Mass discrepancy-acceleration relation: A universal maximum dark matter acceleration and implications for the ultralight scalar dark matter model

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Recent analysis of the rotation curves of a large sample of galaxies with very diverse stellar properties reveals a relation between the radial acceleration purely due to the baryonic matter and the one inferred directly from the observed rotation curves. Assuming the dark matter (DM) exists, this acceleration relation is tantamount to an acceleration relation between DM and baryons. This leads us to a universal maximum acceleration for all halos. Using the latter in DM profiles that predict inner cores implies that the central surface density $\mu_{DM} = \rho_s r_s$ must be a universal constant, as suggested by previous studies of selected galaxies, revealing a strong correlation between the density ρ_s and scale r_s parameters in each profile. We then explore the consequences of the constancy of μ_{DM} in the context of the ultralight scalar field dark matter model (SFDM). We find that for this model $\mu_{DM} = 648 M_\odot \, \mathrm{pc}^{-2}$ and that the so-called WaveDM soliton profile should be a universal feature of the DM halos. Comparing with the data from the Milky Way and Andromeda satellites, we find that they are all consistent with a boson mass of the scalar field particle of the order of 10^{-21} eV/c^2, which puts the SFDM model in agreement with recent cosmological constraints.

I. INTRODUCTION

In the standard cold dark matter (CDM) paradigm, the dark matter (DM) is ~22% of the total matter budget in the Universe [1]. It is assumed to be collisionless and nonrelativistic after decoupling, forming structure hierarchically; i.e., small halos merge to form more massive systems. Several cosmological CDM simulations that exclude the luminous matter have confirmed this formation scenario showing that in all scales, halos share a common density profile with a characteristic cusp (divergent density) near their centers [2–5]. Assuming galaxies are formed in these halos allows the comparison with observations. However, detailed comparisons with the dynamics of low-mass galaxies have led to some long-standing discrepancies, e.g., cusp-core problem, the satellite abundance, and too big to fail [6–9]. Galaxies are then one of the greatest challenges for the CDM paradigm.

One quantity that summarizes the properties of the rotation curves in galaxies is the so-called mass discrepancy-acceleration relation (MDAR) [10,11]. This relation refers to both the excess in dynamical mass to baryonic mass and to its anticorrelation with the gravitational acceleration obtained from the (luminous) baryonic component only. The MDAR is observed for a diverse sample of galaxies, from high to low surface brightness galaxies and of different sizes and morphologies. This seeming independence of the relation on the luminous matter suggests that if there is a common origin to the MDAR, it is probably not strongly tied to the baryonic matter. This is a challenge for the CDM paradigm, but interesting arguments exist in which the MDAR could also arise in CDM due to intrinsic galaxy formation processes, including violent baryonic processes (e.g., galactic winds and supernovae feedback) that strongly perturb the potential well and by this also affect the dark matter distribution; see, for instance, [12,13]. These other two common approaches exist to explain the origin of the MDAR: one is to assume that the acceleration relation results from modifying the gravitational force as suggested by the Modified Newtonian Dynamics (MOND) hypothesis [14] (but see, also, [15,16]), and the second is that the MDAR is a direct consequence of the intrinsic properties of the DM [17]. The latter approach will be assumed for the purposes of this paper.

II. THEORY

Considering the equivalent DM halo to explain the MDAR, it can be shown that the gravitational acceleration of the DM can be found from [10]

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$$g_h = \frac{g_{\text{bar}}}{e^{\sqrt{g_{\text{bar}}/g^\dagger}} - 1}, \tag{1}$$

where g_{bar} is the acceleration produced by the baryons in the galaxy, and $g^\dagger = 1.2 \times 10^{-10}$ ms^{-2} is a characteristic acceleration obtained from fitting the data. Apart from the characteristic acceleration $g^\dagger$, there exists a maximal acceleration $g_{h,\text{max}}$ that can be obtained from Eq. (1) or from any other MOND function [18]. Given that Eq. (1) describes various galaxies, it follows that any halo will have a maximum acceleration. A straightforward calculation shows that the maximal acceleration provided by any DM halo must be $g_{h,\text{max}} = 0.65g^\dagger = 7.8 \times 10^{-11}$ ms^{-2} at $g_{\text{bar}} = 2.5g^\dagger = 3 \times 10^{-10}$ ms^{-2}. The existence of this universal maximal acceleration (UMA) can provide constraints on the surface density of some of the most common DM profiles in the literature and, in particular, on the properties of the scalar field (wave) dark matter (SFDM) model [19–27].[1]

For purposes of generality, let us assume that the DM density profile is spherically symmetric and given in the form $\rho(r) = \rho_s f(r/r_s)$, where ρ_s and r_s are the characteristic density and scale radius of the profile, respectively, and $f(r/r_s)$ is any given function of its argument. Notice that in the case of profiles with a core, we expect that $f(0) = 1$ and, therefore, ρ_s is the central density. The magnitude of the (radial) gravitational acceleration produced by the DM halo at a radius r can be calculated from $g_h(r) = GM(r)/r^2$, where G is Newton's constant, and $M(r)$ is the enclosed mass inside a sphere of radius r. Then, given the general form of the density profile $\rho(r)$, the gravitational acceleration can be written as $g_h(r) = G\mu_{\text{DM}}\hat{g}_h(\hat{r})$, where $\hat{g}_h(\hat{r}) = (4\pi/\hat{r}^2) \int_0^{\hat{r}} f(x)x^2 dx$ is a dimensionless quantity, and the radial coordinate has been normalized to the characteristic radius as $\hat{r} = r/r_s$.

We can see then that the gravitational acceleration at any given radius is proportional to the DM central surface density, which we define simply as $\mu_{\text{DM}} = \rho_s r_s$. Furthermore, for any density profile, the derived maximal acceleration is given by

$$\frac{g_{h,\text{max}}}{10^{-11}\ \text{ms}^{-2}} = 0.014 \left(\frac{\mu_{\text{DM}}}{M_\odot\ \text{pc}^{-2}}\right) \hat{g}_{h,\text{max}}. \tag{2}$$

The value of the dimensionless maximal acceleration $\hat{g}_{h,\text{max}}$ can be readily calculated for any density profile $f(\hat{r})$, and then Eq. (2) directly becomes a constraint equation for the DM surface density μ_{DM}.

III. RESULTS AND DISCUSSION

We have selected DM density profiles of common use in the literature and derived the expected value of their central DM surface density μ_{DM} by imposing that each profile satisfies the UMA, Eq. (2), at their corresponding point of maximal acceleration $\hat{g}_{h,\text{max}}$. The left-hand side of Eq. (2) is the result of a mean behavior of various galaxies, and then the derived values of μ_{DM} in Table I will represent the expected overall behavior that the best-fit parameters of individual galaxies should follow. Our predicted values are in agreement with those reported in previous works for the Burkert [9,34–37], multistate SFDM [38,39], pseudoisothermal (PI) [37], and that of Spano et al. [40] profiles. It can be seen that the standard CDM profile, also known as the Navarro-Frenk-White (NFW) profile [2], shows an acceleration that converges to its maximum at the center. That is, the maximum DM acceleration of the NFW profile is predicted to happen at the center of each galaxy, which is also noticed in CDM simulations [12]. In contrast, we find that for all core profiles, the accelerations will reach their maximal value near their scale radius r_s and then drop to zero for smaller radii.

Although the UMA obtained from the MDAR is a universal quantity independent of the DM profile, this is not the case for the surface density μ_{DM} or the dimensionless maximal value $\hat{g}_{h,\text{max}}$, both dependent on the chosen DM profile. Nonetheless, once $\hat{g}_{h,\text{max}}$ is calculated for a given density profile, its associated value of μ_{DM} will be fixed for all halos modeled using the same profile, which then implies the correlation of the two free parameters in the density profile ρ_s and r_s; the latter are allowed to vary from galaxy to galaxy as long as their product $\rho_s r_s$ remains a constant. If the UMA in the DM is valid independent of the baryonic matter content in a galaxy, it implies that all DM profiles in Table I have only one free parameter to fit the rotation curve of any individual galaxy.[2]

Empirical core profiles have parameters that are not necessarily tied to fundamental properties of the DM; however, the profile parameters in models that are theoretically motivated can be related to intrinsic quantities of the model under study. One particularly interesting case that falls in the latter category is that of ultralight SFDM, which assumes that the DM particle is a scalar field of very small mass whose quantum properties appear at galactic scales [19,20,22,25–27]. Although the relativistic theory may be complicated [44,45], the properties of the halo density

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[1]A similar approach has been pursued in Refs. [28–34], and their results are in agreement with ours once the appropriate conversions between physical quantities are taken into account.

[2]The MDAR in Eq. (1) and any other MOND-like relation gives support to the idea that $\mu_{\text{DM}} = $ const and, in turn, for the correlation of the structural parameters $\rho_s \propto r_s^{-1}$. The latter topics have been discussed widely in the literature [34–36], but there is not yet a consensus on them; see, for instance, [32,42,43]. Under this perspective, the results in Table I should be considered as indicative of the correlation between ρ_s and r_s as long as the MDAR Eq. (1) is consistent with galaxy observations.

TABLE I.　Row 1: DM models and their characteristic quantities. Row 2: Dimensionless density profiles. Row 3: Value of the dimensionless radius $\hat{r}_{\max} = r_{\max}/r_s$ at which the DM acceleration is maximal. Row 4: Maximal value of the (dimensionless) radial acceleration $\hat{g}_{\max}$. Row 5: The surface density $\mu_{DM} = \rho_s r_s$ (in units of $M_\odot$ pc^{-2}) obtained from the constraint (2).

	Burkert [35]	Multistate SFDM [38]	PI [41]	Spano *et al.* [40]	WaveDM [23]	NFW [2]
$f(r/r_s)$	$[(1 + r/r_s)(1 + r^2/r_s^2)]^{-1}$	$\sin^2(r/r_s)/(r/r_s)^2$	$(1 + r^2/r_s^2)^{-1}$	$(1 + r^2/r_s^2)^{-3/2}$	$(1 + r^2/r_s^2)^{-8}$	$[(r/r_s)(1 + r/r_s)^2]^{-1}$
$\hat{r}_{\max}$	0.96	1.57	1.51	1.03	0.36	0
$\hat{g}_{\max}$	1.59	4.02	2.89	2.19	0.86	2π
μ_{DM}	350	138	193	254	648	88

profile are dictated by those of the so-called Schrodinger-Poisson (SP) system of equations; see [46,47] and references therein. The soliton profile actually corresponds to the ground state solution of the SP system, and we refer to it as the WaveDM profile to distinguish it from other more general solutions of the SFDM model, e.g., considering multiple states of the field [38] whose analytical profile is also included in Table I (multistate SFDM).

In empirical profiles, the two parameters ρ_s and r_s are treated as independent, and they are not linked to any particular DM property. For the WaveDM profile, however, the parameters ρ_s and r_s are predicted to have the following scaling property: $\rho_s = \lambda^4 m_a^2 m_{Pl}^2/4\pi$ and $r_s = (0.23\lambda m_a)^{-1}$, where m_{Pl} is the Planck mass, m_a is the mass of the boson particle, and λ is a scaling parameter [46,51]. By the elimination of the scaling parameter λ, we then find the following expression for the surface density μ_{DM} in terms of the mass m_a and the soliton radius r_s,

$$\left(\frac{r_s}{\text{pc}}\right)^{-3}\left(\frac{m_a}{10^{-23}\text{ eV}}\right)^{-2} = 4.1 \times 10^{-15}\left(\frac{\mu_{DM}}{M_\odot\text{ pc}^{-2}}\right). \quad (3)$$

The existence of a universal value of the surface density, namely, $\mu_{DM} = 648 M_\odot$ pc^{-2} (see Table I), implies, similar to other core profiles, a close correlation between m_a and r_s.[3] Moreover, within the WaveDM paradigm, the boson mass m_a is a fundamental physical parameter, and, as such, it cannot vary from galaxy to galaxy. If we now consider the universality of the surface density implied by the MDAR, Eq. (3) must also fix the value of the scale radius r_s. In contrast to other empirical profiles, we then find that the MDAR in Eq. (1) ultimately implies the existence of a universal soliton (core) profile within the WaveDM paradigm.

Notably, if we neglect the assumption that the boson mass m_a is fundamental and treat it as a free parameter, then the best-fit parameters modeling the rotation curves of individual galaxies are expected to satisfy Eq. (3).

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[3]Notice that the universality of μ_{DM} in Eq. (3) implies for the WaveDM profile the correlation $m_a \propto r_s^{-3/2}$, which is also reported in Fig. 2 of Ref. [48], although it is erroneously attributed there to a constant core density ρ_s. See, also, Fig. 7 in Ref. [49].

However, this fitting procedure will inevitably lead to a large dispersion in the mass, which is simply a consequence of the diversity in galaxy sizes. Thus, as long as m_a is treated as a free fitting parameter to describe a diverse sample of galaxies, we cannot derive a meaningful constraint of its value. The left panel in Fig. 1 illustrates this point, where we show the values of individual galaxy fits obtained in previous works [48–50] in which m_a and r_s were treated as independent fitting parameters. In general, the fits lie closely along the line of constant surface density indicated by Eq. (3), which is the expected behavior from the universality of the MDAR. The large scatter in m_a is the reflection that the fitting method in [48–50] cannot provide a reliable determination of the boson mass m_a and that the most they can do is to test the reliability of the constraint (3).

Nonetheless, as we shall show, Eq. (3), along with the assumptions that m_a is constant and that all halos are described by the WaveDM profile, are enough to derive a simple estimate of the boson mass, and we only require an independent estimate of r_s. We will use data from dwarf spheroidal (DS) galaxies for our estimate. Being DM dominated systems, we expect the properties of DSs to be similar, albeit with some possible scatter associated to their formation histories [54].

In the right panel of Fig. 1, we show the (theoretical) gravity profile $\hat{g}(r/r_s)$ of the WaveDM model, together with the acceleration values reported in Ref. [52] for the MW DSs. The data correspond to the enclosed mass $M_{1/2}$ within the half-light radius $r_{1/2}$, which were then converted to a gravitational acceleration at the same radius through the equation $g(r_{1/2}) = GM_{1/2}/r_{1/2}^2$. The latter was then normalized as $g/G\mu_{DM}$ for a proper comparison with $\hat{g}$ using the value of $\mu_{DM} = 648 M_\odot$ pc^{-2} found from the MDAR for the WaveDM profile. Hence, the only free parameter to adjust is r_s, which we use to normalize the half-light radius $r_{1/2}$ of each of the classical MW DSs. We chose three different values, namely, $r_s/\text{kpc} = 0.3, 2, 10$, which then correspond to three relative positions of the data points with respect to the theoretical curve. It can be seen that the best option is $r_s = 0.3$ kpc (red error boxes in Fig. 1), which puts the data points on the right-hand side of the point of maximal acceleration, where they even seem to follow the downward trend of the theoretical curve at large radii.
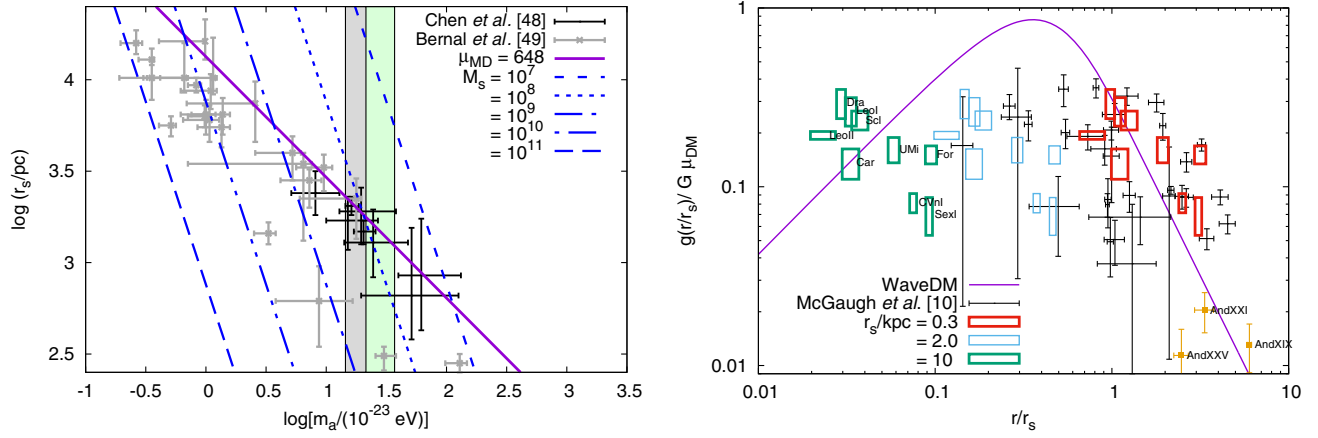
FIG. 1.   (Left) The correlation of the parameters m_a and r_s of the WaveDM profile predicted from the MDAR via Eq. (3) (magenta line). The data points correspond to the *individual* fits of the WaveDM profile to dwarf galaxies as reported in [48] and in [49] in which both m_a and r_s were treated as independent. Notice that the resultant points follow the trend of the correlation dictated by the constraint (3) with $\mu_{DM} = 648 M_\odot$ pc^{-2}. Performing the fits only to the classical Milky Way (MW) dwarf spheroidal (dSph), as done in [48,50], results in an apparent narrow range of the boson mass due to the selection of the sample. In Ref. [48], the estimated boson mass was $m_a = 1.79 \times 10^{-22}$ eV/c^2 (grey shaded region), whereas in Ref. [50], the obtained value was $m_a = 2.4 \times 10^{-22}$ eV/c^2 (green shaded region). From Eq. (3), the scale radius for the aforementioned two values of the boson mass are $r_s = 2.1$ kpc and $r_s = 1.6$ kpc, respectively. The blue lines represent the values of constant soliton mass M_s enclosed within the radius r_s.[4] If the boson mass m_a is allowed to freely vary, the sample of galaxies suggests a variation in m_a (M_s) by 2 (4) orders of magnitude. (Right) Normalized gravitational acceleration $\hat{g} = g/(G\mu_{DM})$ inferred from the classical MW DSs according to the determination of the mass inside the half-light radius $r_{1/2}$ reported in [52] (represented by the error boxes) compared to the expected theoretical curve of WaveDM. Three values of r_s were chosen for the normalization of the half-light radii of each galaxy. Notice that the MW data seem to prefer small values of r_s (red error boxes), for which the data seem to follow the downward trend of the curve, whereas the values of $r_s \geq 2$ kpc (blue and green error boxes) look inappropriate. Also plotted are all the MW and Andromeda satellites with high-quality data from [11] using the same scale radius $r_s = 0.3$ kpc. In particular, AndXIX, AndXXI, and AndXXV, previously considered outliers and regarded as low mass for their size in Ref. [53], seem to agree well with the downward trend of the theoretical curve.

Using the same value of $r_s = 0.3$ kpc, we scale a bigger sample of satellite galaxies [11,53]. In Fig. 1, we include only the dwarf galaxies with high-quality observations selected in [11]. Surprisingly, we find that the data follow the theoretical curve reasonably well after the point of maximal acceleration, the largest scatter coming from the galaxies with the large observational uncertainties. Even more, we see that satellites AndXIX, AndXXI, and AndXXV, which were labeled as outliers in the study of Ref. [53], mostly because of their low mass as compared to their size, seem to vindicate the trend of the theoretical curve of the WaveDM profile at the lowest values of the gravitational acceleration.

For our estimated $r_s = 0.3$ kpc, the constraint equation (3) implies the boson mass $m_a = 1.2 \times 10^{-21}$ eV, and the soliton mass $M_s = 1.8 \times 10^7 M_\odot$, which is consistent with the uniformity of the mass estimates within 300 pc made in Refs. [55,56]. Notice that the boson mass is somehow unexpected, as it is much larger than the values commonly reported in the literature for dwarf galaxies [23,48,50,51].

However, this new and larger value of the boson mass is constant for all halos, as demanded by the hypothesis of the SFDM model and avoids the stringent constraints coming from cosmological observations [27,50,57–61].

Because the MDAR implies a single value of the scale radius r_s in the WaveDM profile, there should be a universal soliton profile present in all galaxy halos, but due to the diversity of galaxy sizes, this implies the WaveDM profile alone will prove unable to describe all DM halos, in particular, large ones [39]. This does not rule out the SFDM model, but it does rule out the possibility that the soliton profile represents all the DM halos; hence, a more general profile than the WaveDM profile is required that extends to larger radii.

One natural approach that has been proposed is to account for the superposition of excited states in the SFDM halo, which mostly affect the outer parts of the halo leaving the characteristic imprint of *wiggles or oscillations* in the density and rotation curve profiles (see the multistate profile in Table I) [38,39,62]. A more *ad hoc* approach deals with smoothly matching the soliton to a NFW profile describing the outer part of the halo which adds a second parameter to the fitting density profile [49–51]; this is motivated by the results of numerical simulations of the SP system [23,63–67].

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[4]The soliton mass is calculated from the formula $M_s = 7.7 \times 10^{13} M_\odot (m_a/10^{-23} \text{ eV})^{-2}(r_s/\text{pc})^{-1}$; see, for instance, [46,48,51].

We have performed numerical calculations on this more general profile (following the prescription in Ref. [49]) that models the full SFDM halo and applied to it the MDAR constraint Eq. (2). The resultant SFDM surface density is found to lie in the range $\mu_{\mathrm{DM}} = (575\text{–}648) M_\odot \ \mathrm{pc}^{-2}$; these values are of the same order of magnitude as in the case of the soliton profile alone, and they would similarly imply that $r_s \sim 0.3$ kpc. We have found that considering this general profile leaves the main results about the unique soliton and the estimated boson mass roughly unchanged and opens the possibility of fitting a diverse sample of galaxies to their outermost radii with a single boson mass. At the same time, it also strengthens the possibility of a universal soliton profile with a total mass of $10^7 \ M_\odot$ in the center of all galaxy halos [55,56,68].

IV. CONCLUSIONS

Summarizing, we have shown that the MDAR implies, in general, a universal value of the surface density for any given DM density profile, which translates into a strong correlation between their central density ρ_s and scale radius r_s. We have explored the consequences of such correlation in the case of the SFDM model, which leads to the conclusion that all galaxy halos should have a similar central core structure formed due to the Heisenberg uncertainty principle. Comparison with the data of satellite galaxies of the MW and Andromeda suggests a boson mass in the SFDM model of 10^{-21} eV$/c^2$, which is in agreement with current cosmological and astrophysical constraints, while still having a distinguishable history of structure formation and halo density distribution from the standard CDM model [57,58,65].

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