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Heptaquarks with two heavy antiquarks in a simple chromomagnetic model

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We investigate the symmetry property and the stability of the heptaquark containing two identical heavy antiquarks using color-spin interaction. We construct the wave function of the heptaquark from the Pauli exclusion principle in the SU(3) breaking case. The stability of the heptaquark against the strong decay into one baryon and two mesons is discussed in a simple chromomagnetic model. We find that $q^2 s^3 \bar{s}^2$ with $I = 0, S = \frac{5}{2}$ is the most stable heptaquark configuration that could be probed by reconstructing the $\Lambda + \phi + \phi$ invariant mass.

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I. INTRODUCTION

Multiquark hadrons made of more than three quarks became a theme of interest since Jaffe predicted their existence using the bag model [1–3]. Unfortunately, an extensive experimental search ruled out the existence of a deeply bound H-dibaryon, and the initial excitement about the finding of $\Theta^+(1540)$ [4] faded away as further experimental study could not confirm it [5–14].

On the other hand, there is a renewed interest in the subject triggered by the discovery of the $X(3872)$ by the Belle Collaboration in $B^+ \to K^{\pm}\pi^+\pi^+ J/\psi$ [15], which was subsequently confirmed by several other experiments [16–18]. Also, in the dibaryon sector, a resonance structure was finally observed in the $I(J^P) = 0(3^+)$ channel by the WASA-at-COSY Collaboration [19,20]. Furthermore, the LHCb Collaboration has recently observed hidden-charm pentaquark states in the $J/\psi p$ invariant mass spectrum in the $\Lambda_b^0 \to J/\psi K^- p$ process [21]. Subsequently, these states were studied using many theoretical approaches, such as the QCD sum rules [22–24], the molecular approach [25–28], and the quark model [29,30]. These experimental findings led to the interest in the study of multiquark hadron states containing heavy quarks. In fact, recent lattice calculations show that the H-dibaryon becomes bound in the massive pion cases [31,32].

Multiquark configurations with heavy quarks were studied before. Silvestre-Brac and Leandri searched stable q^6, $q^5 Q$, and $q^4 Q Q'$ systems in the framework of a pure chromomagnetic Hamiltonian [33–35]. Heavy pentaquarks with heavy quarks were studied in quark models with color-spin [36,37] and flavor-spin interactions [38,39]. Tetraquark states with two heavy quarks were also found to be stable against strong decay if the heavy quark mass was taken to be sufficiently large [40].

Within the constituent quark model, stable multiquark configurations arise from a large attraction in the color-spin interaction when more light quarks can interact with each other in a compact configuration. However, at the same time, bringing additional quarks into a compact configuration will generate additional kinetic energy compared to having isolated hadrons. If the additional quarks or antiquarks are heavy, the additional kinetic energy can be made small while keeping the enhanced color-spin interaction among light quarks large, which can be effectively understood as additional diquark correlation [41], compared to separated hadrons. To further probe configurations with heavy quarks in yet another multiquark configuration, we will consider heptaquarks with two heavy antiquarks.

Heptaquarks composed of five quarks and two antiquarks have been studied by only a few researchers [42–44]. The authors of Ref. [44] suggested that as long as there is a stable meson state composed of two heavy quarks and two light quarks, there will be a stable heptaquark state within the chiral soliton model. In fact, within a constituent quark model, there will be a stable tetraquark state with two heavy quarks or antiquarks [40]. Hence, such configurations are also part of the configurations to be probed in this work using a constituent quark model with color-spin interaction.

To study the possible existence of compact exotic hadrons, one first has to inspect the configuration with the most attractive color-spin interaction. For example, the color-spin interaction in a H-dibaryon is more attractive than that from the two separate $\Lambda\Lambda$ systems because the former allows for the three most attractive diquark configurations while the later has two separated diquark configurations. The most attractive diquark configuration is the maximally antisymmetric configuration in terms of color $\otimes$ flavor $\otimes$ spin. In terms of two quark configurations, there are four states satisfying the Pauli exclusion principle. We represent them together with the matrix elements for the invariant appearing in the color-spin interaction in Table I, which can be obtained from

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$$-\sum_{i<j}^{N}\lambda_i\lambda_j\sigma_i\cdot\sigma_j = \left[\frac{4}{3}N(N-6)+4I(I+1)+\frac{4}{3}S(S+1)+2C_c\right], \quad (1)$$

where N is the total number of quarks, $C_c=\frac{1}{4}\lambda^2$ the color Casimir operator, S the spin, and I the isospin of the system. Using $C_c=\frac{4}{3},\frac{10}{3}$ for the color antitriplet and sextet, respectively, one notes, as given in the table, that while the most attractive channel has spin 0, the spin 1 state also has an attractive combination. For two antiquarks, we can use the same table simply by replacing $\bar{\mathbf{3}}$ and $\mathbf{6}$ with $\mathbf{3}$ and $\bar{\mathbf{6}}$ for the color and flavor state, respectively. For the quark-antiquark configuration, we can construct a similar table as given in the lower part of Table I. Typically, the color-spin interaction is inversely proportional to the two constituent quark masses involved, $1/(m_i m_j)$. Hence, in forming a multiquark configuration, if the addition involves a light quark and a light antiquark, the addition will just fall apart into a meson state. On the other hand, if the addition is composed of a light quark and a heavy antiquark, it could become energetically favorable to be in a compact configuration. To probe such a possibility systematically, we investigate the symmetry property and the stability of the heptaquark containing two identical heavy antiquarks in a simple chromomagnetic model.

This paper is organized as follows. We first explain why kinetic energy favors compact heptaquarks containing heavy flavors in Sec. II. In Sec. III, we represent the color and spin basis functions of the heptaquark configuration. In Sec. IV, we construct the flavor ⊗ color ⊗ spin part of the wave function of the heptaquark in order to satisfy the Pauli exclusion principle. In Sec. V, we represent the color-spin interaction part of the Hamiltonian. In Sec. VI, we calculate the binding potential of the heptaquark and plot the results

TABLE I. The classification of two quarks and quark-antiquark color-spin interaction, with λ_i and σ_i, respectively, representing the color and spin matrix of the i quark. The two-quark state is determined to satisfy the Pauli exclusion principle. We denote antisymmetric and symmetric state as A and S, respectively. In the parentheses, the multiplet state is represented.

	qq			
Color	$A(\bar{3})$	$S(6)$	$A(\bar{3})$	$S(6)$
Flavor	$A(\bar{3})$	$A(\bar{3})$	$S(6)$	$S(6)$
Spin	$A(1)$	$S(3)$	$S(3)$	$A(1)$
$-\lambda_i\lambda_j\sigma_i\cdot\sigma_j$	-8	$-\frac{4}{3}$	$\frac{8}{3}$	4

	$q\bar{q}$			
Color	(1)	(8)	(1)	(8)
Spin	$A(1)$	$A(1)$	$S(3)$	$S(3)$
$-\lambda_i\lambda_j\sigma_i\cdot\sigma_j$	-16	2	$\frac{16}{3}$	$-\frac{2}{3}$

as a function of the light/heavy quark mass ratio parameter η. Finally, we summarize our results in Sec. VII.

II. WHY HEAVY HEPTAQUARK?

In this work, we investigate the stability of the heptaquark using a hyperfine potential. Even if the hyperfine potential is attractive for a given heptaquark configuration, it cannot form a compact stable state if the additional repulsion from kinetic energy is large. Hence, we need to consider which flavor state makes the additional kinetic energy lower. In the remaining part of this paper, we consider only the hyperfine potential, but we can simply estimate the additional kinetic energy using the following coordinate system and simple Gaussian spatial function.

If we label the five light quarks as ($i=1$–5) and the two heavy antiquarks as ($i=6$, 7), then we can choose the Jacobi coordinate system as follows:

$$\sum_{i=1}^{7}\frac{1}{2}m_i\dot{\mathbf{r}}_i^2-\frac{1}{2}M\dot{\mathbf{r}}_{CM}^2=\sum_{i=1}^{6}\frac{1}{2}M_i\dot{\mathbf{x}}_i^2, \quad \text{where}$$

$$M=\sum_{i=1}^{7}m_i, \qquad M_1=M_2=m_u,$$

$$M_3=M_4=\frac{2m_u m_Q}{m_u+m_Q}, \qquad M_5=\frac{5m_u(m_u+m_Q)}{2(4m_u+m_Q)}$$

$$M_6=\frac{7(m_u+m_Q)(4m_u+m_Q)}{10(5m_u+2m_Q)}$$

$$\mathbf{r}_{CM}=\frac{1}{M}\sum_{i=1}^{7}m_i\mathbf{r}_i \quad (2)$$

$$\mathbf{x}_1=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2)$$

$$\mathbf{x}_2=\sqrt{\frac{2}{3}}\left(\frac{1}{2}\mathbf{r}_1+\frac{1}{2}\mathbf{r}_2-\mathbf{r}_3\right)$$

$$\mathbf{x}_3=\frac{1}{\sqrt{2}}(\mathbf{r}_4-\mathbf{r}_6)$$

$$\mathbf{x}_4=\frac{1}{\sqrt{2}}(\mathbf{r}_5-\mathbf{r}_7)$$

$$\mathbf{x}_5=\sqrt{\frac{6}{5}}\left(\frac{1}{3}\mathbf{r}_1+\frac{1}{3}\mathbf{r}_2+\frac{1}{3}\mathbf{r}_3-\frac{m_u}{m_u+m_Q}\mathbf{r}_4-\frac{m_Q}{m_u+m_Q}\mathbf{r}_6\right)$$

$$\mathbf{x}_6=\sqrt{\frac{10}{7}}\left\{\frac{1}{4m_u+m_Q}(m_u\mathbf{r}_1+m_u\mathbf{r}_2+m_u\mathbf{r}_3+m_u\mathbf{r}_4+m_Q\mathbf{r}_6)-\frac{1}{m_u+m_Q}(m_u\mathbf{r}_5+m_Q\mathbf{r}_7)\right\}. \quad (3)$$

This coordinate system can describe the decay mode of the heptaquark consisting of one baryon and two mesons. In this coordinate system, $\mathbf{x}_1$, $\mathbf{x}_2$ describe the baryon system, while $\mathbf{x}_3$, $\mathbf{x}_4$ represent the relative quark distances for the two mesons, respectively. If we chose a simple Gaussian form as the spatial function, then we can calculate the kinetic energy of the heptaquark as follows:

$$R = e^{-a_1\mathbf{x}_1^2 - a_2\mathbf{x}_2^2 - a_3\mathbf{x}_3^2 - a_4\mathbf{x}_4^2 - a_5\mathbf{x}_5^2 - a_6\mathbf{x}_6^2}, \tag{4}$$

$$T = \sum_{i=1}^{6} \frac{\mathbf{p}_i^2}{2M_i} = \sum_{i=1}^{6} \frac{3\hbar^2}{2M_i} a_i. \tag{5}$$

From the above expression, we can find that the additional kinetic energy of the heptaquark is $\frac{3\hbar^2}{2M_5}a_5 + \frac{3\hbar^2}{2M_6}a_6$, corresponding to the additional kinetic energy from bringing in the two-meson type of quark-antiquark pair into a compact configuration. In the heavy quark limit, $M_6 \to \infty$, making one of the additional kinetic terms zero; hence there is no penalty in the kinetic energy, while the extra light quark might contribute attractively to the pentaquark configuration. On the other hand, if we chose only one antiquark to be heavy, then the reduced masses become $M_5 = m_u$ and $M_6 = \frac{7m_u(m_u+m_Q)}{2(6m_u+m_Q)}$, so that both of the additional terms survive in the heavy quark mass limit. Therefore, we can conclude that if we want to make the additional kinetic energy of the heptaquark sufficiently small, one needs to include at least two heavy quarks. However, one still needs to weigh in the attraction from the color-spin interaction to determine which combination generates the most attractive heptaquark configuration, which is the subject of this work. It should be noted that if we want to make both additional terms zero, then we have to replace one more quark with a heavy quark; such configurations with three heavy quarks, however, are experimentally quite difficult to produce and will not be considered here.

III. COLOR AND SPIN BASIS FUNCTIONS

A. Color basis function

We can represent the color state of the heptaquark using color decomposition in the $SU(3)$ fundamental representation as follows:

$$\begin{aligned}
&(\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3})\otimes(\mathbf{3}\otimes\bar{\mathbf{3}})\otimes(\mathbf{3}\otimes\bar{\mathbf{3}})\\
&\quad=(\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10})\otimes(\mathbf{1}\oplus\mathbf{8})\otimes(\mathbf{1}\oplus\mathbf{8})\\
&\quad=(\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10})\\
&\qquad\otimes(\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}\oplus\bar{\mathbf{10}}\oplus\mathbf{27}).
\end{aligned} \tag{6}$$

Among the above states, two $(\mathbf{1}\otimes\mathbf{1})$, eight $(\mathbf{8}\otimes\mathbf{8})$, and one $(\mathbf{10}\otimes\bar{\mathbf{10}})$ state can form the color singlet state. Therefore, there are 11 color basis functions for the heptaquark. However, there is a more efficient way to represent the color basis of the heptaquark using the Young-Yamanouchi basis.

The color state of two antiquarks is a triplet or an antisextet. Therefore, in order to construct the color singlet heptaquark state, the color state of five quarks should be an antitriplet or a sextet. For five quarks, the numbers of the Young-Yamanouchi basis of antitriplets and sextets are five and six, respectively. Therefore, we can represent 11 color basis functions for heptaquark as follows:

$$\begin{aligned}
&|C_1\rangle = \left(\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5\\\cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6}\\\hline \bar{7}\\\hline\end{array}\right),\;
|C_2\rangle = \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline 5\\\cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6}\\\hline \bar{7}\\\hline\end{array}\right),\;
|C_3\rangle = \left(\begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline 4\\\cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6}\\\hline \bar{7}\\\hline\end{array}\right),\;
|C_4\rangle = \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline 4\\\cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6}\\\hline \bar{7}\\\hline\end{array}\right),\;
|C_5\rangle = \left(\begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3\\\cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6}\\\hline \bar{7}\\\hline\end{array}\right),\\
&|C_6\rangle = \left(\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1} 5\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right),\;
|C_7\rangle = \left(\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1} 5\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right),\;
|C_8\rangle = \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1} 5\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right),\;
|C_9\rangle = \left(\begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right),\\
&|C_{10}\rangle = \left(\begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right),\;
|C_{11}\rangle = \left(\begin{array}{|c|c|c|}\hline 1&4&5\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\\hline\end{array}\right).
\end{aligned} \tag{7}$$

In the Appendix A, we present the color basis of the heptaquark using the tensor notation. The expectation value of all the color operators for the heptaquarks can be obtained using this color basis.

B. Spin basis function

Seven-quark systems can have spin $\frac{7}{2}, \frac{5}{2}, \frac{3}{2}$, and $\frac{1}{2}$. We represent the spin basis functions in terms of the Young-Yamanouchi basis for each spin value.

(i) $S=\frac{7}{2}$: One basis function with Young tableau [7],

$$|S_1^{\frac{7}{2}}\rangle = \young(1234567) \tag{8}$$

(ii) $S=\frac{5}{2}$: Six basis functions with Young tableau [6,1],

$$\begin{aligned} |S_1^{\frac{5}{2}}\rangle &= \young(123456,7), \quad |S_2^{\frac{5}{2}}\rangle = \young(123457,6), \\ |S_3^{\frac{5}{2}}\rangle &= \young(123467,5), \quad |S_4^{\frac{5}{2}}\rangle = \young(123567,4), \\ |S_5^{\frac{5}{2}}\rangle &= \young(124567,3), \quad |S_6^{\frac{5}{2}}\rangle = \young(134567,2). \end{aligned} \tag{9}$$

(iii) $S=\frac{3}{2}$: 14 basis functions with Young tableau [5,2],

$$\begin{aligned} |S_1^{\frac{3}{2}}\rangle &= \young(12345,67), \quad |S_2^{\frac{3}{2}}\rangle = \young(12346,57), \quad |S_3^{\frac{3}{2}}\rangle = \young(12356,47), \\ |S_4^{\frac{3}{2}}\rangle &= \young(12456,37), \quad |S_5^{\frac{3}{2}}\rangle = \young(13456,27), \quad |S_6^{\frac{3}{2}}\rangle = \young(12347,56), \\ |S_7^{\frac{3}{2}}\rangle &= \young(12357,46), \quad |S_8^{\frac{3}{2}}\rangle = \young(12457,36), \quad |S_9^{\frac{3}{2}}\rangle = \young(13457,26), \\ |S_{10}^{\frac{3}{2}}\rangle &= \young(12367,45), \quad |S_{11}^{\frac{3}{2}}\rangle = \young(12467,35), \quad |S_{12}^{\frac{3}{2}}\rangle = \young(13467,25), \\ |S_{13}^{\frac{3}{2}}\rangle &= \young(12567,34), \quad |S_{14}^{\frac{3}{2}}\rangle = \young(13567,24). \end{aligned} \tag{10}$$

(iv) $S=\frac{1}{2}$: 14 basis functions with Young tableau [4,3],

$$\begin{aligned} |S_1^{\frac{1}{2}}\rangle &= \young(1234,567), \quad |S_2^{\frac{1}{2}}\rangle = \young(1235,467), \quad |S_3^{\frac{1}{2}}\rangle = \young(1245,367), \\ |S_4^{\frac{1}{2}}\rangle &= \young(1345,267), \quad |S_5^{\frac{1}{2}}\rangle = \young(1236,457), \quad |S_6^{\frac{1}{2}}\rangle = \young(1246,357), \\ |S_7^{\frac{1}{2}}\rangle &= \young(1346,257), \quad |S_8^{\frac{1}{2}}\rangle = \young(1256,347), \quad |S_9^{\frac{1}{2}}\rangle = \young(1356,247), \\ |S_{10}^{\frac{1}{2}}\rangle &= \young(1237,456), \quad |S_{11}^{\frac{1}{2}}\rangle = \young(1247,356), \quad |S_{12}^{\frac{1}{2}}\rangle = \young(1347,256), \\ |S_{13}^{\frac{1}{2}}\rangle &= \young(1257,346), \quad |S_{14}^{\frac{1}{2}}\rangle = \young(1357,246). \end{aligned} \tag{11}$$

IV. WAVE FUNCTION

There are two ways of constructing the wave function of the heptaquark. First, we can consider the flavor state of five quarks in SU(3) flavor symmetry. In our previous work [45], we have already classified all the possible flavor states for five light quarks. There are five possible flavor states for five quarks as follows:

$$\begin{aligned} [\bar{3}]_F &= \yng(2,2,1), \quad [6]_F = \yng(3,1,1), \quad [\overline{15}]_F = \yng(3,2), \\ [24]_F &= \yng(4,1), [21]_F = \yng(5). \end{aligned} \tag{12}$$

For a given isospin and spin, we can choose the possible flavor states and construct the remaining part of the wave function using color and spin symmetry. In this description, the wave function should be antisymmetric for five light quarks and for two heavy antiquarks, respectively.

Second, we can calculate the wave function of the heptaquark in the SU(3) breaking case, fixing the position of strange quarks. In the SU(3) flavor symmetry breaking case, we have to construct the wave function to be antisymmetric separately for the u, d quarks, s quarks, and two heavy antiquarks, due to the Pauli exclusion principle.

Both approaches can be shown to give the same result [46]. In this work, we follow the second method for convenience of calculation. To do this, we assume the spatial function to be symmetric such that the rest of the wave function represented by flavor ⊗ color ⊗ spin should be antisymmetric. Using the color-spin coupling scheme [45,47], we represent the wave function of the heptaquark by flavor ⊗ color-spin coupling basis.

A. $q^5\bar{Q}^2$: $\{12345\}\{67\}$

Here, we calculate the wave function of the heptaquark in the flavor $SU(3)$ breaking case and fix the position of each quark on $q(1)q(2)q(3)q(4)q(5)\bar{Q}(6)\bar{Q}(7)$. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry $\{12345\}\{67\}$:

$$\psi_{FCS} = \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline 4\\ \hline 5\\ \hline \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{FCS} \tag{13}$$

(i) $I = \frac{5}{2}$

$$\psi_{FCS} = \left(\begin{array}{|c|c|c|c|c|}\hline 1&2&3&4&5\\ \hline \end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline \end{array} \right)_F \otimes \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline 4\\ \hline 5\\ \hline \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{CS} = F_1 \otimes CS_1 \tag{14}$$

(ii) $I = \frac{3}{2}$

$$\begin{aligned} \psi_{FCS} = \frac{1}{2}\Bigg\{ & \left(\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\ \hline 5\\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline \end{array} \right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&5\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} 4\\ \cline{1-1} \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{CS} - \left(\begin{array}{|c|c|c|c|}\hline 1&2&3&5\\ \hline 4\\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline \end{array} \right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&4\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} 5\\ \cline{1-1} \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{CS} \\ & + \left(\begin{array}{|c|c|c|c|}\hline 1&2&4&5\\ \hline 3\\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline \end{array} \right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&3\\ \hline 2\\ \cline{1-1} 4\\ \cline{1-1} 5\\ \cline{1-1} \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{CS} - \left(\begin{array}{|c|c|c|c|}\hline 1&3&4&5\\ \hline 2\\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline \end{array} \right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&2\\ \hline 3\\ \cline{1-1} 4\\ \cline{1-1} 5\\ \cline{1-1} \end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline \end{array} \right)_{CS} \Bigg\} \\ = & \frac{1}{2}(F_1 \otimes CS_4 - F_2 \otimes CS_3 + F_3 \otimes CS_2 - F_4 \otimes CS_1) \end{aligned} \tag{15}$$

(iii) $I = \frac{1}{2}$

$$\begin{aligned}\psi_{FCS} = \frac{1}{\sqrt{5}}\Bigg\{ & \left(\begin{ytableau}1&2&3\\4&5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&4\\2&5\\3\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} - \left(\begin{ytableau}1&2&4\\3&5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&3\\2&5\\4\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} + \left(\begin{ytableau}1&3&4\\2&5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&2\\3&5\\4\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} \\ & + \left(\begin{ytableau}1&2&5\\3&4\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&3\\2&4\\5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} - \left(\begin{ytableau}1&3&5\\2&4\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&2\\3&4\\5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} \Bigg\} \\ = {} & \frac{1}{\sqrt{5}}(F_1 \otimes CS_5 - F_2 \otimes CS_4 + F_3 \otimes CS_3 + F_4 \otimes CS_2 - F_5 \otimes CS_1)\end{aligned} \tag{16}$$

B. $q^4 s\bar{Q}^2$: {1234}5{67}

We fix the position of each quark on $q(1)q(2)q(3)q(4)s(5)\bar{Q}(6)\bar{Q}(7)$. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry $\{1234\}5\{67\}$ because there is no symmetry between the u,d quarks and s quark in the $SU(3)$ breaking case:

$$\psi_{FCS} = \left(\begin{ytableau}1\\2\\3\\4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{FCS} \tag{17}$$

(i) $I = 2$

$$\psi_{FCS} = \left(\begin{ytableau}1&2&3&4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1\\2\\3\\4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} = F_1 \otimes CS_1 \tag{18}$$

(ii) $I = 1$

$$\begin{aligned}\psi_{FCS} = \frac{1}{\sqrt{3}}\Bigg\{ & \left(\begin{ytableau}1&2&3\\4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&4\\2\\3\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} - \left(\begin{ytableau}1&2&4\\3\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&3\\2\\4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} \\ & + \left(\begin{ytableau}1&3&4\\2\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&2\\3\\4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} \Bigg\} \\ = {} & \frac{1}{\sqrt{3}}(F_1 \otimes CS_3 - F_2 \otimes CS_2 + F_3 \otimes CS_1)\end{aligned} \tag{19}$$

(iii) $I = 0$

$$\begin{aligned}\psi_{FCS} = \frac{1}{\sqrt{2}}\Bigg\{ & \left(\begin{ytableau}1&2\\3&4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&3\\2&4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} - \left(\begin{ytableau}1&3\\2&4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}&\bar{7}\end{ytableau}\right)_F \otimes \left(\begin{ytableau}1&2\\3&4\end{ytableau}, \begin{ytableau}5\end{ytableau}, \begin{ytableau}\bar{6}\\\bar{7}\end{ytableau}\right)_{CS} \Bigg\} \\ = {} & \frac{1}{\sqrt{2}}(F_1 \otimes CS_2 - F_2 \otimes CS_1)\end{aligned} \tag{20}$$

C. $q^3s^2\bar{Q}^2$: {123}{45}{67}

We fix the position of each quark on $q(1)q(2)q(3)s(4)s(5)\bar{Q}(6)\bar{Q}(7)$. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry $\{123\}\{45\}\{67\}$:

$$\psi_{FCS} = \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{FCS} \tag{21}$$

(i) $I = \frac{3}{2}$

$$\psi_{FCS} = \left(\begin{array}{|c|c|c|}\hline 1&2&3\\ \hline\end{array}, \begin{array}{|c|c|}\hline 4&5\\ \hline\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline\end{array}\right)_F \otimes \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{CS} = F_1 \otimes CS_1 \tag{22}$$

(ii) $I = \frac{1}{2}$

$$\begin{aligned}\psi_{FCS} &= \frac{1}{\sqrt{2}}\left\{\left(\begin{array}{|c|c|}\hline 1&2\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline 4&5\\ \hline\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline\end{array}\right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&3\\ \hline 2\\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{CS} - \left(\begin{array}{|c|c|}\hline 1&3\\ \hline 2\\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline 4&5\\ \hline\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline\end{array}\right)_F \otimes \left(\begin{array}{|c|c|}\hline 1&2\\ \hline 3\\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{CS}\right\} \\ &= \frac{1}{\sqrt{2}}(F_1 \otimes CS_2 - F_2 \otimes CS_1)\end{aligned} \tag{23}$$

D. $s^3q^2\bar{Q}^2$: {123}{45}{67}

We fix the position of each quark on $s(1)s(2)s(3)q(4)q(5)\bar{Q}(6)\bar{Q}(7)$ for convenience in calculation. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry $\{123\}\{45\}\{67\}$:

$$\psi_{FCS} = \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{FCS} \tag{24}$$

(i) $I = 1$: The wave function is the same as in the case of $q^3s^2\bar{Q}^2$ with $I = \frac{3}{2}$.
(ii) $I = 0$

$$\begin{aligned}\psi_{FCS} &= \left(\begin{array}{|c|c|c|}\hline 1&2&3\\ \hline\end{array}, \begin{array}{|c|}\hline 4\\ \hline 5\\ \hline\end{array}, \begin{array}{|c|c|}\hline \bar{6}&\bar{7}\\ \hline\end{array}\right)_F \otimes \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}, \begin{array}{|c|c|}\hline 4&5\\ \hline\end{array}, \begin{array}{|c|}\hline \bar{6}\\ \hline \bar{7}\\ \hline\end{array}\right)_{CS} \\ &= F_1 \otimes CS_1\end{aligned} \tag{25}$$

E. $s^4q\bar{Q}^2$: {1234}5{67}

We fix the position of each quark on $s(1)s(2)s(3)s(4)q(5)\bar{Q}(6)\bar{Q}(7)$. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry $\{1234\}\ 5\ \{67\}$:

$$\psi_{FCS} = \left(\begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ \boxed{4} \end{array}, \boxed{5}, \begin{array}{c} \boxed{\bar{6}} \\ \boxed{\bar{7}} \end{array} \right)_{FCS} \tag{26}$$

(i) $I = \frac{1}{2}$: The wave function is the same as in the case of $q^4 s \bar{Q}^2$ with $I = 2$.

F. $s^5\bar{Q}^2$: {12345}{67}

We fix the position of each quark on $s(1)s(2)s(3)\times s(4)s(5)\bar{Q}(6)\bar{Q}(7)$. In this case, the flavor ⊗ color ⊗ spin wave function should satisfy the symmetry {12345}{67}:

$$\psi_{FCS} = \left(\begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ \boxed{4} \\ \boxed{5} \end{array}, \begin{array}{c} \boxed{\bar{6}} \\ \boxed{\bar{7}} \end{array} \right)_{FCS} \tag{27}$$

TABLE II. All the possible heptaquark states containing two heavy antiquarks with the corresponding multiplicity. M represents the multiplicity of the color ⊗ flavor ⊗ spin state.

	Isospin	Spin	M		Isospin	Spin	M
$q^5\bar{Q}^2$	$\frac{5}{2}$	$\frac{3}{2}$	1	$q^3s^2\bar{Q}^2$	$\frac{3}{2}$	$\frac{7}{2}$	1
		$\frac{1}{2}$	1			$\frac{5}{2}$	3
	$\frac{3}{2}$	$\frac{5}{2}$	1			$\frac{3}{2}$	8
		$\frac{3}{2}$	3			$\frac{1}{2}$	7
		$\frac{1}{2}$	3		$\frac{1}{2}$	$\frac{7}{2}$	1
	$\frac{1}{2}$	$\frac{7}{2}$	1			$\frac{5}{2}$	5
		$\frac{5}{2}$	3			$\frac{3}{2}$	13
		$\frac{3}{2}$	4			$\frac{1}{2}$	13
		$\frac{1}{2}$	3	$q^2s^3\bar{Q}^2$	1	$\frac{7}{2}$	1
$q^4s\bar{Q}^2$	2	$\frac{5}{2}$	1			$\frac{5}{2}$	3
		$\frac{3}{2}$	4			$\frac{3}{2}$	8
		$\frac{1}{2}$	4			$\frac{1}{2}$	7
	1	$\frac{7}{2}$	1		0	$\frac{5}{2}$	3
		$\frac{5}{2}$	5			$\frac{3}{2}$	6
		$\frac{3}{2}$	10			$\frac{1}{2}$	7
		$\frac{1}{2}$	10	$qs^4\bar{Q}^2$	$\frac{1}{2}$	$\frac{5}{2}$	1
	0	$\frac{7}{2}$	1			$\frac{3}{2}$	4
		$\frac{5}{2}$	3			$\frac{1}{2}$	4
		$\frac{3}{2}$	7	$s^5\bar{Q}^2$	0	$\frac{5}{2}$	1
		$\frac{1}{2}$	6			$\frac{3}{2}$	3
						$\frac{1}{2}$	3

(i) $I = 0$: The wave function is the same as in the case of $q^5\bar{Q}^2$ with $I = \frac{5}{2}$.

We show all the possible heptaquark states for each flavor, isospin, and spin with the corresponding multiplicity in Table II.

V. COLOR-SPIN INTERACTION

In this article, we investigate the stability of the heptaquark configurations using the hyperfine potential given as

$$H = -A \sum_{i<j} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j, \tag{28}$$

where m_i's are the constituent quark masses, and $\lambda_i^c/2$ are the color operator of the ith quark for the color SU (3), and A is taken to be a constant determined from its contribution to the proton mass using the comprehensive Hamiltonian [45]. The expectation value of the hyperfine potential of a proton is approximately -160 MeV. Since $-\sum \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$ for a proton is -8, we extract the value $A/m_u^2 = 20$ MeV. While this value depends on the wave function of a multiquark state, we take this value to search for possible stable multiquark configurations that can potentially be stable against strong decays. In this work, for a given flavor and quantum number of a heptaquark configuration, we calculate the matrix elements for Eq. (28) for all possible color-spin flavor bases and then diagonalize the matrix to obtain the configuration with the lowest hyperfine interaction strength.

VI. RESULTS

A heptaquark can decay into one baryon and two mesons. The differences in the confining and coulomb potentials are proportional to the two-body color force $\lambda_i \lambda_j$. As far as the compact heptaquark, baryon, and mesons are taken to occupy the same size, the difference in these energies between the heptaquark and the sum of the baryon and two mesons are negligible. This is so because if the heptaquark, baryon, and mesons have the same size, the confining potential will just be proportional to a common value and the sum of all the two-body interactions $\sum \lambda_i \lambda_j$, which are equal to $-56/3$, -8, $-16/3$ for the heptaquark, baryon, and meson, respectively. The main difference comes from the difference in the color-spin potential. Therefore, we define the binding potential of the heptaquark as the difference in the color-spin interaction between the heptaquark and the sum of the baryon and two mesons as follows:

$$V_B = H_{\text{heptaquark}} - H_{\text{baryon}} - H_{\text{meson1}} - H_{\text{meson2}}. \tag{29}$$

To search for possible stable configurations, we plot the binding potential of the heptaquark as a function of the heavy quark mass using a variable η defined as follows:

$$\eta = 1 - \frac{m_u}{m_Q}. \quad (30)$$

Here, we fix the strange quark mass to 632 MeV, which comes from our previous work [45]. When the flavor of antiquarks is strange, charm, and bottom, η values are approximately 0.46, 0.82, and 0.93, respectively. Decay channels that give the lowest potential can change as η varies. Hence, in some figures, there are graphs with turning points that have a sudden change in the slope.

A. $q^5\bar{Q}^2$: {12345}{67}

As we can see in Figs. 1–3, there is no possibility of a stable heptaquark except $I = \frac{1}{2}$ and $S = \frac{3}{2}$ when the antiquarks are $\bar{u}$ or $\bar{d}$ quarks. However, the absolute value of the binding potential is very small, so it cannot be compact when we consider the total Hamiltonian including the kinetic term.

B. $q^4s\bar{Q}^2$: {1234}5{67}

In the case of $q^4s\bar{Q}^2$ with $I = 2$ in Fig. 4, there is no possibility of a stable heptaquark. However, in the case with $I = 1$ and $S = \frac{5}{2}$ in Fig. 5, there can be a stable heptaquark when the antiquarks are light quarks. But, the absolute value of the binding potential is still small. In contrast, as can be seen in Fig. 6, the heptaquark configuration with $I = 0$ and $S = \frac{1}{2}, \frac{3}{2}$ can be stable when the mass of antiquarks becomes very large.

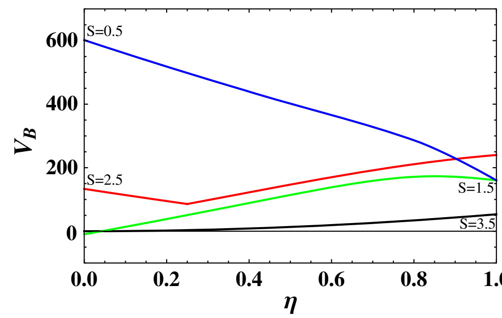

FIG. 3. V_B of $q^5\bar{Q}^2$ with $I = \frac{1}{2}$ (units of MeV).

C. $q^3s^2\bar{Q}^2$: {123}{45}{67}

The heptaquark containing two strange quarks with $I = \frac{3}{2}$ in Fig. 7 shows no possibility of a stable heptaquark. In the case of $q^3s^2\bar{Q}^2$ with $I = \frac{1}{2}$ and $S = \frac{5}{2}$, as shown in Fig. 8, there is a configuration with a slight negative binding potential when the antiquarks are light quarks. Furthermore, for $I = \frac{1}{2}$ and $S = \frac{1}{2}, \frac{3}{2}$ configurations, the potential becomes attractive when the mass of antiquarks becomes very large. We represent the expectation values of the hyperfine potential for the heptaquark

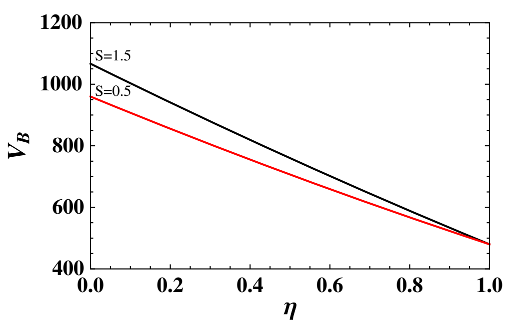

FIG. 1. V_B of $q^5\bar{Q}^2$ with $I = \frac{5}{2}$ (units of MeV).

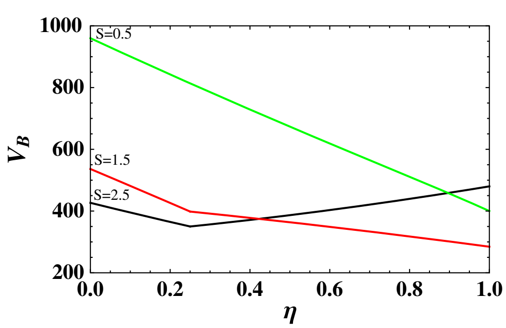

FIG. 2. V_B of $q^5\bar{Q}^2$ with $I = \frac{3}{2}$ (units of MeV).

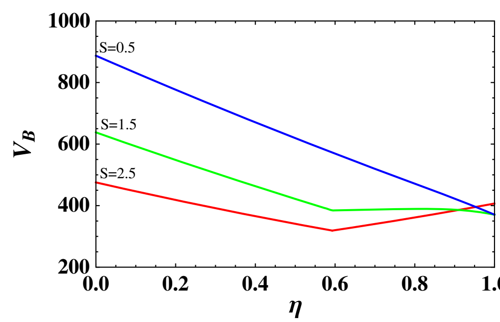

FIG. 4. V_B of $q^4s\bar{Q}^2$ with $I = 2$ (units of MeV).

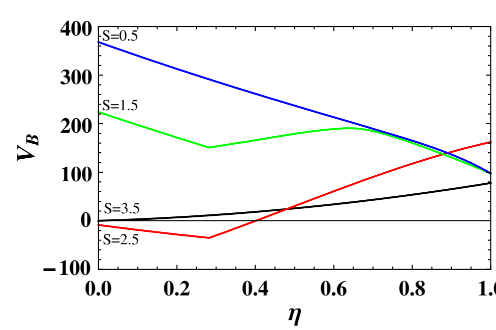

FIG. 5. V_B of $q^4s\bar{Q}^2$ with $I = 1$ (units of MeV).

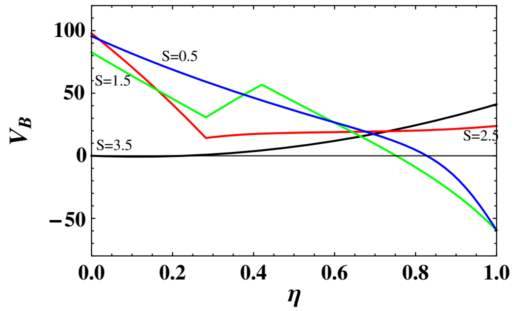

FIG. 6. V_B of $q^4 s\bar{Q}^2$ with $I = 0$ (units of MeV).

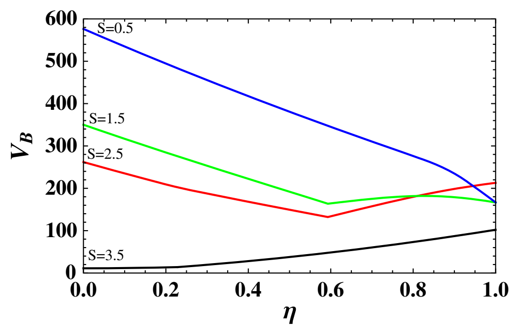

FIG. 7. V_B of $q^3 s^2 \bar{Q}^2$ with $I = \frac{3}{2}$ (units of MeV).

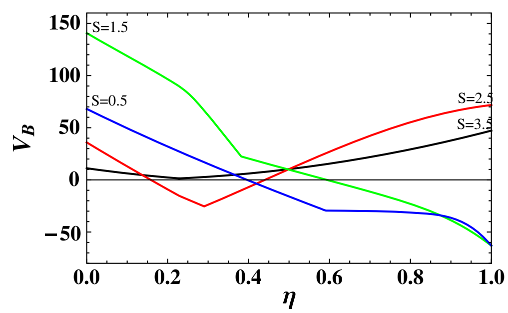

FIG. 8. V_B of $q^3 s^2 \bar{Q}^2$ with $I = \frac{1}{2}$ (units of MeV).

configuration with $S = \frac{1}{2}$ and the lowest decay mode in Table III. We take the charm quark mass to be 1930 MeV as extracted from fits to the heavy baryon masses using the variational method [45]. For the $q^3 s^2 \bar{s}^2$ $(I = \frac{1}{2}, S = \frac{1}{2})$ case, there is an additional interaction between the u quarks as compared to the isolated baryon meson states. However, the strength of the interaction between the u quarks and s quarks is reduced. When the antiquarks are heavy quarks, there is an additional repulsion between the s quarks, while there is also an additional attraction between the u quarks and s quarks, making the binding potential negative.

As we mentioned in the Introduction, there is a possibility of a stable heptaquark state as long as there is a stable meson state composed of two heavy quarks and two light quarks within the chiral soliton model [44]. It is well known that T_{cc} with $J^P = 1^+$, $I = 0$, could be a stable tetraquark state [40,41]. Therefore, taking the result in Ref. [44] to be valid, there should be a stable configuration composed of five light quarks and two heavy antiquarks with $S = \frac{3}{2}$ or $S = \frac{1}{2}$ and $I = \frac{1}{2}$. It should be noted that although our results for $q^5 \bar{Q}^2$ with $I = \frac{1}{2}$ do not support a stable heptaquark, the configuration with $q^3 s^2 \bar{Q}^2$ with $S = \frac{3}{2}, \frac{1}{2}$ and $I = \frac{1}{2}$ indeed may be a stable heptaquark state. As we can see in Table II, since these two states have large multiplicities compared to the other states, it may lead to a low binding potential.

TABLE III. The expectation values of the hyperfine potential divided by constant factor A for $q^3 s^2 \bar{Q}^2$ $(I = \frac{1}{2}, S = \frac{1}{2})$ and the corresponding lowest decay mode.

Heptaquark	The lowest decay mode
$q^3 s^2 \bar{s}^2$ $(I = \frac{1}{2}, S = \frac{1}{2})$	$\Xi + K + K$
$-\frac{2.51}{m_u^2} - \frac{34.22}{m_u m_s} - \frac{5.98}{m_s^2}$	$-\frac{128}{3 m_u m_s} + \frac{8}{3 m_s^2}$
$q^3 s^2 \bar{c}^2$ $(I = \frac{1}{2}, S = \frac{1}{2})$	$\Lambda + D + D_s$
$-\frac{4.3}{m_u^2} - \frac{11.7}{m_u m_s} + \frac{3.01}{m_s^2}$	$-\frac{8}{m_u^2} - \frac{16}{m_u m_c} - \frac{16}{m_s m_c}$
$-\frac{15.04}{m_u m_c} - \frac{16.99}{m_s m_c} + \frac{3.07}{m_c^2}$	

D. $s^3 q^2 \bar{Q}^2$: {123}{45}{67}

In this study, the heptaquark with three strange quarks and two antiquarks leads to the most stable configuration. In the case of $s^3 q^2 \bar{Q}^2$ with $I = 1$ in Fig. 9, there is no possibility of a stable heptaquark. As we can see in Fig. 10, however, there is a large negative binding energy with $I = 0$ and $S = \frac{5}{2}$. In Table IV, we can see that there is a considerable amount of additional attraction between the u and s quarks for $s^3 q^2 \bar{s}^2$ $(I = 0, S = \frac{5}{2})$ compared to the corresponding lowest decay mode. However, when the antiquarks are heavy quarks, there is an additional repulsion between the s quarks, so it makes the binding potential smaller.

In Table V, we present the additional kinetic energy and binding potential of the heptaquark for the two most stable cases. Here, we calculate the additional kinetic energy in

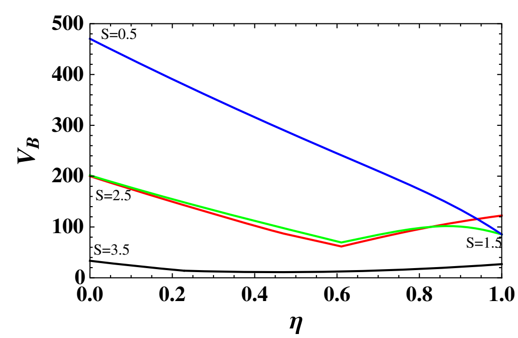

FIG. 9. V_B of $s^3 q^2 \bar{Q}^2$ with $I = 1$ (units of MeV).

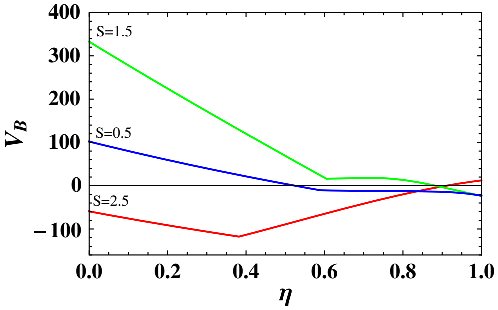

FIG. 10. V_B of $s^3q^2\bar{Q}^2$ with $I=0$ (units of MeV).

Eq. (4) with $a_5 = a_6 = 2.5\ \mathrm{fm}^{-2}$, which assumes that the interquark distance of a heptaquark is similar to that of a proton. As we can see in the table, when the antiquark is a heavy quark, the additional kinetic energy is reduced for both cases.

For the $q^3s^2\bar{Q}^2$ $(I=\frac{1}{2}, S=\frac{1}{2})$ case, when the antiquarks are heavy quarks, the binding potential is also reduced. However, the additional kinetic energy is still much larger than the absolute value of the binding potential.

For the $q^2s^3\bar{Q}^2$ $(I=0, S=\frac{5}{2})$ case, when the antiquarks are light quarks, the expectation value of the binding potential is largest and becomes smaller when the antiquarks are heavy quarks. This is so because the interaction between u, d quarks and antiquarks is reduced due to the $1/m$ factor. As sizable repulsion comes from the interaction between the two strange quarks for this quantum number, replacing the strange quark with the heavy quark might lead to a stable heptaquark state.

It should be noted, however, that the numbers for the additional kinetic energy shown in Table V are obtained assuming that one brings the additional quarks into a compact size of around $\langle r^2\rangle^{1/2} = a_4^{-1/2} \sim 0.632$ fm. Assuming that the size becomes larger by a factor of 2, the additional kinetic energy would be reduced by a factor of 4. Then the $q^3s^2\bar{b}^2$ $(I=\frac{1}{2}, S=\frac{1}{2})$ and the $q^2s^3\bar{s}^2$ $(I=0, S=\frac{5}{2})$ configurations could become stable. These states will have masses of around 11949 MeV and 3572 MeV, respectively, within our model. In particular, the $q^2s^3\bar{s}^2$ $(I=0, S=\frac{5}{2})$ state could decay into $\Lambda + \phi + \phi$, which is easy to reconstruct. The exact values of the mass and the additional kinetic energy depend on the model employed as we explain the case for the MIT bag model in the Appendix B. However, the attraction coming from the color-spin interaction will be common to all models and hence the most attractive configurations will point to the possible stable heptaquark state.

TABLE IV. The expectation values of the hyperfine potential divided by constant factor A for $s^3q^2\bar{Q}^2$ $(I=0, S=\frac{5}{2})$ and the corresponding lowest decay mode.

Heptaquark	The lowest decay mode
$s^3q^2\bar{s}^2$ $(I=0, S=\frac{5}{2})$	$\Lambda+\phi+\phi$
$-\frac{5.97}{m_u^2} - \frac{12.16}{m_u m_s} + \frac{9.28}{m_s^2}$	$-\frac{8}{m_u^2} + \frac{32}{3m_s^2}$
$s^3q^2\bar{c}^2$ $(I=0, S=\frac{5}{2})$	$\Lambda + D_s^* + D_s^*$
$-\frac{6.83}{m_u^2} - \frac{4.71}{m_u m_s} + \frac{8.25}{m_s^2}$	$-\frac{8}{m_u^2} + \frac{32}{3m_s m_c}$
$-\frac{4.24}{m_u m_c} - \frac{1.32}{m_s m_c} + \frac{2.74}{m_c^2}$	

TABLE V. The additional kinetic energy (ΔK) and binding potential (V_B) of the heptaquark. The first table is for $q^3s^2\bar{Q}^2$ $(I=\frac{1}{2}, S=\frac{1}{2})$ and the second one is for $q^2s^3\bar{Q}^2$ $(I=0, S=\frac{5}{2})$. In the third table, we represent the parameters used to calculate the additional kinetic energy. The units of ΔK and V_B are MeV.

$I=\frac{1}{2}, S=\frac{1}{2}$	$q^3s^2\bar{s}^2$		$q^3s^2\bar{c}^2$		$q^3s^2\bar{b}^2$	
ΔK	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$
	388.75	294.73	210.03	139.51	163.97	65.08
V_B	−9.84		−31.75		−40.69	

$I=0, S=\frac{5}{2}$	$q^2s^3\bar{s}^2$		$q^2s^3\bar{c}^2$		$q^2s^3\bar{b}^2$	
ΔK	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$	$\frac{3\hbar^2}{2M_5}a_5$	$\frac{3\hbar^2}{2M_6}a_6$
	271.57	245.82	201.34	135.18	162.46	63.89
V_B	−98.84		−16		3.13	

m_u	m_s	m_c	m_b	a_5	a_6
343 MeV	632 MeV	1930 MeV	5305 MeV	2.5 fm^{-2}	2.5 fm^{-2}

Another interesting possibility is that the string tension in the compact heptaquark configuration will be smaller than those in the usual hadrons. Such a possibility has been discussed in Ref. [48] in relation to a stable dibaryon. The nonperturbative gauge field configuration for generating the confining potential may change in the presence of other color sources and lead to a smaller string tension in a heptaquark or dibaryon configuration. In such cases, even if the heptaquark, baryon, and mesons have the same size, the contributions from the confining potential in the heptaquark will be smaller than the sum of the baryon and mesons. Furthermore, due to a smaller string tension, the wave function of the heptaquark will be more extended, leading to a smaller additional kinetic energy. These two effects could lead to a more stable and strongly bound heptaquark configuration.

E. $s^4q\bar{Q}^2$: {1234}5{67}

The wave function of $s^4q\bar{Q}^2$ is the same as $q^4s\bar{Q}^2$ with $I=2$. The only difference is the mass factor in the hyperfine potential. As we can see in Fig. 11, there is no stable heptaquark with four strange quarks. Additionally, the plot of $s^5\bar{Q}^2$ is the same as $q^5\bar{Q}^2$ with $I=\frac{5}{2}$.

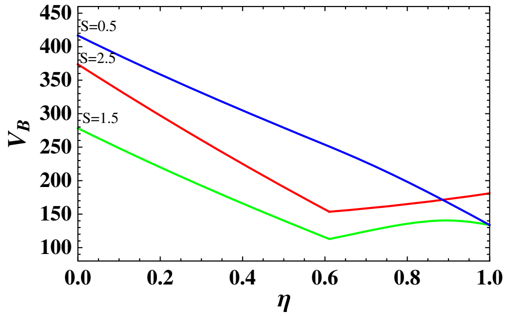

FIG. 11. V_B of $s^4 q\bar{Q}^2$ with $I = \frac{1}{2}$ (units of MeV).

VII. SUMMARY

In this work, we investigated the symmetry property and the stability of the heptaquark containing two identical heavy antiquarks. We constructed the flavor ⊗ color ⊗ spin wave function satisfying the Pauli principle in the flavor SU(3) breaking case. We then searched for the heptaquark configuration with the lowest color-spin interaction, and found that the $s^3 q^2 \bar{s}^2$ configuration with $I = 0$, $S = \frac{5}{2}$ is the most stable state. For this quantum number, when seven quarks form a compact configuration, the interaction between u and d quarks is reduced compared to that in the Λ, but the additional interaction between the light quarks and the s quarks results in the additional attracting that could make the heptaquark state stable. This state could be probed by reconstructing the $\Lambda + \phi + \phi$ invariant mass or by its weak decay products if it is strongly bound.

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APPENDIX A: COLOR BASIS OF THE HEPTAQUARK

Here, we present the color basis of the heptaquark using the tensor form. The expectation value of all the color operators for the heptaquarks can be obtained using this basis:

$$|C_1\rangle = \frac{1}{\sqrt{6}}\left\{-\frac{\sqrt{3}}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(3)\varepsilon^{lmn}q^m(4)q^n(5) + \frac{1}{4\sqrt{6}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(4)\varepsilon^{lmn}q^m(3)q^n(5)\right.$$
$$-\frac{1}{2\sqrt{6}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(5)\varepsilon^{lmn}q^m(3)q^n(4) - \frac{1}{2\sqrt{6}}\varepsilon^{ijk}q^i(1)q^j(3)q^k(4)\varepsilon^{lmn}q^m(2)q^n(5)$$
$$\left.+\frac{1}{\sqrt{6}}\varepsilon^{ijk}q^i(1)q^j(3)q^k(4)\varepsilon^{lmn}q^m(2)q^n(5)\right\}\varepsilon_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$|C_2\rangle = \frac{1}{\sqrt{6}}\left\{\frac{1}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(3)\varepsilon^{lmn}q^m(4)q^n(5) - \frac{1}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(4)\varepsilon^{lmn}q^m(3)q^n(5)\right.$$
$$\left.+\frac{1}{2\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(5)\varepsilon^{lmn}q^m(3)q^n(4)\right\}\varepsilon_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$|C_3\rangle = \frac{1}{\sqrt{6}}\left\{\frac{1}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(3)\varepsilon^{lmn}q^m(4)q^n(5) - \frac{1}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(4)\varepsilon^{lmn}q^m(3)q^n(5)\right.$$
$$\left.+\frac{1}{2\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(3)q^k(4)\varepsilon^{lmn}q^m(2)q^n(5)\right\}\varepsilon_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$|C_4\rangle = \frac{1}{\sqrt{6}}\left\{\frac{\sqrt{3}}{4\sqrt{2}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(4)\varepsilon^{lmn}q^m(3)q^n(5) - \frac{1}{4\sqrt{6}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(3)\varepsilon^{lmn}q^m(4)q^n(5)\right\}\varepsilon_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$|C_5\rangle = \frac{1}{\sqrt{6}}\left\{\frac{1}{2\sqrt{3}}\varepsilon^{ijk}q^i(1)q^j(2)q^k(3)\varepsilon^{lmn}q^m(4)q^n(5)\right\}\varepsilon_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$|C_6\rangle = \frac{1}{\sqrt{6}}\left\{\frac{1}{\sqrt{30}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(4)d^{lmn}q^m(3)q^n(5) - \frac{1}{\sqrt{30}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(5)d^{lmn}q^m(3)q^n(4)\right.$$
$$+\frac{1}{\sqrt{30}}\varepsilon^{ijk}q_i(1)q_j(3)q_k(4)d^{lmn}q^m(2)q^n(5) - \frac{1}{\sqrt{30}}\varepsilon^{ijk}q_i(1)q_j(3)q_k(5)d^{lmn}q^m(2)q^n(4)$$
$$\left.+\frac{\sqrt{3}}{\sqrt{10}}\varepsilon^{ijk}q_i(1)q_j(4)q_k(5)d^{lmn}q^m(2)q^n(3)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7)$$

$$
\begin{aligned}
|C_7\rangle =& \frac{1}{\sqrt{6}}\left\{\frac{\sqrt{3}}{4\sqrt{5}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(3)d^{lmn}q^m(4)q^n(5) + \frac{1}{4\sqrt{15}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(4)d^{lmn}q^m(3)q^n(5)\right.\\
&-\frac{1}{\sqrt{15}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(5)d^{lmn}q^m(3)q^n(4) - \frac{1}{2\sqrt{15}}\varepsilon^{ijk}q_i(1)q_j(3)q_k(4)d^{lmn}q^m(2)q^n(5)\\
&\left.+\frac{2}{\sqrt{15}}\varepsilon^{ijk}q_i(1)q_j(3)q_k(5)d^{lmn}q^m(2)q^n(4)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7)\\
|C_8\rangle =& \frac{1}{\sqrt{6}}\left\{-\frac{1}{4\sqrt{5}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(3)d^{lmn}q^m(4)q^n(5) - \frac{1}{4\sqrt{5}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(4)d^{lmn}q^m(3)q^n(5)\right.\\
&\left.+\frac{1}{\sqrt{5}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(5)d^{lmn}q^m(3)q^n(4)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7)\\
|C_9\rangle =& \frac{1}{\sqrt{6}}\left\{\frac{1}{4}\varepsilon^{ijk}q_i(1)q_j(2)q_k(3)d^{lmn}q^m(4)q^n(5) - \frac{1}{4}\varepsilon^{ijk}q_i(1)q_j(2)q_k(4)d^{lmn}q^m(3)q^n(5)\right.\\
&\left.+\frac{1}{2}\varepsilon^{ijk}q_i(1)q_j(3)q_k(4)d^{lmn}q^m(2)q^n(5)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7)\\
|C_{10}\rangle =& \frac{1}{\sqrt{6}}\left\{-\frac{1}{4\sqrt{3}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(3)d^{lmn}q^m(4)q^n(5) + \frac{\sqrt{3}}{4}\varepsilon^{ijk}q_i(1)q_j(2)q_k(4)d^{lmn}q^m(3)q^n(5)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7)\\
|C_{11}\rangle =& \frac{1}{\sqrt{6}}\left\{\frac{1}{\sqrt{6}}\varepsilon^{ijk}q_i(1)q_j(2)q_k(3)d^{lmn}q^m(4)q^n(5)\right\}d_{lpr}\bar{q}_p(6)\bar{q}_r(7),
\end{aligned}
\tag{A1}
$$

where the nonvanishing d^{abc} and d_{abc} constants are

$$
\begin{aligned}
&d^{111} = d_{111} = d^{222} = d_{222} = d^{333} = d_{333} = 1\\
&d^{412} = d_{412} = d^{421} = d_{421} = d^{523} = d_{523} = d^{532}\\
&\quad = d_{532} = d^{613} = d_{613} = d^{631} = d_{631} = \frac{1}{\sqrt{2}}.
\end{aligned}
\tag{A2}
$$

APPENDIX B: KINETIC ENERGY

Consider the simple MIT bag model mass formula for a hadron composed of $N = N_1 + N_2$ quarks [49–51] in an S wave:

$$
E_N = N\frac{\omega}{R} + B\frac{4}{3}\pi R^3 - \frac{Z_0}{R}. \tag{B1}
$$

Here, $\omega \sim 2.04$, R is the bag radius, and B is the pressure. The last term was originally introduced as the zero point energy or Casimir energy effect but is understood to be taking care of the center of mass motion of the hadrons composed of N quarks [52]. If the hypothetical hadron decays into two color singlet hadrons of N_1 and N_2 quarks, respectively, their masses will also follow the same formula as Eq. (B1) after replacing the number of quarks with either N_1 or N_2. For each hadron, the bag radius R is determined by minimizing the mass with respect to R. However, comparing the mass of the multiquark composed of N quarks to the sum of two hadrons, one notices that the multiquark state has one less factor of the center of mass term. This difference is the additional kinetic energy needed to bring the $N_1 + N_2$ quarks into a compact configuration compared to two isolated hadrons.

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