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Photonic chiral vortical effect

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Circularly polarized photons have the Berry curvature in the semiclassical regime. Based on the kinetic equation for such chiral photons, we derive the (non)equilibrium expression of the photon current in the direction of the vorticity. We briefly discuss the relevance of this "photonic chiral vortical effect" in pulsars and rotating massive stars and its possible realization in semiconductors.

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I. INTRODUCTION

Recently, the effect of the Berry curvature for photons has attracted great interest in optics and photonics. This effect, originating from the helical nature of circularly polarized photons, leads to remarkable topological transport phenomena. One canonical example of such phenomena is the quantum spin Hall effect of light [1,2], which had also previously been known as the "optical Magnus effect" [3].

In this paper, we develop a kinetic framework for right- and left-handed circularly polarized photons, similar to the one for spin-1/2 chiral fermions (known as the chiral kinetic theory) [4–7]. Based on the kinetic equation, we derive a new type of topological transport phenomenon of photons in a rotation—the *photonic chiral vortical effect* (CVE). This is the photon current along the direction of the vorticity. A similar CVE is known to appear in chiral matter that includes chiral fermions [8–11] and has been intensively studied due to its possible relevance to quark matter in heavy ion collisions [9,10] and neutrino matter in supernovae [12]. We argue that the photonic CVE provides a hitherto neglected contribution to the photon emission from pulsars and rotating massive stars. We also discuss a possible realization of this effect with a *nonzero* chemical potential in semiconductors. We emphasize that our work enlarges the chiral transport phenomena so far limited to (nearly) massless chiral fermions to a drastically wider area of physical systems involving massless photons.

II. QUANTUM MECHANICS FOR PHOTONS

A. Wave equation

We first briefly recapitulate the wave equation for photons (see, e.g., Ref. [13] for a review). To keep quantum mechanical and relativistic nature apparent, we will write explicitly $\hbar$ and c in this section.

The wave function of photons must satisfy the following requirements:

(a) It is linear, such that it can be superposed to have interference effects.

(b) The coefficients are constants unrelated to the specific motion of photons: $\hbar$ and/or c.

(c) It satisfies the relation $\omega = ck$, with ω the frequency and $k \equiv |\boldsymbol{k}|$ the wave number.

The first two conditions account for the wave nature of photons similar to the Schrödinger equation for electrons, and the third condition ensures the dispersion relation of massless photons or electromagnetic waves.

Before proceeding further, let us recall the basic properties of the two polarizations of photons (which we denote as $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$) propagating in the direction specified by the wave vector $\boldsymbol{k}$:

$$\boldsymbol{e}_1 \cdot \boldsymbol{e}_2 = 0, \qquad \boldsymbol{k} \cdot \boldsymbol{e}_1 = \boldsymbol{k} \cdot \boldsymbol{e}_2 = 0, \tag{1}$$

$$\boldsymbol{k} \times \boldsymbol{e}_1 = k\boldsymbol{e}_2, \qquad \boldsymbol{k} \times \boldsymbol{e}_2 = -k\boldsymbol{e}_1, \tag{2}$$

or equivalently,

$$k\boldsymbol{e}_{\pm} = \pm i\boldsymbol{k} \times \boldsymbol{e}_{\pm}, \tag{3}$$

$$\boldsymbol{k} \cdot \boldsymbol{e}_{\pm} = 0, \tag{4}$$

where we defined $\boldsymbol{e} \equiv \boldsymbol{e}_1 \pm i\boldsymbol{e}_2$.

Now we take the wave functions of right- and left-handed photons, $\boldsymbol{\psi}_{\pm}(t,\boldsymbol{x})$, to be proportional to the complex polarizations $\boldsymbol{e}_{\pm}$. (Note that $\boldsymbol{\psi}_{\pm}$ are three-component wave functions.) Recalling the properties (3) and (4), it turns out that the following equations satisfy all the requirements for the wave equation above:

$$i\partial_t \boldsymbol{\psi}_{\pm} = \pm c\boldsymbol{\nabla} \times \boldsymbol{\psi}_{\pm}, \tag{5}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\psi}_{\pm} = 0, \tag{6}$$

for right- and left-handed photons, respectively. Equation (5) can be rewritten in the form of the Schrödinger-type equation as

$$i\hbar\partial_t \boldsymbol{\psi}_{\pm} = \pm c\left(\boldsymbol{S} \cdot \frac{\hbar}{i}\boldsymbol{\nabla}\right)\boldsymbol{\psi}_{\pm}, \tag{7}$$

where $(S_i)_{jk} = -i\epsilon_{ijk}$ with the indices i, j, k running over 1,2,3.

The corresponding Hamiltonian of chiral photons is thus given by

$$H = \pm c\boldsymbol{S}\cdot\boldsymbol{p}, \qquad (8)$$

where $\boldsymbol{p}$ is the momentum. Note that 3×3 matrix S_i ($i = 1, 2, 3$) satisfies the commutation relations, $[S_i, S_j] = i\epsilon_{ijk}S_k$. This should be contrasted with the Hamiltonian of chiral fermions, $H = \pm c\boldsymbol{\sigma}\cdot\boldsymbol{p}$, where σ_i is the 2×2 Pauli matrix that satisfies the commutation relations, $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$.

In the medium where Lorentz symmetry is explicitly broken, the Hamiltonian (8) is modified to

$$H = \pm v\boldsymbol{S}\cdot\boldsymbol{p}, \qquad (9)$$

where $v = 1/\sqrt{\epsilon\mu}$ is the velocity in medium with ϵ and μ being permittivity and permeability, respectively.

B. Path integral formulation

In the following, we use the natural units $\hbar = c = 1$ for simplicity. In order to derive the semiclassical theory for chiral photons with Berry curvature effects, we consider the path integral formulation for the Hamiltonian (8). Let us start with the path integral for right-handed photons $\boldsymbol{\psi}_+$,

$$Z = \int \mathcal{D}x\mathcal{D}p\mathcal{P}e^{iI}, \qquad I = \int \mathrm{d}t(\boldsymbol{p}\cdot\dot{\boldsymbol{x}} - \boldsymbol{S}\cdot\boldsymbol{p}), \qquad (10)$$

where $\mathcal{P}$ denotes the path-ordered product of the matrices $\exp(-i\boldsymbol{S}\cdot\boldsymbol{p}\Delta t)$ over the path in the phase space. The following argument can be similarly applied for left-handed photons $\boldsymbol{\psi}_-$ as well.

The eigenvalues of the 3×3 matrix $\boldsymbol{S}\cdot\boldsymbol{p}$ are found to be $\pm|\boldsymbol{p}|$ and 0. One can diagonalize this matrix using a unitary matrix $V_{\boldsymbol{p}}$, such that

$$V_{\boldsymbol{p}}^\dagger \boldsymbol{S}\cdot\boldsymbol{p}V_{\boldsymbol{p}} = \begin{pmatrix} |\boldsymbol{p}| & 0 & 0 \\ 0 & -|\boldsymbol{p}| & 0 \\ 0 & 0 & 0 \end{pmatrix} \equiv |\boldsymbol{p}|\lambda_3, \qquad (11)$$

where $\lambda_3 = \mathrm{diag}(1, -1, 0)$ is one of the SU(3) generators. The eigenstate of the eigenvalue $-|\boldsymbol{p}|$ has negative energy and should be regarded as unphysical for photons. This is to be contrasted with the case of chiral fermions, where negative energy states, corresponding to antiparticles, are possible. The eigenstate of the eigenvalue 0 is given by $\hat{\boldsymbol{p}} \equiv \boldsymbol{p}/|\boldsymbol{p}|$ (multiplied by any nonzero proportionality constant) and is longitudinal with respect to $\boldsymbol{p}$. Due to the additional constraint (6), this state is forbidden to appear and is unphysical as well; hence, $\boldsymbol{\psi}_+$ has only one physical eigenstate with the eigenvalue $|\boldsymbol{p}|$, corresponding to the positive helicity state $h = +1$.

Following Ref. [5], one can rewrite the path integral (10) by inserting $1 = V_{\boldsymbol{p}}V_{\boldsymbol{p}}^\dagger$ between the exponential factors, so that the matrix in the exponential factor is diagonalized at each point of the trajectory as follows:

$$\begin{aligned} &\cdots \exp(-i\boldsymbol{S}\cdot\boldsymbol{p}_2\Delta t)\exp(-i\boldsymbol{S}\cdot\boldsymbol{p}_1\Delta t)\cdots \\ &= \cdots V_{\boldsymbol{p}_2}V_{\boldsymbol{p}_2}^\dagger \exp(-i\boldsymbol{S}\cdot\boldsymbol{p}_2\Delta t)V_{\boldsymbol{p}_2}V_{\boldsymbol{p}_2}^\dagger \\ &\quad\times V_{\boldsymbol{p}_1}V_{\boldsymbol{p}_1}^\dagger \exp(-i\boldsymbol{S}\cdot\boldsymbol{p}_1\Delta t)V_{\boldsymbol{p}_1}V_{\boldsymbol{p}_1}^\dagger\cdots \\ &= \cdots V_{\boldsymbol{p}_2}\exp(-i|\boldsymbol{p}_2|\lambda_3\Delta t)V_{\boldsymbol{p}_2}^\dagger \\ &\quad\times V_{\boldsymbol{p}_1}\exp(-i|\boldsymbol{p}_1|\lambda_3\Delta t)V_{\boldsymbol{p}_1}^\dagger\cdots. \end{aligned} \qquad (12)$$

Taking $\Delta\boldsymbol{p} \equiv \boldsymbol{p}_2 - \boldsymbol{p}_1$ to be sufficiently small, the factor $V_{\boldsymbol{p}_2}^\dagger V_{\boldsymbol{p}_1}$ between the two exponential factors in Eq. (12) can be expressed as

$$V_{\boldsymbol{p}_2}^\dagger V_{\boldsymbol{p}_1} \approx \exp(-i\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\Delta\boldsymbol{p}) = \exp(-i\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}}\Delta t), \qquad (13)$$

where $\hat{\boldsymbol{a}}_{\boldsymbol{p}} \equiv iV_{\boldsymbol{p}}^\dagger\boldsymbol{\nabla}_{\boldsymbol{p}}V_{\boldsymbol{p}}$.

We now take the semiclassical limit where off-diagonal components of $\hat{\boldsymbol{a}}_{\boldsymbol{p}}$ are negligible. (We will discuss the validity of this approximation later.) Focusing on the positive energy state, we arrive at the semiclassical action for right-handed photons,

$$I = \int \mathrm{d}t(\boldsymbol{p}\cdot\dot{\boldsymbol{x}} - \boldsymbol{a}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}} - \epsilon_{\boldsymbol{p}}), \qquad (14)$$

where $\epsilon_{\boldsymbol{p}} = |\boldsymbol{p}|$ is the energy dispersion (in the vacuum) and $\boldsymbol{a}_{\boldsymbol{p}} \equiv [\hat{\boldsymbol{a}}_{\boldsymbol{p}}]_{11}$ is the gauge field in momentum space, called the Berry connection. The corresponding field strength, called the Berry curvature, is defined as $\boldsymbol{\Omega}_{\boldsymbol{p}} \equiv \boldsymbol{\nabla}_{\boldsymbol{p}}\times\boldsymbol{a}_{\boldsymbol{p}}$. Similarly, one can obtain the semiclassical action for left-handed photons (or negative helicity state $h = -1$) by repeating the similar argument for $\boldsymbol{\psi}_-$.

From the definition of $\boldsymbol{a}_{\boldsymbol{p}}$ above, one finds that

$$\boldsymbol{\Omega}_{\boldsymbol{p}} = \pm\frac{\hat{\boldsymbol{p}}}{|\boldsymbol{p}|^2}, \qquad (15)$$

for right- and left-handed photons with $h = \pm 1$, respectively. This is the fictitious magnetic field of the magnetic monopole (in momentum space) with the monopole charge,

$$k = \frac{1}{4\pi}\int \boldsymbol{\Omega}_{\boldsymbol{p}}\cdot\mathrm{d}\boldsymbol{S} = \pm 1, \qquad (16)$$

where the area integral is taken over the surface of the sphere with radius $|\boldsymbol{p}|$. Note that the Berry curvature of chiral photons in Eq. (15) is twice larger than that of chiral fermions in Refs. [4–6].

Let us discuss the applicability of the semiclassical description for photons above. For the off-diagonal components of $\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}}$ to be negligible to obtain Eq. (14) from Eq. (13), they must be much smaller than the energy gap $2|\boldsymbol{p}|$ between the two eigenstates with the eigenvalues $\pm|\boldsymbol{p}|$. As $|\hat{\boldsymbol{a}}_{\boldsymbol{p}}| \sim 1/|\boldsymbol{p}|$, this condition amounts to

$$|\dot{\boldsymbol{p}}| \ll |\boldsymbol{p}|^2, \tag{17}$$

meaning that $\boldsymbol{p}$ must be sufficiently away from the level crossing point $\boldsymbol{p} = \boldsymbol{0}$.

C. Semiclassical equations of motion

Let us look into the consequences of the Berry curvature corrections for chiral photons. From the action (14), one obtains the equation of motion for $\dot{\boldsymbol{x}}$,

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}. \tag{18}$$

The second term on the right-hand side of Eq. (18) is the "Lorentz force" in momentum space, originally known as the optical Magnus effect [3]. This term has been found to induce the spin Hall effect of light [1,2]: the trajectory of the circularly polarized photon is shifted perpendicularly to the direction of $\dot{\boldsymbol{p}}$. For example, in an inhomogeneous medium with coordinate-dependent permittivity $\epsilon(\boldsymbol{x})$, the equation of motion is $\dot{\boldsymbol{p}} = -(\boldsymbol{\nabla} v)|\boldsymbol{p}|$, where $v = 1/\sqrt{\epsilon}$. In this case, the shift of the trajectory is perpendicular to the direction of $\boldsymbol{\nabla}\epsilon$ [1–3].

III. PHOTONIC CHIRAL VORTICAL EFFECT

As we will discuss from now on, the Berry curvature correction of photons also leads to a new type of topological transport phenomenon—the photonic CVE.

Let us consider the response of a system with right- or left-handed photons to a global rotation or a local vorticity $\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{v}$, where $\boldsymbol{v}$ is the local fluid velocity.[1] For this purpose, let us go to the comoving frame rotating with angular velocity $\boldsymbol{\omega}$ with respect to the laboratory frame, similarly to Ref. [5]. In this frame, photons experience the noninertial Coriolis force to the linear order in $\boldsymbol{\omega}$. We will be interested in sufficiently small $|\boldsymbol{\omega}|$ so that the centrifugal force of order $O(\boldsymbol{\omega}^2)$ is negligible. Then the equation of motion for $\dot{\boldsymbol{p}}$ is given by

$$\dot{\boldsymbol{p}} = 2|\boldsymbol{p}|\dot{\boldsymbol{x}} \times \boldsymbol{\omega} + O(\boldsymbol{\omega}^2), \tag{19}$$

where we assumed a homogeneous medium for simplicity. The right-hand side of Eq. (19) can be understood as a relativistic generalization of the Coriolis force $2m\dot{\boldsymbol{x}} \times \boldsymbol{\omega}$ for a nonrelativistic particle with mass m. Substituting Eq. (19) into Eq. (18), we have

$$\sqrt{G}\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + 2\boldsymbol{\omega}|\boldsymbol{p}|(\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}}), \tag{20}$$

where $G = (1 + 2|\boldsymbol{p}|\boldsymbol{\omega} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})^2$ is the determinant of the 6×6 matrix of the coefficients in Eqs. (18) and (19) for $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{p}}$.

Using the distribution function of right- or left-handed photons in the phase space, $n_{\boldsymbol{p}}$, the photon current density is given by

$$\boldsymbol{j} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \sqrt{G}\dot{\boldsymbol{x}} n_{\boldsymbol{p}}, \tag{21}$$

where we took into account the fact that the invariant phase space measure becomes $\sqrt{G}\mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{p}/(2\pi)^3$ instead of $\mathrm{d}^3\boldsymbol{x}\mathrm{d}^3\boldsymbol{p}/(2\pi)^3$ due to the modification in Eq. (20). This modification is similar to that of chiral fermions in a magnetic field [4–6] (see also Refs. [15,16]).

From the Berry-curvature corrections in Eq. (20), one finds the photon current proportional to the vorticity,

$$\boldsymbol{j}_{\mathrm{CVE}} = 2\boldsymbol{\omega} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} |\boldsymbol{p}|(\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}}) n_{\boldsymbol{p}}. \tag{22}$$

This is the nonequilibrium expression of the photonic CVE. Note that, although the expression itself seems the same as the CVE for chiral fermions in Ref. [5], this is indeed different from the latter: the Berry curvature $\boldsymbol{\Omega}_{\boldsymbol{p}}$ in Eq. (15) is twice larger than the one in Ref. [5] and $n_{\boldsymbol{p}}$ is the bosonic distribution function unlike the fermionic one in Ref. [5].

These differences can be clearly seen in the thermal equilibrium state where $n_{\boldsymbol{p}}$ takes the Bose-Einstein distribution characterized by temperature T and chemical potential μ,[2]

$$n_{\boldsymbol{p}} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p - \mu)} - 1}, \tag{23}$$

with $\beta \equiv 1/T$. In this case, the photonic CVE becomes

$$\boldsymbol{j}^{\pm}_{\mathrm{CVE}} = \pm\boldsymbol{\omega} \int_0^\infty \frac{\mathrm{d}p}{\pi^2} \frac{p}{\mathrm{e}^{\beta(p-\mu)} - 1} = \pm\frac{1}{\pi^2} F(2, -\beta\mu) T^2 \boldsymbol{\omega}, \tag{24}$$

for right- and left-handed photons, respectively, where $p \equiv |\boldsymbol{p}|$ and

$$F(s, \alpha) \equiv \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^{x+\alpha} - 1} \mathrm{d}x = \sum_{n=1}^\infty \frac{\mathrm{e}^{-n\alpha}}{n^s} \tag{25}$$

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[1] In the case of a system in a global rotation $\boldsymbol{\omega}$, the size of the system of interest is to be understood as satisfying $r < 1/|\boldsymbol{\omega}|$. Otherwise, the velocity of the boundary exceeds the speed of light, leading to unphysical results [8] (see also Ref. [14]).

[2] Usually the chemical potential of photons is vanishing, $\mu = 0$, because the number of photons can vary without any constraint. However, this is not always true when photons are in chemical equilibrium with the excitations of matter with a nonzero chemical potential; e.g., chemical equilibrium between photons and electron-hole pairs in a light-emitting diode can lead to a photon medium with $\mu \neq 0$ [17]; see also Sec. IV. Here we consider the most generic case with nonzero T and μ.

is the Bose-Einstein integral with $\Gamma(s)$ the gamma function. Apparently, the transport coefficient in Eq. (22) is different from that for chiral fermions in Ref. [5]. This is one of the main results in this paper.

In particular, when $\mu = 0$, the expression of the photonic CVE is further simplified by using $F(2,0) = \pi^2/6$ as

$$\boldsymbol{j}^{\pm}_{\rm CVE} = \pm \frac{T^2}{6}\boldsymbol{\omega}. \tag{26}$$

Incidentally, the transport coefficient in this case is the same as that for chiral matter with a single chiral fermion. This is a consequence of two modifications compared with chiral fermions, which cancel with each other: one is the fact that the helicity of photons, $h = \pm 1$, is twice larger than the helicity of chiral fermions, $h = \pm 1/2$ (and so is the Berry curvature), and the other is that the contribution of antiparticles is absent for photons.

Notice that the coefficients of the photonic CVE have the opposite signs for thermalized right- and left-handed photons. Hence, in a system with *both* right- and left-handed photons in a rotation, they tend to move in the opposite direction. The corresponding axial current at finite T and μ is expressed as

$$\boldsymbol{j}^{\rm A} \equiv \boldsymbol{j}^{+} - \boldsymbol{j}^{-} = \frac{2}{\pi^2}F(2,-\beta\mu)T^2\boldsymbol{\omega}. \tag{27}$$

As a result, right- and left-handed photons are separated along the rotation. This is the *photonic chiral separation effect*.

One might think that the argument leading to Eq. (24) above would not be completely justified because the integration over momentum space in Eq. (24) includes the singular point $\boldsymbol{p} = \boldsymbol{0}$, where the semiclassical description for photons breaks down. In fact, the condition (17), together with the equations of motion (18) and (19), requires that $|\boldsymbol{\omega}| \ll |\boldsymbol{p}|$. However, one can show that the contribution around the singular point with the region $|\boldsymbol{p}| \leq \Delta$ (with Δ satisfying $|\boldsymbol{\omega}| \ll \Delta \ll T$) to the integral in Eq. (24) is vanishingly small; when $\mu \neq 0$, we have

$$\int_0^{\Delta} \frac{\mathrm{d}p}{\pi^2}\frac{p}{\mathrm{e}^{\beta(p-\mu)}-1} \sim \Delta^2 \ll T^2, \tag{28}$$

and when $\mu = 0$,

$$\int_0^{\Delta} \frac{\mathrm{d}p}{\pi^2}\frac{p}{\mathrm{e}^{\beta p}-1} \sim T\Delta \ll T^2, \tag{29}$$

where we used $\mathrm{e}^{\beta p} \simeq 1 + \beta p$ for $p \ll T$. Hence, the support of the integrand in Eq. (24) comes from the region $p > \Delta$, where the semiclassical treatment is valid.

IV. DISCUSSIONS

In this paper, we developed a kinetic description for right- and left-handed circularly polarized photons. Using the kinetic equation, we derived the expression of the (non) equilibrium photon number current along the direction of a vorticity. The nonequilibrium and equilibrium photonic chiral vortical effects are given by Eqs. (22) and (24), respectively.

Among others, the photonic CVE may provide a novel contribution to the photon emission from pulsars and rotating massive stars.[3] This effect is remarkable in that photons emitted along the rotational axis of a star have a dependence on the circular polarizations: only right-handed photons are emitted from one of the poles while only left-handed ones from the other. As the photonic CVE becomes larger as the temperature and angular velocity increase [see Eq. (26)], we expect this effect to be most significant in the accretion-powered millisecond pulsars. A more detailed analysis in this direction will be reported elsewhere. One can extend the conventional framework of radiation hydrodynamics for matter-photon coupled astrophysical systems to include this effect by using the chiral kinetic theory for photons considered in this paper [19].

The photonic CVE with a *nonzero* chemical potential may also be realized in table-top experiments. In fact, thermalized photon media with finite temperature and chemical potential can be produced in semiconductors (e.g., light-emitting diodes) in an external electric field due to the chemical reactions of electrons and holes with photons [17].[4] Then, such a system in a rotation should exhibit the photonic chiral separation effect: right- and left-handed circularly polarized photons are separated along the rotation.

As the CVE appears not only for the spin-$1/2$ chiral fermions, but also for spin-1 chiral photons as we argued in this paper, one expects the same is also the case for higher-spin massless chiral particles (such as chiral gravitons). It would also be interesting to study possible new collective modes induced by the photonic CVE (see Refs. [20–25] for the case of chiral fermions). Finally, one may be able to derive the chiral kinetic theory for photons from the microscopic quantum field theory in a way similar to the one for chiral fermions [6,26–28].

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[3] We note that an attempt to associate the usual *fermionic* CVE with astrophysical jets was made in Ref. [18].

[4] This situation is somewhat similar to left-handed neutrinos at the core of supernovae; even weakly interacting neutrinos can be thermalized and have nonzero chemical potential due to the weak equilibrium with nucleons and electrons in dense nuclear matter, where the CVE for neutrinos is relevant to the evolution of core-collapse supernovae [12].

Note added.—Recently, we learned that A. Avkhadiev and A. V. Sadofyev [29] and V. A. Zyuzin [30] have independently studied the chiral vortical effect for bosons by different approaches. We note, however, that our derivation and results, based on the kinetic theory with Berry-curvature corrections, are more generic than those of Refs. [29,30] in that the former is applicable to the *nonequilibrium* state (and equilibrium state with finite T and μ), while the latter is limited to the *equilibrium* state without μ.

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