

Tetraquarks in holographic QCD

 Thomas Gutsche,¹ Valery E. Lyubovitskij,^{1,2,3,4} and Ivan Schmidt²

¹*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

²*Departamento de Física y Centro Científico Tecnológico de Valparaíso-CCTVal, Universidad Técnica Federico Santa María, Casilla 110-V Valparaíso, Chile*

³*Department of Physics, Tomsk State University, 634050 Tomsk, Russia*

⁴*Laboratory of Particle Physics, Tomsk Polytechnic University, 634050 Tomsk, Russia*
(Received 27 June 2017; published 30 August 2017)

Using a soft-wall AdS/QCD approach we derive the Schrödinger-type equation of motion for the tetraquark wave function, which is dual to the dimension-4 AdS bulk profile. The latter coincides with the number of constituents in the leading Fock state of the tetraquark. The obtained equation of motion is solved analytically, providing predictions for both the tetraquark wave function and its mass. A low mass limit for possible tetraquark states is given by $M \geq 2\kappa = 1$ GeV, where $\kappa = 0.5$ GeV is the typical value of the scale parameter in soft-wall AdS/QCD. We confirm results of the COMPASS Collaboration recently reported on the discovery of the $a_1(1414)$ state, interpreted as a tetraquark state composed of light quarks and having $J^{PC} = 1^{++}$. Our prediction for the mass of this state, $M_{a_1} = \sqrt{2}$ GeV ≈ 1.414 GeV, is in good agreement with the COMPASS result $M_{a_1} = 1.414^{+0.015}_{-0.013}$ GeV. Next we included finite quark mass effects, which are essential for the tetraquark states involving heavy quarks.

DOI: [10.1103/PhysRevD.96.034030](https://doi.org/10.1103/PhysRevD.96.034030)

I. INTRODUCTION

This work addresses the problem of constructing hadronic wave functions of tetraquark states using the AdS and light-front QCD correspondence [1]. The idea for such a correspondence works very well for conventional hadrons—mesons and baryons [1–7]. In particular, from the matching of matrix elements for physical processes one can relate the bulk profile of the AdS field in a holographic dimension to the transverse part of the hadronic light-front wave function (LFWF) for the case of massless quarks [1]. The LFWF was generalized in Ref. [8] for the case of a two-parton state by the explicit inclusion of the constituent quark masses in the LF kinetic energy $\sum_i (\mathbf{k}_{\perp i}^2 + m_i^2)/x_i$ while the introducing of quark masses for tetraquark states was done in Ref. [9]. In the LFWF the inclusion of quark masses corresponds to the introduction of the longitudinal wave function (WF), which was done in this particular case through the so-called Brodsky-Huang-Lepage (BHL) or *Gaussian ansatz* [10,11]. In Refs. [3,5,6] we studied the problem of the longitudinal part of the LFWF following the ideas of Ref. [8]. In particular, in Ref. [5] we derived the longitudinal part of the LFWF, using constraints of chiral symmetry in the sector of light quarks, and heavy quark effective theory in the sector of heavy quarks. The idea that explicit breaking of chiral symmetry is a property of the longitudinal part of the hadronic LFWF and that it can be induced via the current quark mass dependence of the longitudinal LFWF, had been proposed before in large N_c two-dimensional QCD [12]. This mechanism was later used in the context of the two-dimensional massive

Schwinger model [13–15] and was reexamined in Refs. [16,17].

In the present paper we extend the soft-wall AdS/QCD model [18] to the description of tetraquarks—compact exotic states composed of two quarks and two antiquarks (for reviews see e.g. Ref. [19–29]). In Refs. [30–32] tetraquarks have been analyzed in the context of relativistic quark models. Discussions on tetraquark production can be found, e.g. in Refs. [33–36]. The formalism for tetraquark production consistent with quark counting rules has been developed in Refs. [33,34,36]. In Ref. [37] tetraquarks were studied in the large N_c limit and it was correctly stressed that in the context of $SU(N_c)$ color symmetry the quark q^a and antiquark \bar{q}^a fields can be replaced by $N_c - 1$ antiquark and quarks, respectively, because of similar transformations concerning the $SU(N_c)$ color group. It means that baryons and multi-quark states emerge from quark-antiquark mesons under the replacements $\bar{q}^{a_1} \rightarrow \epsilon^{a_1 \dots a_n} q^{a_2} \dots q^{a_n}$ and $q^{a_1} \rightarrow \epsilon^{a_1 \dots a_n} \bar{q}^{a_2} \dots \bar{q}^{a_n}$ as

Mesons $\rightarrow N_c$ Baryons:

$$\bar{q}^a q^a \rightarrow \epsilon^{ab_2 \dots b_n} q^a q^{b_2} \dots q^{b_n},$$

Mesons $\rightarrow 2(N_c - 1)$ Multi-quarks:

$$\bar{q}^a q^a \rightarrow \epsilon^{aa_2 \dots a_n} \epsilon^{ab_2 \dots b_n} q^{a_2} \dots q^{a_n} \bar{q}^{b_2} \dots \bar{q}^{b_n}. \quad (1)$$

In the case of QCD we have $N_c = 3$ and arrive at a picture where mesons, baryons and tetraquarks appear as fundamental color singlet states. This is consistent with arguments of superconformal symmetry where mesons, baryons

with $L = 0$ and $L = 1$, tetraquarks are classified as members of a superquadruplet [9].

First applications of AdS/QCD to the description of tetraquarks have been performed in Refs. [9,38–42]. In particular, in Refs. [38,39] the masses of scalar tetraquarks with quantum numbers $n = L = 0$ have been predicted: 0.995–2.981 GeV [38] using hard-wall AdS/QCD (here a variation of masses corresponds to a variation of the Λ_{QCD} parameter from 100 MeV to 300 MeV) and 1.960 GeV [39] using soft-wall AdS/QCD. In Ref. [40] the effective action for light scalar tetraquarks was derived. Based on this action the equation of motion for the wave functions and mass spectrum M^2 of scalar tetraquarks are derived. The spectrum results in

$$M^2 = 4\kappa^2(n + 3), \quad (2)$$

where $\kappa \sim 500$ MeV is the dilaton scale parameter in soft-wall AdS/QCD and n is the radial quantum number. From Eq. (2) it follows that the lower bound for the ground-state mass of the tetraquark is set by $2\kappa\sqrt{3}$. In Ref. [9] the tetraquark state was introduced as a partner of the supersymmetric quadruplet consisting of two baryon states with positive and negative chirality, meson state and tetraquark, which it is consistent with the ideas of Ref. [37]. In this vein, it was shown in Ref. [9] that the lowest-lying light-quark tetraquark $f_0(980)$, with isospin $I = 0$ and spin-parity $J^{PC} = 0^{++}$, is a partner of the $b_1(1235)$ with $J^{PC} = 0^{+-}$ and the nucleon, with $J^P = \frac{1}{2}^+$, while possible more heavier tetraquarks, the axial state $a_1(1260)$ with $J^{PC} = 1^{++}$, could be a partner of the $\Delta(1230)$ with $J^P = \frac{3}{2}^+$ and the $a_2(1320)$ with $J^{PC} = 2^{++}$. The mass formula for tetraquarks with quantum numbers n, L, S (radial quantum number, orbital angular momentum and internal spin) derived in Ref. [9], in the limit of massless quarks reads

$$M^2 = 4\kappa^2 \left(n + L + 1 + \frac{S}{2} \right). \quad (3)$$

The formalism for the study of tetraquarks proposed in Ref. [41] contains a basic error, because of the use of the conformal dimension for the two-partonic states. This gives misleading results for the twist-scaling for small values of the holographic coordinate of the tetraquark wave function and results in a underestimate of the tetraquark mass spectrum. E.g. the lower bound for the tetraquark mass is incorrect. In our consideration we derive the action for the tetraquarks with adjustable quantum numbers of n, J, L . Then we derive the equation of motion for the tetraquark wave function and the resulting mass spectrum M^2

$$M^2 = 4\kappa^2 \left(n + \frac{L + J}{2} + 1 \right). \quad (4)$$

Our equation of motion and solution for the tetraquark mass spectrum is different from the ones derived in Ref. [40], because of the different choice for the dimension of the AdS fields dual to tetraquarks. In our case the conformal dimension is $\Delta = \tau = N + L$, where $\tau_{L=0} = N = 4$ is the leading twist-dimension of the tetraquark Fock state (corresponding to the number of partons in the leading Fock state) and L is the maximal magnitude of the orbital angular momentum in the four-quark configuration. Such choice for the conformal dimension guarantees the correct power scaling of hadronic form factors at large values of the Euclidean transfer momentum squared Q^2 . In Ref. [40], the conformal dimension was chosen as $\Delta = 6$ for the light scalar tetraquark, which differs from our choice $\Delta = 4, 5$ at $N = 4$ and $L = 0, 1$. Moreover, our mass formula is different from the result of Ref. [9] due to a correction which takes into account the spontaneous breaking of superconformal symmetry with

$$\Delta M^2 = 2\kappa^2(J - L - S). \quad (5)$$

Next we include the finite masses of the constituents which form the tetraquark states. Note that the first inclusion of finite quark mass corrections to the tetraquark spectroscopy has been done in Ref. [9]. In our case we proceed in analogy with quark-antiquark mesons (see details in Ref. [5]), where we derived full wave functions containing transverse wave functions matched from AdS/QCD and longitudinal wave functions encoding the mass effects of tetraquark constituents. In particular, we consider two possibilities: (1) tetraquarks are bound states of two quarks and two antiquarks (color diquark-antidiquark configuration); (2) tetraquarks are bound states of two mesons (hadronic molecules).

The paper is organized as follows. In Sec. II we present our formalism: derive the master equation of motion for the full wave function of the tetraquark, providing solutions for the mass spectrum including quark and diquark mass effects, present numerical results and give our final conclusions.

II. APPROACH

The starting point for a discussion of tetraquarks in soft-wall AdS/QCD is the action for spin- J boson fields $\Phi_J = \Phi_{M_1 \dots M_J}(x, z)$, with a negative dilaton in the exponential prefactor, proposed in Refs. [3,43]

$$S_J = \int d^d x dz \sqrt{g} e^{-\varphi(z)} [\partial_M \Phi_J \partial^M \Phi^J - (\mu_J^2 + \hat{V}_J(z)) \Phi_J \Phi^J]. \quad (6)$$

This expression is fully equivalent to the action with a positive dilaton [44] after appropriate dilaton-dependent redefinition of the spin- J boson fields.

The AdS metric is specified as $ds^2 = e^{2A(z)} \times (dx_\mu dx^\mu - dz^2)$, $g = e^{5A(z)}$, $A(z) = \log(R/z)$ and R is the AdS radius. Here Φ_J is the symmetric, traceless tensor classified by the representation $D(E_0, J/2, J/2)$ with energy $E_0 = \Delta = \tau.E_0$ is related to the bulk mass μ_J as $\mu_J^2 R^2 = (E_0 - J)(E_0 - 4 + J)$ where $\tau = N + L$ is the twist-dimension and N is number of partons; $\hat{V}_J(z)$ is the effective dilaton potential, which has an analytical expression in terms of the field $\varphi(z)$ and the “metric” field $A(z)$, without referring to any specific form of their z profiles:

$$\begin{aligned}\hat{V}_J(z) &= e^{-2A(z)} \hat{U}_J(z), \\ \hat{U}_J(z) &= \varphi''(z) + (d-1-2J)\varphi'(z)A'(z).\end{aligned}\quad (7)$$

In order to study the bound-state problem it is convenient to make a dilaton-dependent redefinition of the bulk field, $\Phi_J \rightarrow \Phi_J e^{\varphi(z)/2}$. In the case of the action with positive dilaton profile the dilaton-dependent redefinition of the bulk field is $\Phi_J \rightarrow \Phi_J e^{-\varphi(z)/2}$. After the redefinitions $\Phi_J \rightarrow \Phi_J e^{\pm\varphi(z)/2}$ both versions of the soft-wall model reduce to the same no-wall action

$$S_J = \int d^d x dz \sqrt{g} [\partial_M \Phi_J \partial^M \Phi^J - (\mu_J^2 + V_J(z)) \Phi_J \Phi^J], \quad (8)$$

where

$$\begin{aligned}V_J(z) &= e^{-2A(z)} U_J(z), \\ U_J(z) &= \frac{1}{2} \left(\varphi''(z) + \frac{(\varphi'(z))^2}{2} + (d-1-2J)\varphi'(z)A'(z) \right).\end{aligned}\quad (9)$$

At $d = 4$, $A(z) = \log(R/z)$ and $\varphi(z) = \kappa^2 z^2$ one gets

$$U_J(z) = \kappa^4 z^2 + 2\kappa^2(J-1). \quad (10)$$

The potential $U_J(z)$ is the confinement potential which breaks both conformal and chiral invariance spontaneously.

Next, using the Kaluza-Klein decomposition

$$\begin{aligned}\Phi_J(x, z) &= \sum_n \phi_{nJ}(x) \Phi_{n\tau}(z), \\ \Phi_{n\tau}(z) &= e^{-A(z)(d-1)/2} \varphi_{n\tau}(z),\end{aligned}\quad (11)$$

we derive the Schrödinger-type equation for the bulk profiles $\varphi_{n\tau}(z)$

$$\left[-\frac{d^2}{dz^2} + U_J(z) + W_\tau(z) \right] \varphi_{n\tau}(z) = M_{nJ\tau}^2(z) \varphi_{n\tau}(z), \quad (12)$$

where

$$W_\tau(z) = \frac{4\Delta(\Delta-d) + d^2 - 1}{4z^2} = \frac{4\tau(\tau-d) + d^2 - 1}{4z^2} \quad (13)$$

is the centrifugal potential. For $d = 4$ and in the case of quark-antiquark mesons ($\tau = 2 + L$), tetraquarks ($\tau = 4 + L$) and six quarks ($\tau = 6 + L$), this potential reads

$$\begin{aligned}\text{Mesons} \quad W_{2+L}(z) &= \frac{4L^2 - 1}{4z^2}, \\ \text{Tetraquarks} \quad W_{4+L}(z) &= \frac{4(L+2)^2 - 1}{4z^2}, \\ \text{Sixquarks} \quad W_{6+L}(z) &= \frac{4(L+4)^2 - 1}{4z^2},\end{aligned}\quad (14)$$

The hadronic wave functions are identified with the profiles of the AdS modes $\varphi_{n\tau}(z)$ in the z direction:

$$\varphi_{n\tau}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2} L_n^{\tau-2}(\kappa^2 z^2). \quad (15)$$

They possess the correct behavior in both the ultraviolet and infrared limits, with

$$\begin{aligned}\Phi_{n\tau}(z) &\sim z^{3/2} \varphi_{n\tau}(z) \rightarrow z^\tau \text{ at small } z, \\ \Phi_{n\tau}(z) &\rightarrow 0 \text{ at large } z\end{aligned}\quad (16)$$

and are normalized according to the condition

$$\int_0^\infty dz \varphi_{n\tau}^2(z) = 1. \quad (17)$$

The mass spectrum of multi-quark meson states is given by

$$M_{nJ\tau}^2 = 4\kappa^2 \left[n + \frac{\tau + J - 2}{2} \right] = 4\kappa^2 \left[n + \frac{L + J}{2} + 1 \right]. \quad (18)$$

For $z \rightarrow 0$ the scaling of the bulk profile is identified with the scaling of the corresponding mesonic interpolating operator τ . As we mentioned in the Introduction, τ depends on L (instead of J as in conformal field theory), because we are modeling QCD and therefore should reproduce the scaling of hadronic form factors. As we stressed before, the dependence on L reflects the a spontaneous breaking of chiral invariance, which is expected, since after the introduction of the dilaton field we break the conformal or gauge invariance acting in AdS space. As we noted in Ref. [5], the chiral group is isomorphic to a subgroup of $SO(4, 2)$.

The next step is to include the longitudinal part of the LFWF following the approach used in our paper on mesons [5]. In the case of mesons the first step in this direction was taken in Refs. [8] for mesons and in Ref. [9] for tetraquarks. These authors proposed a factorized form for the mesonic two-parton wave function as a product of transverse $\phi_{nL}(\zeta)$, longitudinal $f(x, m_1, m_2)$ and angular $e^{im\phi}$ modes and depending on variables ζ (transverse variable), x (light-cone coordinate), m (magnetic quantum number), ϕ (azimuthal angle), and quark masses m_1 and m_2 . In Ref. [3] we presented a more convenient form, factorizing the additional factor $\sqrt{x(1-x)}$, which is the Jacobian of the coordinate transformation $\zeta \rightarrow |\mathbf{b}_\perp|$, with:

$$\psi_{q_1\bar{q}_2}(x, \zeta, m_1, m_2) = \frac{\phi_{nL}(\zeta)}{\sqrt{2\pi\zeta}} f(x, m_1, m_2) e^{im\phi} \sqrt{x(1-x)}. \quad (19)$$

In Ref. [3] we used a *Gaussian ansatz* for the longitudinal part of the LFWF and m_1 and m_2 as constituent quark masses. In Ref. [5] we followed Refs. [12,13] and considered current quark masses. By an appropriate choice of the longitudinal wave function $f(x, m_1, m_2)$, we got consistency with QCD in both sectors of light and heavy quarks. In this way we generate the masses of light pseudoscalar mesons in agreement with the scheme resulting from explicit breaking of chiral symmetry—in the leading order of the chiral expansion the masses of pseudoscalar mesons are linear in the current quark mass [45]. We also guarantee that the pseudoscalar π , K , and η meson masses M_π^2 , M_K^2 , M_η^2 satisfy the Gell-Mann-Oakes-Renner

$$M_\pi^2 = 2\hat{m}B \quad (20)$$

and the Gell-Mann-Okubo

$$4M_K^2 = M_\pi^2 + 3M_\eta^2 \quad (21)$$

relations, where $\hat{m} = (m_u + m_d)/2$ is the average mass of u and d quarks, $B = |\langle 0|\bar{u}u|0\rangle|/F_\pi^2$ is the quark condensate parameter, and F_π is the leptonic decay constant. In the sector of heavy quarks we set agreement with both heavy quark effective theory and potential models of heavy quarkonia. In the heavy quark mass limit $m_Q \rightarrow \infty$ we obtain the correct scaling of the leptonic decay constants for both heavy-light mesons $f_{Q\bar{q}} \sim 1/\sqrt{m_Q}$ and heavy quarkonia $f_{Q\bar{Q}} \sim \sqrt{m_Q}$ and $f_{c\bar{b}} \sim m_c/\sqrt{m_b}$ at $m_c \ll m_b$. In this limit we also generated the correct expansion of heavy meson masses

$$\begin{aligned} M_{Q\bar{q}} &= m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q), \\ M_{Q\bar{Q}} &= 2m_Q + E + \mathcal{O}(1/m_Q), \end{aligned} \quad (22)$$

where $\bar{\Lambda}$ is the approximate difference between the masses of the heavy-light meson and the heavy quark, E is the binding energy in heavy quarkonia. The corresponding mass splittings, e.g. between vector and pseudoscalar states of heavy-light mesons, become

$$M_{Q\bar{q}}^V - M_{Q\bar{q}}^P \sim \frac{1}{m_Q}. \quad (23)$$

We chose the longitudinal wave function in the form

$$f(x, m_1, m_2) = Nx^{\alpha_1}(1-x)^{\alpha_2} \quad (24)$$

where N is the normalization constant fixed from

$$1 = \int_0^1 dx f^2(x, m_1, m_2) \quad (25)$$

and α_1, α_2 are parameters that have been fixed in order to get consistency with QCD.

Our master formula for the mass spectrum M_{nJ}^2 of quark-antiquark mesons in terms of the arbitrary longitudinal wave function $f(x, m_1, m_2)$ is given by the expression

$$\begin{aligned} M_{2q;nJ}^2 &= 4\kappa^2 \left(n + \frac{L+J}{2} \right) \\ &+ \int_0^1 dx \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) f^2(x, m_1, m_2). \end{aligned} \quad (26)$$

Using our ansatz for the $f(x, m_1, m_2)$ (24) we get an analytic expression for the correction to the mass spectrum [3]:

$$\begin{aligned} M_{2q;nJ}^2 &= 4\kappa^2 \left(n + \frac{L+J}{2} \right) \\ &+ (1 + 2\alpha_1 + 2\alpha_2) \left(\frac{m_1^2}{2\alpha_1} + \frac{m_2^2}{2\alpha_2} \right). \end{aligned} \quad (27)$$

In the case of the tetraquark states we have two possibilities: (1) tetraquarks are bound states of two quarks and two antiquarks (color diquark-antidiquark configuration); (2) tetraquarks are bound states of two mesons (hadronic molecules).

When tetraquarks have the hadronic molecular configuration the mass formula reads

$$\begin{aligned} (M_{4q;nJ}^{\text{HM}})^2 &= 4\kappa^2 \left(n + \frac{L+J}{2} + 1 \right) \\ &+ \int_0^1 dx \left(\frac{M_1^2}{x} + \frac{M_2^2}{1-x} \right) f^2(x, M_1, M_2) \\ &= 4\kappa^2 \left(n + \frac{L+J}{2} + 1 \right) \\ &+ (1 + 2\alpha_1 + 2\alpha_2) \left(\frac{M_1^2}{2\alpha_1} + \frac{M_2^2}{2\alpha_2} \right), \end{aligned} \quad (28)$$

where M_1 and M_2 are the masses of the quark-antiquark clusters forming the hadronic molecule. Note that M_1 and

M_2 are a bit smaller than the physical masses of the corresponding mesons because of the separation of the contribution of the longitudinal and transverse wave function of tetraquarks. Somehow, one can call M_1 and M_2 bare meson masses, because the transverse part of the wave function gives an additional contribution due to the confinement potential. In the following consideration of the tetraquarks with hadronic molecular configuration we will use more detailed notation $M_{H_{q_1\bar{q}_2}}$ for the masses M_1 , M_2 , where H will specify the spin-parity J^P of the quark cluster ($q_1\bar{q}_2$), e.g. $P = 0^-, V = 1^-, A = 1^+$, and q_1 and \bar{q}_2 will specify flavors of quark and antiquark. In numerical calculations we use the masses of the pseudoscalar quark-antiquark clusters as input parameters fitted from data, while the masses of other spin-parity states are calculated according to the mass splittings

$$\begin{aligned} M_{V_{c\bar{q}}} &= M_{P_{c\bar{q}}} + M_{D^*} - M_D, & M_{V_{c\bar{s}}} &= M_{P_{c\bar{s}}} + M_{D_s^*} - M_{D_s}, \\ M_{V_{q\bar{b}}} &= M_{P_{q\bar{b}}} + M_{B^*} - M_B, \end{aligned} \quad (29)$$

etc.

We will now consider the four-quark structure of tetraquark states. Note that in Ref. [9] the mass formula for the tetraquark states, in terms of quark degrees of freedom, was obtained using superconformal algebra as

$$M_{4q;nJ}^2 = 4\kappa^2 \left(n + L + \frac{S}{2} + 1 \right) + \frac{\kappa^4}{F(\kappa^2)} \frac{dF(\kappa^2)}{d\kappa^2}, \quad (30)$$

where

$$F(\kappa^2) = \int_0^1 dx_1 \dots \int_0^1 dx_4 \delta \left(\sum_{i=1}^4 x_i - 1 \right) \exp \left[- \sum_{i=1}^4 \frac{m_i^2}{x_i \kappa^2} \right]. \quad (31)$$

The difference between the two formulas (28) and (30) in the zero mass limit for the constituents is due to the term which spontaneously breaks superconformal symmetry

$$\Delta M_{4q;nJ}^2 = 2\kappa^2 (J - L - S), \quad (32)$$

which is zero for tetraquark systems with $J = L + S$.

Notice that our formula (18) gives, in the zero mass limit, an natural explanation of the tetraquark state $a(1420)$ discovered by the COMPASS Collaboration [46]. In our approach, for $n = 0$, $J = L = 1$, we get

$$M_{a(1420)}^2 = 8\kappa^2 \quad (33)$$

or $M_{a(1420)} = 2\kappa\sqrt{2}$ resulting in $M_{a(1420)} = \sqrt{2} \text{ GeV} \simeq 1.414 \text{ GeV}$ at $\kappa = 0.5 \text{ GeV}$, which agrees perfectly with the experimental result of $M_{a(1420)} = 1.414_{-0.013}^{+0.015} \text{ GeV}$. The analysis of Ref. [47] within QCD sum rules disfavors assigning the $a_1(1420)$ to an axial-vector tetraquark state and proposes that it is a mixed state of the

$a_1(1260)$ meson and the tetraquark state with the configuration $[su]_{S=1}[\bar{s}\bar{d}]_{S=0} + [su]_{S=0}[\bar{s}\bar{d}]_{S=1}$. On the other hand, the QCD sum rules analysis performed in Ref. [48] confirmed the existence of $a_1(1420)$ as a tetraquark state. Questions related to the nature of the $a_1(1420)$ meson have also been addressed in other papers. In particular, it has been proposed that this state is a consequence of rescattering effects, and in fact in Ref. [49] the $a_1(1420)$ was interpreted as a dynamical effect due to a singularity (branch point) in the triangle diagram formed by the processes $a_1(1260) \rightarrow K^*\bar{K}$, $K^* \rightarrow K\pi$ and $K\bar{K} \rightarrow f_0(980)$. In Ref. [50] it was shown that a single $I = 1$ spin-parity $J^{PC} = 1^{++}$ a_1 resonance can manifest itself as two separated mass peaks. One decays into an S -wave $\rho\pi$ system and the second decays into a P -wave $f_0(980)\pi$ system, with a rapid increase of the phase difference between their amplitudes, arising mainly from the structure of the diffractive production process. In Ref. [51] it was claimed that resonances such as the $a_1(1420)$ could be produced due to the so-called ‘‘anomalous triangle singularity’’, if it is located in a specific kinematical region. In Ref. [52] $a_1(1420)$ was considered as peak in the $a_1(1260) \rightarrow \pi f_0(980)$ decay mode. In Ref. [53] it was proposed to test the possible rescattering nature of the $a_1(1420)$ in heavy meson decays.

Our approach gives also a lower limit for the masses of the tetraquarks with $n = J = L = S = 0$

$$M_{4q} \geq 2\kappa = 1 \text{ GeV} \quad (34)$$

for $\kappa = 0.5 \text{ GeV}$. It means that the lightest possible tetraquark is composed of two diquarks forming a state with spin parity $J^P = 0^+$, and then a possible candidate is the $f_0(980)$ consistent with the conclusion of Ref. [9]. Note that in the massless limit we consider the same universal dilaton parameter κ for the constituents of all states forming tetraquarks. When the masses of the tetraquark constituents are taken into account, we assume that the numerical value of κ for tetraquarks could change from its original value of 0.5 GeV .

Next we include quark mass effects, following the approach presented in our paper on applications of holographic QCD to mesons [5]. We proposed the following form for the longitudinal wave function containing quark masses

$$f(x_1, \dots, x_4, m_1, \dots, m_4) = N x_1^{\alpha_1} \dots x_4^{\alpha_4}, \quad (35)$$

where N is a normalization constant fixed from the condition

$$\begin{aligned} 1 &= \int_0^1 dx_1 \dots \int_0^1 dx_4 \delta \left(\sum_{i=1}^4 x_i - 1 \right) \\ &\times f^2(x_1, \dots, x_4, m_1, \dots, m_4). \end{aligned} \quad (36)$$

The contribution of the longitudinal wave function (35) to the mass spectrum is

$$\begin{aligned} \Delta M_{4q;nJ}^2 &= \int_0^1 dx_1 \cdots \int_0^1 dx_4 \delta\left(\sum_{i=1}^4 x_i - 1\right) \\ &\times f^2(x_1, \dots, x_4, m_1, \dots, m_4) \sum_{i=1}^4 \frac{m_i^2}{x_i} \\ &= \left(3 + 2 \sum_{i=1}^4 \alpha_i\right) \sum_{i=1}^4 \frac{m_i^2}{2\alpha_i}. \end{aligned} \quad (37)$$

It should be stressed, as in case of mesons, that by an appropriate choice of the α_i parameters we can guarantee the correct behavior of the tetraquark spectrum in both the

light and heavy quark sectors. In particular, tetraquarks composed of light nonstrange (u, d) quarks receive the following quark mass correction

$$\Delta M_{4q;nJ}^2([q\bar{q}]^2) = 2\left(\frac{3}{\alpha_q} + 8\right)\hat{m}^2, \quad (38)$$

where $\hat{m} = m_u = m_d$ in the isospin limit. The parameter $\alpha_q = 3\hat{m}/(2B)$ is fixed from the condition

$$\Delta M_{4q;nJ}^2([q\bar{q}]^2) = 4\hat{m}B + \mathcal{O}(\hat{m}^2) \simeq 2M_\pi^2. \quad (39)$$

By analogy, for light tetraquarks containing single, two and three strange quarks/antiquarks using $\alpha_q = 3\hat{m}/(2B)$ and $\alpha_s = 3m_s/(2B)$ we get

$$\begin{aligned} \Delta M_{4q;nJ}^2([s\bar{q}][q\bar{q}]) &= \Delta M_{4q;nJ}^2([q\bar{s}][q\bar{q}]) = (3 + 6\alpha_q + 2\alpha_s) \left(\frac{3\hat{m}^2}{2\alpha_q} + \frac{m_s^2}{2\alpha_s}\right) \\ &= B(3\hat{m} + m_s) + \mathcal{O}(\hat{m}^2, m_s^2, \hat{m}m_s) \simeq M_\pi^2 + M_K^2, \end{aligned} \quad (40)$$

$$\begin{aligned} \Delta M_{4q;nJ}^2([q\bar{s}]^2) &= \Delta M_{4q;nJ}^2([s\bar{q}]^2) = \Delta M_{4q;nJ}^2([s\bar{q}][q\bar{s}]) = (3 + 4\alpha_q + 4\alpha_s) \left(\frac{\hat{m}^2}{\alpha_q} + \frac{m_s^2}{\alpha_s}\right) \\ &= 2B(\hat{m} + m_s) + \mathcal{O}(\hat{m}^2, m_s^2, \hat{m}m_s) \simeq 2M_K^2, \end{aligned} \quad (41)$$

$$\begin{aligned} \Delta M_{4q;nJ}^2([q\bar{s}][s\bar{s}]) &= \Delta M_{4q;nJ}^2([s\bar{q}][s\bar{s}]) = (3 + 2\alpha_q + 6\alpha_s) \left(\frac{\hat{m}^2}{2\alpha_q} + \frac{3m_s^2}{2\alpha_s}\right) \\ &= B(\hat{m} + 3m_s) + \mathcal{O}(\hat{m}^2, m_s^2, \hat{m}m_s) \simeq 3M_K^2 - M_\pi^2, \end{aligned} \quad (42)$$

and

$$\begin{aligned} \Delta M_{4q;nJ}^2([s\bar{s}]^2) &= 2\left(\frac{3}{\alpha_s} + 8\right)m_s^2 \\ &= 4m_sB + \mathcal{O}(m_s^2) \simeq 4M_K^2 - 2M_\pi^2. \end{aligned} \quad (43)$$

We will now derive quark mass corrections for tetraquarks containing single, two, three and four heavy quarks/anti-quarks $Q = b, c$ (in the following index q denotes all light quarks u, d, s , and for simplicity we consider heavy and light quarks each of the same flavor):

$$\begin{aligned} \Delta M_{4q;nJ}^2([Q\bar{q}][q\bar{q}]) &= \Delta M_{4q;nJ}^2([q\bar{Q}][q\bar{q}]) = (3 + 2\alpha_Q + 6\alpha_q) \left(\frac{m_Q^2}{2\alpha_Q} + \frac{3m_q^2}{2\alpha_q}\right), \\ \Delta M_{4q;nJ}^2([Q\bar{Q}][q\bar{q}]) &= \Delta M_{4q;nJ}^2([Q\bar{q}][q\bar{Q}]) = (3 + 4\alpha_Q + 4\alpha_q) \left(\frac{m_Q^2}{\alpha_Q} + \frac{m_q^2}{\alpha_q}\right), \\ \Delta M_{4q;nJ}^2([Q\bar{q}][Q\bar{Q}]) &= \Delta M_{4q;nJ}^2([q\bar{Q}][Q\bar{Q}]) = (3 + 6\alpha_Q + 2\alpha_q) \left(\frac{3m_Q^2}{2\alpha_Q} + \frac{m_q^2}{2\alpha_q}\right), \\ \Delta M_{4q;nJ}^2([Q\bar{Q}]^2) &= 16m_Q^2 \left(1 + \frac{3}{8\alpha_Q}\right). \end{aligned} \quad (44)$$

To get the correct scaling of heavy tetraquarks for $m_Q \rightarrow \infty$ with

$$\begin{aligned}
\Delta M_{4q;nJ}^2([Q\bar{q}][q\bar{q}]) &= (m_Q + \bar{\Lambda}_{3q} + \mathcal{O}(1/m_Q))^2, \\
\Delta M_{4q;nJ}^2([Q\bar{Q}][q\bar{q}]) &= \Delta M_{4q;nJ}^2([Q\bar{q}][q\bar{Q}]) = (2m_Q + \bar{\Lambda}_{2q} + \mathcal{O}(1/m_Q))^2, \\
\Delta M_{4q;nJ}^2([Q\bar{Q}][Q\bar{q}]) &= \Delta M_{4q;nJ}^2([q\bar{Q}][Q\bar{Q}]) = (3m_Q + \bar{\Lambda}_q + \mathcal{O}(1/m_Q))^2, \\
\Delta M_{4q;nJ}^2([Q\bar{Q}][Q\bar{Q}]) &= (4m_Q + E + \mathcal{O}(1/m_Q))^2,
\end{aligned} \tag{45}$$

where $\bar{\Lambda}_{nq}$ is the approximate difference between the mass of the heavy-light tetraquark and the sum of the masses of its $(4-n)$ constituent heavy quarks, we fix the α_Q parameters as follows: $\alpha_Q = \alpha$ (independent on the heavy quark flavor) in the case of heavy-light tetraquarks and

$$\alpha_Q = \frac{3m_Q}{4E} \left(1 - \frac{E}{8m_Q}\right) \tag{46}$$

in the case of tetraquarks composed only of heavy quarks. The parameter for light quarks α_q occurring in the case of heavy-light tetraquarks is fixed as

$$\alpha_q \equiv \alpha_q^{(n)} = \frac{2\alpha \bar{\Lambda}_{nq}}{n m_Q} \left(1 + \frac{1}{2(4-n)} \frac{\bar{\Lambda}_{nq}}{m_Q}\right) - \frac{3}{2n} \tag{47}$$

where n is the number of light quark/antiquarks in a tetraquark. In particular,

$$\alpha_q^{(3)} = \frac{2\alpha \bar{\Lambda}_{3q}}{3 m_Q} \left(1 + \frac{\bar{\Lambda}_{3q}}{2m_Q}\right) - \frac{1}{2} \tag{48}$$

in the case of $n = 3$,

$$\alpha_q^{(2)} = \alpha \frac{\bar{\Lambda}_{2q}}{m_Q} \left(1 + \frac{\bar{\Lambda}_{2q}}{4m_Q}\right) - \frac{3}{4} \tag{49}$$

in the case of $n = 2$,

$$\alpha_q^{(1)} = 2\alpha \frac{\bar{\Lambda}_q}{m_Q} \left(1 + \frac{\bar{\Lambda}_q}{6m_Q}\right) - \frac{3}{2} \tag{50}$$

in the case of $n = 1$.

In the case of tetraquarks with a hadronic molecular configuration it is worthwhile to test the prediction of Ref. [54] about the structure of the $Z_b(10610) = (B\bar{B}^* + \text{H.c.})$ and $Z_b(10650) = (B^*\bar{B}^*)$ resonances, which are supposed to be $J^P = 1^+$ states. In our approach their masses are given by

$$\begin{aligned}
M_{Z_b(10610)}^2 &= 8\kappa^2 + (1 + 2\alpha_{P_{q\bar{b}}} + 2\alpha_{V_{q\bar{b}}}) \left(\frac{M_{P_{q\bar{b}}}^2}{2\alpha_{P_{q\bar{b}}}} + \frac{M_{V_{q\bar{b}}}^2}{2\alpha_{V_{q\bar{b}}}} \right), \\
M_{Z_b(10650)}^2 &= 8\kappa^2 + (1 + 4\alpha_{V_{q\bar{b}}}) \frac{M_{V_{q\bar{b}}}^2}{\alpha_{V_{q\bar{b}}}}.
\end{aligned} \tag{51}$$

Fixing $\alpha_{P_{q\bar{b}}}$ and $\alpha_{V_{q\bar{b}}}$ as

$$\alpha_i = \frac{1}{4} \left(\frac{M_i}{\Delta_B} - \frac{1}{4} \right), \quad \Delta_B = M_{V_{q\bar{b}}} - M_{P_{q\bar{b}}} \equiv M_{B^*} - M_B \tag{52}$$

results in the expressions

$$\begin{aligned}
M_{Z_b(10610)}^2 &= 8\kappa^2 + 4\bar{M}_B^2 \left[\left(1 + \frac{\Delta_B}{2\bar{M}_B}\right)^2 + \mathcal{O}\left(\frac{\Delta_B^3}{\bar{M}_B^3}\right) \right], \\
M_{Z_b(10650)}^2 &= 8\kappa^2 + 4\bar{M}_B^2 \left[\left(1 + \frac{\Delta_B}{\bar{M}_B}\right)^2 + \mathcal{O}\left(\frac{\Delta_B^3}{\bar{M}_B^3}\right) \right],
\end{aligned} \tag{53}$$

where $\bar{M}_B = \frac{M_{V_{q\bar{b}}} + M_{P_{q\bar{b}}}}{2}$. For the mass splitting $M_{Z_b(10650)}^2 - M_{Z_b(10610)}^2$ we get

$$M_{Z_b(10650)}^2 - M_{Z_b(10610)}^2 = 4\Delta_B \bar{M}_B \left[1 + \mathcal{O}\left(\frac{\Delta_B}{\bar{M}_B}\right) \right] \tag{54}$$

or

$$\Delta = M_{Z_b(10650)} - M_{Z_b(10610)} = \Delta_B \left[1 + \mathcal{O}\left(\frac{\Delta_B}{\bar{M}_B}\right) \right] \tag{55}$$

in agreement with Ref. [54]. By analogy we get a similar conclusion for the related charmed tetraquark states $Z_c(3900) = (D\bar{D}^* + \text{H.c.})$ and $Z_c(4020) = (D^*\bar{D}^*)$.

Another interesting state is the $X(3872)$. In our approach we consider this state in a mixed configuration of the molecular $(D^0\bar{D}^{*0} + \bar{D}^0D^{*0})/\sqrt{2}$ and the charmonia $c\bar{c}$ states following Refs. [55,56] with

$$|X(3872)\rangle = \frac{\cos\theta}{\sqrt{2}} |D^0\bar{D}^{*0} + \bar{D}^0D^{*0}\rangle + \sin\theta c\bar{c}. \tag{56}$$

Using Eq. (56) and

$$\alpha_c = \frac{1}{4} \left(\frac{m_c}{E} - \frac{1}{4} \right), \quad \alpha_i = \frac{1}{4} \left(\frac{M_i}{\Delta_D} - \frac{1}{4} \right), \quad i = V_{c\bar{q}}, P_{c\bar{q}} \tag{57}$$

we get the following mass formula for the $X(3872)$ state

$$\begin{aligned}
M_{X(3872)}^2 &= \cos^2\theta \left[8\kappa^2 + (1 + 2\alpha_{P_{c\bar{q}}} + 2\alpha_{V_{c\bar{q}}}) \left(\frac{M_{P_{c\bar{q}}}^2}{2\alpha_{P_{c\bar{q}}}} + \frac{M_{V_{c\bar{q}}}^2}{2\alpha_{V_{c\bar{q}}}} \right) \right] + \sin^2\theta \left[4\kappa^2 + 4m_c^2 \left(1 + \frac{1}{4\alpha_c} \right) \right] \\
&= 4\kappa^2(1 + \cos^2\theta) + 4\bar{M}_D^2 \left[\left(1 + \frac{\Delta_D}{2\bar{M}_D} \right)^2 + \mathcal{O}\left(\frac{\Delta_D^3}{\bar{M}_D^3}\right) \right] \cos^2\theta \\
&\quad + 4m_c^2 \left[\left(1 + \frac{E}{2m_c} \right)^2 + \mathcal{O}\left(\frac{E^3}{(2m_c)^3}\right) \right] \sin^2\theta, \tag{58}
\end{aligned}$$

where by analogy with the bottom sector we have

$$\bar{M}_D = \frac{M_{V_{c\bar{q}}} + M_{P_{c\bar{q}}}}{2}, \quad \Delta_D = M_{V_{c\bar{q}}} - M_{P_{c\bar{q}}} \equiv M_{D^*} - M_D. \tag{59}$$

Note that the hadronic molecular contribution to the $X(3872)$ state is given by the same expression as for the $Z_c(3900)$ and $Z_c(4020)$ states. The values $m_c = 1.275$ GeV and $E = 0.795$ GeV are taken from Ref. [5]. We find that for $|\theta| = 11^\circ$ we could reproduce the experimental value of the $X(3872)$ mass of $M_{X(3872)} = 3871.69 \pm 0.17$ MeV.

Next we look at the possible $J^P = 0^+$ Y -states as hadronic molecules

$$\begin{aligned}
|Y(3940)\rangle &= \frac{1}{\sqrt{2}} |D^{*+}D^{*-} + D^{*0}\bar{D}^{*0}\rangle, \\
|Y(4140)\rangle &= |D_s^{*+}D_s^{*-}\rangle \tag{60}
\end{aligned}$$

studied by us before using a phenomenological Lagrangian approach in Ref. [57]. Holographic QCD gives the following results for the masses of $Y(3940)$ and $Y(4140)$ states

$$\begin{aligned}
M_{Y(3940)}^2 &= 4\kappa^2 + 4M_{V_{c\bar{q}}}^2 \left(1 + \frac{1}{4\alpha_{Y(3940)}} \right) \\
&= 4\kappa^2 + 4M_{V_{c\bar{q}}}^2 \left[\left(1 + \frac{\Delta_Y}{M_{V_{c\bar{q}}}} \right)^2 + \mathcal{O}\left(\frac{\Delta_Y^3}{M_{V_{c\bar{q}}}^3}\right) \right], \\
M_{Y(4140)}^2 &= 4\kappa^2 + 4M_{V_{c\bar{s}}}^2 \left(1 + \frac{1}{4\alpha_{Y(4140)}} \right) \\
&= 4\kappa^2 + 4M_{V_{c\bar{s}}}^2 \left[\left(1 + \frac{\Delta_Y}{M_{V_{c\bar{s}}}} \right)^2 + \mathcal{O}\left(\frac{\Delta_Y^3}{M_{V_{c\bar{s}}}^3}\right) \right], \tag{61}
\end{aligned}$$

where

$$\alpha_{Y(3940)} = \frac{1}{8} \left(\frac{M_{V_{c\bar{q}}}}{\Delta_Y} - \frac{1}{2} \right), \quad \alpha_{Y(4140)} = \frac{1}{8} \left(\frac{M_{V_{c\bar{s}}}}{\Delta_Y} - \frac{1}{2} \right), \tag{62}$$

and Δ_Y is phenomenological parameter of order of $\mathcal{O}(M_i^0)$. With $\Delta_Y = 96$ MeV we can reproduce the mass data for these states.

Finally, we consider the $Z(4430)^+$ state in the hadronic molecular picture, which has the structure [58]

$$|Z(4430)^+\rangle = \frac{1}{\sqrt{2}} |D_1^+ \bar{D}^{*0} + D^{*+} \bar{D}_1^0\rangle. \tag{63}$$

Its mass squared neglecting isospin breaking corrections is given by

$$M_{Z(4430)}^2 = 8\kappa^2 + (1 + 2\alpha_{A_{c\bar{q}}} + 2\alpha_{V_{c\bar{q}}}) \left(\frac{M_{A_{c\bar{q}}}^2}{2\alpha_{A_{c\bar{q}}}} + \frac{M_{V_{c\bar{q}}}^2}{2\alpha_{V_{c\bar{q}}}} \right). \tag{64}$$

Taking

$$\alpha_i = \frac{1}{2} \left(\frac{M_i}{\Delta_Z} - \frac{1}{8} \right), \quad \Delta_Z = M_{A_{c\bar{q}}} - M_{V_{c\bar{q}}} \equiv M_{D_1} - M_{D^*} \tag{65}$$

we get

$$\begin{aligned}
M_{Z(4430)}^2 &= 8\kappa^2 + (M_{A_{c\bar{q}}} + M_{V_{c\bar{q}}})^2 \\
&\quad \times \left[\left(1 + \frac{\Delta_Z}{2(M_{A_{c\bar{q}}} + M_{V_{c\bar{q}}})} \right)^2 \right. \\
&\quad \left. + \mathcal{O}\left(\frac{\Delta_Z^3}{(M_{A_{c\bar{q}}} + M_{V_{c\bar{q}}})^3}\right) \right]. \tag{66}
\end{aligned}$$

Our predictions for the masses for selected light and heavy tetraquarks are shown in Tables I and II. For some states, suspected as tetraquarks, we include a comparison with data [59], while for other states we make mass predictions. Following derived analytical formulas for tetraquarks with color diquark-antidiquark configuration we neglect by the m_q^2 and $1/m_Q$ corrections, while for the hadronic molecular configuration we neglect by the Δ^3/M^3 corrections. In future work we plan to make a more detailed analysis.

For completeness we also consider two candidates on tetraquarks with color diquark-color diquark configuration— $X(3755)$ and $X(3915)$ states, with $\bar{\Lambda}_{2q} = 1.07$ GeV we reproduce the data for their masses. Also we suppose that $X(3915)$ is the radial excitation of the $X(3755)$ state.

TABLE I. Masses of possible light tetraquarks (in MeV).

Tetraquark	Quark Content	Quantum Numbers (J^P, n, L, S)	κ (MeV)	Mass (Zero Quark Masses)	Mass (Finite Quark Masses)	Data [59]
$f_0(980)$	$[q\bar{s}][s\bar{q}]$	$(0^+, 0, 0, 0)$	351	702	990	990 ± 20
$f_1(1215)$	$[q\bar{s}][s\bar{q}]$	$(1^+, 0, 1, 0)$	351	993	1214	
$f_2(1400)$	$[q\bar{s}][s\bar{q}]$	$(2^+, 0, 2, 0)$	351	1216	1402	
$f_3(1570)$	$[q\bar{s}][s\bar{q}]$	$(3^+, 0, 3, 0)$	351	1404	1568	
$a_1(1420)$	$[q\bar{s}][s\bar{q}]$	$(1^+, 0, 1, 1)$	435	1230	1414	1414^{+15}_{-13} [46]

TABLE II. Masses of heavy tetraquarks (in MeV).

Tetraquark	Quark Content	Quantum Numbers (J^P, n, L, S)	κ (MeV)	Mass	Data [59]
$X(3755)$	$[c\bar{q}][q\bar{c}]$	$(0^+, 0, 0, 0)$	500	3756	
$X(3915)$	$[c\bar{q}][q\bar{c}]$	$(0^+, 1, 0, 0)$	500	3886	3918.4 ± 1.9
$Y(3940)$	$[c\bar{q}][q\bar{c}]$	$(0^+, 0, 0, 0)$	500	3940	$3943.0 \pm 11 \pm 13$
$Y(4140)$	$[c\bar{s}][s\bar{c}]$	$(0^+, 0, 0, 0)$	500	4146	$4143.0 \pm 2.9 \pm 1.2$
$X(3872)$	$\cos\theta[c\bar{d}][d\bar{c}] + \sin\theta c\bar{c}$	$(1^+, 0, 1, 1)$	500	3872	3871.69 ± 0.17
$Z_c(3900)^+$	$[c\bar{d}][u\bar{c}]$	$(1^+, 0, 1, 1)$	500	3886	3886.6 ± 2.4
$Z_c(4020)^+$	$[c\bar{d}][u\bar{c}]$	$(1^+, 0, 1, 1)$	500	4017	4024.1 ± 1.9
$Z(4430)^+$	$[c\bar{d}][u\bar{c}]$	$(1^+, 0, 1, 1)$	500	4468	4478^{+15}_{-18}
$Z_b(10610)^+$	$[b\bar{d}][u\bar{b}]$	$(1^+, 0, 1, 1)$	500	10607	10607.2 ± 2.0
$Z'_b(10650)^+$	$[b\bar{d}][u\bar{b}]$	$(1^+, 0, 1, 1)$	500	10652	10652.2 ± 1.5

In the numerical analysis we use the same set of quark masses as for the conventional quark-antiquark mesons:

$$\begin{aligned}
m_u &= m_d = \hat{m} = 7 \text{ MeV}, \\
m_s &= 24\hat{m} = 168 \text{ MeV}, \\
m_c &= 1.275 \text{ GeV}, \quad m_b = 4.18 \text{ GeV}. \quad (67)
\end{aligned}$$

We use the universal parameters $\kappa = 351/435$ MeV for light and $\kappa = 500$ MeV for heavy-light tetraquarks. We suppose that the inclusion of finite mass effects of the constituents in the light tetraquarks shifts the parameter κ to lower values than 500 MeV. However, in the massless limit for the tetraquark constituents, the parameter κ is universal and the same for all tetraquark states independent on the light/heavy quark/diquark/meson content. The masses of the quark-antiquark clusters for pseudoscalar states are taken as

$$\begin{aligned}
M_{P_{c\bar{q}}} &= 1.669 \text{ GeV}, \quad M_{P_{c\bar{s}}} = 1.772 \text{ GeV}, \\
M_{P_{u\bar{b}}} &= 5.211 \text{ GeV}. \quad (68)
\end{aligned}$$

As we stressed before, the masses of other spin-parity states are calculated according to Eq. (29).

In conclusion, we summarize the main results of the paper. Using a soft-wall AdS/QCD approach we derived formalism for tetraquarks with adjustable quantum numbers and composed of light and heavy quarks. Starting from effective action we derived the Schrödinger-type equation of motion for the tetraquark wave function dual to the bulk

profile of dimension-4 AdS field. Solving this equation analytically we provided predictions for the tetraquark wave function and its mass. We showed that a low mass limit for possible tetraquark states is given by $M \geq 2\kappa = 1$ GeV, where $\kappa = 0.5$ GeV is the typical value of the scale parameter in soft-wall AdS/QCD. We confirmed results of the COMPASS Collaboration on the discovery of the $a_1(1414)$ state, interpreted as a tetraquark state composed of light quarks and having $J^{PC} = 1^{++}$. In particular, our prediction for the mass of this state $M_{a_1} = \sqrt{2}$ GeV ≈ 1.414 GeV is in good agreement with the COMPASS result $M_{a_1} = 1.414^{+0.015}_{-0.013}$ GeV. As the next step, we extended our formulas for the tetraquark wave functions and masses including finite masses of their constituents and using constraints of chiral symmetry in the sector of light quarks, and heavy quark effective theory in the sector of heavy quarks and theoretical developments in exotic states phenomenology. In particular, we derived the mass formulas including finite mass effects of constituents for tetraquarks with hadronic molecular and color diquark-antidiquark configurations. Finally, in Tables I and II we presented numerical results for masses of light and heavy tetraquarks.

ACKNOWLEDGMENTS

The authors thank Stan Brodsky for useful discussions. This work was funded by the German Bundesministerium für Bildung und Forschung (BMBF) under Project

05P2015—ALICE at High Rate (BMBF-FSP 202): “Jet- and fragmentation processes at ALICE and the parton structure of nuclei and structure of heavy hadrons”; by CONICYT (Chile) PIA/Basal FB0821, by CONICYT (Chile) Research Projects No. 80140097, and under Grants No. 7912010025, 1140390; by Tomsk State

University Competitiveness Improvement Program and the Russian Federation program “Nauka” (Contract No. 0.1764.GZB.2017). The research is carried out at Tomsk Polytechnic University within the framework of Tomsk Polytechnic University Competitiveness Enhancement Program Grant No. VIU-FTI-72/2017.

-
- [1] S. J. Brodsky and G. F. de Teramond, Light-front dynamics and AdS/QCD correspondence: The pion form factor in the space- and time-like regions, *Phys. Rev. D* **77**, 056007 (2008).
- [2] S. J. Brodsky and G. F. de Teramond, Light-front dynamics and AdS/QCD correspondence: Gravitational form factors of composite hadrons, *Phys. Rev. D* **78**, 025032 (2008).
- [3] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Light and heavy mesons in a soft-wall holographic approach, *Phys. Rev. D* **82**, 074022 (2010).
- [4] S. J. Brodsky, F. G. Cao, and G. F. de Teramond, Meson transition form factors in light-front holographic QCD, *Phys. Rev. D* **84**, 075012 (2011).
- [5] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Chiral symmetry breaking and meson wave functions in soft-wall AdS/QCD, *Phys. Rev. D* **87**, 056001 (2013).
- [6] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Light-front potential for heavy quarkonia constrained by the holographic soft-wall model, *Phys. Rev. D* **90**, 096007 (2014).
- [7] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Light-front quark model consistent with Drell-Yan-West duality and quark counting rules, *Phys. Rev. D* **89**, 054033 (2014); Erratum, *Phys. Rev. D* **92**, 019902 (2015); Nucleon structure in a light-front quark model consistent with quark counting rules and data, *Phys. Rev. D* **91**, 054028 (2015).
- [8] S. J. Brodsky and G. F. de Teramond, AdS/CFT and light-front QCD, *Subnuclear series* **45**, 139 (2009).
- [9] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and C. Lorce, Universal effective hadron dynamics from superconformal algebra, *Phys. Lett. B* **759**, 171 (2016).
- [10] S. J. Brodsky, T. Huang, and G. P. Lepage, Hadronic and nuclear interactions in QCD, *Springer Tracts Mod. Phys.* **100**, 81 (1982).
- [11] T. Huang, B.-Q. Ma, and Q.-X. Shen, Analysis of the pion wave function in light cone formalism, *Phys. Rev. D* **49**, 1490 (1994).
- [12] G. 't Hooft, A two-dimensional model for mesons, *Nucl. Phys.* **B75**, 461 (1974).
- [13] H. Bergknoff, Physical particles of the massive Schwinger model, *Nucl. Phys.* **B122**, 215 (1977).
- [14] Y. Ma and J. R. Hiller, Numerical solution of the one pair equation in the massive Schwinger model, *J. Comput. Phys.* **82**, 229 (1989).
- [15] Y. Z. Mo and R. J. Perry, Basis function calculations for the massive Schwinger model in the light front Tamm-Dancoff approximation, *J. Comput. Phys.* **108**, 159 (1993).
- [16] S. S. Chabysheva and J. R. Hiller, Dynamical model for longitudinal wave functions in light-front holographic QCD, *Ann. Phys. (Berlin)* **337**, 143 (2013).
- [17] J. R. Forshaw and R. Sandapen, An AdS/QCD Holographic Wavefunction for the Rho Meson and Diffractive Rho Meson Electroproduction, *Phys. Rev. Lett.* **109**, 081601 (2012).
- [18] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, Linear confinement and AdS/QCD, *Phys. Rev. D* **74**, 015005 (2006); S. J. Brodsky and G. F. de Teramond, Hadronic Spectra and Light-Front Wavefunctions in Holographic QCD, *Phys. Rev. Lett.* **96**, 201601 (2006); O. Andreev, $1/q^2$ corrections and gauge/string duality, *Phys. Rev. D* **73**, 107901 (2006).
- [19] M. Nielsen, F. S. Navarra, and S. H. Lee, New charmonium states in QCD sum rules: A concise review, *Phys. Rep.* **497**, 41 (2010).
- [20] G. V. Pakhlova, P. N. Pakhlov, and S. I. Eidelman, *Usp. Fiz. Nauk* **180**, 225 (2010) [Exotic charmonium, *Phys. Usp.* **53**, 219 (2010)].
- [21] N. Drenska, R. Faccini, F. Piccinini, A. Polosa, F. Renga, and C. Sabelli, New hadronic spectroscopy, *Riv. Nuovo Cimento* **33**, 633 (2010).
- [22] N. Brambilla *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
- [23] S. L. Olsen, A new hadron spectroscopy, *Front. Phys.* **10**, 121 (2015).
- [24] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni, and A. D. Polosa, Four-quark hadrons: An updated review, *Int. J. Mod. Phys. A* **30**, 1530002 (2015).
- [25] C. Z. Yuan, Recent progress on the study of the charmoniumlike states, *Int. J. Mod. Phys. A* **29**, 1430046 (2014).
- [26] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, *Phys. Rep.* **639**, 1 (2016).
- [27] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-quark QCD exotica, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [28] A. Esposito, A. Pilloni, and A. D. Polosa, Multi-quark resonances, *Phys. Rep.* **668**, 1 (2017).
- [29] A. Ali, J. S. Lange, and S. Stone, Exotics: Heavy pentaquarks and tetraquarks, [arXiv:1706.00610](https://arxiv.org/abs/1706.00610).
- [30] D. Ebert, R. N. Faustov, and V. O. Galkin, Masses of heavy tetraquarks in the relativistic quark model, *Phys. Lett. B* **634**, 214 (2006); Excited heavy tetraquarks with hidden charm, *Eur. Phys. J. C* **58**, 399 (2008); Masses of light tetraquarks and scalar mesons in the relativistic quark model, *Eur. Phys. J. C* **60**, 273 (2009); Masses of tetraquarks

- with open charm and bottom, *Phys. Lett. B* **696**, 241 (2011); D. Ebert, R.N. Faustov, V.O. Galkin, and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, *Phys. Rev. D* **76**, 114015 (2007).
- [31] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Korner, P. Santorelli, and G. G. Saidullaeva, One-photon decay of the tetraquark state $X(3872) \rightarrow \gamma + J/\psi$ in a relativistic constituent quark model with infrared confinement, *Phys. Rev. D* **84**, 014006 (2011); S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, and J. G. Korner, Quark model description of the tetraquark state $X(3872)$ in a relativistic constituent quark model with infrared confinement, *Phys. Rev. D* **81**, 114007 (2010).
- [32] T. Gutsche, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij, Isospin-violating strong decays of scalar single-heavy tetraquarks, *Phys. Rev. D* **94**, 094012 (2016); F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and P. Santorelli, Four-quark structure of the $Z_c(3900)$, $Z(4430)$, and $X_b(5568)$ states, *Phys. Rev. D* **94**, 094017 (2016); F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij, $Z_b(10610)$ and $Z'_b(10650)$ decays in a covariant quark model, [arXiv:1707.00539](https://arxiv.org/abs/1707.00539).
- [33] S. J. Brodsky, D. S. Hwang, and R. F. Lebed, Dynamical Picture for the Formation and Decay of the Exotic XYZ Mesons, *Phys. Rev. Lett.* **113**, 112001 (2014); S. J. Brodsky and R. F. Lebed, QCD dynamics of tetraquark production, *Phys. Rev. D* **91**, 114025 (2015).
- [34] M. B. Voloshin, Constituent counting rule for exclusive production of heavy quarkoniumlike exotic resonance and a light hadron, *Phys. Rev. D* **94**, 074042 (2016).
- [35] F. K. Guo, U. G. Meißner, and W. Wang, On the constituent counting rules for hard exclusive processes involving multi-quark states, *Chin. Phys. C* **41**, 053108 (2017).
- [36] S. J. Brodsky, R. F. Lebed, and V. E. Lyubovitskij, QCD compositeness as revealed in exclusive vector boson reactions through double-photon annihilation: $e^+e^- \rightarrow \gamma\gamma^* \rightarrow \gamma V^0$ and $e^+e^- \rightarrow \gamma^*\gamma^* \rightarrow V^0V^0$, *Phys. Lett. B* **764**, 174 (2017).
- [37] C. K. Chow, From tetraquark to hexaquark: A systematic study of heavy exotics in the large $N(c)$ limit, *Phys. Rev. D* **51**, 6327 (1995); Semileptonic decays of heavy tetraquarks, *Phys. Rev. D* **51**, 3541 (1995).
- [38] A. Vega and I. Schmidt, Scalar hadrons in $AdS_5 \times S^5$, *Phys. Rev. D* **78**, 017703 (2008).
- [39] A. Vega and I. Schmidt, Hadrons in AdS/QCD correspondence, *Phys. Rev. D* **79**, 055003 (2009).
- [40] H. Forkel, Light scalar tetraquarks from a holographic perspective, *Phys. Lett. B* **694**, 252 (2010); Tetraquarks in AdS/QCD, *Proc. Sci.*, QNP (2012) 117 [[arXiv:1206.5745](https://arxiv.org/abs/1206.5745)].
- [41] Z. Guo, T. Liu, and B. Q. Ma, Light-front holographic QCD with generic dilaton profile, *Phys. Rev. D* **93**, 076010 (2016).
- [42] A. Vega and A. Ibaez, Analysis of soft wall AdS / QCD potentials to obtain melting temperature of scalar hadrons, [arXiv:1706.01994](https://arxiv.org/abs/1706.01994).
- [43] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Dilaton in a soft-wall holographic approach to mesons and baryons, *Phys. Rev. D* **85**, 076003 (2012).
- [44] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Light-front holographic QCD and emerging confinement, *Phys. Rep.* **584**, 1 (2015).
- [45] S. Weinberg, Phenomenological Lagrangians, *Physica A (Amsterdam)* **96A**, 327 (1979); J. Gasser and H. Leutwyler, Chiral perturbation theory to one loop, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
- [46] C. Adolph *et al.* (COMPASS Collaboration), Observation of a New Narrow Axial-Vector Meson $a_1(1420)$, *Phys. Rev. Lett.* **115**, 082001 (2015).
- [47] Z. G. Wang, Light axial-vector tetraquark state candidate: $a_1(1420)$, [arXiv:1401.1134](https://arxiv.org/abs/1401.1134).
- [48] H. X. Chen, E. L. Cui, W. Chen, T. G. Steele, X. Liu, and S. L. Zhu, $a_1(1420)$ resonance as a tetraquark state and its isospin partner, *Phys. Rev. D* **91**, 094022 (2015).
- [49] M. Mikhasenko, B. Ketzner, and A. Sarantsev, Nature of the $a_1(1420)$, *Phys. Rev. D* **91**, 094015 (2015).
- [50] J. L. Basdevant and E. L. Berger, Peak Locations and Relative Phase of Different Decay Modes of the a_1 Axial Vector Resonance in Diffractive Production, *Phys. Rev. Lett.* **114**, 192001 (2015).
- [51] X. H. Liu, M. Oka, and Q. Zhao, Searching for observable effects induced by anomalous triangle singularities, *Phys. Lett. B* **753**, 297 (2016).
- [52] F. Aceti, L. R. Dai, and E. Oset, $a_1(1420)$ peak as the $\pi f_0(980)$ decay mode of the $a_1(1260)$, *Phys. Rev. D* **94**, 096015 (2016).
- [53] W. Wang and Z. X. Zhao, Production of a_1 in heavy meson decays, *Eur. Phys. J. C* **76**, 59 (2016).
- [54] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Heavy quark spin structure in Z_b resonances, *Phys. Rev. D* **84**, 054010 (2011).
- [55] E. S. Swanson, Short range structure in the $X(3872)$, *Phys. Lett. B* **588**, 189 (2004).
- [56] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Estimate for the $X(3872) \rightarrow \gamma J/\psi$ decay width, *Phys. Rev. D* **77**, 094013 (2008); $J/\psi\gamma$ and $\psi(2S)\gamma$ decay modes of the $X(3872)$, *J. Phys. G* **38**, 015001 (2011); Y. Dong, A. Faessler, T. Gutsche, S. Kovalenko, and V. E. Lyubovitskij, $X(3872)$ as a hadronic molecule and its decays to charmonium states and pions, *Phys. Rev. D* **79**, 094013 (2009).
- [57] T. Branz, T. Gutsche, and V. E. Lyubovitskij, Hadronic molecule structure of the $Y(3940)$ and $Y(4140)$, *Phys. Rev. D* **80**, 054019 (2009).
- [58] T. Branz, T. Gutsche, and V. E. Lyubovitskij, Hidden-charm and radiative decays of the $Z(4430)$ as a hadronic $D_1\bar{D}^*$ bound state, *Phys. Rev. D* **82**, 054025 (2010).
- [59] C. Patrignani *et al.* (Particle Data Group Collaboration), Review of particle physics, *Chin. Phys. C* **40**, 100001 (2016).