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Interference phenomena in the decay $D_s^+ \to \eta\pi^0\pi^+$ induced by the $a_0^0(980) - f_0(980)$ mixing

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Using the data on the decay $D_s^+ \to f_0(980)\pi^+ \to K^+K^-\pi^+$, we estimate the amplitude of the process $D_s^+ \to [f_0(980) \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980)]\pi^+ \to \eta\pi^0\pi^+$, caused by the mixing of $a_0^0(980)$ and $f_0(980)$ resonances that breaks the isotopic invariance due to the K^+ and K^0 meson mass difference. Effects of the interference of this amplitude with the amplitudes of the main mechanisms responsible for the decay $D_s^+ \to \eta\pi^0\pi^+$ are analyzed. As such mechanisms, we examine the transition $D_s^+ \to \eta\rho^+ \to \eta\pi^0\pi^+$, which is observed in experiment, and the possible transition $D_s^+ \to (a_0^0(980)\pi^+ + a_0^+(980)\pi^0) \to \eta\pi^0\pi^+$. It is shown that the rapidly varying phase of the $a_0^0(980) - f_0(980)$ transition amplitude strongly influences the interference curves.

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I. INTRODUCTION

A threshold phenomenon known as the mixing of $a_0^0(980)$ and $f_0(980)$ resonances appreciably breaks the isotopic invariance since the effect is proportional to $\sqrt{2(M_{K^0} - M_{K^+})/M_{K^0}} \approx 0.13$ in the modulus of the amplitude [1]; see also Ref. [2]. This effect appears as the narrow (with the width of about $2(M_{K^0} - M_{K^+}) \approx$ 8 MeV) resonant peak between the K^+K^- and $K^0\bar{K}^0$ thresholds owing to the transition $a_0^0(980) \to K\bar{K} \to f_0(980)$ or vice versa $f_0(980) \to K\bar{K} \to a_0^0(980)$. There are many proposals in the literature concerning both the searching of the $a_0^0(980) - f_0(980)$ mixing and estimating the effects related with this phenomenon; the detailed list of references may be found, for example, in Ref. [3].

Recently, this phenomenon has been discovered experimentally and studied with the help of detectors VES in Protvino in π^-N collisions [4,5] and BESIII in Beijing in J/ψ decays [6–8]. As a result, it has become clear [3,9,10] that the similar isospin breaking effect can appear not only due to the $a_0^0(980) - f_0(980)$ mixing, but also due to any mechanism of the production of the $K\bar{K}$ pairs with the definite isospin in the S wave, $X \to K\bar{K} \to f_0(980)/a_0^0(980)$ [11]. Thus, a new tool emerged to study the production mechanism and nature of light scalars.

In the present work, we discuss, for the first time, the possibility of the $a_0^0(980) - f_0(980)$ mixing detection in three-body hadronic decay of the D_s^+ mesons into $\eta\pi^0\pi^+$. We pay attention to the fact that the manifestation of the isospin-breaking amplitude $f_0(980) \to K\bar{K} \to a_0^0(980)$ can be enhanced in this decay owing to its interference with the amplitudes of other mechanisms. The sharp and large variation of the phase of the $f_0(980) - a_0^0(980)$ transition amplitude (by about 90° in the region between K^+K^- and $K^0\bar{K}^0$ thresholds) plays an important role in the interference phenomenon. So far, this characteristic feature of the $a_0^0(980) - f_0(980)$ mixing has remained in the shadows [12–14]. By our estimates, the decay $D_s^+ \to \eta\pi^0\pi^+$ has potential for the $a_0^0(980) - f_0(980)$ mixing detection.

II. THE $a_0^0(980) - f_0(980)$ MIXING IN $D_s^+ \to \eta\pi^0\pi^+$

A. The case of two mechanisms

Figure 1 shows the *BABAR* data [15] on the S-wave mass spectrum of the K^+K^- system produced in the decay $D_s^+ \to K^+K^-\pi^+$. Its shape, as well as the shape of the S-wave $\pi^+\pi^-$ spectrum in $D_s^+ \to \pi^+\pi^-\pi^+$ [16], is

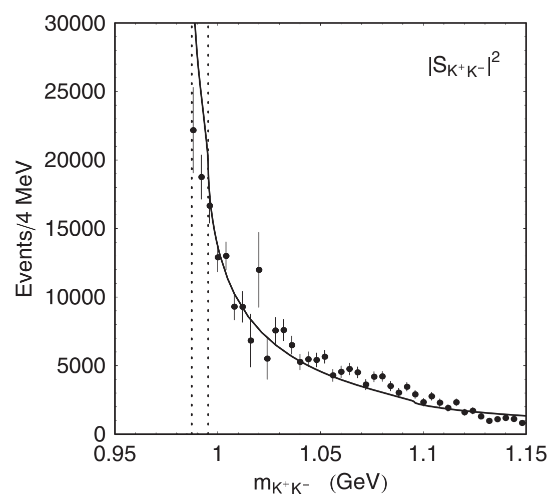

FIG. 1. The *BABAR* data [15] on the S-wave K^+K^- mass spectrum in the decay $D_s^+ \to K^+K^-\pi^+$. The data correspond to the modulus squared of the transition amplitude without the phase space factor of the K^+K^- system in $D_s^+ \to K^+K^-\pi^+$. The dotted vertical lines show the locations of the K^+K^- and $K^0\bar{K}^0$ thresholds. The solid curve corresponds to the $f_0(980)$ resonance contribution described in the text.

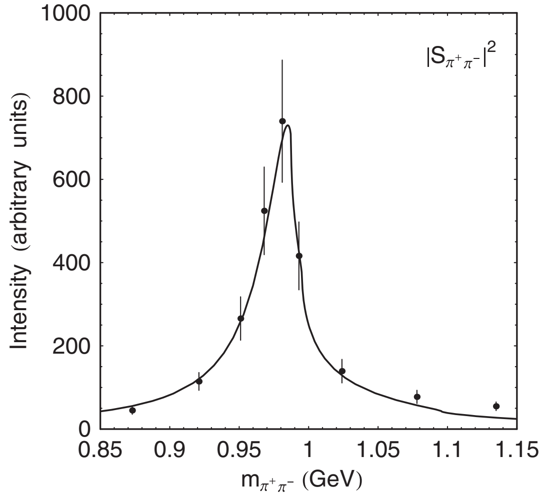

FIG. 2. The *BABAR* data [16] on the *S*-wave $\pi^+\pi^-$ mass spectrum in the decay $D_s^+ \to \pi^+\pi^-\pi^+$; see also Ref. [17]. The shape of the curve corresponds to the $f_0(980)$ resonance contribution described in the text.

approximated by the $f_0(980)$ resonance contribution (see Figs. 1 and 2 and Ref. [17]).

The solid curves in Figs. 1 and 2 are proportional to the modulus squared of the $f_0(980)$ resonance propagator, i.e., $|S_{K^+K^-}|^2 \sim 1/|D_{f_0}(m^2_{K^+K^-})|^2$, where $m_{K^+K^-}$ is the invariant mass of K^+K^- in the region above the K^+K^- threshold, and $|S_{\pi^+\pi^-}|^2 \sim 1/|D_{f_0}(m^2_{\pi^+\pi^-})|^2$, where $m_{\pi^+\pi^-}$ is the invariant mass of $\pi^+\pi^-$, respectively. Here the $f_0(980)$ propagator, $1/D_{f_0}$, was taken from Ref. [3] without any changes.

The Particle Data Group (PDG) gives [18]

$$\mathrm{BR}(D_s^+ \to f_0(980)\pi^+ \to K^+K^-\pi^+) = (1.15 \pm 0.32)\%. \tag{1}$$

This value and its accuracy require further careful study (see discussions of the assumptions made by *BABAR* [15] and CLEO [19] with the treatment of the initial data). In fact, in the original *BABAR* [15] and CLEO [19] analyses a possible presence of the $a_0^0(980)$ resonance has been neglected so that the number given in Eq. (1) effectively corresponds to a sum of the $f_0(980)$ and $a_0(980)$ contributions in the decays of the D_s^+ mesons. Therefore, we consider the results of our analysis as some guide and hope that the detection of the $a_0^0(980) - f_0(980)$ mixing may shed extra light on the mechanisms of the $f_0(980)$ and $a_0^0(980)$ production in D_s^+ decays.

Using Eq. (1), together with the values of the $f_0(980)$ and $a_0^0(980)$ resonance parameters (see Appendix), obtained in Ref. [3] by analyzing the BESIII data [6] on the intensity of the $a_0^0(980) - f_0(980)$ mixing in the decays $J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi\eta\pi$ and $\psi' \to \gamma\chi_{c1} \to \gamma a_0(980)\pi^0 \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0$, we find the following estimate for the branching ratio of the decay $D_s^+ \to \eta\pi^0\pi^+$ induced by the $a_0^0(980) - f_0(980)$ mixing:

$$\mathrm{BR}(D_s^+ \to [f_0(980) \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980)]\pi^+ \to \eta\pi^0\pi^+) = 4.1 \times 10^{-4}. \tag{2}$$

The relevant amplitude of the transition $D_s^+ \to [f_0(980) \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980)]\pi^+ \to \eta\pi^0\pi^+$ is presented just below in Eq. (6).

The available data on the decay $D_s^+ \to \eta\pi^0\pi^+$ [18,20,21] show that it proceeds predominantly via the $\eta\rho^+$ intermediate state:

$$\mathrm{BR}(D_s^+ \to \eta\rho^+ \to \eta\pi^0\pi^+) = (8.9 \pm 0.8)\%, \tag{3}$$

$$\mathrm{BR}(D_s^+ \to \eta\pi^0\pi^+) = (9.2 \pm 1.2)\%. \tag{4}$$

Let us denote the $D_s^+ \to \eta\rho^+ \to \eta\pi^0\pi^+$ and $D_s^+ \to [f_0(980) \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980)]\pi^+ \to \eta\pi^0\pi^+$ transition amplitudes as $A_{\eta\rho^+}$ and $A_{f_0a_0^0}$, respectively. For the description of their dependence on the mass variables, we use the following expressions:

$$A_{\eta\rho^+} \equiv A_{\eta\rho^+}(m^2_{\eta\pi^0}, m^2_{\eta\pi^+}, m^2_{\pi^0\pi^+}) \equiv A_{\eta\rho^+}(s,t,u) = C_{D_s^+\eta\rho^+}\frac{s-t}{D_{\rho^+}(u)}F_\rho(u)\sqrt{\frac{g^2_{\rho\pi\pi}}{16\pi}}, \tag{5}$$

$$A_{f_0a_0^0} \equiv A_{f_0a_0^0}(m^2_{\eta\pi^0}) \equiv A_{f_0a_0^0}(s) = C_{D_s^+f_0\pi^+}\frac{\Pi_{a_0^0f_0}(s)}{D_{a_0^0}(s)D_{f_0}(s) - \Pi^2_{a_0^0f_0}(s)}\sqrt{\frac{g^2_{a_0^0\eta\pi^0}}{16\pi}}, \tag{6}$$

where $s = m^2_{\eta\pi^0}$, $t = m^2_{\eta\pi^+}$, and $u = m^2_{\pi^0\pi^+}$ are the invariant masses squared of the indicated meson pairs in the decay $D_s^+ \to \eta\pi^0\pi^+$ ($\Sigma = s + t + u = m^2_{D_s} + 2m^2_\pi + m^2_\eta$, and here we neglect the π^0 and π^+ mass difference and put $m_\pi = 0.135$ GeV); $D_{\rho^+}(u)$, $D_{a_0^0}(s)$, $D_{f_0}(s)$, and $\Pi_{a_0^0f_0}(s)$ are the inverse propagators of ρ^+, $a_0^0(980)$, $f_0(980)$ resonances and the amplitude of the $a_0^0(980) \to (K^+K^- + K^0\bar{K}^0) \to f_0(980)$ transition, respectively, $F_\rho(u)$ is the centrifugal barrier penetration factor (formulas for all these quantities are presented in Appendix); $g_{\rho\pi\pi}$ and $g_{a_0^0\eta\pi^0}$ are the coupling constants (see also the Appendix), $C_{D_s^+\eta\rho^+}$ and $C_{D_s^+f_0\pi^+}$ are the invariant amplitudes of the decays $D_s^+ \to \eta\rho^+$ and $D_s^+ \to f_0(980)\pi^+$, respectively. In so doing, the effective vertices $D_s^+ \to \eta\rho^+$ and $\rho^+ \to \pi^0\pi^+$ are taken in the form

$$V_{D_s^+\eta\rho^+} = C_{D_s^+\eta\rho^+}(\epsilon^*_{\rho^+}, p_{D_s^+} + p_\eta), \tag{7}$$

$$V_{\rho^+\pi^0\pi^+} = g_{\rho\pi\pi}(\epsilon_{\rho^+}, p_{\pi^+} - p_{\pi^0}), \tag{8}$$

where ϵ_{ρ^+} is the polarization four-vector of the ρ^+ meson, $p_{D_s^+}$, p_η, p_{π^0}, and p_{π^+} are the four-momenta of the D_s^+, η, π^0, and π^+ mesons in the decay $D_s^+ \to \eta\pi^0\pi^+$. Hence the kinematical factor $s-t$ in Eq. (5) is $(p_{D_s^+}+p_\eta, p_{\pi^0}-p_{\pi^+})$. The amplitude $A_{f_0\pi}$, responsible for the decay $D_s^+\to f_0(980)\pi^+\to K^+K^-\pi^+$ [see Eq. (1)], is given by

$$A_{f_0\pi}\equiv A_{f_0\pi}(m^2_{K^+K^-})\equiv A_{f_0\pi}(s) = C_{D_s^+f_0\pi^+}\frac{1}{D_{f_0}(s)}\sqrt{\frac{g^2_{f_0K^+K^-}}{16\pi}}. \quad (9)$$

Each invariant amplitude $C_{D_s^+\eta\rho^+}$ and $C_{D_s^+f_0\pi^+}$ is represented by two real numbers, a modulus and a phase, which are independent of the mass variables, i.e., $C_{D_s^+\eta\rho^+}=a_1e^{i\varphi_1}$ and $C_{D_s^+f_0\pi^+}=a_2e^{i\varphi_2}$. Such an approximation of the amplitudes of heavy quarkonium decays with the participation of light resonances in intermediate states is commonly used in the data treatments (fits to experimental distributions in the Dalitz plots), see, for example, Refs. [15,16,19]. We use this approximation for our estimates.

Taking into account Eqs. (2) and (3), we present in Figs. 3(a) and 3(b) the $\eta\pi^0$ and $\pi^0\pi^+$ mass spectra in the decay $D_s^+\to\eta\pi^0\pi^+$ for the case of the incoherent sum of the contributions from the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ and $D_s^+\to[f_0(980)\to(K^+K^-+K^0\bar K^0)\to a_0^0(980)]\pi^+\to\eta\pi^0\pi^+$ mechanisms. The sharp peak with the width of about $2(m_{K^0}-m_{K^+})\approx 8$ MeV in Fig. 3(a) in the region of K^+K^- and $K^0\bar K^0$ thresholds arises owing to the $a_0^0(980)-f_0(980)$ mixing. Figures 3(c) and 3(d) show, as an example, the $s-u$ and $s-t$ Dalitz plots for approximately 10^4 $D_s^+\to\eta\pi^0\pi^+$ Monte Carlo events generated for the above hypothetical case of the incoherent sum of two mechanisms. As seen from Eq. (5), the $s-u$ and $s-t$ distributions for the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ decay mechanism vanish on the dashed lines $u=m^2_{D_s}+2m^2_\pi+m^2_\eta-2s$ and $t=s$ shown in Figs. 3(c) and 3(d), respectively. These lines divide the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ events into two equal parts. The events caused by the $a_0^0(980)-f_0(980)$ mixing concentrate in the vicinity of $s=m^2_{\eta\pi^0}\approx 4m^2_K$ on the $s-u$ and $s-t$ Dalitz plots. They make up about one-hundredth of a half of the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ events [see Eqs. (2) and (3)]. This is large for the isospin breaking contribution which, at the first sight, could be naturally expected to have the magnitude at the level of $(m_d-m_u)/\bar m$ (where $m_d, m_u, \bar m=(m_d+m_u)/2$ are the constituent-quark masses) or $\alpha=e^2/4\pi$ (electromagnetic constant) in the reaction amplitude and thus at the level of 10^{-4} in the amplitude squared.

Figures 3(e) and 3(f) show four variants of the $\eta\pi^0$ mass spectrum in the region of the K^+K^- and $K^0\bar K^0$ thresholds with taking into account the interference of the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ transition amplitude $A_{\eta\rho^+}$ and the amplitude $A_{f_0a_0^0}$ caused by the $a_0^0(980)-f_0(980)$ mixing,

$$\frac{dN_{\eta\pi^0}}{dm_{\eta\pi^0}}=\int_{a_-(s)}^{a_+(s)}|A_{\eta\rho^+}+A_{f_0a_0^0}|^2 2m_{\eta\pi^0}dm^2_{\pi^0\pi^+}. \quad (10)$$

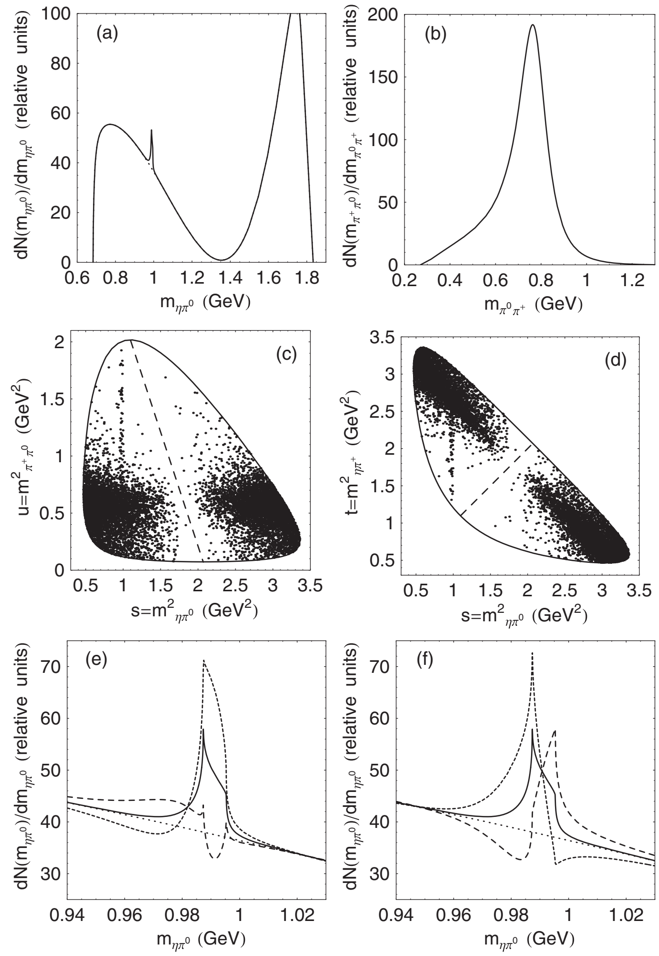

FIG. 3. The illustration of the $a_0^0(980)-f_0(980)$ mixing manifestation in the decay $D_s^+\to\eta\pi^0\pi^+$ against the mechanism $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$. The solid curves in (a) and (b) show, respectively, the $\eta\pi^0$ and $\pi^0\pi^+$ mass spectra in the decay $D_s^+\to\eta\pi^0\pi^+$ for the case of the incoherent sum of the contributions from the $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ and $D_s^+\to[f_0(980)\to(K^+K^-+K^0\bar K^0)\to a_0^0(980)]\pi^+\to\eta\pi^0\pi^+$ mechanisms. The $s-u$ and $s-t$ Monte Carlo Dalitz plot distributions for this case are shown in (c) and (d), respectively. Plots (e) and (f) show the $\eta\pi^0$ mass spectra in the region of the K^+K^- and $K^0\bar K^0$ thresholds for four variants of the interference between the amplitudes $D_s^+\to\eta\rho^+\to\eta\pi^0\pi^+$ and $D_s^+\to[f_0(980)\to(K^+K^-+K^0\bar K^0)\to a_0^0(980)]\pi^+\to\eta\pi^0\pi^+$ in comparison with the incoherent case; the curves are described in the text.

Here the integration is made over the physical region of the variable $m^2_{\pi^0\pi^+}=u$ from $a_-(s=m^2_{\eta\pi^0})$ to $a_+(s=m^2_{\eta\pi^0})$, where

$$a_\pm(s)=\frac{1}{2}\left(\Sigma-s-\frac{(m^2_{D_s}-m^2_\pi)(m^2_\eta-m^2_\pi)}{s}\right) \pm\frac{2m_{D_s}}{\sqrt{s}}p(s)q(s), \quad (11)$$

$$p(s)=\sqrt{m_{D_s}^4-2m_{D_s}^2(s+m_\pi^2)+(s-m_\pi^2)^2}/(2m_{D_s}), \quad (12)$$

$$q(s)=\sqrt{s^2-2s(m_\eta^2+m_\pi^2)+(m_\eta^2-m_\pi^2)^2}/(2\sqrt{s}). \quad (13)$$

Using the data from Eqs. (1) and (3), we find $C_{D_s^+f_0\pi^+}/C_{D_s^+\eta\rho^+}=(a_2/a_1)\xi\approx(4.5\text{ GeV})\xi$, where $\xi=e^{i\varphi_{21}}$ and $\varphi_{21}=\varphi_2-\varphi_1$ is the relative phase of the amplitudes $C_{D_s^+f_0\pi^+}$ and $C_{D_s^+\eta\rho^+}$. This phase is unknown, and to illustrate the possible interference patterns we put $\varphi_{21}=0°, \pm 90°$, and $180°$ (respectively, $\xi=1, \pm i$, and -1). The short and long dashed curves in Fig. 3(e) show the $\eta\pi^0$ mass spectra for $\xi=1$ and $\xi=-1$, respectively. The dotted curve in this figure shows the contribution from the amplitude $A_{\eta\rho^+}$ only, and the solid curve corresponds to the above case of the incoherent sum of two mechanisms. The solid and dotted curves in Fig. 3(f) show the same as in Fig. 3(e), and the short and long dash curves illustrate the interference patterns corresponding to $\xi=i$ and $\xi=-i$, respectively.

Note that the interference of $A_{f_0a_0^0}$ with the other contributions will be practically always essential (see Figs. 3(e) and 3(f)) in consequence of the sharp change of the phase of the $a_0^0(980)-f_0(980)$ transition amplitude $\Pi_{a_0^0f_0}(s)$ by about 90° in the region between K^+K^- and $K^0\bar{K}^0$ thresholds [3,13,14], where the modulus of $\Pi_{a_0^0f_0}(s)$ is maximal and approximately constant (see Appendix for details).

B. The case of three mechanisms

In principle, the decay $D_s^+\to\eta\pi^0\pi^+$ can proceed not only via the $\eta\rho^+$ intermediate state but also via the $(a_0(980)\pi)^+$ production, $D_s^+\to[a_0^+(980)\pi^0+a_0^0(980)\pi^+]\to\eta\pi^0\pi^+$. However, such a transition should be expected to be small. Based on the data quoted in Eqs. (3) and (4), we put $\text{BR}(D_s^+\to(a_0(980)\pi)^+\to\eta\pi^0\pi^+)\approx 1\%$ as a very rough upper estimate. Note that by our estimate the relevant upper limit for $\text{BR}(D_s^+\to a_0^0(980)\pi^+\to K^+K^-\pi^+)$ is $\approx 0.1\%$. This estimate consists with the initial dominance of the $f_0(980)$ resonance in the decay $D_s^+\to f_0(980)\pi^+\to K^+K^-\pi^+$ (see Eq. (1) and the discussion after it, and also Ref. [22]).

Thus, we have three interfering mechanisms of the decay $D_s^+\to\eta\pi^0\pi^+$. The corresponding decay amplitude is

$$A_{D_s^+\to\eta\pi^0\pi^+}=A_{\eta\rho^+}+A_{f_0a_0^0}+A_{a_0\pi}, \quad (14)$$

where the amplitude $A_{a_0\pi}$ describes the transition $D_s^+\to(a_0(980)\pi)^+\to\eta\pi^0\pi^+$. Like $A_{\eta\rho^+}$ [see Eq. (5)], the amplitude $A_{a_0\pi}$ has to be antisymmetric with respect to permutation of the s and t variables [23]. Taking this into account, we approximate the amplitude $A_{a_0\pi}$ by the following expression,

$$A_{a_0\pi}\equiv A_{a_0\pi}(m_{\eta\pi^0}^2,m_{\eta\pi^+}^2)\equiv A_{a_0\pi}(s,t)$$
$$=C_{D_s^+a_0^0\pi^+}\left[\frac{1}{D_{a_0^0}(s)}-\frac{1}{D_{a_0^+}(t)}\right]\sqrt{\frac{g_{a_0\eta\pi^0}^2}{16\pi}}, \quad (15)$$

where the production amplitude $C_{D_s^+a_0^0\pi^+}=a_3e^{i\varphi_3}$ is assumed to be the s- and t-independent complex constant. Note that any coherent sum of the amplitudes $A_{\eta\rho^+}$ and $A_{a_0\pi}$ gives the symmetric distribution of the $\eta\pi^0\pi^+$ events in the $s-t$ Dalitz plot relative to the $t=s$ line. The isospin-breaking amplitude $A_{f_0a_0^0}=A_{f_0a_0^0}(s)$ caused by the $a_0^0(980)-f_0(980)$ mixing depends exclusively on s and therefore is responsible for the asymmetry of the distribution of the $\eta\pi^0\pi^+$ events in the $s-t$ Dalitz plot (relative to the $t=s$ line).

By our estimate $C_{D_s^+a_0^0\pi^+}/C_{D_s^+\eta\rho^+}=(a_3/a_1)\xi'\approx(1.65\text{ GeV})\xi'$, where $\xi'=e^{i\varphi_{31}}$ and $\varphi_{31}=\varphi_3-\varphi_1$ is an unknown relative phase of the amplitudes $C_{D_s^+a_0^0\pi^+}$ and $C_{D_s^+\eta\rho^+}$. We examined 16 variants of the interference patterns corresponding to different combinations of the relative phase values $\varphi_{21}=0°, \pm 90°, 180°$ and $\varphi_{31}=0°, \pm 90°, 180°$ or parameters $\xi=1, \pm i, -1$ and $\xi'=1, \pm i, -1$. To illustrate possible manifestations of the $a_0^0(980)-f_0(980)$ mixing effect, we chose 4 of them with $(\xi,\xi')=(i,-1)$, $(-1,1)$,

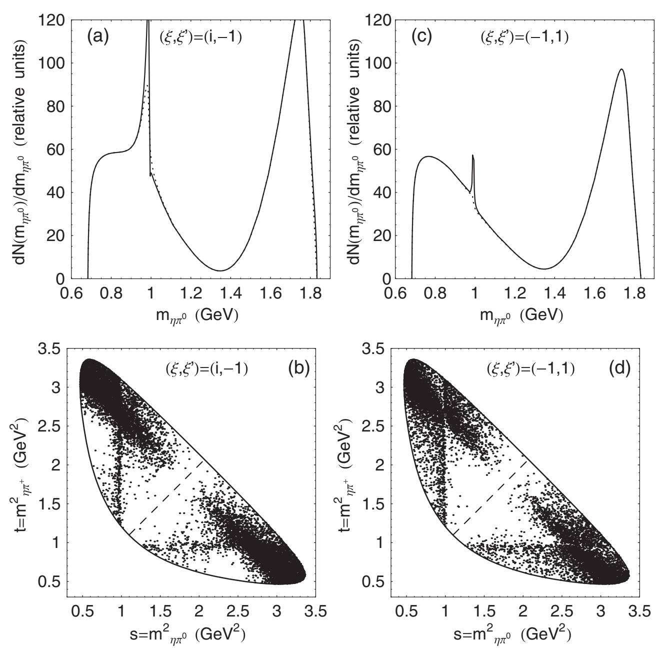

FIG. 4. The illustration of possible manifestations of the $a_0^0(980)-f_0(980)$ mixing effect in the decay $D_s^+\to\eta\pi^0\pi^+$ for the case of three interfering mechanisms. The solid curves in (a) and (c) show the mass spectra $dN_{\eta\pi^0}/dm_{\eta\pi^0}$ calculated with the use of Eqs. (14) and (16) for two sets of the relative phases indicated in the plots. The corresponding $s-t$ Monte Carlo Dalitz plot distributions ($\sim|A_{D_s^+\to\eta\pi^0\pi^+}|^2$) are shown in (b) and (d). The mass spectra without the contribution of the amplitude $A_{f_0a_0^0}$ are shown in (a) and (c) by the dotted curves (see Fig. 5 for details).

$(1,i)$, and $(1,-1)$. The solid curves in Figs. 4(a) and 4(c) show the $\eta\pi^0$ mass spectra,

$$\frac{dN_{\eta\pi^0}}{dm_{\eta\pi^0}} = \int_{a_-(s)}^{a_+(s)} |A_{D_s^+\to\eta\pi^0\pi^+}|^2 2m_{\eta\pi^0} dm^2_{\pi^0\pi^+}, \quad (16)$$

calculated with the use of Eqs. (5), (6), (11)–(15). The corresponding distributions of the Monte Carlo events ($\sim|A_{D_s^+\to\eta\pi^0\pi^+}|^2$) in the $s-t$ Dalitz plots are shown in Figs. 4(b) and 4(d). The variant represented in Figs. 4(a) and 4(b) corresponds to combination $(\xi,\xi')=(i,-1)$ for which the influence of the $a_0^0(980)-f_0(980)$ mixing seems most appreciable. The variant represented in Figs. 4(c) and 4(d) corresponds to combination $(\xi,\xi')=(-1,1)$. In this case, the $\eta\pi^0$ mass spectrum demonstrates a small narrow peak located on the smooth background in the region of the $K\bar{K}$ thresholds [see Fig. 4(c)]. Nevertheless, the asymmetry effect is clearly visible in the Dalitz plot [see Fig. 4(d)] (though it almost collapses in the $\eta\pi^0$ projection). The mass spectra $dN_{\eta\pi^0}/dm_{\eta\pi^0}$ in the $a_0^0(980)$ resonance region are presented in more detail in Fig 5 for variants with $(\xi,\xi')=(i,-1)$, $(-1,1)$, $(1,i)$, and $(1,-1)$. The dotted curves in Figs. 4(a), 4(c), and 5 correspond to the mass spectra $dN_{\eta\pi^0}/dm_{\eta\pi^0}$ without the contribution of the amplitude $A_{f_0a_0^0}$. Note that the asymmetry in the $s-t$ Dalitz plot distributions relative to the $t=s$ line (see Fig. 4) manifests itself in all considered 16 variants.

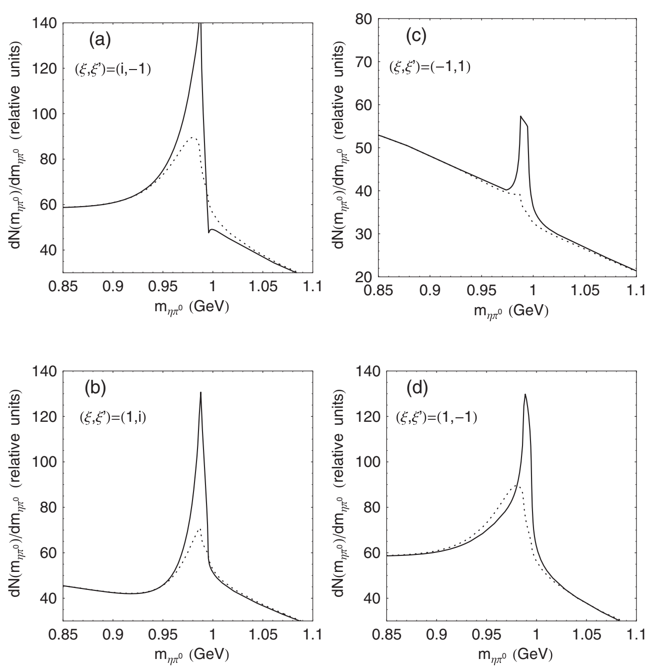

FIG. 5. The solid curves in plots (a), (c) and (b), (d) show the $\eta\pi^0$ mass spectra in the $a_0^0(980)$ resonance region corresponding to the interference variants with $(\xi,\xi')=(i,-1)$, $(-1,1)$ [see Figs. 4(a) and 4(c)] and $(1,i)$, $(1,-1)$, respectively. The dotted curves correspond the mass spectra without the contribution of the amplitude $A_{f_0a_0^0}$.

Detecting signs of the $D_s^+\to[a_0^0(980)\pi^+ + a_0^+(980)\pi^0]\to\eta\pi^0\pi^+$ decay mechanisms is one of the interesting problems both for the weak hadronic decay physics of the D_s^+ meson and for the physics of the light scalar $a_0(980)$ and $f_0(980)$ mesons. At present, intensive investigations in these lines are realized by the LHCb, *BABAR*, CLEO, Belle, and BESIII Collaborations (see, for example, recent reviews [18,24–26]).

III. CONCLUSION AND DISCUSSION

Light meson spectroscopy from hadronic charm meson decays (in particular, study of the $a_0^0(980)$ and $f_0(980)$ resonances) is one of the main lines of the LHCb program on charm physics [24,25]. It is hoped that the measurements of the D_s^+ meson decays with huge statistics, really reachable at LHCb, will allow us to reveal the isospin breaking effect caused by the $a_0^0(980)-f_0(980)$ mixing in the $D_s^+\to\eta\pi^0\pi^+$ channel and obtain new information on the production mechanisms and nature of the light scalar mesons.

Note that the investigations of the $a_0^0(980)-f_0(980)$ mixing in three-body decays of the D^0 meson, such as $D^0\to K_S^0\pi^+\pi^-$, $D^0\to K_S^0\eta\pi^0$, $D^0\to\bar{K}^0K^-K^+$, $D^0\to K^-K^+\pi^0$, and $D^0\to\pi^+\pi^-\pi^0$, are also promising and interesting. These decays differ appreciably from those of the D_s^+ meson. We hope to present detailed estimates for the case of the D^0 decays elsewhere in the near future.

Note also that the $a_0^0(980)-f_0(980)$ mixing in the semileptonic decays $D_s^+\to[\pi^0\eta,\pi\pi]e^+\nu$ has been discussed recently in Ref. [27].

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APPENDIX: PROPAGATORS AND $a_0^0(980)$–$f_0(980)$ MIXING AMPLITUDE

The inverse propagator of the ρ^+ meson in Eq. (5) is

$$D_{\rho^+}(u) = m_\rho^2 - u - i\sqrt{u}\Gamma_\rho(u), \quad (A1)$$

where $\Gamma_\rho(u)=(m_\rho^2/u)\Gamma_\rho[q(u)/q(m_\rho^2)]^3F_\rho^2(u)$, $F_\rho^2(u)=[1+r_\rho^2q^2(m_\rho^2)]/[1+r_\rho^2q^2(u)]$, $r_\rho=5\ \mathrm{GeV}^{-1}$, $q(u)=\sqrt{u-4m_\pi^2}/2$, $m_\rho=0.775$ GeV, $\Gamma_\rho=0.148$ GeV, $g^2_{\rho\pi\pi}/(4\pi)=2.8$ [18].

The $a_0^0(980)-f_0(980)$ mixing amplitude in Eq. (6), caused by the diagrams shown in Fig. 6, has the form

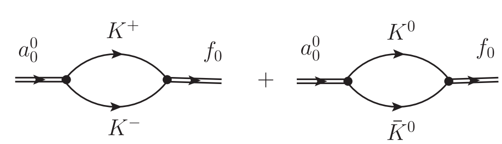

FIG. 6. The $K\bar{K}$ loop mechanism of the $a_0^0(980)-f_0(980)$ mixing.

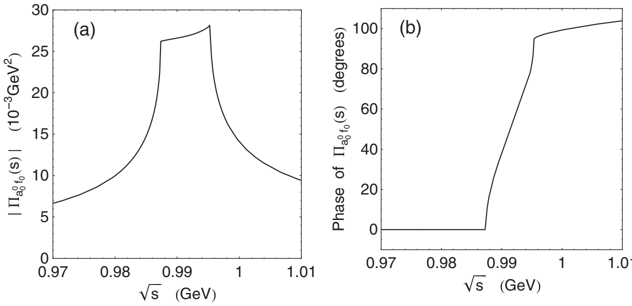

FIG. 7. (a) An example of the modulus of the $a_0^0(980)-f_0(980)$ mixing amplitude. (b) The phase of the $a_0^0(980)-f_0(980)$ mixing amplitude.

$$\Pi_{a_0^0 f_0}(s) = \frac{g_{a_0^0 K^+K^-} g_{f_0 K^+K^-}}{16\pi}\Bigg[i(\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)) - \frac{\rho_{K^+K^-}(s)}{\pi}\ln\frac{1+\rho_{K^+K^-}(s)}{1-\rho_{K^+K^-}(s)} + \frac{\rho_{K^0\bar{K}^0}(s)}{\pi}\ln\frac{1+\rho_{K^0\bar{K}^0}(s)}{1-\rho_{K^0\bar{K}^0}(s)}\Bigg], \quad \text{(A2)}$$

where s (the square of the invariant virtual mass of scalar resonances) $\geq 4m^2_{K^0}$ and $\rho_{K\bar{K}}(s) = \sqrt{1-4m_K^2/s}$; in the region $0 \leq s \leq 4m_K^2$, $\rho_{K\bar{K}}(s)$ should be replaced by $i|\rho_{K\bar{K}}(s)|$. The modulus and the phase of $\Pi_{a_0^0 f_0}(s)$ are shown in Fig. 7. Since $\Pi_{a_0^0 f_0}(s)$ is not small between the $K\bar{K}$ thresholds, all orders of the $a_0^0(980)-f_0(980)$ mixing has been taken into account in Eq. (6) for the amplitude $A_{f_0 a_0}$ [1–3].

In Eq. (6), $D_r(s)$ is the inverse propagator of the unmixed resonance r $[r = a_0^0(980), f_0(980)]$ with the mass m_r,

$$D_r(s) = m_r^2 - s + \sum_{ab}[\mathrm{Re}\Pi_r^{ab}(m_r^2) - \Pi_r^{ab}(s)], \quad \text{(A3)}$$

$ab = (\eta\pi^0, K^+K^-, K^0\bar{K}^0, \eta'\pi^0)$ for $r = a_0^0(980)$ and $ab = (\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0, \eta\eta)$ for $r = f_0(980)$; $\Pi_r^{ab}(s)$ stands for the diagonal matrix element of the polarization operator of the resonance r corresponding to the contribution of the ab intermediate state [28].

At $s > (m_a + m_b)^2$,

$$\Pi_r^{ab}(s) = \frac{g^2_{rab}}{16\pi}\Bigg[\frac{m_{ab}^{(+)} m_{ab}^{(-)}}{\pi s}\ln\frac{m_b}{m_a} + \rho_{ab}(s) \times\left(i - \frac{1}{\pi}\ln\frac{\sqrt{s-m_{ab}^{(-)2}} + \sqrt{s-m_{ab}^{(+)2}}}{\sqrt{s-m_{ab}^{(-)2}} - \sqrt{s-m_{ab}^{(+)2}}}\right)\Bigg], \quad \text{(A4)}$$

where g_{rab} is the coupling constant of r with ab, $\rho_{ab}(s) = \sqrt{s-m_{ab}^{(+)2}}\sqrt{s-m_{ab}^{(-)2}}/s$, $m_{ab}^{(\pm)} = m_a \pm m_b$, and $m_a \geq m_b$; $\mathrm{Im}\Pi_r^{ab}(s) = \sqrt{s}\Gamma_{r\to ab}(s) = (g^2_{rab}/16\pi)\rho_{ab}(s)$. At $m_{ab}^{(-)2} < s < m_{ab}^{(+)2}$

$$\Pi_r^{ab}(s) = \frac{g^2_{rab}}{16\pi}\Bigg[\frac{m_{ab}^{(+)} m_{ab}^{(-)}}{\pi s}\ln\frac{m_b}{m_a} - \rho_{ab}(s)\left(1 - \frac{2}{\pi}\arctan\frac{\sqrt{m_{ab}^{(+)2} - s}}{\sqrt{s - m_{ab}^{(-)2}}}\right)\Bigg], \quad \text{(A5)}$$

where $\rho_{ab}(s) = \sqrt{m_{ab}^{(+)2} - s}\sqrt{s - m_{ab}^{(-)2}}/s$. At $s \leq m_{ab}^{(-)2}$

$$\Pi_r^{ab}(s) = \frac{g^2_{rab}}{16\pi}\Bigg[\frac{m_{ab}^{(+)} m_{ab}^{(-)}}{\pi s}\ln\frac{m_b}{m_a} + \rho_{ab}(s)\frac{1}{\pi}\ln\frac{\sqrt{m_{ab}^{(+)2} - s} + \sqrt{m_{ab}^{(-)2} - s}}{\sqrt{m_{ab}^{(+)2} - s} - \sqrt{m_{ab}^{(-)2} - s}}\Bigg], \quad \text{(A6)}$$

where $\rho_{ab}(s) = \sqrt{m_{ab}^{(+)2} - s}\sqrt{m_{ab}^{(-)2} - s}/s$.

The propagators $1/D_{a_0^0}(s)$ and $1/D_{f_0}(s)$ constructed with taking into account the finite width corrections [see Eqs. (A3)–(A6)] satisfy the Källén-Lehman representation in the wide domain of coupling constants of the scalar mesons with two-particle states and, due to this fact, provide the normalization of the total decay probability to unity: $\sum_{ab}\mathrm{BR}(r\to ab) = 1$ [29].

Here we use the numerical estimates of the coupling constants $g^2_{f_0 ab}/(16\pi)$ and $g^2_{a_0^0 ab}/(16\pi)$ obtained in Ref. [3]

$$\frac{g^2_{f_0\pi\pi}}{16\pi} \equiv \frac{3}{2}\frac{g^2_{f_0\pi^+\pi^-}}{16\pi} = 0.098\ \text{GeV}^2, \quad \text{(A7)}$$

$$\frac{g^2_{f_0 K\bar{K}}}{16\pi} \equiv 2\frac{g^2_{f_0 K^+K^-}}{16\pi} = 0.4\ \text{GeV}^2, \quad \text{(A8)}$$

$$\frac{g^2_{a_0^0\eta\pi^0}}{16\pi} = 0.2\ \text{GeV}^2, \quad \text{(A9)}$$

$$\frac{g^2_{a_0^0 K\bar{K}}}{16\pi} \equiv 2\frac{g^2_{a_0^0 K^+K^-}}{16\pi} = 0.5\ \text{GeV}^2. \quad \text{(A10)}$$

As in Ref. [3], we fix $m_{a_0^0} = 0.985$ GeV, $m_{f_0} = 0.985$ GeV and set $g^2_{a_0^0\eta'\pi^0} = g^2_{a_0^0\eta\pi^0}$ and $g^2_{f_0\eta\eta} = g^2_{f_0K^+K^-}$ by the $q^2\bar{q}^2$ model, see, e.g., Refs. [2,30].

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[22] Nevertheless, we must note that if the $f_0(980)$ is produced as a solitary resonance without any background in the $\pi^+\pi^-$ and K^+K^- channels, then the CLEO data [21] on the branching ratio $\mathrm{BR}(D_s^+ \to \pi^+\pi^-\pi^+) = (1.11 \pm 0.04 \pm 0.04)\%$ (see also Refs. [16,18]) contradict the *BABAR* [15] and CLEO [19] assumption of the $f_0(980)$ dominance in the decay $D_s^+ \to f_0(980)\pi^+ \to K^+K^-\pi^+$. Remember that the PDG average $\mathrm{BR}(D_s^+ \to f_0(980)\pi^+ \to K^+K^-\pi^+) = (1.15 \pm 0.32)\%$ [see Eq. (1)]. The situation requires clarification. By our estimate, $\mathrm{BR}(D_s^+ \to f_0(980)\pi^+ \to \pi^+\pi^-\pi^+) \approx 3.4\%$ is in agreement with Eq. (1) and the curve in Fig. 2. Let us emphasize once again that the detection of the $a_0^0(980) - f_0(980)$ mixing can provide a unique opportunity to clarify the puzzling mechanisms of the $f_0(980)$ and $a_0^0(980)$ production and decay in the three-body hadronic decays of the D_s^+ meson. The branching ratio of the solitary $f_0(980)$ resonance decay into $\pi^+\pi^-$ is larger than into K^+K^- because of the phase space factor of the K^+K^- system that suppresses the $f_0(980) \to K^+K^-$ decay near the threshold.

[23] Note that the direct production of the $(a_0(980)\pi)^+$ system via the $D_s^+ \to W^+ \to (a_0^+(980)\pi^0 + a_0^0(980)\pi^+)$ transition (the annihilation amplitude) is impossible, i.e., the matrix element of the isovector axial vector current $\langle 0|j_\mu^A|(a_0(980)\pi)^+\rangle = 0$. The matter is that the isovector axial vector current has the negative G parity in Standard Model, whereas the G parity of the $(a^0(980)\pi)^+$ system is positive. The final $(a_0(980)\pi)^+$ system is produced in the weak D_s^+ decay in the state with isospin $I = 1$. This implies that the effective interaction Lagrangian responsible for the transition $D_s^+ \to (a_0(980)\pi)^+_{I=1}$ is proportional to $(a_0^+(980)\pi^0 - a_0^0(980)\pi^+)$. Thus, the amplitude $A_{a_0\pi}$, like $A_{\eta\rho^+}$, is antisymmetric with respect to permutation of the s and t variables.

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