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Removing the Ostrogradski ghost from degenerate gravity theories

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The Ostrogradski ghost problem that appears in a higher-derivative system is considered for theories with constraints. A new prescription for removal of the ghost-creating momenta that come along the constrained systems is described based on the Dirac's constraint analysis. It is shown how one can make the canonical Hamiltonian bounded from below by systematically removing the constraints appearing in the system, thereby reducing the effective dimension of the phase space. To show the effect of higher-derivative terms, we consider the singularity-free Gauss-Bonnet theory coupled via a matter field to the Einstein-Hilbert action. Finally, we construct the canonical Hamiltonian for the theory that is bounded from below.

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I. INTRODUCTION

Higher-derivative (HD) theories have persuaded the physics community for their usefulness in the field theoretic context. By higher derivative, we mean that the fields appearing in the action have a time derivative more than 1. Historically, these HD terms were added to the Lagrangian for renormalization [1–3], perturbative corrections [4]. Also, they frequently appear in diverse fields like relativistic particle [5], string theory [6] and general relativity [7–10].

Right from the inception of the higher-derivative theories, they were diagnosed with a problem called the Ostrogradski ghost problem [11,12]. The ghost fields are nothing but some unphysical field arising in the theory that gives rise to the negative norm states while quantizing the theory. More precisely, in the canonical Hamiltonian, there may appear terms linear in momenta of the higher-derivative fields. Because of this, the Hamiltonian is not bounded from below, which exactly creates the problem as the negative energy states propagate the whole phase space and give rise to instabilities. Although Ostrogradski pointed out that only nondegenerate theories will have the ghost problem, here we show that, at the classical level, degenerate theories will also consist of terms in the canonical Hamiltonian that cause the instabilities. So, in the case of both degenerate and nondegenerate theories, the Ostrogradski ghost problem can appear at the classical level. These instabilities sometimes are of tachyonic nature depending upon the wave function, if it is oscillatory.

Over many decades, there were numerous attempts to solve the problem of these negative norm states by different authors. For example, one possible way to remove the ghost fields by applying some boundary conditions was suggested; e.g., in Ref. [13], the ghost free version was obtained by applying the Neumann boundary condition in the wave function. Also, Bender and Manheim showed in Ref. [14] that for a specific class of theories with PTs symmetry the ghost fields behave as usual fields and give positive PT norms. For the theories with no constraints, it was suggested in Ref. [15] to consider some external relations between the phase space variables, thereby decreasing the number of degrees of freedom. Recently, the authors in Ref. [16] showed that for massive and bimetric gravity theories there appear two second-class constraints which in turn help to eliminate the ghost field and its corresponding momenta. There has been much work done very recently to find ghost-free massive gravity theories [17]. The ghost fields are not always "bad;" in some cases, despite that they are present in the theory, they do not pollute it. As in Ref. [18], the massive gravity theory was having a negative energy state, but it was from a disjoint branch and hence cannot communicate to the positive energy states, thus leading to no instability. There are also attempts to remove the ghosts by considering an infinite set of higher-derivative terms, in particular, in the form of an exponential [19]. In Ref. [20], the ghost-free states were found for the linearized gravity with the Gauss-Bonnet couplings in the Randal-Sundurum picture. For degenerate gravity theories, Ostrogradski ghost removal was discussed in Ref. [21].

The Einstein-Hilbert action by definition contains higher-derivative terms of the field $g_{\mu\nu}$, but it is easy to point out that these higher-derivative terms actually are surface terms that can be neglected while considering the integrations. On the other hand, usual gravity theories are not renormalizable unless higher-derivative terms are added [1]. For that reason, the higher-derivative terms are inevitable in gravity theories. There are two ways by which HD terms enter into the gravitational action. One is directly as a function of higher curvatures like $f(R)$ or via the matter fields. This type of models of adding matter fields is a direct implication that one can get from the compactification of Kaluza-Klein theories. In this paper, we will restrict ourselves only in the higher curvatures. For that, as a viable HD term, we consider the Gauss-Bonnet gravity. The Gauss-Bonnet term is a special combination of the

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curvature squared terms (R^2, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$), which is actually a surface term added to the action. But it is very important for explaining dark energy [22], the inflationary scenario [23], the bouncing universe [24], implications of massive gravity [25], brane world gravity [26], etc. [27]. Very recently, the coupling parameters for the Gauss-Bonnet terms were proposed based on the results of BICEP2 and Planck [28]. Discussions on the stability conditions of the vacua in Gauss-Bonnet gravity can be found in Ref. [29].

In this paper, we first describe, in detail, how the ghosts appear via the canonical Hamiltonian and thereafter describe a procedure for removing these ghosts. To apply this method in models, we first consider the Einstein-Hilbert action and then add a Gauss-Bonnet term that is coupled via a field-dependent coupling parameter [30,31]. It is worth mentioning that, although these two models are independent of the ghosts, at first glance, the ghost-creating momenta appear in the canonical Hamiltonian and make the theory appear as if it is ghost dependent. The method described in this paper is quite capable of removing these ghosts from the system. The Gauss-Bonnet model has a singularity-free solution with conditions on the coupling parameter. With the Friedmann-Robertson-Lamitre-Walker (FRLW) background, we found the metric in the preferred minisuperspace version after Arnowitt-Deser-Misner (ADM) decomposition [32–34]. Since it is a higher-derivative system, we adopt the first-order formalism [35,36] and rename all the field variables to apply the Hamiltonian formalism. We found out that the system has constraints and therefore followed the Dirac constraint analysis to find out all the constraints in the system [37,38]. We construct the canonical Hamiltonian and find that there are terms linear in momenta of the fields. The momenta that correspond to the higher-derivative fields can give rise to negative norm states. To get rid of them, we eliminate these momenta from the canonical Hamiltonian. Accordingly, we remove the momenta appearing in the constraints by solving the second-class constraints. It is to be noted that the ghost-creating momenta will be appearing in some of the constraints, which may be first class or second class in nature. If they are second class, we can solve the constraints for the momenta and replace it in the canonical Hamiltonian. On the other hand, if the momenta appear in the first-class constraints, we need to introduce gauge conditions. For this model, all the constraints obtained are found to be second class in nature. Essentially, after solving the constraints, all the Poisson brackets in the system have to be replaced by Dirac brackets, which, during quantization, will play the role of commutators. Based on this concept, for theories with constraints, we showed how to systematically construct the ghost-free Hamiltonian that is independent of the linear dependence of the momenta corresponding to the higher-derivative field(s). The prescription presented here may be useful in models from different fields, although a general proof is warranted to value the method more.

The paper is organized as described. In Sec. II, we describe the general procedure for how the momenta appear by default in the canonical Hamiltonian for a higher-derivative theory. We also describe, for a constraint theory, how to remove these fields by considering the second-class constraints in the reduced phase space. In Sec. III, we consider the Einstein-Hilbert action with the FRLW background spacetime. Adopting the first-order formalism, we perform the Hamiltonian formulation and finally find out the ghost-free version of the canonical Hamiltonian. In Sec. IV, we add the Gauss-Bonnet term via a matter field to the Einstein-Hilbert action and construct the canonical Hamiltonian in the first-order formalism. We solve the second-class constraints appearing in the system to find out the final form of the ghost-free canonical Hamiltonian. Also, we compute the corresponding Dirac brackets between the canonical variables in the reduced phase space.

II. OPEN-ENDED HIGHER-DERIVATIVE THEORIES: GENERAL PRESCRIPTION

The Lagrangian of a HD theory of an nth-order derivative in time is generally written as

$$L = f(q, \dot{q}, \ddot{q}, \dots q^{(n)}). \tag{1}$$

The Lagrangian is a function in the configuration space, which consists of the field (q) and its time derivative(s). We convert the Lagrangian into first-order theory by incorporating the variables

$$q_1 = q, \qquad q_2 = \dot{q}\dots \qquad q_n = q^{(n-1)}. \tag{2}$$

So, the Lagrangian in the first order is written as

$$L' = F(q_1, q_2, q_3 \dots . \dot{q}_n) + \sum_{i=1}^{n} \lambda_i (q_{i+1} - \dot{q}_i). \tag{3}$$

Here, these λ_i are the Lagrange multipliers multiplied to the constraints that appeared due to the redefinition. Consequently, the dimension of the configuration space is increased. The momenta in this formalism can be written as

$$p_i = \frac{\partial L'}{\partial \dot{q}_i} = \frac{\partial F}{\partial \dot{q}_i} - \lambda_i, \qquad i = 1, 2 \dots (n-1) \tag{4}$$

$$p_{\lambda_i} = \frac{\partial L'}{\partial \dot{\lambda}_i} = 0. \tag{5}$$

The constraint p_{λ_i} is obvious and will be generated every time while defining the new fields. However, we should not bother about this as these fields are nondynamical and can eventually be set up to zero at the end. The character of p_i's is important. The function $\frac{\partial F}{\partial \dot{q}_i}$ is zero other than for $i = n$

depending on the nature of $F(q_1, q_2 ... \dot{q}_n)$. The Poisson brackets between the canonical variables $Q_i \equiv \{q_i, q_{\lambda_i}\}$ and their corresponding momenta $P_i \equiv \{P_i, P_{\lambda_i}\}$ are

$$\{q_i, p_j\} = \delta_{ij}. \tag{6}$$

Surely, Eqs. (4) and (5) lead to primary constraints, and hence the momenta will not be a function of the derivative of the corresponding field. Thus, the primary constraints are written generally as

$$\Phi_{i1} : p_{\lambda_i} \approx 0, \qquad \Phi_{i2} : p_i - \frac{\partial F}{\partial \dot{q}_i} + \lambda_i \approx 0. \tag{7}$$

The canonical Hamiltonian is

$$\begin{aligned} H_c &= p_1 \dot{q}_1 + p_2 \dot{q}_2 + p_\lambda \dot{q}_\lambda - L' \\ &= \sum_{i=1}^{n-1} p_i q_{i+1} + h(q_1, q_2 ... q_n). \end{aligned} \tag{8}$$

In the canonical Hamiltonian (8), the first term consists of (p,q), but the rest of the terms will also be simplified to make the Hamiltonian a function of the phase space variable once we consider the explicit form of the functions $h(q_1, q_2 ... q_n)$. What is important to look for in the Hamiltonian is that there are terms proportional to p_i. These terms can span in the whole phase-space region and consequently make the canonical Hamiltonian negative. This states that the Hamiltonian is not bounded from below and can range between both the positive or negative axis, i.e., with an open-ended solution. Therefore, in the quantum picture of the theory, there appear negative norm states that are known as ghost states. This is essentially a manifestation of the Ostrogradski theorem. It is interesting to see that renaming the variable to look at it as if a first-order theory and also by defining the momenta in the usual way does not help us to get rid of the negative Hamiltonian. This general discussion reveals that the appearance of the HD momenta making the canonical Hamiltonian negative is an inherent character for any HD theory.

A. Removing ghost degrees of freedom using constraints

What have we understood until now is that in the HD theory the Ostrogradski ghosts will eventually appear, and that is evident from the canonical Hamiltonian. As mentioned in the Introduction, for their removal, there are several methods available in the literature for the nondegenerate theories. But for degenerate theories, a less introspected way may be followed when the ghost-creating momenta are involved in the constraints. In Eq. (7), we get the set of primary constraints in which the set of momenta p_i appear. Also, we notice that this set of momenta p_i appearing in the canonical Hamiltonian are responsible for creating the negative norm states. A way out can be removal of these momenta by solving the constraints as at the end in the physical phase space the constraints will eventually be removed. For that, it is essential that we should get the full constraint structure. Let us write down a general form for the total Hamiltonian, which is given as

$$H_T = H_c + \sum_{i=1}^{n} u_i \Phi_{i1} + \sum_{i=1}^{n-1} u_i \Phi_{i2}. \tag{9}$$

The evolution of the primary constraints may give rise to secondary constraints, and in a similar way, we get tertiary constraints and so on. Once we have all the constraints, we can further categorize them as first class or second class. This division is essential, as we can know about the gauge symmetries of the system also [35,37]. Solving the constraints, let us reduce the dimension of the phase space, a way out mentioned in Ref. [15] for nondegenerate HD theories. The same can be done at the expense of the second-class constraints. Now, there may arise two conditions:

(i) The canonical Hamiltonian is a first-class constraint. This means there will be no more generation of the constraint. Since the theory has first-class constraints at the primary level, there is a gauge degree of freedom. The dimension of the phase space can be reduced by incorporating external condition on an *ad hoc* basis. These are called gauge conditions. These gauge conditions make the first-class constraints second class. Next, we solve all these second-class constraints and incorporate Dirac brackets. The canonical Hamiltonian becomes free from the momenta that were the source of instabilities.

(ii) The canonical Hamiltonian is a second-class constraint. In this case, there will be more constraint arising in the theory. The reduced dimensionality of the phase space can be obtained by setting these second-class constraints to zero. Then all the Poission brackets (PB) of the theory is replaced by the Dirac brackets.

So, reduction of the degrees of freedom is a must. In the squeezed phase space, the proper choice of the canonical variables can be made by inspecting the corresponding brackets. Thereafter, it will be a ghost-free theory. Between any two functions f and g of the canonical variables, Dirac brackets are defined as

$$\{f, g\}_D = \{f, g\} - \{f, \Psi_i\} \Delta_{ij}^{-1} \{\Psi_j, g\}, \tag{10}$$

where Δ_{ij} is the Poisson bracket matrix of the second-class constraints.

III. EINSTEIN-HILBERT ACTION IN MINISUPERSPACE

We consider the metric of the FRLW kind as

$$ds^2 = -dt^2 + da^2 + a^2 d\Omega_3^2, \tag{11}$$

where $d\Omega_3^2$ is the metric for the unit 3-sphere. We parametrize the brane using the parameter τ as

$$x^\mu = X^\mu(\xi^a) = (t(\tau), a(\tau), \chi, \theta, \phi), \tag{12}$$

and $a(\tau)$ is known as the scale factor.

After ADM decomposition with spacelike unit normals [$N(\tau) = \sqrt{\dot{t}^2 - \dot{a}^2}$ is the lapse function],

$$n_\mu = \frac{1}{N}(-\dot{a}, \dot{t}, 0, 0, 0), \tag{13}$$

the induced metric on the world volume is given by

$$ds^2 = -N^2 d\tau^2 + a^2 d\Omega_3^2. \tag{14}$$

Computation of the Ricci scalar is straightforward and is given by

$$\mathcal{R} = \frac{6\dot{t}}{a^2 N^4}(a\ddot{a}\,\dot{t} - a\dot{a}\,\ddot{t} + N^2\dot{t}). \tag{15}$$

The Lagrangian corresponding to the standard Einstein-Hilbert action with the nonzero cosmological constant is

$$\mathcal{L} = \sqrt{-g}\left(\frac{\alpha}{2}\mathcal{R} - \Lambda\right). \tag{16}$$

The Lagrangian in terms of the fields with arbitrary parameter τ can be written as [33,34]

$$\begin{aligned} L(N, N', a, a', a'') &= \frac{a}{N(\tau)^2}(-3aa'N'(\tau) + 3N(\tau)(aa'' + a'^2) \\ &\quad + N(\tau)^3(\Lambda a^2 + 3)). \end{aligned} \tag{17}$$

Here, "prime" refers to differentiation with respect to parameter τ. Note that the Lagrangian (17) contains higher-derivative terms of the field a. However, we can write it as [33]

$$L = -\frac{aa'^2}{N} + aN(1 - a^2 H^2) + \frac{d}{d\tau}\left(\frac{a^2 a'}{N}\right). \tag{18}$$

The above Lagrangian (18) has total derivative term that actually vanishes while performing the integrations, but we will keep this term, as it carries information about the entropy of the system. With the redefinition of the fields

$$a'(\tau) = A(\tau), \tag{19}$$

we obtain the first-order Lagrangian, which is given by

$$\begin{aligned} \mathcal{L} &= \frac{a}{N(\tau)^2}(3N(\tau)(aA' + A^2) - 3aAN'(\tau) \\ &\quad + N(\tau)^3(\Lambda a^2 + 3)) + \lambda_a(A - a'). \end{aligned} \tag{20}$$

Here, we incorporated the constraint due to field redefinition via the Lagrange multiplier $\lambda_a(\tau)$. The Euler-Lagrange equations of motion are

$$a(\tau)\colon\ \frac{6aA'}{N(\tau)} - \frac{6aAN'(\tau)}{N(\tau)^2} + \lambda_a' + 3N(\tau)(\Lambda a^2 + 1) + \frac{3A^2}{N(\tau)} = 0 \tag{21}$$

$$A(\tau)\colon\ \frac{6a(A - a') + N(\tau)\lambda_a}{N(\tau)} = 0 \tag{22}$$

$$N(\tau)\colon\ \frac{a(6Aa' + N(\tau)^2(\Lambda a^2 + 3) - 3A^2)}{N(\tau)^2} = 0 \tag{23}$$

$$\lambda_a(\tau)\colon\ A - a' = 0. \tag{24}$$

We construct the phase space in the next subsection for Hamiltonian formulation.

A. Constructing ghost-free Hamiltonian

The phase space is constructed out of the variables $\{N(\tau), a(\tau), A(\tau), \lambda_a(\tau)\}$, and their corresponding momenta are $\{\Pi_N(\tau), \Pi_a(\tau), \Pi_A(\tau), \Pi_{\lambda_a}(\tau)\}$. Since the Lagrangian is in first-order form, the momenta defined in the usual way are

$$\Pi_N = -\frac{3a^2 A}{N(\tau)^2}, \qquad \Pi_a = -\lambda_a,$$
$$\Pi_A = \frac{3a^2}{N(\tau)}, \qquad \Pi_{\lambda_a} = 0.$$

None of the momenta here is invertible with respect to the corresponding velocity; hence, we can construct the primary constraints as

$$\Phi_1 = \Pi_N + \frac{3a^2 A}{N(\tau)^2} \approx 0, \qquad \Phi_2 = \Pi_a + \lambda_a \approx 0,$$
$$\Phi_3 = \Pi_A - \frac{3a^2}{N(\tau)} \approx 0, \qquad \Phi_4 = \Pi_{\lambda_a} \approx 0.$$

The nonzero Poisson brackets between the primary constraints are

$$\{\Phi_1, \Phi_2\} = \frac{6aA}{N(\tau)^2}, \qquad \{\Phi_2, \Phi_3\} = \frac{6a}{N(\tau)}, \qquad \{\Phi_2, \Phi_4\} = 1.$$

It seems that all the primary constraints are second class in nature. If we replace Φ_1 by the combination $\xi_1 = \Phi_1 + \frac{6aA}{N(\tau)^2}\Phi_4$, we get ξ_1 as first class at this level. But we are not interested in exploring the gauge symmetries, so let us make use of these second-class constraints. Next, we construct the canonical Hamiltonian, which is given by

$$H_{\rm can} = -\frac{3aA^2}{N(\tau)} + A\Pi_a - aN(\tau)(\Lambda a^2 + 3). \qquad (25)$$

Surely, this Hamiltonian is not bounded from below due to the existence of the momenta Π_a. Now, the total Hamiltonian is given by

$$H_T = H_{\rm can} + \Lambda_1\xi_1 + \Lambda_2\Phi_2 + \Lambda_3\Phi_3 + \Lambda_4\Phi_4. \qquad (26)$$

To see the time evolution of the primary constraints, we compute the Poisson brackets between the primary constraints and total Hamiltonian, which give

$$\{\xi_1, H_T\} = a\left(\Lambda a^2 + \frac{3A^2}{N^2} + 3\right), \qquad (27)$$

$$\{\Phi_2, H_T\} = \frac{3}{N(\tau)}(2\Lambda_3 a + N(\tau)^2(\Lambda a^2 + 1) + A^2) \times N(\tau) + \Lambda_4 \qquad (28)$$

$$\{\Phi_3, H_T\} = -\frac{6a(\Lambda_2 - A)}{N(\tau)} + \lambda_a \qquad (29)$$

$$\{\Phi_4, H_T\} = -\Lambda_2 + A_2. \qquad (30)$$

Equating (30) them to zero, we get $\Lambda_2 = A_2$. Now, demanding equating (29) and (27) to zero and using $\Lambda_2 = A_2$, we get two secondary constraints:

$$\Psi_1 = a\left(\Lambda a^2 + \frac{3A^2}{N(\tau)^2} + 3\right) \approx 0, \qquad (31)$$

$$\Psi_2 = \lambda_a \approx 0. \qquad (32)$$

Nonzero Poisson brackets between the primary and secondary constraints are

$$\{\Phi_1, \Psi_1\} = -\frac{6aA^2}{N(\tau)^3},$$
$$\{\Phi_1, \Psi_2\} = -\frac{6aA}{N(\tau)^2}$$
$$\{\Phi_2, \Psi_1\} = 3\Lambda a^2 + \frac{3A^2}{N(\tau)^2} + 3,$$
$$\{\Phi_3, \Psi_1\} = \frac{6aA}{N(\tau)^2},$$
$$\{\Phi_4, \Psi_1\} = -1.$$

Time conservation of the secondary constraints (31) and (32) gives

$$\{\Psi_1, H_T\} = \frac{6\Lambda_1 aA^2}{N(\tau)^3} - \frac{6\Lambda_3 aA}{N(\tau)^2} + \Lambda_2\left(-3\Lambda a^2 - \frac{3A^2}{N(\tau)^2} - 3\right),$$
$$\{\Psi_2, H_T\} = \frac{6\Lambda_1 aA}{N(\tau)^2} + \Lambda_4. \qquad (33)$$

Using the equations in (33) and (28), one can in principle solve Λ_1, Λ_3, and Λ_4. But it turns out that they are not independent, and so Λ_1 remains undetermined. The solution is

$$\Lambda_3 = \frac{\Lambda_1 A}{N(\tau)} - \frac{\Lambda a^2 N(\tau)^2 + A^2 + N(\tau)^2}{2a},$$
$$\Lambda_4 = -\frac{6\Lambda_1 aA}{N(\tau)^2}.$$

Thus, the constraint chain stops here. The existence of the undetermined multiplier Λ_1 signals that there is gauge symmetry present in the system, which we know as the diffeomorphism. The primary first-class constraint can emerge if we take the field redefinition ($N = \sqrt{\dot{t}^2 - \dot{a}^2}$) as worked out in Ref. [34]. But here we only need the second-class constraint to go through the process, as first-class constraints also can be made second class by incorporating the gauge conditions. Now, we solve the constraints Φ_2 and Φ_4 to remove the unphysical variables λ_a and its corresponding momenta Π_{λ_a}. We solve the second-class constraint Φ_2 using Ψ_2 form (32) and get

$$\Pi_a = 0. \qquad (34)$$

As the second-class constraints are directly related to the degrees of freedom count, we should always remove them in pairs. So, we also choose Ψ_1 for removal. The canonical Hamiltonian after solving Ψ_2 is

$$H_{\rm ghost\text{-}free} = -\frac{3aA^2}{N(\tau)} - aN(\tau)(\Lambda a^2 + 3). \qquad (35)$$

From the above equation (35), we can clearly see that the Hamiltonian is free from the unwanted momenta and hence does not have any negative norm states.

The expression for the Hamiltonian is interesting, as it matches with Ψ_1 (31). This is expected, as it is the very well-known Hamiltonian constraint. One should remember that at this stage we have to consider the Dirac brackets to replace all calculations involving Poisson brackets. Since all the constraints have become second class, they are just identities, and hence one can be replaced with respect to the others, to have a viable representation of the theory. To compute the Dirac brackets between the variables, we list below all the second-class constraints [after removing the Lagrange multiplier fields $(\lambda_a, \Pi_{\lambda_a})$]:

$$S_1 = \frac{3a^2 A}{N(\tau)^2} + \Pi_N \qquad S_2 - = \Pi_A - \frac{3a^2}{N(\tau)}$$

$$S_3 = -\Lambda a(\tau)^2 - \frac{3A(\tau)^2}{N(\tau)^2} - 3 \qquad S_4 = \Pi_a(\tau).$$

A degrees-of-freedom count is necessary for the fact to validate the theory. In this theory, the degrees of freedom in the reduced phase space are 2× (total number of phase space variables)—(2× number of first-class constraints + number of second-class constraints) = 2×4−(2×0+6)=2.

This degrees-of-freedom count agrees with the standard gravitational results. The matrix $\Delta_{ij} = \{S_i, S_j\}$ is given by

$$\begin{bmatrix} 0 & 0 & 0 & \frac{6aA}{N(\tau)^2} \\ 0 & 0 & -\Pi_a & -\frac{6a}{N(\tau)} \\ 0 & \Pi_a & 0 & -2\Lambda a^2 N(\tau) \\ -\frac{6aA}{N(\tau)^2} & \frac{6a}{N(\tau)} & 2\Lambda a^2 N(\tau) & 0 \end{bmatrix}.$$

Below, we list all the nonzero Dirac brackets (10) between the fields:

$$\begin{aligned}
\{N(\tau), \Pi_N\}_D &= 1 \\
\{a, \Pi_N\}_D &= -\frac{1}{N(\tau)^5}(3a^2(2a(2A^2(\Lambda N(\tau)^4 + 3) + 3N(\tau)^2) + AN(\tau)^3\Pi_a)) \\
\{a, \Pi_a\}_D &= \frac{1}{N(\tau)^4}(6aAN(\tau)^3\Pi_a + 36a^2(A^2 + N(\tau)^2) + 4\Lambda^2 a^4 N(\tau)^6 + N(\tau)^4) \\
\{a, \Pi_A\}_D &= 2a^2\left(3aA\left(2\Lambda + \frac{3}{N(\tau)^4}\right) - \Lambda N(\tau)\Pi_a\right) \\
\{A, \Pi_N\}_D &= \frac{6aA^2\Pi_a}{N(\tau)^2} \\
\{A, \Pi_a\}_D &= -2\Lambda a^2 N(\tau)\Pi_a \\
\{A, \Pi_A\}_D &= -\frac{6aA\Pi_a}{N(\tau)} + \Pi_a^2 + 1 \\
\{N(\tau), a\}_D &= \frac{6aA}{N(\tau)^2} \\
\{a, A\}_D &= A\Pi_a + \frac{6a}{N(\tau)} \\
\{\Pi_N, \Pi_a\}_D &= \frac{6\Pi_a(\Lambda a^4 N(\tau)^2 - 6a^2 A^2)}{N(\tau)^3} \\
\{\Pi_N, \Pi_A\}_D &= \frac{3a^2\Pi_a(6aA - N(\tau)\Pi_a)}{N(\tau)^3} \\
\{\Pi_A, \Pi_a\}_D &= \frac{6a\Pi_a(6aA - N(\tau)\Pi_a)}{N(\tau)^2}.
\end{aligned}$$

The Dirac brackets obtained thus can be used for quantization of the system in the reduced phase space.

IV. SINGULARITY-FREE GAUSS-BONNET GRAVITY

In this section, we consider the matter field added to the Einstein action along with a Gauss-Bonnet term. The action is

$$S = \int \sqrt{-g}\left(\frac{R}{2} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \alpha\xi(\phi)(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right)d^4x.$$

In the language of the metric components for the minisuperspace universe (14), the required Lagrangian is

$$L = \frac{r^2 \sin\theta}{2\sqrt{1-Kr^2}N(\tau)^4}(6a(KN(\tau)^5 - N(\tau)^3 a'^2) + a^3N(\tau)^5\phi'^2 + 48\alpha\xi(\phi)(KN(\tau)^2 - a'^2) \times (N(\tau)a'' - a'N'(\tau)) + 6a^2N(\tau)^2(a'N'(\tau) - N(\tau)a'')). \tag{36}$$

To get a singularity-free model of (36), we take $\xi(\phi) = \phi^2$ and $\alpha = 1/32$ as in Ref. [31] for a negatively curved universe ($K = -1$). Immediately, we can write down the first-order Lagrangian with the redefinition (19)

$$\mathcal{L} = \frac{1}{N(\tau)^4}(-12a^2N(\tau)^2(N(\tau)A' - AN'(\tau)) - 12aN(\tau)^3(A^2 + N(\tau)^2) + 2a^3N(\tau)^5\phi'^2 - 3\phi^2(A^2 + N(\tau)^2)(N(\tau)A' - AN'(\tau))) + \lambda_a(A - a'). \tag{37}$$

Here, this λ_a is the Lagrange multiplier corresponding to compensating the redefinition of the variables in (19). While deriving the form of the Lagrangian (37), we did not consider terms proportional to (r, θ), as they can be integrated out, which eventually will not be effective in the equation of motion. The Euler-Lagrange equations for this Lagrangian are given by

$$a(\tau)\colon\ \frac{1}{N(\tau)^2}(N(\tau)(-24aA' + L\lambda_a' + 6N(\tau)^2(a^2\phi'^2 - 2)) + 24aAN'(\tau) - 12A^2N(\tau)) = 0,$$

$$A(\tau)\colon\ \frac{1}{N(\tau)^3}(-24aN(\tau)^2(A - a') + N(\tau)^3\lambda_a + 6A^2\phi\phi' + 6N(\tau)^2\phi\phi') = 0,$$

$$N(\tau)\colon\ \frac{1}{N(\tau)^4}(2(-6aN(\tau)^2(2Aa' - A^2 + N(\tau)^2) + a^3N(\tau)^4\phi'^2 - 3A\phi(A^2 + N(\tau)^2)\phi')) = 0,$$

$$\phi\colon\ \frac{1}{N(\tau)^4}(6\phi(A^2 + N(\tau)^2)(AN'(\tau) - N(\tau)A')) - 12a^2N(\tau)a'\phi' - 4a^3N'(\tau)\phi' - 4a^3N(\tau)\phi'' = 0,$$

$$\lambda_a(\tau)\colon\ A - a' = 0.$$

A. Hamiltonian formulation and ghost-free Hamiltonian

In the phase space, momenta defined by Eqs. (4) and (5) are given by

$$\Pi_L = \frac{1}{N(\tau)^4}(12a^2AN(\tau)^2 + 3A\phi^2(A^2 + N(\tau)^2)),$$

$$\Pi_a = -\lambda_a,$$

$$\Pi_A = \frac{1}{N(\tau)^4}(-12a^2N(\tau)^3 - 3N(\tau)\phi^2(A^2 + N(\tau)^2)),$$

$$\Pi_\phi = 4a^3N(\tau)\phi',$$

$$\Pi_{\lambda_a} = 0.$$

These momenta give the following primary constraints:

$$\Phi_1 = \Pi_N(\tau) - \frac{1}{N(\tau)^4}(12a^2AN(\tau)^2 + 3A\phi^2(A^2 + N(\tau)^2) \approx 0, \tag{38}$$

$$\Phi_2 = \lambda_a + \Pi_a \approx 0, \tag{39}$$

$$\Phi_3 = \Pi_A + \frac{1}{N(\tau)^4}(12a^2N(\tau)^3 + 3N(\tau)\phi^2(A^2 + N(\tau)^2) \approx 0. \tag{40}$$

$$\Phi_4 = \Pi_{\lambda_a} \approx 0. \tag{41}$$

We list here the nonzero PBs between the primary constraints:

$$\{\Phi_1, \Phi_2\} = -\frac{24aA}{N(\tau)^2},$$

$$\{\Phi_2, \Phi_3\} = -\frac{24a}{N(\tau)},$$

$$\{\Phi_2, \Phi_4\} = 1.$$

A careful redefinition of $\Phi_1 \to \Phi_1' = \Phi_1 - \frac{24aA}{N(\tau)^2}\Phi_4$ gives more compact PBs, which are (only nonzero components are shown)

$$\{\Phi_2, \Phi_3\} = -\frac{24a}{N(\tau)},$$

$$\{\Phi_2, \Phi_4\} = 1.$$

We can immediately write down the canonical Hamiltonian as

$$H_{\mathrm{can}} = \frac{12a(A(\tau)^2 + N(\tau)^2)}{N(\tau)} + A\Pi_a + \frac{\Pi_\phi^2}{8a^3N(\tau)}. \tag{42}$$

The canonical Hamiltonian (42) contains a term linear in momenta Π_a. This signals the presence of the Ostrgradski ghost. This is in conformity with the general equation (8), which says that corresponding to each higher-derivative field there will be at least one term linear in momenta. The total Hamiltonian is

$$H_{\text{tot}} = H_{\text{can}} + \Lambda_1\Phi_1' + \Lambda_2\Phi_2 + \Lambda_3\Phi_3 + \Lambda_4\Phi_4. \quad (43)$$

Equating the PBs of the total Hamiltonian with the Φ_4 to zero, we get $\Lambda_2 = 0$. Also, the PBs of the total Hamiltonian with Φ_1' and Φ_3 identical give secondary constraints, respectively, as

$$\Psi_1 = 96a^4N(\tau)^3(A^2 + N(\tau)^2) + \Pi_\phi(12A\phi(A^2 + N(\tau)^2) - N(\tau)^3\Pi_\phi), \quad (44)$$

$$\Psi_2 = 2a^3N(\tau)^4\Pi_a - 3\phi\Pi_\phi(A^2 + N(\tau)^2). \quad (45)$$

On the other hand, PBs of the total Hamiltonian with Φ_2 in (39) give the following equation:

$$\frac{3}{8N(\tau)}\left(32(2\Lambda_3 a + A^2 + N(\tau)^2) - \frac{\Pi_\phi{}^2}{a^4}\right) - \Lambda_4 = 0. \quad (46)$$

To get the tertiary constraints, it is necessary to see the PBs of the total Hamiltonian with the secondary constraints. Poisson bracket of Ψ_1 and Ψ_2 with the total Hamilonian give the following equations:

$$\begin{aligned}
&-384a^6N(\tau)^7(A+\Lambda_2)(A^2+N(\tau)^2) - 96a^7N(\tau)^6(\Lambda_1(3A^2+5N(\tau)^2) + 2\Lambda_3AN(\tau))\\
&\quad - 3AN(\tau)^3(A^2+N(\tau)^2)\Pi_\phi^2 + 3a^3(-24A\phi^2(A^2+N(\tau)^2)^2(\Lambda_1A - \Lambda_3N(\tau))\\
&\quad - 4N(\tau)^3\phi\Pi_\phi(\Lambda_1A(N(\tau)^2 - A^2) + 2\Lambda_3N(\tau)(2A^2+N(\tau)^2)) + \Lambda_1N(\tau)^6\Pi_\phi^2) = 0\\
&96a^6N(\tau)^6(-2\Lambda_1aA + 2\Lambda_3aN(\tau) + A^2N(\tau) + N(\tau)^3) + 72a^3\phi^2(A^2+N(\tau)^2)^2(\Lambda_1A - \Lambda_3N(\tau))\\
&\quad + 8a^5N(\tau)^7\lambda_a(4\Lambda_1a + 3N(\tau)(A+\Lambda_2)) + 24a^3N(\tau)^4\phi\Pi_\phi(\Lambda_3A + \Lambda_1N(\tau))\\
&\quad + 3N(\tau)^3\Pi_\phi^2(-a^2N(\tau)^4 + A^2 + N(\tau)^2) = 0.
\end{aligned} \quad (47)$$

From these two equations of (47) along with (46), we can easily solve Λ_1, Λ_2, and Λ_3. So, the constraint chain ends here.

As we got the full constraint structure, we notice that the ghost-creating momenta appear in (45). Following the prescription described in the earlier section, we solve the secondary constraints Ψ_1 and Ψ_2 to remove the variables ϕ and Π_a, respectively. Thus, the ghost-free canonical Hamiltonian for (36) is given by the simple form as

$$H_{\text{ghost-free}} = \frac{12a(A(\tau)^2 + N(\tau)^2)}{N(\tau)} + A\Pi_a + \frac{\Pi_\phi^2}{8a^3N(\tau)}. \quad (48)$$

The momenta Π_ϕ appearing here are of quadratic power and hence are bounded from below. The above result is very interesting and gives back the ghost-free canonical Hamiltonian that does not contain any term linear in any of the momenta. As we have solved the second-class constraints (46), we need to give up the Poisson brackets. Notice that to obtain the ghost-free version, which was our sole aim, is achieved just by solving Ψ_1 and Ψ_2.

The Dirac bracket structure can be obtained by solving the constraints Φ_1, Φ_2, Ψ_1, and Ψ_2. Reduction of Φ_1 and Φ_2 is trivial, as they do not modify the Poisson brackets. Also, by solving these two constraints, we get rid of the unphysical degrees of freedoms λ_a and Π_{λ_a}. To reduce the phase space further, we obtain the Poisson brackets between Ψ_1 and Ψ_2, which is given by

$$\{\Psi_1, \Psi_2\} = 6N(\tau)^3(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2). \quad (49)$$

To compute the Dirac brackets (10) between two canonical functions, we shall use (49). Below, we give all the nonzero Dirac brackets between the variables in the phase space:

$$\begin{aligned}
\{N, \Pi_N\}_D &= 1\\
\{a, \Pi_N\}_D &= -\frac{a^3N(\tau)^2(32a^4(3A^2N(\tau) + 5N(\tau)^3) + \Pi_\phi(8A\phi - N(\tau)\Pi_\phi))}{(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2)}\\
\{a, \Pi_a\}_D &= \frac{\Pi_\phi^2}{\Pi_\phi^2 - 128a^6N(\tau)^4}\\
\{a, \Pi_A\}_D &= -\frac{4a^3N(\tau)(16a^4AN(\tau)^3 + \phi(3A^2 + N(\tau)^2)\Pi_\phi)}{(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2)}\\
\{a, \Pi_\phi\}_D &= -\frac{4a^3AN(\tau)\Pi_\phi}{128a^6N(\tau)^4 - \Pi_\phi^2}
\end{aligned}$$

$$\{\phi, \Pi_N(\tau)\}_D = \frac{1}{6N(\tau)(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2)}(288a^4\phi(8A^2N(\tau)^2 + 3A^4 + 5N(\tau)^4)$$
$$+ 16a^3N(\tau)\Pi_a(6AN(\tau)^2\phi + 6A^3\phi - N(\tau)^3\Pi_\phi) + 3\phi(N(\tau)^2 - 3A^2)\Pi_\phi^2)$$
$$\{\phi, \Pi_a\}_D = \frac{2a^2(N(\tau)\Pi_a(6AN(\tau)^2\phi + 6A^3\phi - N(\tau)^3\Pi_\phi) + 96a\phi(A^2 + N(\tau)^2)^2)}{(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2)}$$
$$\{\phi, \Pi_A\}_D = \frac{2\phi(48a^4AN(\tau)^3(A^2 + N(\tau)^2) + \Pi_\phi(AN(\tau)^3\Pi_\phi + 6A^2N(\tau)^2\phi + 3A^4\phi + 3N(\tau)^4\phi))}{N(\tau)^3(A^2 + N(\tau)^2)(128a^6N(\tau)^4 - \Pi_\phi^2)}$$
$$\{\phi, \Pi_\phi\}_D = \frac{128a^6N(\tau)^4}{128a^6N(\tau)^4 - \Pi_\phi^2}.$$

One should keep in mind that now we are working in the reduced phase space and, due to the two second-class constraint reductions, the dimensionality of the phase space has also reduced by 2.

V. CONCLUSION

Quantization of the gravitational fields is one of the most important challenges for this era of theoretical physicists. The problem is that the gravity theories are not renormalizable. Although, adding higher-derivative fields can make them renormalizable [1], but not all combinations of higher-derivative terms are allowed [2]. With these restrictions, higher-derivative gravity theories are still considered strong candidates for developing quantum gravity. The problem that comes along frequently while quantizing the higher-derivative theories is that there appear negative norm states that we refer to as ghost states [12]. The origin of these ghost fields can be traced back in the canonical Hamiltonian in which the momenta corresponding to ghost field appear linearly [19]. For the higher-derivative theories only, the Ostrogradsky theorem itself tells us that the degenerate theories contains ghosts while the nondegenerate theories are secretly stable. This issue of removing the ghosts that appear the canonical momenta was considered here, and we showed how one can remove the ghost-creating momenta one by one tactfully by considering the constraints only.

In this paper, we took two models simultaneously: one consists of only the gravity theory, i.e., the Einstein-Hilbert action, whereas in the other model, we considered the Gauss-Bonnet gravity along with a matter field coupled to the Einstein-Hilbert action [30,31]. The reason for the inclusion of the matter field is to confirm that the algorithm provided here does not break down even in the presence of matter fields. Following the Hamiltonian formulation, we found out all the constraints in the theory. The canonical Hamiltonian, as usual, contains the linear momenta that source the instability. We notice that these momenta also appear in the second-class constraints. So, to remove them from the canonical Hamiltonian, we solved the second-class constraints and found out the canonical Hamiltonian that became independent of the any ghost-creating momenta. Thus, following the very effective method of Dirac's constraint analysis, we constructed the ghost-free canonical Hamiltonian [37]. Further, we computed the Dirac brackets between the canonical fields by solving the second-class constraints that contain the ghost-creating momenta. It should be mentioned that one always need to solve an even number of second-class constraints to compute Dirac brackets. The degrees of freedom count was done from the number of constraints, and this agreed with the expected results.

Because of this, one might inquire about the system with first-class constraints. The first-class constraints can be made second class by incorporating gauge conditions. In fact, in Ref. [34], we, with other coauthors, discussed the Einstein-Hilbert action in which there exists a primary first-class constraint and a gauge condition was proposed to remove it. The Hamiltonian obtained thereby was free from the linear momenta. For future projects, the method followed in this paper can be utilized while quantizing theories with more complicated actions. In this regard, one should first check if the canonical Hamiltonian contains ghost-creating momentum or not, and if it does, the constraints can be removed as described in this paper. This is so because by construction the momenta corresponding to the higher-derivative fields will appear in some of the primary constraints and thus appear in the canonical Hamiltonian.

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