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Universality of multiplicity distribution in proton-proton and electron-positron collisions

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It is argued that the multiplicity distribution in proton-proton (pp) collisions, which is often parametrized by the negative binomial distribution, may result from the multiplicity distribution measured in electron-positron (e^+e^-) collisions, once the fluctuating energy carried by two leading protons in pp is taken into account.

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I. INTRODUCTION

The charged particle multiplicity distribution is one of the most basic observables in high energy collisions. Although there is an abundance of experimental results (see e.g., [1,2]), on the theory side this problem is poorly understood.

The multiplicity distribution measured in proton-proton (pp) collisions is often parametrized by the negative binomial (NB) distribution [1–5], which is characterized by two parameters: the mean number of particles $\langle n\rangle$ and k, which measures the deviation from the Poisson distribution.[1] NB distribution works reasonably well, with certain limitations [1,6], for a broad range of energies and in total and limited phase-space rapidity bins. For completeness we add that k is a decreasing function of energy.

Interestingly, similar experimental observations were made in electron-positron (e^+e^-) collisions; see e.g., [2,7]. NB works relatively well for total and limited phase-space bins in rapidity and k decreases with energy.

There are many similarities between pp and e^+e^-, as far as the soft particle production is concerned, but there are also important differences. At the same $\sqrt{s}$, the mean number of particles and k are significantly larger in e^+e^- than in pp.[2]

As pointed out in Refs. [8–10] some differences between pp and e^+e^- can be easily understood. In pp collisions a large fraction of initial energy, given by $\sqrt{s}$, is carried away by two leading protons and is not available for particle production. This explains larger mean multiplicity in e^+e^-, where the leading proton effect is not present. This leads to the striking relation between the total (full phase-space) mean number of charged particles in pp and e^+e^- interactions [2] (see also [9,11–18]):

$$N_{\mathrm{pp}}(\sqrt{s}) = N_{\mathrm{ee}}(K\sqrt{s}) + 2; \qquad K = 0.35; \tag{1}$$

that is, the mean number of particles in pp at a given $\sqrt{s}$ is given by the mean number of particles in e^+e^- at $K\sqrt{s}$, plus two leading protons. It turns out that the coefficient of inelasticity, K, present in Eq. (1) is approximately energy independent (see Fig. 10 in Ref. [2]) and Eq. (1) works surprisingly well from 30 to 1800 GeV. It remains to be verified at the LHC energy. More recently, certain similarities between e^+e^- and ultrarelativistic heavy-ion collisions at RHIC and the LHC were reported [18,19]. See further discussion in Sec. IV.

Equation (1) is suggestive of a universal mechanism of particle production (or more precisely, a universal mechanism of hydronization) in both systems, controlled mainly by the actual energy deposited into particle creation [8]. In e^+e^- all initial energy is consumed by produced particles, whereas in pp the *effective* energy available for particle production is given by

$$E_{\mathrm{eff}}^2 = (p_1 + p_2 - q_1 - q_2)^2 \approx s(1 - x_1)(1 - x_2), \tag{2}$$

where p_i and q_i are the incoming and the leading proton momenta, respectively. x_i is a fraction of the longitudinal momentum carried by a leading proton, $x_i = q_{i,z}/p_{i,z}$, and $s = (p_1 + p_2)^2$.

We note that a universal hadronization mechanism in e^+e^- and pp collisions is strongly supported by the success of the statistical hadronization model [20–22], which provides a very good description of hadronic multiplicities with a common hadronization temperature.

In this paper we show that Eq. (1) can be naturally extended to the whole multiplicity distribution. In particular, we demonstrate that the broad multiplicity distributions measured in pp collisions naturally result from relatively narrow multiplicity distributions observed in e^+e^- interactions once the effective energy, E_{eff}, in pp is properly taken into account.

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[1] For NB $\langle n^2\rangle - \langle n\rangle^2 = \langle n\rangle[1 + \frac{\langle n\rangle}{k}]$, which goes to Poisson if $k \to \infty$ (at fixed $\langle n\rangle$).

[2] For example, at $\sqrt{s} = 200$ GeV in pp collisions $k \approx 5$ (full phase space) in comparison to $k \approx 22$ in e^+e^- at $\sqrt{s} \approx 100$ GeV, or $k \approx 16$ when extrapolated to $\sqrt{s} = 200$ GeV.

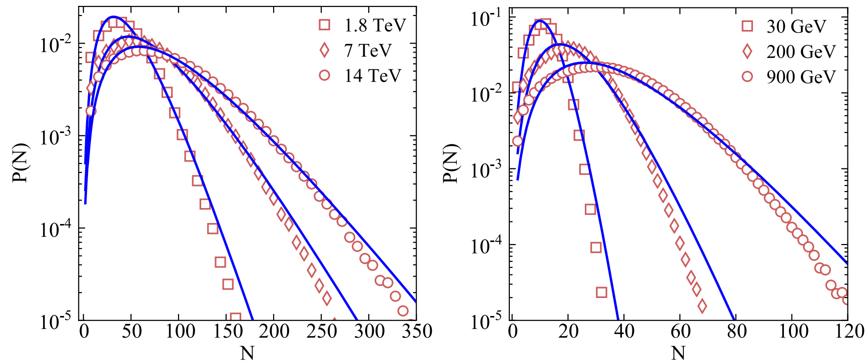

FIG. 1. Calculated full phase-space multiplicity distributions in proton-proton collisions (open points) with NB distribution fits (lines). For clarity we show every eighth (second) point in the left (right) plot.

II. LEADING PROTONS

The problem of multiplicity distribution is naturally more complicated than the mean number of particles; see, e.g., [23]. To proceed we need to specify the leading proton x distribution.[3] We choose the beta distribution

$$f(x) \propto x^{\lambda}(1-x)^{\mu}. \quad (3)$$

It is supported by rather limited experimental evidence [24,25]; however, it seems a natural first choice. We note that our discussion is of qualitative character and certain refinements concerning Eq. (3) are certainly possible. Having (3) we obtain

$$\langle x\rangle = \frac{1+\lambda}{2+\lambda+\mu}, \quad (4)$$

which is the average momentum fraction taken by a leading proton.

Next we would like to clarify how $f(x)$ is related to Eq. (1). We obtain[4]

$$N_{\rm pp}(\sqrt{s}) = \int f(x_1)f(x_2)N_{\rm ee}(E_{\rm eff})dx_1dx_2 + 2. \quad (5)$$

Taking

$$N_{\rm ee}(\sqrt{s}) = a + b\cdot s^{\alpha}, \quad (6)$$

where $\alpha \approx 0.17$ [2] (see Sec. IV for further discussion) we arrive at

$$N_{\rm pp}(\sqrt{s}) = N_{\rm ee}(\langle(1-x)^{\alpha}\rangle^{1/\alpha}\sqrt{s}) + 2. \quad (7)$$

It means that 0.35 from Eq. (1) is not related to $\langle 1-x\rangle$, as naively expected, but to $\langle(1-x)^{\alpha}\rangle^{1/\alpha}$. The latter can be calculated analytically leading to the following equation,

$$\langle(1-x)^{\alpha}\rangle = \frac{\Gamma(1+\alpha+\mu)\Gamma(2+\lambda+\mu)}{\Gamma(1+\mu)\Gamma(2+\alpha+\lambda+\mu)} = 0.35^{\alpha}, \quad (8)$$

which constrains possible parameters of the beta distribution. Assuming $\langle x\rangle = 0.4$ [25] [see Eq. (4)], we obtain $\lambda \simeq -0.8$ and $\mu = -0.7$, which fully determines $f(x)$.

III. CALCULATIONS AND RESULTS

The multiplicity distribution in pp collisions, $P_{\rm pp}(n)$, is related to the multiplicity distribution in e^+e^- interactions, $P_{\rm ee}(n)$, as

$$P_{\rm pp}(n;\sqrt{s}) = \int f(x_1)f(x_2)P_{\rm ee}(n-2;E_{\rm eff})dx_1dx_2, \quad (9)$$

where $n \geq 2$. This equation is a straightforward generalization of Eq. (5). Instead of directly calculating the integral (9) we performed our calculations as follows.

First we sampled x_1 and x_2 of two leading protons from the beta distribution, $f(x)$, with $\langle x\rangle = 0.4$ and $\lambda = -0.8$, and calculated the effective energy,[5] $E_{\rm eff}$, available for particle production in pp.[6] Our choice of $\langle x\rangle$ and λ ensures that Eq. (1) is satisfied with the right coefficient 0.35. Next we sampled the number of particles from the multiplicity distribution measured in e^+e^- collisions at $\sqrt{s} = E_{\rm eff}$. Clearly we do not know $P_{\rm ee}(n)$ for all energies and thus we assume NB with the mean given by Eq. (6), where $a = -2.65$, $b = 5.01$, $\alpha = 0.17$, and $k^{-1} = c + d\ln(\sqrt{s})$,

[3] In other words, we assume that the energy deposited into particle production fluctuates from event to event.

[4] In Ref. [10] it was found that the x's of two leading protons are uncorrelated.

[5] In our calculations we use the exact formula for $E^2_{\rm eff}$ given by $(\sqrt{s} - \sqrt{m^2+(x_1p_z)^2} - \sqrt{m^2+(x_2p_z)^2})^2 - p_z^2(x_1-x_2)^2$, where $p_z^2 = s/4 - m^2$ and m is a proton mass.

[6] We accept only these events where $E_{\rm eff} > 0.3$ GeV so that at least two pions can be produced.

TABLE I. Calculated mean number of charged particles in pp collisions (full phase space) compared with the experimental data.

$\sqrt{s}$ (GeV)	N_{pp} (model)	N_{pp} (data)
30.4	11.4	10.54 ± 0.14
200	21.1	21.4 ± 0.6
900	34.6	35.6 ± 1.1
1800	43.7	45 ± 1.5
7000	69.1	no data
14000	87.5	no data

TABLE II. k parameters [see Eq. (10)] of calculated multiplicity distributions in pp collisions compared with the experimental data.

$\sqrt{s}$ (GeV)	k (model)	k (data)
30.4	12.7	9.2 ± 0.9
200	5.8	4.8 ± 0.4
900	4.2	3.7 ± 0.3
1800	3.8	3.1 ± 0.1
7000	3.3	no data
14000	3.0	no data

where $c = -0.066$ and $d = 0.024$ [2].[7] On top of that we add two particles corresponding to the leading protons.

We performed our calculations at $\sqrt{s} = 30$, 200, 900, 1800, 7000, and 14000 GeV. The results are shown in Fig. 1, where the calculated full phase-space multiplicity distributions in pp collisions (open symbols) are compared with NB fits. We repeat that Eq. (1) is satisfied by construction so our multiplicity distributions have the correct mean values; see Table I. The crucial test of our approach is the value of k, which we calculate as

$$k = \frac{\langle N \rangle^2}{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle}, \tag{10}$$

where $\langle N \rangle = N_{\mathrm{pp}}$. In Table II we list the obtained values of k and compare them with available data. Taking into account the simplicity of our approach, the agreement is satisfactory.

IV. DISCUSSION

Several comments are in order.

(i) We do not offer any explanation of multiplicity distributions in e^+e^- collisions. Our goal was to demonstrate that the problem of multiplicity distributions in pp could be reduced to e^+e^- once the fluctuating energy carried away by two leading protons in pp collisions is taken into account. We provided new evidence in favor of the hypothesis that the number of produced particles in both systems (also possibly in heavy-ion collisions) is mostly driven by the amount of effective energy deposited into particle production, which naturally varies from event to event, and certain microscopic differences between the two systems are of lesser importance. In the literature this problem is extensively discussed in the context of the average number of particles. The fact that the similar connection holds between the widths of the full multiplicity distributions in pp and e^+e^- is new and not *a priori* expected.

(ii) In this paper we focused on the total phase-space multiplicity distributions. It is plausible that the total number of particles is determined (mostly) by the amount of available energy. This is not obvious (expected) for limited phase-space bins since the distribution of particles in transverse momentum or rapidity may be modified by some nontrivial dynamics. This problem is much more difficult to tackle and any considerations would be strongly model dependent. For example, interesting collective effects were recently discovered in pp collisions (see, e.g., [26]) and their origin is still under debate [27]. A possible parton rescattering (cascade, hydrodynamics) or other sources of correlations are not expected to significantly change the total number of produced particles.

(iii) The starting point of our analysis is the experimental observation summarized in Eq. (1). The coefficient of inelasticity, $K = 0.35$, was found [2] to be practically energy independent from $\sqrt{s} = 30$ to $\sqrt{s} = 1800$ GeV in contrast to certain dynamical models [2]; see, e.g., Refs. [13,15,28]. The value of $K = 0.35$ is often interpreted as a manifestation of the three-quark structure of the nucleon; see, e.g., Refs. [14,16–18]. In a typical (minimum-bias) pp collision roughly one constituent quark per nucleon interacts and this corresponds to an average inelasticity of $K \approx 1/3$. In heavy-ion collisions a nucleon usually undergoes more collisions and thus more quarks per nucleon are involved in particle production, leading to a higher value of K [19]. In fact, the average number of particles produced in heavy-ion collisions is quite well described in a wounded quark or quark-diquark model; see, e.g., Refs. [29–32]. In this paper we argue that an event-by-event fluctuation of K can naturally connect the multiplicity distributions measured in e^+e^- and pp collisions and it would be interesting to investigate the full multiplicity distributions in proton-nucleus (pA) and nucleus-nucleus (AA) collisions [18,33].

(iv) The main uncertainty of our approach is the leading proton x distribution given in Eq. (3). This form is partly supported by existing data, but at rather limited energies and ranges of x. Thus it should be treated as

[7]Negative k is rounded to the integer value. NB with a negative integer k becomes a binomial distribution with the number of trials $-k$ and the Bernoulli success probability $-N_{\mathrm{ee}}/k$. We also checked that the Poisson distribution for $k < 0$ leads to practically the same results.

an educated guess, which hopefully is not far from reality. In addition, we assumed that $f(x)$ is energy independent, which is not proven experimentally. A mild energy dependence is indicated by theoretical studies of [15]. The agreement between the model and the data presented in Table II suggests that the assumed leading proton x distribution might be an acceptable first approximation.

(v) To calculate the multiplicity distribution in pp collisions one needs, as an input, the multiplicity distribution in e^+e^- at all energies; see Eq. (9). In this paper we assumed that e^+e^- follows a NB distribution [with the mean given by Eq. (6) and k^{-1} discussed in Sec. III], which should be a reasonable approximation for our semiquantitative study. As seen in Fig. 1, the obtained multiplicity distributions in pp collisions are close to NB with certain deviations. For example, the NB fits overestimate calculated multiplicity distributions for higher values of N. Interestingly, similar trends are seen in experimental data; see, e.g., Fig. 6 in Ref. [2] or Figs. 3–5 in Ref. [34]. Also it is known that for higher energies NB seems to fail for both e^+e^- and pp collisions [2,7]. This is not in contradiction to our study. In fact, if the multiplicity distribution is revealing a new structure at a given energy in e^+e^-, we expect the same phenomena to appear in pp collisions but at different (higher) energies. In this paper we focus on the width of the multiplicity distribution, being the first step after the mean number of particles, and thus detailed questions regarding an exact shape of the multiplicity distributions are not fully addressed in this paper.

(vi) We extrapolated $N_{\mathrm{ee}}(\sqrt{s})$ into higher energies using the 3NLO QCD result [2,35], which is almost identical to the NLO QCD fit (see, e.g., Fig. 10 in Ref. [2]) and Eq. (6). For k^{-1} we assumed it is a linear function of $\ln(\sqrt{s})$ up to the LHC energies.

(vii) There are many sophisticated Monte Carlo models (PYTHIA [36], HIJING [37], EPOS [38] etc.) that are used to describe the multiplicity distributions in various colliding systems. However, we are not aware of any Monte Carlo model that would naturally explain Eq. (1), which, as discussed earlier, is usually interpreted in the constituent quark picture. It would be very interesting to investigate this problem in detail, in particular, to see to what extent the multiplicity distribution in pp is related to the multiplicity distribution in e^+e^- interactions.

(viii) The particle production in pp and AA collisions can be successfully described in the color glass condensate (CGC) approach; see, e.g., [39–41]. In Ref. [39] it was shown that the production of gluons from glasma color flux tubes follows the negative binomial distribution. In Ref. [40] the measured pp multiplicity distributions were described within the CGC multiparticle production framework. In Ref. [41], the authors argue that the mean number of particles can be described with an input from jet production in e^+e^- annihilation. It would be interesting to see if the full multiplicity distribution in pp can be described in a similar manner.

V. CONCLUSIONS

In conclusion, we argued that the full phase-space multiplicity distribution in pp collisions is directly related to the multiplicity distribution in e^+e^- interactions, once the leading proton effect in pp is properly accounted for. In pp a large fraction of initial energy, roughly $1/2$ on average, is carried away by two leading protons and is not available for particle production. This component fluctuates from event to event, which results in a significantly broader multiplicity distribution in pp than in e^+e^-. We provide a new argument in favor of a common mechanism of soft particle production in both systems, which is mainly driven by the amount of energy available for particle production.

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