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New proposal for a holographic boundary conformal field theory

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We propose a new holographic dual of conformal field theory defined on a manifold with boundaries, i.e., boundary conformal field theory (BCFT). Our proposal can apply to general boundaries and agrees with Takayanagi [Phys. Rev. Lett. **107**, 101602 (2011)] for the special case of a disk and half-plane. Using the new proposal of AdS/BCFT, we successfully obtain the expected boundary Weyl anomaly, and the obtained boundary central charges naturally satisfy a c-like theorem holographically. We also investigate the holographic entanglement entropy of BCFT and find that the minimal surface must be normal to the bulk spacetime boundaries when they intersect. Interestingly, the entanglement entropy depends on the boundary conditions of BCFT and the distance to the boundary. The entanglement wedge has an interesting phase transition that is important for the self-consistency of AdS/BCFT.

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I. INTRODUCTION

Conformal field theory (CFT) and boundary conformal field theory (BCFT) [1] are crucial to the description of critical phenomena and quantum phase transitions in many-body condensed matter systems and quantum field theory, and are also fundamental building blocks in string theory. The AdS/CFT correspondence [2,3] is a concrete realization of holography. The duality has not only opened the door to previously intractable strongly coupled nonperturbative systems in quantum field theories (QFTs), but has also offered many useful insights into the fundamental properties of quantum gravity. In this regard, it is interesting to extend the AdS/CFT correspondence to BCFT in order to get a new handle on tackling some of the difficult dynamical problems in BCFT. The presence of a boundary in the QFT will also offer new twists in the realization of the AdS/CFT correspondence, and should lead to a deeper understanding of the holographic principle.

Consider a BCFT defined on a manifold M with a boundary P. Takayanagi [4] proposed to extend the d-dimensional manifold M to a $(d+1)$–dimensional asymptotically AdS space N so that $\partial N = M \cup Q$, where Q is a d-dimensional manifold which satisfies $\partial Q = \partial M = P$. See Fig. 1 for an example. The gravitational action for a holographic BCFT is [4,5]

$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_M \sqrt{g}K + 2\int_Q \sqrt{h}(K - T) + 2\int_P \sqrt{\sigma}\theta, \tag{1}$$

where $\theta = \arccos(n_M \cdot n_Q)$ is the supplementary angle between the boundaries M and Q, and is needed for a well-defined variational principle for the joint P [6]. We have taken $16\pi G_N = 1$. Note that here we have allowed in the action a constant term T on Q. T can be regarded as the holographic dual of boundary conditions of BCFT since it affects the boundary entropy [and also the boundary central charges, see Eqs. (17) and (18) below], which is closely related to the boundary conditions (BCs) [4,5].

A central issue in the construction of the AdS/BCFT is the determination of the location of Q in the bulk. Imposing a Dirichlet BC on M and P, $\delta g_{ij}|_M = \delta\sigma_{ab}|_P = 0$, we get the variation of the on-shell action

$$\delta I = -\int_Q \sqrt{h}(K^{\alpha\beta} - (K - T)h^{\alpha\beta})\delta h_{\alpha\beta}. \tag{2}$$

Interestingly, Takayanagi [4] proposed to impose a Neumann BC on Q,

$$K_{\alpha\beta} - (K - T)h_{\alpha\beta} = 0, \tag{3}$$

to fix the position of Q. For more general boundary conditions that break boundary conformal invariance, Takayanagi [4] proposed to add matter fields on Q and replace Eq. (3) by

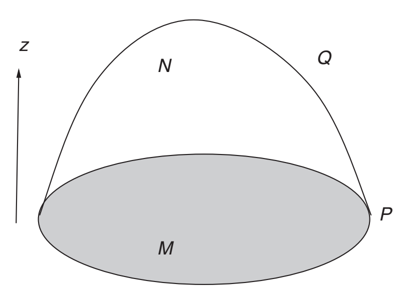

FIG. 1. BCFT on M and its dual N.

$$K_{\alpha\beta} - Kh_{\alpha\beta} = \frac{1}{2}T^{Q}_{\alpha\beta}, \tag{4}$$

where we have included $2Th_{\alpha\beta}$ in the matter stress tensor $T^{Q}_{\alpha\beta}$. For a geometrical shape of M with high symmetry, such as the case of a disk or half-plane, Eq. (3) fixes the location of Q and produces many elegant results for BCFT [4,5,7]. However since Q is of codimension 1 and its shape is determined by a single embedding function, Eq. (3) gives too many constraints and there is no solution in a given metric such as AdS generally. On the other hand, of course, there should exist a well-defined BCFT with general boundaries. As motivated in [4,5], Eqs. (3) and (4) are natural from the viewpoint of a braneworld scenario. However, from a practical point of view, it is not entirely satisfactory, because one has a large freedom to choose the matter fields as long as they satisfy various energy conditions. As a result, it seems one can put the boundary Q at almost any position one likes. In addition, it is not appealing that the holographic dual depends on the details of matter on Q. In this paper, we propose a new holographic dual of BCFT with Q determined by a new condition (7). This condition is consistent and provides a unified treatment for general shapes of P. Besides, as we will show below, it yields the expected boundary contributions to the Weyl anomaly.

II. NEW PROPOSAL FOR HOLOGRAPHIC BCFT

Instead of imposing the Neumann BC (3), we propose to impose on Q the mixed BCs $\Pi_{-\alpha\beta}^{\alpha'\beta'}\delta h_{\alpha'\beta'} = 0$ and

$$(K^{\alpha\beta} - (K-T)h^{\alpha\beta})\Pi_{+\alpha\beta}{}^{\alpha'\beta'} = 0. \tag{5}$$

Here $\Pi_{\pm}$ are projection operators satisfying $\Pi_{+\alpha\beta}{}^{\alpha'\beta'} + \Pi_{-\alpha\beta}{}^{\alpha'\beta'} = \delta^{\alpha'}_{\alpha}\delta^{\beta'}_{\beta}$ and $\Pi_{\pm\alpha\beta}{}^{\alpha'\beta'}\Pi_{\pm\alpha'\beta'}{}^{\alpha_1\beta_1} = \Pi_{\pm\alpha\beta}{}^{\alpha_1\beta_1}$. Since we could impose at most one condition to fix the location of the codimension-1 surface Q, we require Π_+ to be of the form $\Pi_{+\alpha\beta}{}^{\alpha'\beta'} = A_{\alpha\beta}B^{\alpha'\beta'}$. $\Pi_+\Pi_+ = \Pi_+$ then implies $\mathrm{tr}AB^T = 1$. The mixed boundary condition (5) becomes

$$(K^{\alpha\beta} - (K-T)h^{\alpha\beta})A_{\alpha\beta} = 0, \tag{6}$$

where $A_{\alpha\beta}$ are to be determined. It is natural to require that Eq. (6) be linear in K so that it is a second-order differential equation for the embedding. In this work we propose the choice $A_{\alpha\beta} = h_{\alpha\beta}$, and the condition for Q becomes the traceless condition

$$T_{BY}{}^{\alpha}{}_{\alpha} = 2(1-d)K + 2dT = 0, \tag{7}$$

where $T_{\mathrm{BY}\alpha\beta} = 2K_{\alpha\beta} - 2(K-T)h_{\alpha\beta}$ is the Brown-York stress tensor on Q. In general, it could also depend on the intrinsic curvatures which we will treat in [8]. The condition (7) is consistent and provides a unified treatment for general shapes of P. Besides, as we will show below, it yields the expected boundary contributions to the Weyl anomaly. We will also show below that there are problems with the other choices such as

$$A_{\alpha\beta} = \lambda_1 h_{\alpha\beta} + \lambda_2 K_{\alpha\beta} + \lambda_3 R_{\alpha\beta} + \cdots, \qquad \lambda_1, \lambda_2 \neq 0. \tag{8}$$

A few remarks on Eq. (7) are in order. (i) We note that the condition (4) is similar in form to the junction condition for a thin shell with spacetime on both sides [6]. However, here Q is the boundary of spacetime and not a thin shell, so there is no need to consider the junction condition. (ii) For the same reason, it is expected that Q has no backreaction on the geometry, just as for the boundary M. (iii) Equation (7) implies that Q is a constant mean curvature surface, which is, just as is the minimal surface, of great interest in both mathematics and physics. (iv) Equation (7) reduces to the proposal by [4] for a disk and half-plane, and it can reproduce all the results in [4,5,7]. (v) Equation (7) is a purely geometric equation and has solutions for arbitrary shapes of boundaries and arbitrary bulk metrics. (vi) In general, there could be more than one self-consistent boundary condition for a theory [9], so the proposal of [4] and our proposal have, in principle, no contradiction. Very importantly, our proposal gives a nontrivial boundary Weyl anomaly, which solves the difficulty met in [4,5].

Let us recall that in the presence of a boundary, the Weyl anomaly of CFT generally picks up a boundary contribution $\langle T^a_a\rangle_P$ in addition to the usual bulk term $\langle T^i_i\rangle_M$, i.e., $\langle T^i_i\rangle = \langle T^i_i\rangle_M + \delta(x_\perp)\langle T^a_a\rangle_P$, where $\delta(x_\perp)$ is a delta function with support on the boundary P. Our proposal yields the expected boundary Weyl anomaly for 3D and 4D BCFTs [10–12],

$$\langle T^a_a\rangle_P = c_1\mathcal{R} + c_2\mathrm{Tr}\bar{k}^2, d = 3, \tag{9}$$

$$\langle T^a_a\rangle_P = \frac{a}{16\pi^2}E_4^{\mathrm{bdy}} + b_1\mathrm{Tr}\bar{k}^3 + b_2 C^{ac}{}_{bc}\bar{k}^b{}_a, \qquad d = 4, \tag{10}$$

where c_1, c_2, b_1, b_2 are boundary central charges, and $a = 2\pi^2$ is the bulk central charge for 4D CFTs dual to Einstein gravity. Here $\mathcal{R}$ and $\bar{k}_{ab}$ are the intrinsic curvature and the traceless part of the extrinsic curvature of P, C_{abcd} is the pullback of the Weyl tensor of M to P (it is $C^{ac}{}_{bc} = -C^{an}{}_{bn}$), and

$$E_4^{\mathrm{bdy}} = 4\left(2\mathrm{Tr}(k\mathcal{R}) - k\mathcal{R} + \frac{2}{3}\mathrm{Tr}k^3 - k\mathrm{Tr}k^2 + \frac{1}{3}k^3\right) \tag{11}$$

is the boundary term of the Euler density $E_4 = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$. E_4^{bdy} (11) is needed in order to preserve the topological invariance of E_4 on the manifold with boundaries. Since Q is not a minimal surface in our case, our results [(17) and (18)] are nontrivial generalizations of the Graham-Witten anomaly [13] for the submanifold.

III. HOLOGRAPHIC BOUNDARY WEYL ANOMALY

A. Action method

One way to derive the Weyl anomaly is to obtain it from the logarithmic divergent terms of the gravitational action [14]. For our purpose, we focus below only on the boundary contributions to the Weyl anomaly.

Consider the asymptotically AdS metric in the Fefferman-Graham gauge,

$$ds^2 = \frac{dz^2 + g_{ij}dx^i dx^j}{z^2}, \tag{12}$$

where $g_{ij} = g^{(0)}_{ij} + z^2 g^{(1)}_{ij} + \cdots$, $g^{(0)}_{ij}$ is the metric of BCFT on M, and $g^{(1)}_{ij}$ can be fixed by the Penrose-Brown-Henneaux (PBH) transformation [15]

$$g^{(1)}_{ij} = -\frac{1}{d-2}\left(R^{(0)}_{ij} - \frac{R^{(0)}}{2(d-1)} g^{(0)}_{ij}\right). \tag{13}$$

Note that the curvatures in our notation differ from those of [15] by a minus sign. Without loss of generality, we choose the Gauss normal coordinates for the metric $g^{(0)}_{ij}$,

$$ds_0^2 = dx^2 + (\sigma_{ab} + 2xk_{ab} + x^2 q_{ab} + \cdots)dy^a dy^b, \tag{14}$$

where P is located at $x = 0$ and y^a are the coordinates along P. The bulk boundary Q is given by $x = X(z, y)$. Expanding it in z,

$$x = a_1 z + a_2 z^2 + \cdots + (b_{d+1} \ln z + a_{d+1}) z^{d+1} + \cdots, \tag{15}$$

where the coefficients a and b are functions of y. Substituting Eqs. (12)–(15) into the boundary condition Eq. (7), we obtain that

$$T = (d-1)\tanh\rho, \qquad a_1 = \sinh\rho,$$
$$a_2 = -\frac{\cosh^2\rho \mathrm{Tr}k}{2(d-1)}, \tag{16}$$

where we have reparametrized the constant T. It is worth noting that the other choices (8) of $A_{\alpha\beta}$ gives the same T, a_1, a_2 but different $a_3, a_4, \ldots$. In other words, the results (16) are independent of the choices of $A_{\alpha\beta}$ in the boundary condition (6) [8]. In fact since $K^\alpha_\beta = \frac{a_1}{\sqrt{1+a_1^2}}\delta^\alpha_\beta + O(z)$, one obtains from (6) that $(1-d)\frac{a_1}{\sqrt{1+a_1^2}} + T = 0$ as long as $A^\alpha_\alpha \neq 0$. This gives the first two terms in Eq. (16). As for the coefficient a_2, according to [16], the embedding function Eq. (15) is highly constrained by the asymptotic symmetry of AdS, and it can be fixed by PBH transformations up to some conformal tensors. Adapting the method of [16] to the present case, one can indeed prove the universality of a_2 in the Gauss normal coordinates [8]. In this way, we obtain $a_2 = -\frac{\cosh^2\rho \mathrm{Tr}k}{2(d-1)}$, which agrees with the result obtained in [16] for the special case of $a_{\mathrm{odd}} = \rho = 0$.

Now we are ready to derive the boundary Weyl anomaly. For simplicity, we focus on the case of 3D and 4D BCFTs. Substituting Eqs. (12)–(16) into the action (1) and selecting the logarithmic divergent terms after the integral along x and z, we can obtain the boundary Weyl anomaly. We note that I_M and I_P do not contribute to the logarithmic divergent term in the action since they have at most singularities in powers of z^{-1} but there is no integration along z; thus, there is no way for them to produce $\log z$ terms. We also note that only a_2 appears in the final results. The terms including a_3 and a_4 automatically cancel each other out. This is also the case for the holographic Weyl anomaly and universal terms of entanglement entropy for 4D and 6D CFTs [17,18]. After some calculations, we obtain the boundary Weyl anomaly for 3D and 4D BCFTs as

$$\langle T^a_a\rangle_P = \sinh\rho\mathcal{R} - \sinh\rho\mathrm{Tr}\bar{k}^2, \tag{17}$$

$$\langle T^a_a\rangle_P = \frac{1}{8}E_4^{\mathrm{bdy}} + \left(\cosh(2\rho) - \frac{1}{3}\right)\mathrm{Tr}\bar{k}^3 - \cosh(2\rho)C^{ac}{}_{bc}\bar{k}^b_a, \tag{18}$$

which takes the expected form Eqs. (9) and (10). It is remarkable that the coefficient of E_4^{bdy} takes the correct value to preserve the topological invariance of E_4. This is a nontrivial check of our results. In addition, the boundary charges c_1 in Eq. (9) is expected to satisfy a c-like theorem [5,19,20]. As was shown in [4,7], the null-energy condition on Q implies ρ decreases along renormalization group (RG) flow; this is also true for us. As a result, Eq. (17) indeed obey the c theorem for boundary charges, which is another support for our results. Most importantly, our confidence is based on the above universal derivations that there is no need to make any assumption about $A_{\alpha\beta}$ in the boundary condition (6).

We remark that, based on the results of free CFTs [11] and the variational principle, it has been suggested that the coefficient of Ck in Eq. (18) is universal for all 4D BCFTs [12]. Here we provide evidence, based on holography, against this suggestion: our results agree with the suggestion of [12] for the trivial case $\rho = 0$, though they disagr ee generally. As argued in [20], the proposal of [12] is suspicious since it means that there could be no independent boundary central charge related to the Weyl invariant $\sqrt{\sigma}C^{ac}{}_{bc}\bar{k}^b{}_a$. However, in general, every Weyl invariant should correspond to an independent central charge, such as the case for 2D, 4D, and 6D CFTs. In addition, we notice that the law obeyed by free CFTs usually does not apply to strongly coupled CFTs; see [21–24] for examples.

In the above, we have proven that, by using the method of Ref. [14], all the possible boundaries Q allowed by Eq. (6) produce the same boundary Weyl anomaly for 3D and 4D BCFTs. Thus, this method cannot distinguish the proposal (7) from the other choices (8).

B. Stress-tensor method

To resolve the above ambiguity, let us use the holographic stress tensor [25] to study the boundary Weyl anomaly, as this method needs the information of $(a_3, a_4, \ldots)$ and, hence, can distinguish the different choices of the BC (6). For simplicity, we focus on the case of 3D BCFT.

The first step of the method [25] is to find a finite action by adding suitable covariant counterterms. We obtain

$$I_{\rm ren} = \int_N \sqrt{G}(R-2\Lambda) + 2\int_M \sqrt{g}\left(K - 2 - \frac{1}{2}R_M\right) + 2\int_Q \sqrt{h}(K-T) + 2\int_P \sqrt{\sigma}(\theta - \theta_0 - K_M), \quad (19)$$

where we have included in M the usual counterterms in holographic renormalization [25,26], $\theta_0 = \theta(z=0)$ is a constant [5], and K_M, the extrinsic curvature of P, is the Gibbons-Hawking term for R_M on M. Notice that there is no freedom to add other counterterms, except for some finite terms that are irrelevant to the Weyl anomaly. For example, we may add terms like $\sqrt{\sigma}\mathcal{R}$ and $\sqrt{\sigma}K_M^2$ to I_P. However, such terms are invariant under constant Weyl transformations, so they do not contribute to the boundary Weyl anomaly. In conclusion, the renormalized action (19) is unique up to some irrelevant finite counterterms. From Eq. (19), it is straightforward to derive the Brown-York stress tensor on P,

$$B_{ab} = 2(K_{Mab} - K_M\sigma_{ab}) + 2(\theta - \theta_0)\sigma_{ab}. \quad (20)$$

In the spirit of [5,25,26], the boundary Weyl anomaly is given by

$$\langle T^a_a\rangle_P = \lim_{z\to 0}\frac{B^a_a}{z^2} = \lim_{z\to 0}\frac{4(\theta-\theta_0) - 2K_M}{z^2}, \quad (21)$$

where $\theta = \cos^{-1}\frac{x'}{\sqrt{g^{xx}+x'^2}} + O(z^3)$, $\theta_0 = \cos^{-1}(\tanh\rho)$, and $K_M = z\frac{\partial_x(\sqrt{g}\sqrt{g^{xx}})}{\sqrt{g}} + O(z^3)$. Substituting Eqs. (12)–(16) into Eq. (21), we get

$$\langle T^a_a\rangle_P = -\frac{\mathrm{sech}^2\rho}{4}[48a_3 + \sinh 3\rho(2q - 3k^2 - 4\mathrm{Tr}\bar{k}^2) + \sinh\rho(2\mathcal{R} + 6q - 6k^2 - 6\mathrm{Tr}\bar{k}^2)], \quad (22)$$

where q is the trace of q_{ab}. This gives the correct boundary Weyl anomaly (17) if and only if

$$a_3 = \frac{1}{48}\sinh\rho[\cosh(2\rho)(-2\mathcal{R} - 4q + k^2 + 10\mathrm{Tr}k^2) - 4\mathcal{R} - 8q + 3k^2 + 12\mathrm{Tr}k^2], \quad (23)$$

which is just the solution to our proposed boundary condition (7). One can check that the other choices (8) give different a_3 and, thus, can be excluded. Following the same approach, we can also derive the boundary Weyl anomaly for 4D BCFT [8], which agrees with the correct result (18) iff a_3 and a_4 are given by the solutions to Eq. (7). This is very strong support for the boundary condition (7) we proposed.

IV. HOLOGRAPHIC ENTANGLEMENT ENTROPY

Following [27,28], it is not difficult to derive the holographic entanglement entropy for BCFT; this is also given by the area of minimal surface

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}, \quad (24)$$

where A is a subsystem on M and γ_A denotes the minimal surface that ends on ∂A. What is new for BCFT is that the minimal surface could also end on the bulk boundary Q, when A is close to the boundary P. It turns out that γ_A is always orthogonal to Q when they intersect. See Fig. 1 for an example.

One way to see the orthogonality condition is to keep the endpoints of extreme surfaces γ'_A freely on Q, and select the one with minimal area as γ_A. It follows immediately that γ_A is orthogonal to the boundary Q when they intersect,

$$n^a_{\gamma_A}\cdot n_Q|_{\gamma_A\cap Q} = 0. \quad (25)$$

Here n_Q is the normal vector of Q and $n^a_{\gamma_A}$ are the two independent normal vectors of γ_A. Another way to see the condition (25) is that, otherwise there will arise problems in the holographic derivations of entanglement entropy when using the replica trick. In the replica method, one considers the n-fold cover M_n of M and then extends it to the bulk as N_n. It is important that N_n is a smooth bulk solution. As a result, the Einstein equation should be smooth on surface γ_A. Now the metric near γ_A is given by [28]

$$ds^2 = \frac{1}{r^{2\epsilon}}(dr^2 + r^2 d\tau^2) + (g_{ij} + 2\mathcal{K}_{aij}x^a + O(r^2))dy^i dy^j,$$

where $\epsilon \equiv 1 - \frac{1}{n}$, r is coordinate normal to the surface, $\tau \sim \tau + 2\pi n$ is the Euclidean time, y^i are coordinates along the surface, $x^a = (r\cos\tau, r\sin\tau)$, and $\mathcal{K}_{aij}$ are the two extrinsic curvature tensors. Going to complex coordinates $z = re^{i\tau}$, the zz component of Einstein equations

$$R_{zz} = -\mathcal{K}_z \frac{\epsilon}{z} + \cdots \tag{26}$$

is divergent unless the trace of extrinsic curvatures vanish, $\mathcal{K}_a = 0$. This gives the condition for a minimal surface [28]. Labeling the boundary Q by $f(z, \bar{z}, y) = 0$, we obtain the extrinsic curvature of Q as

$$K \sim \epsilon \partial_z f \partial_{\bar{z}} f \left(\frac{\partial_z f}{\bar{z}} + \frac{\partial_{\bar{z}} f}{z} \right) + \cdots. \tag{27}$$

Thus, the boundary condition (7) is smooth only if $\partial_z f|_{\gamma_A \cap Q} = \partial_{\bar{z}} f|_{\gamma_A \cap Q} = 0$, which is exactly the orthogonal condition (25). As a summary, the holographic entanglement entropy for BCFT is given by the Ryu-Takayanagi (RT) formula (24) together with the orthogonal condition (25).

V. BOUNDARY EFFECTS ON ENTANGLEMENT

Let us take a simple example to illustrate the boundary effects on entanglement entropy. Consider the Poincaré metric of AdS_3, $ds^2 = (dz^2 + dx^2 - dt^2)/z^2$, where P is at $x = 0$. For simplicity, in what follows we focus on $T = \tanh \rho \geq 0$. Solving Eq. (7) for Q, we get $x = \sinh(\rho) z$. We choose A as an interval $d \leq x \leq d + 2l$. Because of the presence of a boundary, there are now two kinds of minimal surfaces: one ends on Q and the other one does not. Determining which one has the smaller area depends on d. From Eqs. (24) and (25), we obtain

$$S_A = \begin{cases} \frac{1}{2G_N} \log(\frac{2l}{\epsilon}), & d \geq d_c, \\ \frac{\rho}{2G_N} + \frac{1}{4G_N} \log\left(\frac{4d(d+2l)}{\epsilon^2}\right), & d \leq d_c, \end{cases} \tag{28}$$

where $d_c = l\sqrt{e^{-2\rho} + 1} - l$ is the critical distance. It is remarkable that entanglement entropy (28) depends on the distance d and the boundary condition (ρ) when it is close enough to the boundary. This behavior is expected from the viewpoint of BCFT, because correlation functions in BCFT generally depend on the distance to the boundary [29].

To extract the effects of the boundary on the entanglement entropy, let us define the following quantity when A does not intersect the boundary P:

$$I_A \equiv S_A^{\mathrm{CFT}} - S_A^{\mathrm{BCFT}}. \tag{29}$$

The complementary situation, where the entangling surface intersects the boundary P, is discussed in [4]. Here, in Eq. (29) S_A^{CFT} is the entanglement entropy when the boundary disappears or is at infinity. In holographic language, it is given by the area of minimal surface that does not end on Q. Thus, S_A^{CFT} is equal to or bigger than S_A^{BCFT} and I_A is always non-negative. It is expected that the boundary does not affect the divergent parts of the entanglement entropy when $A \cap P = 0$, so that all the divergences cancel in Eq. (29). As a result, I_A is not only non-negative but also finite. Physically, I_A measures the decrease in the entanglement of the subsystem A with the environment when a boundary is introduced. For the example discussed above, we find

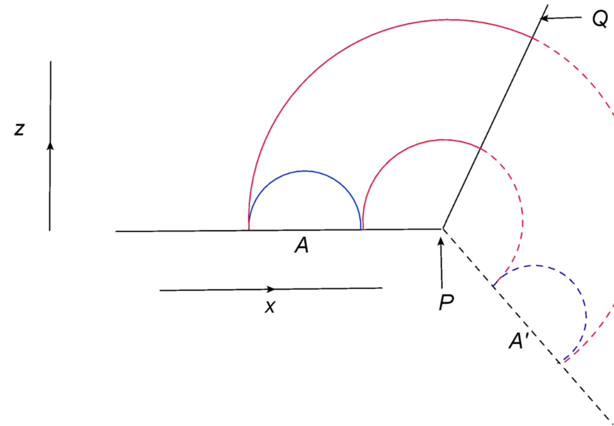

FIG. 2. Subsystem A and its mirror image A'.

$$I_A = \begin{cases} 0, & d \geq d_c \\ \frac{1}{4G} \log\left(\frac{l^2}{d(d+2l)}\right) - \frac{\rho}{2G}, & 0 < d < d_c, \end{cases} \tag{30}$$

which is indeed both non-negative and finite. Note that I_A depends both on the distance from the boundary and the boundary condition when $d < d_c$, but becomes independent of them when $d \geq d_c$. This represents some kind of phase transition. It is also intriguing to note that, in this simple example, I_A is just one half of the mutual information between A and its mirror image A', so it must be non-negative and finite. See Fig. 2 for an example.

VI. ENTANGLEMENT WEDGE

According to [30,31], a subregion A on the AdS boundary is dual to an entanglement wedge $\mathcal{E}_A$ in the bulk where all the bulk operators within $\mathcal{E}_A$ can be reconstructed by using only the operators of A. The entanglement wedge is defined as the bulk domain of dependence of any achronal bulk surface between the minimal surface γ_A and the subsystem A.

It is interesting to study the entanglement wedge in AdS/BCFT. For simplicity, let us focus on the static spacetime and constant time slice. A key observation is that the entanglement wedge exhibits a phase transition and becomes much larger than that in AdS/CFT, when A gets bigger and approaches the boundary (see Fig. 3). This phase transition is important for the self-consistency of holographic BCFT. If there is no phase transition, then $\mathcal{E}_A$ is always given by the first kind (as shown in left-hand side of Fig. 3) and there will be a large space left outside even when A fills up the whole M and P. This would mean that there are operators in the bulk that cannot be reconstructed from the operators on the boundary. Thanks to the phase

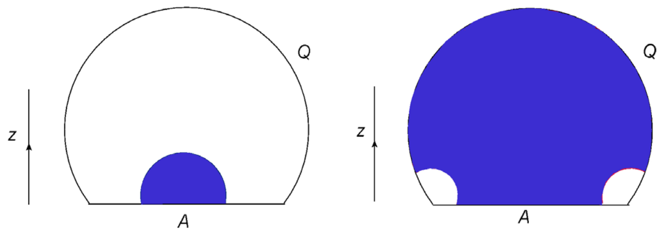

FIG. 3. Entanglement wedge for small A and large A.

transition, $\mathcal{E}_A$ for large A is given by the second kind of wedge (right-hand side of Fig. 3). As a result, all the bulk operators can be reconstructed by using the boundary operators.

VII. CONCLUSIONS AND DISCUSSIONS

In this paper, we have proposed a new holographic dual of BCFT that can accommodate all possible shapes of the boundary P in a unified prescription. The key idea is to impose the mixed boundary condition (7) so that there is only one constraint for the codimension-1 boundary Q. In general there could be more than one self-consistent boundary condition for a theory [9], so our proposal and that of [4] do not, in principle, contradict each other. However, the proposal of [4] is too restrictive to include the general BCFT. The main advantage of our proposal is that we can easily deal with all shapes of the boundary P. It is appealing that the bulk boundary Q is given by a constant mean curvature surface, which is a natural generalization of the minimal surface.

Applying the new AdS/BCFT, we obtain the expected boundary Weyl anomaly, and the obtained boundary central charges naturally satisfy a c-like theorem holographically. As a by-product, we give a holographic argument against the proposal of [12] and clarify the validity of the $S_{RE} = S_{EE}$ conjecture [32], which is based on [12] and depends sensitively on the boundary conditions of nonfree BCFTs. In addition, we find the holographic entanglement entropy is given by the RT formula together with the condition that the minimal surface must be orthogonal to Q if they intersect. The presence of boundaries leads to many interesting effects, e.g., the phase transition of the entanglement wedge. Of course, many things are left to be explored, for instance, the edge modes [33,34], the shape dependence of entanglement [35,36], the applications to condensed matter, and the relation between BCFT and quantum information [37]. Finally, it is straightforward to generalize our work to Lovelock gravity, higher dimensions, and general boundary conditions.

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