin(\varphi+\varphi_S)} = -k_{\perp M}^3 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dT}^{\perp 2} + D_{+3dT}^{\perp 4}), \tag{3.37}$$

$$z^2 W_T^{\sin(3\varphi-\varphi_S)} = k_{\perp M}^3 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dT}^{\perp 2} - D_{+3dT}^{\perp 4}), \tag{3.38}$$

$$z^2 \tilde{W}_{T2}^{\cos(\varphi-\varphi_S)} = 8k_{\perp M} \kappa_M^2 c_1^e c_3^q G_{3T}^{\perp}/z, \tag{3.39}$$

$$z^2 \tilde{W}_T^{\cos(\varphi+\varphi_S)} = k_{\perp M}^3 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dT}^{\perp 2} + D_{+3dT}^{\perp 4}), \tag{3.40}$$

$$z^2 \tilde{W}_T^{\cos(3\varphi-\varphi_S)} = -k_{\perp M}^3 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dT}^{\perp 2} - D_{+3dT}^{\perp 4}), \tag{3.41}$$

$$z^2 W_{LL2} = 8\kappa_M^2 c_1^e c_1^q D_{3LL}/z, \tag{3.42}$$

$$z^2 W_{LL}^{\cos 2\varphi} = -2k_{\perp M}^2 \kappa_M^2 c_1^e c_1^q \mathrm{Re} D_{-3dLL}^{\perp}, \tag{3.43}$$

$$z^2 \tilde{W}_{LL}^{\sin 2\varphi} = -2k_{\perp M}^2 \kappa_M^2 c_1^e c_3^q \mathrm{Im} D_{-3dLL}^{\perp}, \tag{3.44}$$

$$z^2 W_{LT2}^{\cos(\varphi-\varphi_{LT})} = -8k_{\perp M} \kappa_M^2 c_1^e c_1^q D_{3LT}^{\perp}/z, \tag{3.45}$$

$$z^2 W_{LT}^{\cos(\varphi+\varphi_{LT})} = k_{\perp M}^3 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dLT}^{\perp 2} - D_{+3dLT}^{\perp 4}), \tag{3.46}$$

$$z^2 W_{LT}^{\cos(3\varphi-\varphi_{LT})} = k_{\perp M}^3 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dLT}^{\perp 2} + D_{+3dLT}^{\perp 4}), \tag{3.47}$$

$$z^2 \tilde{W}_{LT2}^{\sin(\varphi-\varphi_{LT})} = -8k_{\perp M} \kappa_M^2 c_1^e c_3^q G_{3LT}^{\perp}/z, \tag{3.48}$$

$$z^2 \tilde{W}_{LT}^{\sin(\varphi+\varphi_{LT})} = k_{\perp M}^3 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dLT}^{\perp 2} - D_{+3dLT}^{\perp 4}), \tag{3.49}$$

$$z^2 \tilde{W}_{LT}^{\sin(3\varphi-\varphi_{LT})} = k_{\perp M}^3 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dLT}^{\perp 2} + D_{+3dLT}^{\perp 4}), \tag{3.50}$$

$$z^2 W_{TT2}^{\cos(2\varphi-2\varphi_{TT})} = 8k_{\perp M}^2 \kappa_M^2 c_1^e c_1^q D_{3TT}^{\perp}/z, \tag{3.51}$$

$$z^2 W_{TT}^{\cos 2\varphi_{TT}} = -k_{\perp M}^4 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dTT}^{\perp 2} - D_{+3dTT}^{\perp 4}), \tag{3.52}$$

$$z^2 W_{TT}^{\cos(4\varphi-2\varphi_{TT})} = -k_{\perp M}^4 \kappa_M^2 c_1^e c_1^q \mathrm{Re}(D_{-3dTT}^{\perp 2} + D_{+3dTT}^{\perp 4}), \tag{3.53}$$

$$z^2 \tilde{W}_{TT2}^{\sin(2\varphi-2\varphi_{TT})} = 8k_{\perp M}^2 \kappa_M^2 c_1^e c_3^q G_{3TT}^{\perp}/z, \tag{3.54}$$

$$z^2 \tilde{W}_{TT}^{\sin 2\varphi_{TT}} = -k_{\perp M}^4 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dTT}^{\perp 2} - D_{+3dTT}^{\perp 4}), \tag{3.55}$$

$$z^2 \tilde{W}_{TT}^{\sin(4\varphi-2\varphi_{TT})} = -k_{\perp M}^4 \kappa_M^2 c_1^e c_3^q \mathrm{Im}(D_{-3dTT}^{\perp 2} + D_{+3dTT}^{\perp 4}). \tag{3.56}$$

The rest 36 have only twist-3 contributions

$$z^2 W_{U1}^{\cos\varphi} = 4k_{\perp M} \kappa_M c_3^e c_3^q D^{\perp}, \tag{3.57}$$

$$z^2 W_{U2}^{\cos\varphi} = 2k_{\perp M} \kappa_M c_1^e c_1^q D^{\perp}, \tag{3.58}$$

$$z^2 \tilde{W}_{U1}^{\sin\varphi} = -4k_{\perp M} \kappa_M c_3^e c_1^q G^{\perp}, \tag{3.59}$$

$$z^2 \tilde{W}_{U2}^{\sin\varphi} = -2k_{\perp M} \kappa_M c_1^e c_3^q G^{\perp}, \tag{3.60}$$

$$z^2 W_{L1}^{\sin\varphi} = 4k_{\perp M} \kappa_M c_3^e c_3^q D_L^{\perp}, \tag{3.61}$$

$$z^2 W_{L2}^{\sin\varphi} = 2k_{\perp M}\kappa_M c_1^e c_1^q D_L^\perp, \tag{3.62}$$

$$z^2 \tilde{W}_{L1}^{\cos\varphi} = 4k_{\perp M}\kappa_M c_3^e c_1^q G_L^\perp, \tag{3.63}$$

$$z^2 \tilde{W}_{L2}^{\cos\varphi} = 2k_{\perp M}\kappa_M c_1^e c_3^q G_L^\perp, \tag{3.64}$$

$$z^2 W_{T1}^{\sin\varphi_S} = 4\kappa_M c_3^e c_3^q D_T, \tag{3.65}$$

$$z^2 W_{T2}^{\sin\varphi_S} = 2\kappa_M c_1^e c_1^q D_T, \tag{3.66}$$

$$z^2 W_{T1}^{\sin(2\varphi-\varphi_S)} = 2k_{\perp M}^2\kappa_M c_3^e c_3^q D_T^\perp, \tag{3.67}$$

$$z^2 W_{T2}^{\sin(2\varphi-\varphi_S)} = k_{\perp M}^2\kappa_M c_1^e c_1^q D_T^\perp, \tag{3.68}$$

$$z^2 \tilde{W}_{T1}^{\cos\varphi_S} = 4\kappa_M c_3^e c_1^q G_T, \tag{3.69}$$

$$z^2 \tilde{W}_{T2}^{\cos\varphi_S} = 2\kappa_M c_1^e c_3^q G_T, \tag{3.70}$$

$$z^2 \tilde{W}_{T1}^{\cos(2\varphi-\varphi_S)} = 2k_{\perp M}^2\kappa_M c_3^e c_1^q G_T^\perp, \tag{3.71}$$

$$z^2 \tilde{W}_{T2}^{\cos(2\varphi-\varphi_S)} = k_{\perp M}^2\kappa_M c_1^e c_3^q G_T^\perp, \tag{3.72}$$

$$z^2 W_{LL1}^{\cos\varphi} = 4k_{\perp M}\kappa_M c_3^e c_3^q D_{LL}^\perp, \tag{3.73}$$

$$z^2 W_{LL2}^{\cos\varphi} = 2k_{\perp M}\kappa_M c_1^e c_1^q D_{LL}^\perp, \tag{3.74}$$

$$z^2 \tilde{W}_{LL1}^{\sin\varphi} = -4k_{\perp M}\kappa_M c_3^e c_1^q G_{LL}^\perp, \tag{3.75}$$

$$z^2 \tilde{W}_{LL2}^{\sin\varphi} = -2k_{\perp M}\kappa_M c_1^e c_3^q G_{LL}^\perp, \tag{3.76}$$

$$z^2 \tilde{W}_{LT1}^{\sin\varphi_{LT}} = 4\kappa_M c_3^e c_1^q G_{LT}, \tag{3.77}$$

$$z^2 \tilde{W}_{LT2}^{\sin\varphi_{LT}} = 2\kappa_M c_1^e c_3^q G_{LT}, \tag{3.78}$$

$$z^2 \tilde{W}_{LT1}^{\sin(2\varphi-\varphi_{LT})} = 2k_{\perp M}^2\kappa_M c_3^e c_1^q G_{LT}^\perp, \tag{3.79}$$

$$z^2 \tilde{W}_{LT2}^{\sin(2\varphi-\varphi_{LT})} = k_{\perp M}^2\kappa_M c_1^e c_3^q G_{LT}^\perp, \tag{3.80}$$

$$z^2 W_{LT1}^{\cos\varphi_{LT}} = 4\kappa_M c_3^e c_3^q D_{LT}, \tag{3.81}$$

$$z^2 W_{LT2}^{\cos\varphi_{LT}} = 2\kappa_M c_1^e c_1^q D_{LT}, \tag{3.82}$$

$$z^2 W_{LT1}^{\cos(2\varphi-\varphi_{LT})} = 2k_{\perp M}^2\kappa_M c_3^e c_3^q D_{LT}^\perp, \tag{3.83}$$

$$z^2 W_{LT2}^{\cos(2\varphi-\varphi_{LT})} = k_{\perp M}^2\kappa_M c_1^e c_1^q D_{LT}^\perp, \tag{3.84}$$

$$z^2 \tilde{W}_{TT1}^{\sin(\varphi-2\varphi_{TT})} = 4k_{\perp M}\kappa_M c_3^e c_1^q G_{TT}'^\perp, \tag{3.85}$$

$$z^2 \tilde{W}_{TT2}^{\sin(\varphi-2\varphi_{TT})} = 2k_{\perp M}\kappa_M c_1^e c_3^q G_{TT}'^\perp, \tag{3.86}$$

$$z^2 \tilde{W}_{TT1}^{\sin(3\varphi-2\varphi_{TT})} = 2k_{\perp M}^3\kappa_M c_3^e c_1^q G_{TT}^\perp, \tag{3.87}$$

$$z^2 \tilde{W}_{TT2}^{\sin(3\varphi-2\varphi_{TT})} = k_{\perp M}^3\kappa_M c_1^e c_3^q G_{TT}^\perp, \tag{3.88}$$

$$z^2 W_{TT1}^{\cos(\varphi-2\varphi_{TT})} = 4k_{\perp M}\kappa_M c_3^e c_3^q D_{TT}'^\perp, \tag{3.89}$$

$$z^2 W_{TT2}^{\cos(\varphi-2\varphi_{TT})} = 2k_{\perp M}\kappa_M c_1^e c_1^q D_{TT}'^\perp, \tag{3.90}$$

$$z^2 W_{TT1}^{\cos(3\varphi-2\varphi_{TT})} = 2k_{\perp M}^3\kappa_M c_3^e c_3^q D_{TT}^\perp, \tag{3.91}$$

$$z^2 W_{TT2}^{\cos(3\varphi-2\varphi_{TT})} = k_{\perp M}^3\kappa_M c_1^e c_1^q D_{TT}^\perp. \tag{3.92}$$

As in [41] for SIDIS, we see again the following two distinct features: (1) Structure functions for sine or cosine of even number of azimuthal angles (φ, φ_S, φ_{LT} and/or $2\varphi_{TT}$) have leading-twist and/or twist-4 contributions, while those for sine or cosine of an odd number of azimuthal angles have twist-3 contributions. (2) For the structure functions that have leading twist contributions, there are always twist-4 addenda to them. The leading twist and twist-4 contributions mix up with each other. However, the twist-3 contributions are always separated from the leading twist and/or twist-4 contributions, and all of the twist-3 FFs are corresponding to the azimuthal asymmetries that are absent in leading twist and twist-4 contributions.

C. Azimuthal Asymmetries

Consider the unpolarized case, i.e., summing over the spin of the produced hadron, we have only two twist-3 and two twist-4 azimuthal asymmetries for $e^+e^- \to h\bar{q}X$, i.e.,

$$\langle\cos\varphi\rangle_U = -2k_{\perp M}\kappa_M \frac{D(y)T_2^q(y)D^\perp}{T_0^q(y)zD_1}, \tag{3.93}$$

$$\langle\sin\varphi\rangle_U = -2k_{\perp M}\kappa_M \frac{D(y)T_3^q(y)G^\perp}{T_0^q(y)zD_1}, \tag{3.94}$$

$$\langle\cos 2\varphi\rangle_U = -\frac{1}{2}k_{\perp M}^2\kappa_M^2 \frac{C(y)c_1^e c_1^q \mathrm{Re}D_{-3d}^\perp}{T_0^q(y)zD_1}, \tag{3.95}$$

$$\langle\sin 2\varphi\rangle_U = -\frac{1}{2}k_{\perp M}^2\kappa_M^2 \frac{C(y)c_1^e c_3^q \mathrm{Im}D_{-3d}^\perp}{T_0^q(y)zD_1}, \tag{3.96}$$

where $y = (1+\cos\theta)/2$,

$$T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y), \tag{3.97}$$

$$T_2^q(y) = c_1^e c_1^q B(y) - c_3^e c_3^q, \tag{3.98}$$

$$T_3^q(y) = c_3^e c_1^q - c_1^e c_3^q B(y), \tag{3.99}$$

and $A(y) = (1-y)^2 + y^2 = (1+\cos^2\theta)/2$, $B(y) = 1-2y = -\cos\theta$, $C(y) = 4y(1-y) = \sin^2\theta$, $D(y) = \sqrt{y(1-y)} = \sin\theta/2$. We note that $\langle\cos\varphi\rangle_U$ and $\langle\cos 2\varphi\rangle_U$ are parity conserved, $\langle\sin\varphi\rangle_U$ and $\langle\sin 2\varphi\rangle_U$ are parity violated.

D. Hadron polarizations

We present only results averaged over azimuthal angle φ. For the longitudinal components, we have both leading twist and twist-4 contributions. They are given by

$$\langle\lambda_h\rangle = -\frac{2}{3}\frac{P_q(y)T_0^q(y)G_{1L}}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_L\kappa_M^2), \quad (3.100)$$

$$\langle S_{LL}\rangle = \frac{1}{2}\frac{T_0^q(y)D_{1LL}}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{LL}\kappa_M^2); \quad (3.101)$$

$$\alpha_U = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3dd}-C(y)c_1^ec_1^qD_3}{z^2T_0^q(y)D_1}, \quad (3.102)$$

$$\alpha_L = 4\frac{zP_q(y)T_0^q(y)\mathrm{Re}D_{-3ddL}+C(y)c_1^ec_3^qG_{3L}}{z^2P_q(y)T_0^q(y)G_{1L}}, \quad (3.103)$$

$$\alpha_{LL} = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddLL}-C(y)c_1^ec_1^qD_{3LL}}{z^2T_0^q(y)D_{1LL}}, \quad (3.104)$$

where $P_q(y)$ is the longitudinal polarization of q produced in $e^+e^-\to Z\to q\bar{q}$, $P_q(y)=T_1^q(y)/T_0^q(y)$, $T_1^q(y)=-c_1^ec_3^qA(y)+c_3^ec_1^qB(y)$. Here, we emphasize in particular that the factor $T_0^q(y)$ in the numerator and that in the denominator in Eqs. (3.100)–(3.104) can *not* cancel with each other, since a summation over flavor q is implicit in the numerator and in the denominator, respectively. This also applies to all of the results presented in the following of this paper.

For the transverse components with respect to the lepton-hadron plane, we have

$$\langle S_T^x\rangle = \frac{8}{3}\kappa_M\frac{D(y)T_3^q(y)G_T}{T_0^q(y)zD_1}, \quad (3.105)$$

$$\langle S_T^y\rangle = -\frac{8}{3}\kappa_M\frac{D(y)T_2^q(y)D_T}{T_0^q(y)zD_1}, \quad (3.106)$$

$$\langle S_{LT}^x\rangle = -\frac{8}{3}\kappa_M\frac{D(y)T_2^q(y)D_{LT}}{T_0^q(y)zD_1}, \quad (3.107)$$

$$\langle S_{LT}^y\rangle = \frac{8}{3}\kappa_M\frac{D(y)T_3^q(y)G_{LT}}{T_0^q(y)zD_1}, \quad (3.108)$$

$$\langle S_{TT}^{xx}\rangle = -\frac{1}{3}k_{\perp M}^4\kappa_M^2\frac{C(y)c_1^ec_1^q\mathrm{Re}(D_{-3dTT}^{\perp 2}-D_{+3dTT}^{\perp 4})}{T_0^q(y)zD_1}, \quad (3.109)$$

$$\langle S_{TT}^{xy}\rangle = -\frac{1}{3}k_{\perp M}^4\kappa_M^2\frac{C(y)c_1^ec_3^q\mathrm{Im}(D_{-3dTT}^{\perp 2}-D_{+3dTT}^{\perp 4})}{T_0^q(y)zD_1}. \quad (3.110)$$

We see that $\langle S_T^x\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$ and $\langle S_{LT}^y\rangle$ have only twist-3 contributions while $\langle S_{TT}^{xx}\rangle$ and $\langle S_{TT}^{xy}\rangle$ have only twist-4 contributions.

For the transverse components with respect to the hadron-jet plane, we obtain

$$\langle S_T^n\rangle = \frac{2}{3}k_{\perp M}\frac{T_0^q(y)D_{1T}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_T^n\kappa_M^2), \quad (3.111)$$

$$\langle S_T^t\rangle = -\frac{2}{3}k_{\perp M}\frac{P_q(y)T_0^q(y)G_{1T}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_T^t\kappa_M^2), \quad (3.112)$$

$$\langle S_{LT}^n\rangle = -\frac{2}{3}k_{\perp M}\frac{P_q(y)T_0^q(y)G_{1LT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{LT}^n\kappa_M^2), \quad (3.113)$$

$$\langle S_{LT}^t\rangle = -\frac{2}{3}k_{\perp M}\frac{T_0^q(y)D_{1LT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{LT}^t\kappa_M^2), \quad (3.114)$$

$$\langle S_{TT}^{nn}\rangle = -\frac{2}{3}k_{\perp M}^2\frac{T_0^q(y)D_{1TT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{TT}^{nn}\kappa_M^2), \quad (3.115)$$

$$\langle S_{TT}^{nt}\rangle = \frac{2}{3}k_{\perp M}^2\frac{P_q(y)T_0^q(y)G_{1TT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{TT}^{nt}\kappa_M^2), \quad (3.116)$$

where the α's are similar to those given by Eqs. (3.103)–(3.104) in the longitudinally polarized case, i.e.,

$$\alpha_T^n = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddT}^\perp-C(y)c_1^ec_1^qD_{3T}^\perp}{z^2T_0^q(y)D_{1T}^\perp}, \quad (3.117)$$

$$\alpha_T^t = 4\frac{zP_q(y)T_0^q(y)\mathrm{Re}D_{-3ddT}^{\perp 3}+C(y)c_1^ec_3^qG_{3T}^\perp}{z^2P_q(y)T_0^q(y)G_{1T}^\perp}, \quad (3.118)$$

$$\alpha_{LT}^n = 4\frac{zP_q(y)T_0^q(y)\mathrm{Re}D_{-3ddLT}^{\perp 3}-C(y)c_1^ec_3^qG_{3LT}^\perp}{z^2P_q(y)T_0^q(y)G_{1LT}^\perp}, \quad (3.119)$$

$$\alpha_{LT}^t = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddLT}^\perp-C(y)c_1^ec_1^qD_{3LT}^\perp}{z^2T_0^q(y)D_{1LT}^\perp}, \quad (3.120)$$

$$\alpha_{TT}^{nn} = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddTT}^\perp-C(y)c_1^ec_1^qD_{3TT}^\perp}{z^2T_0^q(y)D_{1TT}^\perp}, \quad (3.121)$$

$$\alpha_{TT}^{nt} = 4\frac{zP_q(y)T_0^q(y)\mathrm{Re}D_{-3ddTT}^{\perp 3}-C(y)c_1^ec_3^qG_{3TT}^\perp}{z^2P_q(y)T_0^q(y)G_{1TT}^\perp}. \quad (3.122)$$

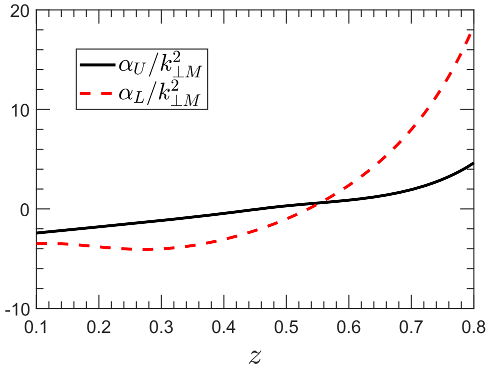

FIG. 3. A rough estimation of the twist-4 contribution factor $\alpha/k_{\perp M}^2$ as a function of z at $y = 0.5$ and $Q = M_Z$.

We see that the transverse components with respect to the hadron-jet plane have both leading and twist-4 contributions. We also note that the leading twist and twist-3 parts are the same as those obtained in [38,39].

If we use the relationships given by Eqs. (2.83)–(2.92) obtained at $g = 0$, we obtain

$$\alpha_U \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y) D_1}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_1}{T_0^q(y) D_1}\right], \quad (3.123)$$

$$\alpha_L \approx -k_{\perp M}^2\left[\frac{\partial \ln P_q(y) T_0^q(y) G_{1L}}{\partial \ln z} - \frac{2C(y) c_1^e c_3^q G_{1L}}{P_q(y) T_0^q(y) G_{1L}}\right], \quad (3.124)$$

$$\alpha_{LL} \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y) D_{1LL}}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_{1LL}}{T_0^q(y) D_{1LL}}\right], \quad (3.125)$$

$$\alpha_T^n \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y) D_{1T}^\perp}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_{1T}^\perp}{T_0^q(y) D_{1T}^\perp}\right], \quad (3.126)$$

$$\alpha_T^t \approx -k_{\perp M}^2\left[\frac{\partial \ln P_q(y) T_0^q(y) G_{1T}^\perp}{\partial \ln z} - \frac{2C(y) c_1^e c_3^q G_{1T}^\perp}{P_q(y) T_0^q(y) G_{1T}^\perp}\right], \quad (3.127)$$

$$\alpha_{LT}^n \approx -k_{\perp M}^2\left[\frac{\partial \ln P_q(y) T_0^q(y) G_{1LT}^\perp}{\partial \ln z} - \frac{2C(y) c_1^e c_3^q G_{1LT}^\perp}{P_q(y) T_0^q(y) G_{1LT}^\perp}\right],$$
$$\alpha_{LT}^t \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y) D_{1LT}^\perp}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_{1LT}^\perp}{T_0^q(y) D_{1LT}^\perp}\right],$$
$$\alpha_{TT}^{nt} \approx -k_{\perp M}^2\left[\frac{\partial \ln P_q(y) T_0^q(y) G_{1TT}^\perp}{\partial \ln z} - \frac{2C(y) c_1^e c_3^q G_{1TT}^\perp}{P_q(y) T_0^q(y) G_{1TT}^\perp}\right], \quad (3.128)$$

$$\alpha_{TT}^{nn} \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y) D_{1TT}^\perp}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_{1TT}^\perp}{T_0^q(y) D_{1TT}^\perp}\right]. \quad (3.129)$$

At present stage, we may use these equations to make rough estimations for twist-4 contributions. To get a feeling of how large they could be, we plot α_U and α_L using the parameterizations of FFs in [47–49]. We see from Fig. 3 that the modifications could be quite significant.

E. Contributions from the four-quark correlator

The calculations presented above are made only for $e^+e^- \to h\bar{q}X$ where only quark-j-gluon-quark correlators are considered. Similar to those in deeply inelastic lepton-nucleon scattering discussed in [41,50], up to twist-4, we have also contributions from diagrams involving the four quark correlator

$$\hat{\Xi}_{(4q)}^{(0)}(k_1, k, k_2) = \frac{g^2}{8}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} e^{-ik_1y + i(k_1-k)y_1 - i(k_2-k)y_2} \times \sum_X \langle 0|\bar{\psi}(y_2)\mathcal{L}^\dagger(0, y_2)\psi(0)|hX\rangle \times \langle hX|\bar{\psi}(y)\mathcal{L}(y, y_1)\psi(y_1)|0\rangle. \quad (3.130)$$

Example of such diagrams are shown in Fig. 4 where we obtain contributions to $e^+e^- \to hgX$ if the cut is given at the middle while they contribute to $e^+e^- \to h\bar{q}X$ if we have the left or right cut. Both of them contribute to $e^+e^- \to h + \text{jet} + X$, so we consider them together.

It can be shown that the collinear expansion can also be applied to this case and the gauge links included in the correlators given by Eq. (3.130) are obtained by taking the

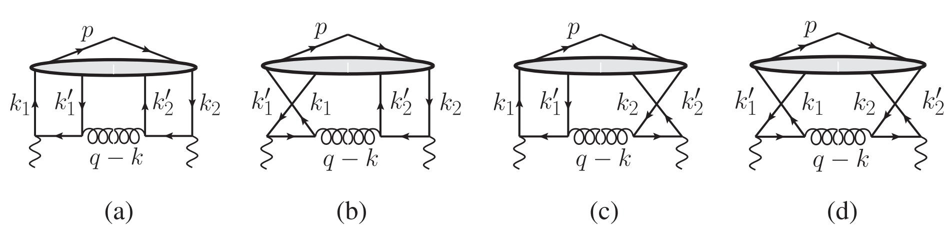

FIG. 4. The first four of the four-quark diagrams where no multiple gluon scattering is involved. In (a), we have $k_1' = k_1 - k$ and $k_2' = k_2 - k$; in (b) we have the interchange of k_1 with k_1'; in (c) we have the interchange of k_2 with k_2'; in (d) we have both interchanges of k_1 with k_1' and k_2 with k_2'.

multiple gluon scattering into account. The hadronic tensor $W^{(g)}_{4q\mu\nu}$ for $e^+e^- \to h+g+X$ and $W^{(q)}_{4q\mu\nu}$ for $e^+e^- \to h+\bar{q}+X$ can be written as the unified form

$$W^{(g/q)}_{4q\mu\nu} = \frac{1}{p\cdot q}\int dz dz_1 dz_2 h^{g/q}_{4q}[(c_1^q g_{\perp\mu\nu} + ic_3^q \varepsilon_{\perp\mu\nu})C_s + (c_3^q g_{\perp\mu\nu} + ic_1^q\varepsilon_{\perp\mu\nu})C_{ps}]. \quad (3.131)$$

Here C_s and C_{ps} are TMD correlation functions given by

$$C_j = \int d^4k_1 d^4k d^4k_2 \delta\left(z - \frac{p^+}{k^+}\right)\delta(k_1^+ z_1 - p^+)\delta(k_2^+ z_2 - p^+) \times (2\pi)^2\delta^2(\vec{k}_\perp + \vec{k}_\perp)\Xi^{(0)}_{(4q)j}(k_1,k,k_2;p,S), \quad (3.132)$$

where $j = s$ or ps and the unintegrated correlation functions $\Xi^{(0)}_{(4q)s}$ and $\Xi^{(0)}_{(4q)ps}$ are defined as

$$\Xi^{(0)}_{(4q)s} = \frac{g^2}{8}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} e^{-ik_1y + i(k_1-k)y_1 - i(k_2-k)y_2} \times \sum_X \{\langle 0|\bar\psi(y_2)\not{n}\psi(0)|hX\rangle\langle hX|\bar\psi(y)\not{n}\psi(y_1)|0\rangle + \langle 0|\bar\psi(y_2)\gamma^5\not{n}\psi(0)|hX\rangle\langle hX|\bar\psi(y)\gamma^5\not{n}\psi(y_1)|0\rangle\}, \quad (3.133)$$

$$\Xi^{(0)}_{(4q)ps} = \frac{g^2}{8}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} e^{-ik_1y + i(k_1-k)y_1 - i(k_2-k)y_2} \times \sum_X \{\langle 0|\bar\psi(y_2)\gamma^5\not{n}\psi(0)|hX\rangle\langle hX|\bar\psi(y)\not{n}\psi(y_1)|0\rangle + \langle 0|\bar\psi(y_2)\not{n}\psi(0)|hX\rangle\langle hX|\bar\psi(y)\gamma^5\not{n}\psi(y_1)|0\rangle\}, \quad (3.134)$$

where we have omitted the gauge links that are the same as those in Eq. (3.130). The $h^{g/q}_{4q}$ are obtained by summing over all the diagrams. For h^g_{4q}, we obtain

$$h^g_{4q} = \frac{zz_B^3\delta(z-z_B)}{(z_1 - z_B + i\epsilon)(z_2 - z_B - i\epsilon)} + \frac{z_B^2/z_1z_2\delta(z-z_B)}{(1/z_1 + i\epsilon)(1/z_2 - i\epsilon)} - \frac{z_B^3/z_2\delta(z-z_B)}{(z_1 - z_B + i\epsilon)(1/z_2 - i\epsilon)} - (1\leftrightarrow 2)^*. \quad (3.135)$$

For h^q_{4q}, we have, $h^q_{4q} = h^{qL}_{4q} + h^{qR}_{4q}$,

$$h^{qL}_{4q} = \frac{zz_B^3\delta(z_1 - z_B)}{(z - z_B - i\epsilon)(z_2 - z_B - i\epsilon)} - \left(\frac{1}{z_2}\to\frac{1}{z}-\frac{1}{z_2}\right) - \frac{zz_B^3\delta(z_1 + z_B - \frac{z_1z_B}{z})}{(z - z_B - i\epsilon)(z_2 - z_B - i\epsilon)} + \left(\frac{1}{z_2}\to\frac{1}{z}-\frac{1}{z_2}\right), \quad (3.136)$$

and $h^{qR}_{4q}(z_1,z,z_2) = h^{qL*}_{4q}(z_2,z,z_1)$. Adding all of them together, we obtain $h_{4q} = h^{qL}_{4q} + h^{qR}_{4q} + h^g_{4q}$. For C_s and C_{ps}, they can be decomposed as

$$z\int dz dz_1 dz_2 h_{4q} C_s = M^2\left(D_{4q} - \frac{\varepsilon_\perp^{kS}}{M}D^\perp_{4qT} + S_{LL}D_{4qLL} + \frac{k_\perp\cdot S_{LT}}{M}D^\perp_{4qLT} + \frac{S^{kk}_{TT}}{M^2}D^\perp_{4qTT}\right), \quad (3.137)$$

$$z\int dz dz_1 dz_2 h_{4q} C_{ps} = M^2\left(\lambda_h G_{4qL} - \frac{k_\perp\cdot S_T}{M}G^\perp_{4qT} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G^\perp_{4qLT} + \frac{S^{\tilde{k}k}_{TT}}{M^2}G^\perp_{4qTT}\right). \quad (3.138)$$

The contributions to the structure functions are given by

$$z^2 W_{4qU1} = -\kappa_M^2 c_1^e c_1^q D_{4q}, \quad (3.139)$$

$$z^2 W_{4qU3} = -2\kappa_M^2 c_3^e c_3^q D_{4q}, \quad (3.140)$$

$$z^2 \tilde{W}_{4qL1} = -\kappa_M^2 c_1^e c_3^q G_{4qL}, \quad (3.141)$$

$$z^2 \tilde{W}_{4qL3} = -2\kappa_M^2 c_3^e c_1^q G_{4qL}, \quad (3.142)$$

$$z^2 W^{\sin(\varphi-\varphi_S)}_{4qT1} = -k_{\perp M}\kappa_M^2 c_1^e c_1^q D^\perp_{4qT}, \quad (3.143)$$

$$z^2 W^{\sin(\varphi-\varphi_S)}_{4qT3} = -2k_{\perp M}\kappa_M^2 c_3^e c_3^q D^\perp_{4qT}, \quad (3.144)$$

$$z^2 \tilde{W}^{\cos(\varphi-\varphi_S)}_{4qT1} = -k_{\perp M}\kappa_M^2 c_1^e c_3^q G^\perp_{4qT}, \quad (3.145)$$

$$z^2 \tilde{W}^{\cos(\varphi-\varphi_S)}_{4qT3} = -2k_{\perp M}\kappa_M^2 c_3^e c_1^q G^\perp_{4qT}, \quad (3.146)$$

$$z^2 W_{4qLL1} = -\kappa_M^2 c_1^e c_1^q D_{4qLL}, \quad (3.147)$$

$$z^2 W_{4qLL3} = -2\kappa_M^2 c_3^e c_3^q D_{4qLL}, \quad (3.148)$$

$$z^2 \tilde{W}^{\sin(\varphi-\varphi_{LT})}_{4qLT1} = k_{\perp M}\kappa_M^2 c_1^e c_3^q G^\perp_{4qLT}, \quad (3.149)$$

$$z^2 \tilde{W}^{\sin(\varphi-\varphi_{LT})}_{4qLT3} = 2k_{\perp M}\kappa_M^2 c_3^e c_1^q G^\perp_{4qLT}, \quad (3.150)$$

$$z^2 W^{\cos(\varphi-\varphi_{LT})}_{4qLT1} = k_{\perp M}\kappa_M^2 c_1^e c_1^q D^\perp_{4qLT}, \quad (3.151)$$

$$z^2 W^{\cos(\varphi-\varphi_{LT})}_{4qLT3} = 2k_{\perp M}\kappa_M^2 c_3^e c_3^q D^\perp_{4qLT}, \quad (3.152)$$

$$z^2 \tilde{W}^{\sin(2\varphi-2\varphi_{TT})}_{4qTT1} = -k^2_{\perp M}\kappa_M^2 c_1^e c_3^q G^\perp_{4qTT}, \quad (3.153)$$

$$z^2 \tilde{W}^{\sin(2\varphi-2\varphi_{TT})}_{4qTT3} = -2k^2_{\perp M}\kappa_M^2 c_3^e c_1^q G^\perp_{4qTT}, \quad (3.154)$$

$$z^2 W_{4qTT1}^{\cos(2\varphi-2\varphi_{TT})} = -k_{\perp M}^2 \kappa_M^2 c_1^e c_1^q D_{4qTT}^{\perp}, \quad (3.155)$$

$$z^2 W_{4qTT3}^{\cos(2\varphi-2\varphi_{TT})} = -2k_{\perp M}^2 \kappa_M^2 c_3^e c_3^q D_{4qTT}^{\perp}. \quad (3.156)$$

We see that they have the same modes as for the leading twist contributions. They lead to twist-4 modifications of hadron polarizations that are given by

$$\alpha_{4qU} = \frac{T_0^q(y) D_{4q}}{z T_0^q(y) D_1},$$

$$\alpha_{4qL} = \frac{P_q(y) T_0^q(y) G_{4qL}}{z P_q(y) T_0^q(y) G_{1L}}, \qquad \alpha_{4qLL} = \frac{T_0^q(y) D_{4qLL}}{z T_0^q(y) D_{1LL}},$$

$$\alpha_{4qT}^t = \frac{P_q(y) T_0^q(y) G_{4qT}^{\perp}}{z P_q(y) T_0^q(y) G_{1T}^{\perp}}, \qquad \alpha_{4qT}^n = \frac{T_0^q(y) D_{4qT}^{\perp}}{z T_0^q(y) D_{1T}^{\perp}},$$

$$\alpha_{4qLT}^n = -\frac{P_q(y) T_0^q(y) G_{4qLT}^{\perp}}{z P_q(y) T_0^q(y) G_{1LT}^{\perp}}, \qquad \alpha_{4qLT}^t = \frac{T_0^q(y) D_{4qLT}^{\perp}}{z T_0^q(y) D_{1LT}^{\perp}},$$

$$\alpha_{4qTT}^{nt} = \frac{P_q(y) T_0^q(y) G_{4qTT}^{\perp}}{z P_q(y) T_0^q(y) G_{1TT}^{\perp}}, \qquad \alpha_{4qTT}^{nn} = \frac{T_0^q(y) D_{4qTT}^{\perp}}{z T_0^q(y) D_{1TT}^{\perp}}.$$

F. Reducing to the inclusive process

By integrating the differential cross section for the semi-inclusive process $e^+e^- \to h\bar{q}X$ over $d^2k'_\perp$, we obtain that for the inclusive process $e^+e^- \to hX$ and correspondingly the inclusive structure functions given by Eqs. (2.44)–(2.52). Among the 19 inclusive structure functions, 6 of them have leading twist contributions, they are given by

$$zF_{U1}^{(\mathrm{in})} = c_1^e c_1^q [\hat{D}_1 - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3dd} + \hat{D}_{4q})/z], \quad (3.157)$$

$$zF_{U3}^{(\mathrm{in})} = 2c_3^e c_3^q [\hat{D}_1 - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3dd} + \hat{D}_{4q})/z], \quad (3.158)$$

$$z\tilde{F}_{L1}^{(\mathrm{in})} = c_1^e c_3^q [\hat{G}_{1L} - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3ddL} + \hat{G}_{4qL})/z], \quad (3.159)$$

$$z\tilde{F}_{L3}^{(\mathrm{in})} = 2c_3^e c_1^q [\hat{G}_{1L} - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3ddL} + \hat{G}_{4qL})/z], \quad (3.160)$$

$$zF_{LL1}^{(\mathrm{in})} = c_1^e c_1^q [\hat{D}_{1LL} - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3ddLL} + \hat{D}_{4qLL})/z], \quad (3.161)$$

$$zF_{LL3}^{(\mathrm{in})} = 2c_3^e c_3^q [\hat{D}_{1LL} - \kappa_M^2 (4\mathrm{Re}\hat{D}_{-3ddLL} + \hat{D}_{4qLL})/z], \quad (3.162)$$

where $\hat{D}$'s and $\hat{G}$'s are the corresponding one-dimensional FFs that can be obtained by integrating their three-dimensional counterparts over $d^2k'_\perp/(2\pi)^2$. We see that these six structure functions correspond to the unpolarized, the longitudinally polarized, and S_{LL}-dependent cases. In Eqs. (2.44)–(2.46), they correspond to the $(1+\cos^2\theta)$ and $\cos\theta$-terms.

There are eight structure functions have twist-3 contributions, and are given by

$$z^2 F_{T1}^{(\mathrm{in})\sin\varphi_S} = 4\kappa_M c_3^e c_3^q \hat{D}_T, \quad (3.163)$$

$$z^2 F_{T2}^{(\mathrm{in})\sin\varphi_S} = 2\kappa_M c_1^e c_1^q \hat{D}_T, \quad (3.164)$$

$$z^2 \tilde{F}_{T1}^{(\mathrm{in})\cos\varphi_S} = 4\kappa_M c_3^e c_1^q \hat{G}_T, \quad (3.165)$$

$$z^2 \tilde{F}_{T2}^{(\mathrm{in})\cos\varphi_S} = 2\kappa_M c_1^e c_3^q \hat{G}_T, \quad (3.166)$$

$$z^2 \tilde{F}_{LT1}^{(\mathrm{in})\sin\varphi_{LT}} = 4\kappa_M c_3^e c_1^q \hat{G}_{LT}, \quad (3.167)$$

$$z^2 \tilde{F}_{LT2}^{(\mathrm{in})\sin\varphi_{LT}} = 2\kappa_M c_1^e c_3^q \hat{G}_{LT}, \quad (3.168)$$

$$z^2 F_{LT1}^{(\mathrm{in})\cos\varphi_{LT}} = 4\kappa_M c_3^e c_3^q \hat{D}_{LT}, \quad (3.169)$$

$$z^2 F_{LT2}^{(\mathrm{in})\cos\varphi_{LT}} = 2\kappa_M c_1^e c_1^q \hat{D}_{LT}. \quad (3.170)$$

They all correspond to the transverse components of hadron polarization, where four of them correspond to the transverse components of the vector polarization with respect to the hadron-lepton plane, and another four correspond to the S_{LT}-dependent part. In Eqs. (2.47)–(2.50) they correspond to the $\sin\theta$ and $\sin 2\theta$-terms.

The rest five of the 19 structure functions have only twist-4 contributions, and they are given by

$$zF_{U2}^{(\mathrm{in})} = 8\kappa_M^2 c_1^e c_1^q \hat{D}_3/z^2, \quad (3.171)$$

$$z\tilde{F}_{L2}^{(\mathrm{in})} = 8\kappa_M^2 c_1^e c_3^q \hat{G}_{3L}/z^2, \quad (3.172)$$

$$zF_{LL2}^{(\mathrm{in})} = 8\kappa_M^2 c_1^e c_1^q \hat{D}_{3LL}/z^2, \quad (3.173)$$

$$z^2 F_{TT}^{(\mathrm{in})\cos 2\varphi_{TT}} = -\kappa_M^2 c_1^e c_1^q \mathrm{Re}(\hat{D}_{-3dTT}^{\perp 2} - \hat{D}_{+3dTT}^{\perp 4}), \quad (3.174)$$

$$z^2 \tilde{F}_{TT}^{(\mathrm{in})\sin 2\varphi_{TT}} = -\kappa_M^2 c_1^e c_3^q \mathrm{Im}(\hat{D}_{-3dTT}^{\perp 2} - \hat{D}_{+3dTT}^{\perp 4}). \quad (3.175)$$

They all correspond to the $\sin^2\theta$-terms in Eqs. (2.44)–(2.46) and Eqs. (2.51)–(2.52).

These results show that in $e^+e^- \to Z \to h + X$, we have leading twist longitudinal polarization and spin alignment with twist-4 addenda. They are given by

$$\langle\lambda_h\rangle^{(\mathrm{in})} = -\frac{2}{3}\frac{P_q(y) T_0^q(y) \hat{G}_{1L}}{T_0^q(y) \hat{D}_1}(1 + \alpha_U^{(\mathrm{in})}\kappa_M^2 - \alpha_L^{(\mathrm{in})}\kappa_M^2), \quad (3.176)$$

$$\langle S_{LL}\rangle^{(\mathrm{in})} = \frac{1}{2}\frac{T_0^q(y) \hat{D}_{1LL}}{T_0^q(y) \hat{D}_1}(1 + \alpha_U^{(\mathrm{in})}\kappa_M^2 - \alpha_{LL}^{(\mathrm{in})}\kappa_M^2); \quad (3.177)$$

$$\alpha_U^{(\mathrm{in})} = \frac{z T_0^q(y)(4\mathrm{Re}\hat{D}_{-3dd} + \hat{D}_{4q}) - 4C(y) c_1^e c_1^q \hat{D}_3}{z^2 T_0^q(y) \hat{D}_1}, \quad (3.178)$$

$$\alpha_L^{(\mathrm{in})} = \frac{z P_q(y) T_0^q(y)(4\mathrm{Re}\hat{D}_{-3ddL} + \hat{G}_{4qL}) + 4C(y) c_1^e c_3^q \hat{G}_{3L}}{z^2 P_q(y) T_0^q(y) \hat{G}_{1L}}, \tag{3.179}$$

$$\alpha_{LL}^{(\mathrm{in})} = \frac{z T_0^q(y)(4\mathrm{Re}\hat{D}_{-3ddLL} + \hat{D}_{4qLL}) - 4C(y) c_1^e c_1^q \hat{D}_{3LL}}{z^2 T_0^q(y) \hat{D}_{1LL}}. \tag{3.180}$$

We have also twist-3 transverse polarization with respect to the lepton-hadron plane given by

$$\langle S_T^x \rangle^{(\mathrm{in})} = \frac{8}{3} \kappa_M \frac{D(y) T_3^q(y) \hat{G}_T}{T_0^q(y) z \hat{D}_1}, \tag{3.181}$$

$$\langle S_T^y \rangle^{(\mathrm{in})} = -\frac{8}{3} \kappa_M \frac{D(y) T_2^q(y) \hat{D}_T}{T_0^q(y) z \hat{D}_1}, \tag{3.182}$$

$$\langle S_{LT}^x \rangle^{(\mathrm{in})} = -\frac{8}{3} \kappa_M \frac{D(y) T_2^q(y) \hat{D}_{LT}}{T_0^q(y) z \hat{D}_1}, \tag{3.183}$$

$$\langle S_{LT}^y \rangle^{(\mathrm{in})} = \frac{8}{3} \kappa_M \frac{D(y) T_3^q(y) \hat{G}_{LT}}{T_0^q(y) z \hat{D}_1}. \tag{3.184}$$

For the S_{TT}-components, we have only twist-4 contributions

$$\langle S_{TT}^{xx} \rangle^{(\mathrm{in})} = -\frac{1}{3} \kappa_M^2 \frac{C(y) c_1^e c_1^q \mathrm{Re}(\hat{D}_{-3dTT}^{\perp 2} - \hat{D}_{+3dTT}^{\perp 4})}{T_0^q(y) z \hat{D}_1}, \tag{3.185}$$

$$\langle S_{TT}^{xy} \rangle^{(\mathrm{in})} = -\frac{1}{3} \kappa_M^2 \frac{C(y) c_1^e c_3^q \mathrm{Im}(\hat{D}_{-3dTT}^{\perp 2} - \hat{D}_{+3dTT}^{\perp 4})}{T_0^q(y) z \hat{D}_1}. \tag{3.186}$$

It is interesting to see that even for the inclusive reaction, we can study the twist-3 and twist-4 FFs by measuring these different components of hadron polarization.

IV. SUMMARY

We present the complete twist-4 results for the semi-inclusive annihilation process $e^+ + e^- \to h + \bar{q}(\mathrm{jet}) + X$. The calculations have been carried out by using the collinear expansion where the multiple gluon scattering has been taken into account, and gauge links are obtained systematically and automatically. We present the cross section in terms of structure functions, and the structure functions are given in terms of the gauge invariant FFs.

Among the 81 structure functions, 18 of them have both leading twist and twist-4 contributions, 27 have only twist-4 contributions, and the rest of the 36 have twist-3 contributions. All of those correspond to the sine or cosine of an even number of azimuthal angles that have leading twist and/or twist-4 contributions; those that correspond to the sine or cosine of an odd number of azimuthal angles have twist-3 contributions. For any structure function that has a leading twist contribution, there is a twist-4 addendum to it.

We also present the results of azimuthal asymmetries and different components of hadron polarization in terms of gauge invariant FFs. In the unpolarized case, for $e^+ + e^- \to h + \bar{q}(\mathrm{jet}) + X$, there are only two twist-3 azimuthal asymmetries, $\langle \cos\varphi \rangle_U$ and $\langle \sin\varphi \rangle_U$, and two twist-4 azimuthal asymmetries, $\langle \cos 2\varphi \rangle_U$ and $\langle \sin 2\varphi \rangle_U$. Two of them (the cosines) are parity conserved, and the other two are parity violated.

For hadron polarization averaged over the azimuthal angle, we have leading twist contributions with twist-4 addenda to the helicity $\langle \lambda_h \rangle$, the spin alignment $\langle S_{LL} \rangle$, and the transverse components with respect to the hadron-jet plane, i.e., $\langle S_T^n \rangle$, $\langle S_T^t \rangle$, $\langle S_{LT}^n \rangle$, $\langle S_{LT}^t \rangle$, $\langle S_{TT}^{nn} \rangle$, and $\langle S_{TT}^{nt} \rangle$. For the transverse components with respect to the lepton-hadron plane, i.e., $\langle S_T^x \rangle$, $\langle S_T^y \rangle$, $\langle S_{LT}^x \rangle$, $\langle S_{LT}^y \rangle$, we have twist-3 contributions, while $\langle S_{TT}^{xx} \rangle$ and $\langle S_{TT}^{xy} \rangle$ only have twist-4 contributions.

The four-quark correlators also contribute at twist-4. The contributions take the same modes as those at the leading twist, hence, just addenda to the corresponding leading twist contributions.

For the inclusive reaction $e^+ + e^- \to h + X$, we have leading twist contributions with twist-4 addenda to the helicity $\langle \lambda_h \rangle^{(\mathrm{in})}$ and the spin alignment $\langle S_{LL} \rangle^{(\mathrm{in})}$. For the transverse components with respect to the lepton-hadron plane, we have twist-3 contributions to $\langle S_T^x \rangle^{(\mathrm{in})}$, $\langle S_T^y \rangle^{(\mathrm{in})}$, $\langle S_{LT}^x \rangle^{(\mathrm{in})}$, and $\langle S_{LT}^y \rangle^{(\mathrm{in})}$; but we only have twist-4 contributions to $\langle S_{TT}^{xx} \rangle^{(\mathrm{in})}$ and $\langle S_{TT}^{xy} \rangle^{(\mathrm{in})}$.

The results are presented for e^+e^--annihilation at the Z-pole, where parity conserved and parity violated structure functions contribute. These results reduce to those for e^+e^--annihilation via virtual photon (γ^*) if we make the replacement of c_V by e_q and $c_A = 0$.

We also suggest a method for a rough estimation of twist-4 contributions based on the leading twist fragmentation functions. From the estimation, we see that the twist-4 contributions could be very significant and have large influences on extracting leading twist FFs from the data.

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