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Top-quark pair-production and decay at high precision

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We present a fully differential and high-precision calculation of top-quark pair-production and decay at the LHC, providing predictions for observables constructed from top-quark leptonic and b-flavored jet final states. The calculation is implemented in a parton-level Monte Carlo and includes an approximation to the next-to-next-to-leading-order (NNLO) corrections to the production and, for the first time, the exact NNLO corrections to the decay subprocesses. The corrections beyond NLO are sizable, and including them is crucial for an accurate description of the cross section constrained by experimental phase-space restrictions. We compare our predictions to published ATLAS and CMS measurements at the LHC, finding improved agreement compared with lower orders in the perturbative expansion.

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I. INTRODUCTION

The presence of top quarks produced in collider experiments is always inferred via the top-quark decay products. The majority of data/theory comparisons are, however, performed at the level of stable tops. Indeed, it is through such comparisons that the top-quark sector of the Standard Model is being carefully scrutinized. High statistics have been collected at all three main center-of-mass energy runs at the LHC and many measurements of "top-quark" inclusive cross sections and distributions are already down to the impressive level of a few percent in uncertainty. On the theoretical side, there has been equally impressive progress with top pair-production [1–8] and single top [9,10] computed fully differentially at next-to-next-to-leading order (NNLO) in perturbative QCD (pQCD) for stable tops. The fixed-order predictions can often be supplemented with various resummations which stabilize the predictions against the effects of large logarithms; see for example Refs. [11–22].

Despite this progress, to compare with stable-top predictions, experiments must, in general, do two things. Firstly, they must extrapolate their measurements from the detector fiducial volumes out to the full phase space, and secondly, they must translate their measurements of final states that they are sensitive to, back to some definition of top-quark "partons." Such extrapolation and unfolding corrections are typically derived from event generators that treat the top decay at leading order (LO), which can lead to inconsistencies when comparing with high-precision predictions. Moreover, the resulting systematic uncertainties can be difficult to estimate—the observed tensions [23] between the ATLAS and CMS measurements of stable-top distributions may just hint at some unknown systematic errors in the above procedure.

To overcome these modeling uncertainties it is evident that experimental measurements of observables constructed directly from top-quark decay products in detector fiducial volumes are the quantities that should be compared with theoretical predictions. In fact, experiments have begun publishing such measurements [24,25] and importantly these often come with smaller systematic errors than measurements of the inclusive cross section [24]. To fully exploit these measurements it is crucial that theoretical predictions describe the decay products fully differentially, and that they are as accurate as possible; i.e. they include higher order perturbative corrections in *both* production and decay subprocesses.

Significant efforts have been made in this direction at next-to-leading order (NLO), both in treating the top-quark propagator in the narrow-width approximation (NWA) [26–30] and in keeping the top quarks off their mass shell [31–44]. Furthermore, frameworks have been recently developed to consistently match both sets of the above NLO calculations to parton showers [45–48]. It is clear that these predictions come much closer to the quantities that are actually measured by experiments.

In this article we focus on the dominant top-quark production mode at the LHC, top-pair ($t\bar{t}$) production, and describe a calculation that goes beyond NLO in pQCD in both the production and decay stages in the NWA. The calculation includes an approximation of the NNLO corrections to the $t\bar{t}$ production subprocess and the exact NNLO corrections to the top and antitop decays. We present the first results of our new calculation, which has been implemented in a parton-level Monte Carlo, making predictions for the $t\bar{t}$ process at the LHC in the dilepton channel, fully differential in the final-state leptons, b-flavored jets (b-jets) and missing energy. This represents a significant improvement to the current state of the art at fixed order in perturbation theory and, as will be shown below, compares very favorably to published fiducial-region measurements by the ATLAS and CMS experiments.

II. DETAILS OF THE CALCULATION

The full technical details of our calculation will be presented in a forthcoming publication; however we now briefly summarize the important building blocks. In the NWA the differential cross section for $t\bar{t}$ production and decay in a particular decay channel (e.g. the dilepton channel) can be written schematically to all orders as

$$d\sigma = d\sigma_{t\bar{t}} \times \frac{d\Gamma_{t\to bl^+\nu_l}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\to\bar{b}l'^-\bar{\nu}_{l'}}}{\Gamma_t}, \qquad (1)$$

where $d\sigma_{t\bar{t}}$, $d\Gamma_{t\to bl^+\nu_l}$ and $d\Gamma_{\bar{t}\to\bar{b}l'^-\bar{\nu}_{l'}}$ are the differential production cross section for a $t\bar{t}$ pair and the differential top- and antitop-quark decay widths (we write the latter as $d\Gamma_t$ and $d\Gamma_{\bar{t}}$ for brevity). Γ_t is the total top-quark width. The $\times$ in Eq. (1) indicates that production and decay are combined in a way that preserves spin correlations. Each term in Eq. (1) has a perturbative expansion in the strong coupling constant α_s,

$$d\sigma_{t\bar{t}} = \alpha_s^2 \sum_{i=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^i d\sigma_{t\bar{t}}^{(i)},$$

$$d\Gamma_{t(\bar{t})} = \sum_{i=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^i d\Gamma_{t(\bar{t})}^{(i)}, \qquad \Gamma_t = \sum_{i=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^i \Gamma_t^{(i)}. \qquad (2)$$

In the expansion in α_s of Eq. (1), we adhere to the convention of a strict expansion; namely, we do not include terms proportional to powers of α_s that are higher than the order that we work to [27,28]. This means that at NLO (NNLO) perturbative contributions proportional to α_s^4 (α_s^5) or higher are not included. In this convention when integrating inclusively over the decay products of the top quarks the cross section for the production of a stable top pair is recovered (multiplied by the appropriate branching fractions for the W-boson decays). This feature constitutes a highly nontrivial check of the implementation of each contribution in the expansion.

In the present calculation we include the exact NNLO corrections (i.e. corrections up to α_s^2) in the expansions of $d\Gamma_{t(\bar{t})}$ and Γ_t and approximate NNLO corrections to $d\sigma_{t\bar{t}}$ (the NLO corrections $d\sigma_{t\bar{t}}^{(1)}$ are included exactly). We denote our best predictions as $\widehat{\text{N}}$NLO (and not NNLO) since we make an approximation to $d\sigma_{t\bar{t}}^{(2)}$ (the exact NNLO corrections to the production where the spin information of the top quarks is kept explicit, required to construct Eq. (1), are not currently known).

The approximation to the exact NNLO corrections in production builds on the work presented in Ref. [49]. The starting point for this approximation is a factorization formula derived in the soft-collinear-effective-theory (SCET) framework [50–52]. It was shown in Ref. [15] that in the soft-gluon limit $z = M_{t\bar{t}}/\hat{s} = (p_t + p_{\bar{t}})^2/\hat{s} \to 1$ ("pair invariant mass" kinematics) the cross section for $t\bar{t}$ production can be written as a convolution of a hard function, containing the effects of virtual corrections, and a soft function, which contains the effects of emissions of soft gluons, together with standard parton distribution functions (PDFs). The factorized structure makes it possible to resum large logarithms of $(1-z)$ and the subsequent expansion to fixed order of the resummed cross section provides an approximation to the exact NNLO. In Ref. [49] this approach was generalized beyond the stable-top approximation and the spin-correlated LO decay of the top quarks was attached to the approximate NNLO production kernels.

Part of the difference between the approximate NNLO and exact NNLO results is due to subleading-power logarithms in $(1-z)$ that are included in the latter but not in the former. Two important sources of these are the higher order terms in the soft expansion of the Altarelli-Parisi splitting functions and the way in which the soft phase space is treated (namely the exact power of z appearing as a soft prefactor; see e.g. Ref. [53]). For our central prediction we take the kernels of Refs. [49] and additionally include the highest subleading-power logarithms in $(1-z)$, that arise from the soft expansion of the splitting functions [54,55]. These are known to bring improvements to the NNLO approximation [54–56], and indeed, when included they lead to an enhancement of the inclusive cross section (of ~5–6%) bringing the approximate NNLO inclusive cross section for stable top quarks into better agreement with the exact NNLO. Our central prediction also treats the soft phase space as in Refs. [15,49].

As discussed in Ref. [49], simply taking the standard envelope of the factorization/renormalization scale variation can lead to underestimating the uncertainty in the approximate NNLO approach. In order to construct a reliable estimate of the theoretical uncertainty, we explicitly use the freedom to include different subleading effects. Specifically, to make this estimate we take the envelope of scale variation together with variations (switching on and off) of subleading corrections of different origin: firstly, from the splitting functions [54,55] and secondly, from the soft phase space (changing the power of the z prefactor by one half, as performed in Ref. [53]).

The quality of the NNLO approximation in the production can ultimately be assessed by comparing to the exact NNLO cross sections for stable tops [2,3]. We find excellent agreement with the NNLO inclusive cross section, for example, for LHC 8 TeV, $m_t = 173.3$ GeV, $\mu_F = \mu_R \in [0.5, 1.0, 2.0]m_t$ and using `MSTW2008` PDFs [57] we have $\sigma(\text{NNLO}_{\text{approx}}) = 239.4^{+5.7}_{-14.0}$ pb, while the exact NNLO cross section computed with `top++` [58] is $\sigma(\text{NNLO}) = 239.2^{+9.2}_{-14.8}$ pb, with equally good agreement for LHC 7 and 13 TeV [59]. Furthermore, as displayed in Fig. 1 for the average transverse momentum of the top and antitop, we also find very good agreement with the exact results at the differential level (similar agreement is also found for the invariant mass of the top-quark pair and the average rapidity of the top and antitop). These validation checks for stable top quarks provide us with confidence

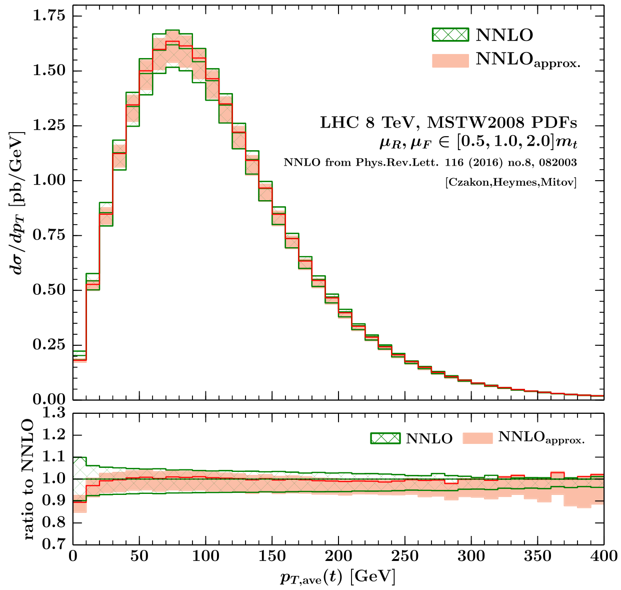

FIG. 1. Comparison of $\text{NNLO}_{\text{approx}}$ with exact NNLO for on-shell, stable $t\bar{t}$ production for the LHC at 8 TeV. Results for the exact NNLO are those published in Ref. [3]. The NNLO uncertainty bands have been obtained via the envelope of variations of μ_F and μ_R, while the uncertainty bands of the $\text{NNLO}_{\text{approx}}$ have been obtained through the envelope of scale variations *and* variations of formally subleading contributions in $(1-z)$. See text for further details.

when using the approximate NNLO kernels for the case when the top quarks are decayed.

The NNLO corrections to the top quark decay are also calculated by retaining full spin correlations between production and decay. Using the SCET-inspired phase-space slicing method presented in Ref. [60], a small cutoff on the invariant mass (m_j) of all QCD partons from the top quark decay is introduced to split the phase space in the computation of $d\Gamma^{(2)}_{t(\bar{t})}$ into resolved and unresolved regions. The resolved region receives contributions from the NLO corrections to the process of top decay plus an additional jet, and can be dealt with straightforwardly. The contribution in the unresolved region can be factorized and calculated using SCET, up to power corrections in m_j^2/m_t^2 [60]. The sum of resolved and unresolved contributions then converges to the exact NNLO correction when the cutoff is sufficiently small. In practice we find that a cutoff of 10^{-5} on m_j^2/m_t^2 is sufficient to ensure the remaining power corrections are negligible for all kinematic distributions considered. We note that the NNLO decay was also computed in Ref. [61].

Finally, the required $\text{NLO} \times \text{NLO}$ production-decay $[\sim d\sigma^{(1)}_{t\bar{t}} \times d\Gamma^{(1)}_{t} \times d\Gamma^{(0)}_{\bar{t}}]$ and decay-decay corrections $[\sim d\sigma^{(0)}_{t\bar{t}} \times d\Gamma^{(1)}_{t} \times d\Gamma^{(1)}_{\bar{t}}]$ have also been computed. Since production and decay subprocesses can be treated separately in the NWA, as far as their singularity structure is concerned, standard NLO techniques [62–64] can be adapted to deal with IR singularities in these contributions.

III. PHENOMENOLOGY

We now apply the calculation outlined above to LHC phenomenology and in particular, to the $t\bar{t}$ process in the dilepton channel. We first focus our attention on the fiducial cross sections measured by the ATLAS experiment for the $e^\pm\mu^\mp$ channel at 7 and 8 TeV [24], and the CMS experiment in the full dilepton channel ($e^\pm\mu^\mp$, e^+e^- and $\mu^+\mu^-$) at 8 TeV [25]. For simplicity we compare to measurements where the indirect decays $W \to \tau \to e(\mu)$ are considered as backgrounds. The corresponding definitions of the fiducial volumes for each experiment, constructed through cuts on final-state leptons (and b-jets, J_b), can be found in Table I.

The inputs for our theoretical predictions are set to

$$m_t = 173.3 \text{ GeV} \qquad \Gamma_t^{(0)} = 1.5048 \text{ GeV}$$
$$m_W = 80.385 \text{ GeV} \qquad \Gamma_W = 2.0928 \text{ GeV}$$
$$m_Z = 91.1876 \text{ GeV} \qquad G_F = 1.166379 \times 10^{-5} \text{ GeV}^{-2}$$

where we note that $\Gamma_t^{(0)}$ is a function of m_t, M_W and G_F. We point out that we have fixed the value of the top mass to the current world average value [66]. The parametric uncertainty on the predictions due to the input value of m_t deserves careful study, particularly in the context of fiducial cross sections. While such a study lies beyond the scope of the work presented here, we mention that the variation of m_t by 1 GeV changes the LO values for the fiducial cross sections presented in Table I by ~2–3% (this dependence should be similar at NLO and NNLO). We use fixed factorization and renormalization scales [67] $\mu = \mu_F = \mu_R \in [0.5, 1.0, 2.0] m_t$ and vary the scale in the NLO and NNLO corrections to the top width $\Gamma_t^{(1,2)}(\mu)$ for consistency. The theoretical uncertainty bands are obtained by taking the envelope of the predictions for each scale. For the approximate NNLO corrections in production, as stated earlier, we additionally take the envelope of predictions computed with different subleading terms in $(1-z)$. We use LO, NLO, NNLO MMHT2014 PDFs [68] with the corresponding value for $\alpha_s(M_Z)$ for our LO, NLO and $\widehat{\text{NNLO}}$ predictions. In the results presented here we treat the W-bosons in the NWA.

Our theoretical predictions of the cross sections for the corresponding fiducial setups of ATLAS and CMS are tabulated in Table I. The ATLAS and CMS measurements are also shown in the same table. The error bars on the experimental data have been obtained by summing the published individual uncertainties (statistical, systematic, luminosity and beam) in quadrature. One observes that for each setup shown, there is a reduction in the uncertainty bands with increasing perturbative order, with the $\widehat{\text{NNLO}}$ bands being roughly half the size of the NLO bands. Additionally the corrections to the cross section going from

TABLE I. Fiducial cross sections for a variety of LHC center-of-mass energies and setups. Theoretical predictions with uncertainties are tabulated at LO, NLO and $\hat{\text{N}}$NLO as are the experimental measurements. The uncertainties on the measured cross sections have been obtained by summing the individual statistical, systematic, beam and luminosity uncertainties in quadrature. δ_{dec} indicates the impact on the cross section of higher-order corrections to the top decay; see Eq. (3). The Monte Carlo uncertainty on all theoretical predictions is better than 1 permil.

		ATLAS setup, $e^\pm\mu^\mp$ channel [24]				
Energy	Fiducial volume	LO (pb)	NLO (pb)	$\hat{\text{N}}$NLO (pb)	δ_{dec}	ATLAS (pb)
7 TeV	$p_T(l^\pm) > 25$ GeV, $\lvert\eta(l^\pm)\rvert < 2.5$	$1.592^{+39.2\%}_{-26.0\%}$	$2.007^{+11.9\%}_{-13.2\%}$	$2.210^{+2.2\%}_{-6.0\%}$	−0.3%	$2.305^{+3.8\%}_{-3.8\%}$
7 TeV	$p_T(l^\pm) > 30$ GeV, $\lvert\eta(l^\pm)\rvert < 2.4$	$1.265^{+39.3\%}_{-26.1\%}$	$1.585^{+11.8\%}_{-13.1\%}$	$1.736^{+2.2\%}_{-6.0\%}$	−0.8%	$1.817^{+3.8\%}_{-3.8\%}$
8 TeV	$p_T(l^\pm) > 25$ GeV, $\lvert\eta(l^\pm)\rvert < 2.5$	$2.249^{+37.9\%}_{-25.5\%}$	$2.855^{+11.9\%}_{-12.9\%}$	$3.130^{+2.3\%}_{-6.0\%}$	−0.3%	$3.036^{+4.1\%}_{-4.1\%}$
8 TeV	$p_T(l^\pm) > 30$ GeV, $\lvert\eta(l^\pm)\rvert < 2.4$	$1.788^{+38.0\%}_{-25.5\%}$	$2.256^{+11.7\%}_{-12.9\%}$	$2.461^{+2.3\%}_{-6.1\%}$	−0.7%	$2.380^{+4.1\%}_{-4.1\%}$
	CMS setup, $e^\pm\mu^\mp, e^+e^-, \mu^+\mu^-$ channel [25], 2 b-jets required (anti-k_t algorithm [65], $R = 0.5$)					
Energy	Fiducial volume	LO (pb)	NLO (pb)	$\hat{\text{N}}$NLO (pb)	δ_{dec}	CMS (pb)
8 TeV	$p_T(l^\pm) > 20$ GeV, $\lvert\eta(l^\pm)\rvert < 2.4$, $p_T(J_b) > 30$ GeV, $\lvert\eta(J_b)\rvert < 2.4$	$3.780^{+37.4\%}_{-25.3\%}$	$4.483^{+9.0\%}_{-11.5\%}$	$4.874^{+2.5\%}_{-6.8\%}$	−8.0%	$4.73^{+4.7\%}_{-4.7\%}$

LO to NLO and from NLO to $\hat{\text{N}}$NLO are also reduced, indicating an improved perturbative convergence. The corrections beyond NLO are significant, around 9-10%, underlining that such corrections are crucial for an accurate description of fiducial regions [69]. This statement is strengthened when a comparison to the experimental measurements is made: for each setup considered, Table I reveals an improvement in the agreement between measurement and data. More precisely, use of the $\hat{\text{N}}$NLO prediction brings the difference between the central values of theory and measurement to within 3-4%.

An important aspect to quantify is the size of the contributions involving corrections to the top decay. This can be done by studying the percent difference between the $\hat{\text{N}}$NLO prediction, which includes higher-order corrections to the decay, and the prediction $\hat{\text{N}}\text{NLO}_{\text{LO dec}}$ that just includes the LO decay ($d\Gamma^{(0)}_{t(\bar{t})}$),

$$\delta_{\text{dec}} = \hat{\text{N}}\text{NLO}/\hat{\text{N}}\text{NLO}_{\text{LO dec}} - 1[\%], \tag{3}$$

also found in Table I. For the ATLAS setup, whose definition of the fiducial volume involves no cuts on the b-jets, we find $\delta_{\text{dec}} < 1\%$; i.e. the contributions from higher orders in the decay are very small. However, this is not the case for the CMS setup, where the constraint $p_T(J_b) > 30$ GeV is in place, and where the prediction that treats the top decay only at LO is 8% larger than the prediction that consistently includes corrections in the decay [70]. Coupled with the comparison to the precise experimental measurements, these findings point to two important conclusions. Firstly, NNLO corrections in general are vital to describe fiducial-region cross sections accurately. Secondly, corrections to the production subprocess alone *do not* uniformly give a good description of the measurements—higher-order corrections to the decay must also be included to see an improved agreement for all setups considered.

Finally, we make a comparison to differential CMS measurements [71] in the dilepton channel. In Fig. 2 we present absolute distributions for the average lepton pseudorapidity, $\eta(l)_{\text{ave.}}$, and the transverse momentum of the lepton pair, $p_T(l^+, l^-)$, and b-jet pair, $p_T(J_b, J_{\bar{b}})$, normalized to the $\hat{\text{N}}$NLO prediction. We have chosen to rescale the published normalized data by the fiducial cross section

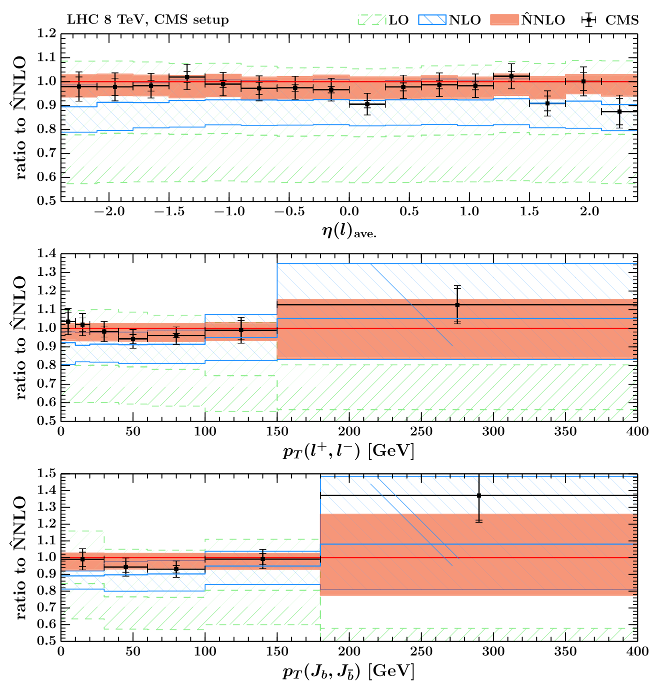

FIG. 2. Distributions of the average pseudorapidity of the leptons, $\eta(l)_{\text{ave.}}$, and the transverse momentum of the lepton-pair, $p_T(l^+, l^-)$, and b-jet pair, $p_T(J_b, J_{\bar{b}})$. The plots show the CMS measurements as well as the LO, NLO and $\hat{\text{N}}$NLO predictions normalized to $\hat{\text{N}}$NLO. The error bars and shaded bands indicate the experimental and theoretical uncertainties respectively. See text for further details.

found in Ref. [25] in order to make the differences between theoretical predictions at different orders more visible. Since there are no published uncertainties for the absolute distributions, we show the experimental points with two error bars—the smaller error bars are those of the normalized cross section while the larger ones are those of the normalized cross section added in quadrature with the uncertainty of the fiducial cross section used for rescaling. Overall, there is again good agreement between the measurements and the $\hat{\text{N}}$NLO predictions—the latter agreeing with the former within uncertainties in all bins. The $\hat{\text{N}}$NLO brings an improvement in the agreement not only in the overall normalization, but also in the shape of each distribution for the bulk of the region of phase space measured. In the last bin of the $p_T(l^+, l^-)$ and $p_T(J_b, J_{\bar{b}})$ distributions the agreement becomes less good; however, in these regions both theoretical and experimental uncertainty bands become large.

IV. CONCLUSIONS AND OUTLOOK

In this article we have presented high-precision results for the fully differential production and decay of a top-quark pair in fiducial regions at the LHC. Our results are based on the NWA and are accurate at approximate NNLO in the production subprocess and exact NNLO in the decay subprocess. The approximation we use in the production does an excellent job at approximating the exact NNLO for stable tops, giving us confidence in the results we present for decayed top quarks.

We have shown that, in general, the NNLO corrections are significant. Moreover, it is vital to include corrections to the decay as well as to the production subprocess for an accurate description of observables constructed from top-quark decay products. The importance of going beyond NLO is clearly seen when comparing theoretical predictions to available ATLAS and CMS fiducial cross section measurements. For different center-of-mass energies and setups we consistently find that the agreement between theory and measurement improves when the $\hat{\text{N}}$NLO predictions are used. Additionally, we see an overall improvement in the agreement, in normalization as well as in shape (for the bulk of the ranges considered) when comparing to distributions constructed from lepton and b-jet final states published by CMS.

We envision that the calculation presented in this work will open up a number of exciting possibilities for the study of top quarks. Given the impressively small experimental uncertainties on the measurements of fiducial cross sections, it would be particularly interesting and timely to use these measurements and exploit this new calculation to perform an extraction of α_s and m_t^{pole}. This would bypass the need to extrapolate measurements to the full phase space and the modeling back to top-quark partons, that affect extractions from the inclusive stable-top cross section. With this calculation at hand it will also be possible to quantify the impact that the exact NNLO top-quark decay corrections have in methods of m_t-extraction sensitive to the decay (see for example Refs. [72,73]).

Data/theory comparisons of top-quark production, at the level of stable tops, have brought numerous impactful applications, for example, constraining the high-x region of the gluon PDF [23]. We advocate that moving towards such applications, but working with observables at the level of the decay products of top quarks, as we have done here, will maximize the impact that current and future top-quark measurements will have both within and beyond the area of top-quark physics.

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[1] P. Bärnreuther, M. Czakon, and A. Mitov, Phys. Rev. Lett. **109**, 132001 (2012).
[2] M. Czakon, P. Fiedler, and A. Mitov, Phys. Rev. Lett. **110**, 252004 (2013).
[3] M. Czakon, D. Heymes, and A. Mitov, Phys. Rev. Lett. **116**, 082003 (2016).
[4] M. Czakon, P. Fiedler, D. Heymes, and A. Mitov, J. High Energy Phys. 05 (2016) 034.
[5] M. Czakon, D. Heymes, and A. Mitov, J. High Energy Phys. 04 (2017) 071.
[6] G. Abelof, A. Gehrmann-De Ridder, and I. Majer, J. High Energy Phys. 12 (2015) 074.
[7] M. Czakon, D. Heymes, and A. Mitov, arXiv:1704.08551.
[8] M. Czakon, D. Heymes, A. Mitov, D. Pagani, I. Tsinikos, and M. Zaro, arXiv:1705.04105.
[9] M. Brucherseifer, F. Caola, and K. Melnikov, Phys. Lett. B **736**, 58 (2014).
[10] E. L. Berger, J. Gao, C. P. Yuan, and H. X. Zhu, Phys. Rev. D **94**, 071501 (2016).
[11] N. Kidonakis, Phys. Rev. D **64**, 014009 (2001).

[12] N. Kidonakis, E. Laenen, S. Moch, and R. Vogt, Phys. Rev. D **64**, 114001 (2001).
[13] N. Kidonakis and R. Vogt, Phys. Rev. D **68**, 114014 (2003).
[14] A. Banfi and E. Laenen, Phys. Rev. D **71**, 034003 (2005).
[15] V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, and L. L. Yang, J. High Energy Phys. 09 (2010) 097.
[16] V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, and L.-L. Yang, J. High Energy Phys. 09 (2011) 070.
[17] M. Beneke, P. Falgari, S. Klein, and C. Schwinn, Nucl. Phys. **B855**, 695 (2012).
[18] M. Cacciari, M. Czakon, M. Mangano, A. Mitov, and P. Nason, Phys. Lett. B **710**, 612 (2012).
[19] A. Ferroglia, B. D. Pecjak, and L. L. Yang, Phys. Rev. D **86**, 034010 (2012).
[20] A. Ferroglia, S. Marzani, B. D. Pecjak, and L. L. Yang, J. High Energy Phys. 01 (2014) 028.
[21] M. Guzzi, K. Lipka, and S.-O. Moch, J. High Energy Phys. 01 (2015) 082.
[22] B. D. Pecjak, D. J. Scott, X. Wang, and L. L. Yang, Phys. Rev. Lett. **116**, 202001 (2016).
[23] M. Czakon, N. P. Hartland, A. Mitov, E. R. Nocera, and J. Rojo, J. High Energy Phys. 04 (2017) 044.
[24] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **74**, 3109 (2014); **76**, 642 (2016).
[25] V. Khachatryan *et al.* (CMS Collaboration), Eur. Phys. J. C **76**, 379 (2016).
[26] W. Bernreuther, A. Brandenburg, Z. G. Si, and P. Uwer, Nucl. Phys. **B690**, 81 (2004).
[27] K. Melnikov and M. Schulze, J. High Energy Phys. 08 (2009) 049.
[28] J. M. Campbell and R. K. Ellis, J. Phys. G **42**, 015005 (2015).
[29] K. Melnikov, M. Schulze, and A. Scharf, Phys. Rev. D **83**, 074013 (2011).
[30] K. Melnikov, A. Scharf, and M. Schulze, Phys. Rev. D **85**, 054002 (2012).
[31] G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos, and M. Worek, J. High Energy Phys. 02 (2011) 083.
[32] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, Phys. Rev. Lett. **106**, 052001 (2011).
[33] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, J. High Energy Phys. 10 (2012) 110.
[34] P. Falgari, A. S. Papanastasiou, and A. Signer, J. High Energy Phys. 05 (2013) 156.
[35] G. Heinrich, A. Maier, R. Nisius, J. Schlenk, and J. Winter, J. High Energy Phys. 06 (2014) 158.
[36] R. Frederix, Phys. Rev. Lett. **112**, 082002 (2014).
[37] F. Cascioli, S. Kallweit, P. Maierhöfer, and S. Pozzorini, Eur. Phys. J. C **74**, 2783 (2014).
[38] P. Falgari, P. Mellor, and A. Signer, Phys. Rev. D **82**, 054028 (2010).
[39] P. Falgari, F. Giannuzzi, P. Mellor, and A. Signer, Phys. Rev. D **83**, 094013 (2011).
[40] A. S. Papanastasiou, R. Frederix, S. Frixione, V. Hirschi, and F. Maltoni, Phys. Lett. B **726**, 223 (2013).
[41] A. Denner and R. Feger, J. High Energy Phys. 11 (2015) 209.
[42] G. Bevilacqua, H. B. Hartanto, M. Kraus, and M. Worek, Phys. Rev. Lett. **116**, 052003 (2016).
[43] G. Bevilacqua, H. B. Hartanto, M. Kraus, and M. Worek, J. High Energy Phys. 11 (2016) 098.
[44] A. Denner and M. Pellen, J. High Energy Phys. 08 (2016) 155.
[45] J. M. Campbell, R. K. Ellis, P. Nason, and E. Re, J. High Energy Phys. 04 (2015) 114.
[46] T. Ježo and P. Nason, J. High Energy Phys. 12 (2015) 065.
[47] R. Frederix, S. Frixione, A. S. Papanastasiou, S. Prestel, and P. Torrielli, J. High Energy Phys. 06 (2016) 027.
[48] T. Ježo, J. M. Lindert, P. Nason, C. Oleari, and S. Pozzorini, Eur. Phys. J. C **76**, 691 (2016).
[49] A. Broggio, A. S. Papanastasiou, and A. Signer, J. High Energy Phys. 10 (2014) 98.
[50] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[51] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).
[52] M. Beneke, A. P. Chapovsky, M. Diehl, and T. Feldmann, Nucl. Phys. **B643**, 431 (2002).
[53] A. Broggio, A. Ferroglia, B. D. Pecjak, A. Signer, and L. L. Yang, J. High Energy Phys. 03 (2016) 124.
[54] M. Kramer, E. Laenen, and M. Spira, Nucl. Phys. **B511**, 523 (1998).
[55] S. Catani, D. de Florian, and M. Grazzini, J. High Energy Phys. 05 (2001) 025.
[56] C. Muselli, M. Bonvini, S. Forte, S. Marzani, and G. Ridolfi, J. High Energy Phys. 08 (2015) 076.
[57] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).
[58] M. Czakon and A. Mitov, Comput. Phys. Commun. **185**, 2930 (2014).
[59] We note that had we used scale variation alone, an uncertainty band of roughly one third of the width would have been obtained—a clear underestimate of the uncertainty.
[60] J. Gao, C. S. Li, and H. X. Zhu, Phys. Rev. Lett. **110**, 042001 (2013).
[61] M. Brucherseifer, F. Caola, and K. Melnikov, J. High Energy Phys. 04 (2013) 059.
[62] S. Catani and M. H. Seymour, Nucl. Phys. **B485**, 291 (1997); **B510**, 503(E) (1998).
[63] S. Catani, S. Dittmaier, M. H. Seymour, and Z. Trocsanyi, Nucl. Phys. **B627**, 189 (2002).
[64] S. Frixione, Z. Kunszt, and A. Signer, Nucl. Phys. **B467**, 399 (1996).
[65] M. Cacciari, G. P. Salam, and G. Soyez, J. High Energy Phys. 04 (2008) 063.
[66] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427.
[67] In future work we intend to explore the effects of dynamical scales, which are known to improve the perturbative stability of the tails of distributions.
[68] L. A. Harland-Lang, A. D. Martin, P. Motylinski, and R. S. Thorne, Eur. Phys. J. C **75**, 204 (2015).
[69] We point out that large NNLO corrections in fiducial regions were also found in single top-quark production [10].
[70] The NNLO-decay corrections alone contribute to a $\sim 3\%$ effect here.
[71] V. Khachatryan *et al.* (CMS Collaboration), Eur. Phys. J. C **75**, 542 (2015).
[72] S. Frixione and A. Mitov, J. High Energy Phys. 09 (2014) 012.
[73] K. Agashe, R. Franceschini, D. Kim, and M. Schulze, Eur. Phys. J. C **76**, 636 (2016).