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Single spin asymmetries in forward p-p/A collisions revisited: The role of color entanglement

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We calculate the single transverse spin asymmetries (SSA) for forward inclusive particle production in pp and pA collisions using a hybrid approach. It is shown that the Sivers type contribution to the SSA drops out due to the color entanglement effect, whereas the fragmentation contribution to the spin asymmetry is not affected by the color entanglement effect. This finding offers a natural solution for the sign mismatch problem.

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I. INTRODUCTION

During the past three decades, the studies of transverse single spin asymmetries in high energy scatterings have greatly deepened our understanding of some central aspects of the quantum chromodynamics (QCD) factorization theorem, among which the universality issue attracted a lot of attention. Within the transverse momentum dependent (TMD) factorization framework [1], a TMD distribution, known as the Sivers function $f_{1T}^{\perp}$ [2] was proposed to account for the observed large SSAs. It has been found that the Sivers function reverses sign between the semi-inclusive deeply inelastic scattering (SIDIS) and the Drell-Yan process [3–5]. The discovery of such a novel and unique universality property has stimulated a lot of theoretical progress over the past decade. The preliminary results from the STAR Collaboration and the COMPASS Collaboration [6,7] seem to confirm the sign change. This is undoubtedly one of the most remarkable achievements in high energy spin physics.

However, the situation with the SSA for the forward inclusive hadron production in pp collisions (denoted as A_N) $p^{\uparrow}p \to hX$ is more complicated. Because of the lack of an additional hard scale, it is more appropriate to compute this observable using the collinear twist-3 approach [8–12] instead of the TMD factorization. Phenomenologically, it was also studied in the generalized parton model [13,14]. The twist-3 effects leading to the SSA can be factorized into various three-parton correlation functions. One of these is the Qiu-Sterman function T_F [9], which can be related to the Sivers function [15],

$$T_F(x,x) = -\int d^2p_\perp \frac{p_\perp^2}{M} f_{1T}^{\perp}(x,p_\perp^2)|_{\mathrm{SIDIS}}, \qquad (1)$$

where M is the nucleon mass. Because of this relation, one can determine T_F using the date on the SSA measured in SIDIS and compared with the Qiu-Sterman function extracted from the inclusive hadron production in pp collisions. Very surprisingly, T_F extracted from these two observables actually differ in sign [16]. To resolve this sign mismatch problem, the authors of Ref. [17] suggested that a genuine twist-3 function $\mathrm{Im}\hat{E}_{\mathrm{F}}$ [11] ($\hat{H}_{\mathrm{FU}}^{\Im}$ in a different notation) instead of T_F gives rise to the dominant contribution to A_N. It is worthy to mention that the data on the SSA in SIDIS [18,19] do not disfavor this point of view because the Sivers function is not well constrained at a large x in SIDIS, allowing flexible parametrizations of T_F. Note that all other possible sources contributing to A_N in the collinear twist-3 approach were shown to be small [20–22].

The study of the SSA for an inclusive hadron production in pA collisions $p^{\uparrow}A \to hX$ could play an important role in pining down the true main cause of A_N since the different sources contributing to A_N are affected by the saturation effect in the different ways. In fact, no strong nuclear suppression was observed in a recent measurement of A_N in forward pA collisions [23]. This implies that the dominant pieces must be these which are not affected by the saturation effect. It is thus of a great interest to take into account the saturation effect on the unpolarized target side. Some earlier work in this direction have been done in Refs. [24–27].

In this paper, we compute A_N using a hybrid approach [28,29], where a target nucleus (or proton) is treated in the color glass condensate (CGC) framework [30], while the collinear twist-3 approach is applied to the transversely polarized projectile. It is a natural and powerful approach to take into account the color entanglement effect that was first discovered in Ref. [31] (for the relevant work, see Refs. [32–34]). Actually, it has been found that the SSAs for the prompt photon production and photon-jet production in pp or pA collisions receive the contribution from the color entanglement effect [28,35]. In contrast, the color entanglement effect is absent in the Drell-Yan process at a low transverse momentum due to the trivial color flow [29].

We notice that the hybrid approach has been used to compute A_N in Refs. [36,37] in the dilute limit. To include the saturation effect, the authors of Ref. [36] derived the Wilson line structure using some heuristic argument, which, however, differs from that we directly derived in the hybrid approach for the Sivers type contribution. To be more explicit, the Wilson line structure we obtained can be cast into the combination $G_{\rm DP} - N_c^2 G_4$, where $G_{\rm DP}$ is the normal dipole type gluon distribution and G_4 is the gluon distribution that arises from the color entanglement effect. Quite dramatically, the relation $G_{\rm DP} = N_c^2 G_4$ held in a quasiclassical model indicates that the Sivers type contribution completely drops out. As explained below, the heuristic argument used in Ref. [37] to work out the Wilson line structure in the fragmentation case is well justified. It is shown [37] that the contribution from the twist-3 fragmentation function related to the moment of the TMD Collins function [3,11] is strongly suppressed by the saturation effect. In view of the recent measurement at RHIC [23], the genuine twist-3 fragmentation function turns out to be the only candidate for the main cause of A_N.

II. THE COMPUTATION OF A_N IN THE HYBRID APPROACH

We start the computation of A_N in the hybrid approach by introducing the relevant kinematics. The dominant partonic channel for the spin independent forward particle production is

$$q_p(xP) + g_A(x'_g \bar{P} + k_\perp) \to q(l_q), \tag{2}$$

which represents a quark q_p from a proton scattering off classical background gluon field g_A inside the target. The light cone momenta are defined as $\bar{P}^\mu = \bar{P}^- n^\mu$ and $P^\mu = P^+ p^\mu$ with the usual light cone vectors, n^μ and p^μ, normalized according to $p \cdot n = 1$.

To generate an imaginary phase necessary for the nonvanishing spin asymmetry, one additional gluon attachment from the remanent of the polarized proton projectile must be taken into account. It is convenient to formulate such a twist-3 calculation in the covariant gauge in which this extra gluon is longitudinally polarized. One then has to sum

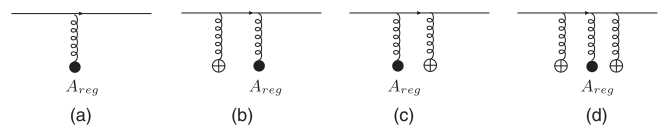

FIG. 1. The contribution from the regular terms to the spin dependent amplitude. A black dot denotes a classical field A_{reg} insertion.

the multiple rescatterings of the incoming quark and the collinear gluon with a small x gluon field inside target to all orders simultaneously.

The incoming quark and gluon with the physical polarization scattering off CGC state can be summed into a Wilson line in the fundamental and adjoint representation, respectively,

$$U(x_\perp) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp)\cdot t\right] \tag{3}$$

$$\tilde{U}(x_\perp) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp)\cdot T\right] \tag{4}$$

with T and t being the generators in the adjoint and fundamental representation. However, the multiple scattering of a longitudinally polarized gluon with the background gluon field of a target can not be simply described by a Wilson line in the CGC formalism. Instead, the expression for the gauge field created through the fusion of a longitudinally polarized gluon from the proton and small x gluons from the target takes a quite complicated form [38]. It contains both the singular terms [proportional to $\delta(z^+)$] and the regular terms: $A^\mu = A^\mu_{reg} + \delta^{\mu -} A^-_{\sin g}$, whose explicit expressions can be found in Refs. [38].

When computing the spin dependent amplitude, all possible insertions of the fields A^μ_{reg} and $A^-_{\sin g}$ on the quark line must be taken into account as illustrated in Fig. 1 and Fig. 2, respectively. We calculate the contributions from Fig. 1 and Fig. 2 following the method outlined in Refs. [28,29]. Note that Figs. 1(c) and 1(d) do not contribute to the amplitude because the two poles are lying on the same half plane. The final expression for the spin dependent amplitude takes the form,

$$\begin{aligned}\mathcal{M} = &-g\int \frac{dk_1^- d^2k_{1\perp} d^2x_\perp d^2x_{1\perp}}{(2\pi)^3} e^{ix_\perp\cdot(k_\perp-k_{1\perp}-p_\perp)} e^{ix_{1\perp}\cdot k_{1\perp}} \bar{u}(l_q) \frac{\not{C}_U/(q,p_\perp)}{q^2+i\epsilon} t^b S_F(l_q-q)\not{n} U(x_\perp) u(xP)[\tilde{U}(x_{1\perp})-1]_{ba} \\ &+ g\int d^2x_\perp e^{i(k_\perp-p_\perp)\cdot x_\perp} \frac{\bar{u}(l_q)\not{n} t^b U(x_\perp) u(xP)}{x_g P^+ + i\epsilon}[\tilde{U}(x_\perp)-1]_{ba} \\ &+ ig\int d^2x_\perp e^{i(k_\perp-p_\perp)\cdot x_\perp} \bar{u}(l_q) t^a \not{p} S_F(l_q - x_g P - p_\perp)\not{n} u(xP)[U(x_\perp)-1],\end{aligned} \tag{5}$$

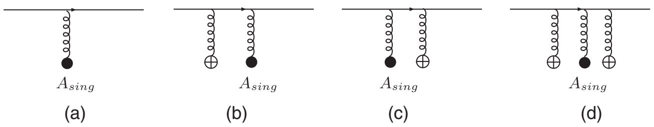

FIG. 2. The contribution from the singular terms to the spin dependent amplitude. A black dot denotes a classical field $A_{\sin g}$ insertion.

where the color index a is associated with the collinear gluon from the polarized projectile, which carries a momentum $x_gP + p_\perp$. $S_F(l_q - q)$ and $S_F(l_q - x_gP - p_\perp)$ denoting the standard quark propagators. The four vector $C_U^\mu(q, p_\perp)$ is defined as

$$C_U^+(q,p_\perp) = -\frac{p_\perp^2}{q^- + i\epsilon}, \qquad C_U^-(q,p_\perp) = \frac{k_{1\perp}^2 - q_\perp^2}{q^+ + i\epsilon},$$
$$C_U^i(q,p_\perp) = -2\mathbf{p}_\perp^i, \tag{6}$$

where $q^\mu = x'_{g1}\bar{P}^\mu + k_{1\perp}^\mu + x_gP^\mu + p_\perp^\mu$. The notation $\mathbf{p}_\perp$ is used to denote the four dimension vector with $p_\perp^2 = -\mathbf{p}_\perp^2$.

It is worthy to point out that the second term in Eq. (5), which describes the interaction between the collinear gluon from the projectile and the color source inside the target is missing in Ref. [36], and the Wilson line structure in the rest two terms are also organized in different ways as compared to that in Ref. [36]. Before computing the twist-3 piece, as a consistency check, let us first have a look at the twist-2 part of the derived amplitude by setting $p_\perp = 0$. The first term vanishes due to $C_U^\mu(q, p_\perp = 0) = 0$. The leading twist contribution of the amplitude is simplified as

$$\mathcal{M}_{\text{twist}-2} = \frac{g}{P^+}\int d^2x_\perp e^{ik_\perp\cdot x_\perp} \times\left\{\left[\mathcal{P}\frac{1}{x_g} + i\pi\delta(x_g)\right][U(x_\perp)-1]\bar{u}(l_q)\not{n}t^a u(xP) - i\pi\delta(x_g)\bar{u}(l_q)\not{n}t^a u(xP)2[U(x_\perp)-1]\right\}. \tag{7}$$

In arriving at the above expression, we used the algebraic identity, $U(x_\perp)t^bU^\dagger(x_\perp) = t^a\tilde{U}_{ba}(x_\perp)$. After integrating out the incoming quark transverse momentum, the contributions proportional to the delta function $\delta(x_g)$ are canceled out between the different cut diagrams. The additional gluon exchange from the proton can be incorporated into the gauge link that appears in the matrix element definition of a quark PDF by carrying out the x_g integration over the principal value part. As expected, the corresponding hard part is just the Born diagram contribution to a quark scattering off CGC state [39]. At this point, one can readily see that it is critical to keep the scattering amplitude gauge invariant by taking into account the initial interaction with the color source inside target. Note that the result derived in Ref. [36] fails to pass this consistency check.

If one applies TMD factorization on the polarized projectile side, the terms proportional to the delta function contribute to the gauge link in the Sivers TMD function. But unlike the photon-jet production [35], such a hybrid approach might not be well justified in the process under consideration because of the lack of an additional hard scale.

We now proceed to compute the spin dependent twist-3 contribution by first isolating the imaginary part from different poles. We start with analyzing the pole structure in the first term in Eq. (5). By carrying out the x_g and k_1^- integration, two propagators are effectively put on shell,

$$q^2 = 0, \qquad (l_q - q)^2 = 0. \tag{8}$$

Three particle lines connected by a quark-gluon vertex being simultaneously on shell implies that three momenta q^μ, $l_q^\mu - q^\mu$, l_q^μ must be collinear to each other. This leads to

$$\bar{u}(l_q)\not{C}_U(q,p_\perp)(\not{l}_q - \not{q}) = -\bar{u}(l_q)\not{l}_q\not{C}_U(q,p_\perp)(1-\beta) = 0, \tag{9}$$

where $q^\mu = \beta l_q^\mu$ for $0 \le \beta \le 1$. When commuting $\not{C}_U$ with $\not{l}_q - \not{q}$ in the above formula, we used the property $C_U^\mu(q,p_\perp)\cdot q_\mu = 0$. One thus concludes that the hard gluon pole (or the soft fermion pole for $\beta = 1$) contribution is completely washed out by the saturation effect. This analysis is in agreement with that made in Ref. [36].

One should notice that the first term in Eq. (5) also contains the soft gluon pole (SGP) contribution, which comes from the minus component of C_U^μ. Combining it with the last two terms in Eq. (5), the SGP contribution is given by

$$\mathcal{M}_{\text{SGP}} = -i\pi g\int\frac{d^2k_{1\perp}d^2x_\perp d^2x_{1\perp}}{(2\pi)^2}e^{ix_\perp\cdot(k_\perp - k_{1\perp} - p_\perp)}e^{ix_{1\perp}\cdot k_{1\perp}} \times\delta(x_gP^+)\frac{k_{1\perp}^2}{q_\perp^2}\bar{u}(l_q)\not{n}t^bU(x_\perp)u(xP)\tilde{U}(x_{1\perp})_{ba} + i\pi g\int d^2x_\perp e^{i(k_\perp - p_\perp)\cdot x_\perp}\delta((l_q - x_gP - p_\perp)^2) \times\bar{u}(l_q)t^a\not{p}(\not{l}_q - x_g\not{P} - \not{p}_\perp)\not{n}u(xP)[U(x_\perp)-1], \tag{10}$$

where the last term gives rise to the so-called derivative term contribution. At this point, we would like to mention that the spin dependent amplitude takes a slightly different form for the left cut diagrams due to the different $p_\perp$ flow. In the collinear twist-3 approach, the spin asymmetry arises from the interference between the imaginary part identified above and the conjugate Born scattering amplitude without an additional gluon attachment from the projectile. It is straightforward to compute the later in the CGC formalism [39]. Following the standard procedure, the next step is to make the $p_\perp$ expansion and factorize the soft part of the polarized proton side into the Qiu-Sterman function.

Finally, in order to express the spin dependent cross section in terms of the known gluon distributions, we simplify the relevant color structure, starting with the one associated with the delta function $\delta(x_g P^+)$,

$$\begin{aligned}&\mathrm{Tr}[t^a U^\dagger(y_\perp) t^b U(x_\perp)]\tilde{U}(x_{1\perp})_{ba}\\&\quad=\frac{-1}{2N_c}\mathrm{Tr}[U^\dagger(y_\perp)U(x_\perp)]\\&\qquad+\frac{1}{2}\mathrm{Tr}[U^\dagger(y_\perp)U(x_{1\perp})]\mathrm{Tr}[U^\dagger(x_{1\perp})U(x_\perp)],\end{aligned}\tag{11}$$

where $U^\dagger(y_\perp)$ is from the conjugate amplitude. Note that the forward scattering amplitude contribution has been neglected as we do so below. The contribution from $[U^\dagger(y_\perp)U(x_\perp)]$ drops out because one can trivially carry out the $x_{1\perp}$ integration, resulting in $k_{1\perp}=0$. In the large N_c approximation, $\langle \mathrm{Tr}[U^\dagger(y_\perp)U(x_{1\perp})]\mathrm{Tr}[U^\dagger(x_{1\perp})U(x_\perp)]\rangle$ can be related to the convolution of two dipole type gluon distributions. After summing the left and right cut diagrams contribution and making the $p_\perp$ expansion, we encounter the following structure:

$$\int d^2k_{1\perp}\left[\frac{l^\alpha_{q\perp}-k^\alpha_{1\perp}}{(l_{q\perp}-k_{1\perp})^2}F(l^2_{q\perp})+\frac{l^\alpha_{q\perp}}{2}\frac{\partial F(l^2_{q\perp})}{\partial l^2_{q\perp}}\right]F(k^2_{1\perp}),\tag{12}$$

where $F(l^2_{q\perp})$ is the Fourier transform of the dipole amplitude whose definition is given below. Using the method introduced in Ref. [37], it is easy to verify that the two terms are completely canceled out in the dilute limit and are strongly suppressed in the saturation regime. One thus can safely neglect the SGP contribution induced by the initial state interaction.

We now turn to discuss the Wilson lines associated with the derivative term contribution, which reads

$$\begin{aligned}\mathrm{Tr}[t^a U^\dagger(y_\perp) t^a U(x_\perp)]&=\frac{1}{2}\mathrm{Tr}[U^\dagger(y_\perp)]\mathrm{Tr}[U(x_\perp)]\\&\quad-\frac{1}{2N_c}\mathrm{Tr}[U^\dagger(y_\perp)U(x_\perp)],\end{aligned}\tag{13}$$

where the nontrivial color structure $\mathrm{Tr}[U^\dagger(y_\perp)]\mathrm{Tr}[U(x_\perp)]$ arises from the color entanglement effect as explained in Refs. [28,29,35]. The extra gluon attachment from the polarized proton plays a crucial role in yielding such a unique structure. With all these calculation recipes, we derive the spin dependent partonic cross section,

$$\begin{aligned}\frac{d\sigma}{dyd^2l_{q\perp}}&=\frac{2\pi^2\alpha_s x x'_g}{N_c(N_c^2-1)}\frac{\epsilon_{\alpha\beta}S^\beta_\perp l^\alpha_{q\perp}}{l^2_{q\perp}}\\&\quad\times\left\{\frac{1}{l^2_{q\perp}}[G_{\mathrm{DP}}(x'_g,l^2_{q\perp})-N_c^2G_4(x'_g,l^2_{q\perp})]x\frac{dT_F(x,x)}{dx}\right.\\&\quad\left.+\frac{\partial[G_{\mathrm{DP}}(x'_g,l^2_{q\perp})-N_c^2G_4(x'_g,l^2_{q\perp})]}{\partial l^2_{q\perp}}T_F(x,x)\right\},\end{aligned}\tag{14}$$

where $S_\perp$ is the transverse spin vector of the proton. The momentum fractions x and x'_g are fixed according to $x=e^y|l_{q\perp}|/\sqrt{s}$ and $x'_g=e^{-y}|l_{q\perp}|/\sqrt{s}$ with y being the outgoing quark rapidity. G_{DP} is the normal dipole type gluon distribution and related to the Fourier transform of the dipole amplitude $x'_g G_{\mathrm{DP}}(x'_g,l^2_{q\perp})=\frac{l^2_{q\perp}N_c}{2\pi^2\alpha_s}F(l^2_{q\perp})$. G_4 introduced in Ref. [28] is the gluon distribution that arises from the color entanglement effect. Their operator definitions are given by

$$\begin{aligned}x'_g G_{\mathrm{DP}}(x'_g,l^2_{q\perp})&=\frac{l^2_{q\perp}N_c}{2\pi^2\alpha_s}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}e^{il_{q\perp}\cdot(x_\perp-y_\perp)}\\&\quad\times\frac{1}{N_c}\langle\mathrm{Tr}[U^\dagger(y_\perp)U(x_\perp)]\rangle\\x'_g G_4(x'_g,l^2_{q\perp})&=\frac{l^2_{q\perp}N_c}{2\pi^2\alpha_s}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}e^{il_{q\perp}\cdot(x_\perp-y_\perp)}\\&\quad\times\frac{1}{N_c^2}\langle\mathrm{Tr}[U^\dagger(y_\perp)]\mathrm{Tr}[U(x_\perp)]\rangle,\end{aligned}\tag{15}$$

which can be evaluated and related to each other in the MV model [28],

$$x'_g G_4(x'_g,l^2_{q\perp})=\frac{1}{N_c^2}x'_g G_{\mathrm{DP}}(x'_g,l^2_{q\perp}).\tag{16}$$

This simple relation leads to a complete cancellation between the contributions from G_{DP} and G_4 in Eq. (14). Therefore, the Sivers type contribution to A_N drops out. Obviously, this conclusion remains true after promoting the partonic spin dependent cross section to the hardron production cross section.

We now comment on the twist-3 fragmentation function contribution to A_N. The derivative term contribution to A_N in pp collisions was first computed in Ref. [11] in the purely collinear twist-3 approach. The complete result was obtained in Ref. [12] (see recent reviews Refs. [40,41]). In order to take into account the multiple gluon rescattering effect on the target side, the similar hybrid approach can also be applied in the fragmentation case [37]. As is well-known, it is highly nontrivial to compute the SGP contribution in the light cone gauge [42,43]. Since the SGP contribution vanishes for the twist-3 fragmentation contribution [44,45], it is more convenient to carry out the calculation in the light cone gauge, where the additional gluon exchange from the twist-3 fragmentation function is physically polarized [37]. A gluon with a physical polarization scattering off the background gluon field can be summarized into a normal Wilson line in the adjoint representation. In this sense, the derivation of the Wilson line structure in Ref. [37] is well justified. If one formulates such a calculation in the covariant gauge, the fact that an imaginary phase from the scattering amplitude is not required in the twist-3 fragmentation case would make an essential difference in deriving the color structure. However, the detailed investigation is beyond the scope of the current work.

We close this section with few further remarks:

(1) Following the standard procedure, one can derive the BK type evolution equation for the gluon distribution G_4, which will be presented in a separate publication. In the large N_c limit, the relation Eq. (16) holds under small x evolution.
(2) The relation Eq. (16) is a model dependent result. In the general case, an incomplete cancellation between two gluon distributions leaves some room for having a tiny spin asymmetry for inclusive jet production in pp or pA collisions [46].
(3) If the G_4 contribution is neglected, Eq. (14) is consistent with the collinear twist-3 result [10] in the dilute limit.
(4) The color entanglement effect is a leading power effect and should be taken into account in the genuine collinear twist-3 approach as well. We plan to redo the calculation in the purely collinear framework by going beyond one gluon exchange approximation on the target side.
(5) T-even objects like the unpolarized twist-2 amplitude, are not affected by the color entanglement effect. The observed color entanglement effect is the consequence of the nontrivial interplay among the T-odd effect, multiple gluon rescattering, and the non-Abelin feature of QCD [28,29,31,35].

III. SUMMARY

Let us now summarize the recent progress on the topic addressed in this paper. The sign mismatch problem was first observed in Ref. [16]. To find a way out, one naturally questions the dominance of the Sivers type contribution to A_N (other possible solutions, see Refs. [47,48]). It was indeed found that the genuine twist-3 fragmentation function could play an important role in generating the spin asymmetry [17]. Later, the authors of Ref. [37] have sorted out the piece of the contribution from the twist-3 fragmentation functions that is not suppressed by the saturation effect using a hybrid approach first developed in Refs. [28,29]. The saturation suppressed fragmentation contribution being the major source of A_N has been ruled out by the recent measurement [23]. In this work, we demonstrate that the Sivers type contribution to the spin asymmetry drops out due to the color entanglement effect. The nuclear independent part of the genuine twist-3 fragmentation contribution turns out to be the only candidate for the main cause of A_N. A recent work [49] shows that it is almost sufficient to account for A_N by taking into account this fragmentation term alone with the input constrained by the Lorentz invariance relation [50]. We thus believe that the sign mismatch problem has been solved.

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