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Calculation of coupling constants and decay width of light meson with kaon from QCD sum rules

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In this research paper, we evaluate the strong coupling constants of the $K^*K\pi$, ϕKK, ϕK^*K^* and ρK^*K^* vertices in the framework of the three-point QCD sum rules. In each vertex two different off-shell particles are considered and the final results are compared to the other existing predictions. We study the decay width of $K^* \to K\pi$ and $\phi \to KK$ and obtain the values $\Gamma(K^* \to K\pi) = 46.8$ and $\Gamma(\phi \to KK) = 5.58$, which have good agreement with experimental values.

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I. INTRODUCTION

Investigation of meson and mesonic vertices particulary at the low-energy region has been one of the most interesting subjects of QCD. Such investigations are carried out by the nonperturbative QCD approach. QCD sum rules have been more reliable than other approaches [1], due to the fact that they are based upon a QCD Lagrangian with no model dependent parameter to be matched with experimental data. QCD sum rules (QCDSR) have also been successful in describing tetraquark states [2–4], molecular states [5–7] and semileptonic decays [8–11], and in calculating form factors, decay constant, strong coupling constant, etc.

Determination of coupling constants helps to better understand the nature of strong interactions and hadronic phenomena. When one of the participating particles is off shell, calculations are first carried out in the QCDSR framework, and then extended to on-shell regions. Using three-point QCDSR, coupling constants corresponding to different mesonic vertices have been calculated such as $D^*D\pi$ [12,13], $DD\rho$ [14], $D^*D^*\rho$ [15], DDJ/ψ [16], D^*DJ/ψ [17], D^*D_sK, D_s^*DK, D_0D_sK, $D_{s0}DK$ [18], D^*D^*P, D^*DV, DDV [19], $D^*D^*\pi$ [20], D^*D^*J/ψ [21], D_sD^*K, D_s^*DK [22], $DD\omega$ [23], $\phi D_{s0}^*D_{s0}^*$, ϕD_sD_s, $\phi D_s^*D_s^*$, $\phi D_{s1}D_{s1}$ [24], $B_{s0}BK$ [25], B_s^*BK [26], $D_s^*D_s\phi$ [27], $D_sDK_0^*$, $B_sBK_0^*$, D_s^*DK, B_s^*BK, $D_s^*DK_1$, and $B_s^*BK_1$ [28].

In this research work, the $K^*K\pi$, ϕKK, ϕK^*K^*, and ρK^*K^* vertices have been investigated by using three-point QCDSR. The interaction Lagrangians between the three mesons are

$$\begin{aligned}\mathcal{L}_{K^*(\phi)K\pi(K)} &= ig_{K^*(\phi)K\pi(K)}K^*(\phi)^\mu[\pi(K)\partial_\mu K - \partial_\mu\pi(K)K],\\ \mathcal{L}_{\phi(\rho)K^*K^*} &= ig_{\phi(\rho)K^*K^*}\{\phi(\rho)^\mu[K^{*\nu}\partial_\mu K^*_\nu - \partial_\mu K^{*\nu}K^*_\nu] + K^{*\mu}[\partial_\mu\phi(\rho)^\nu K^*_\nu - \phi(\rho)^\nu\partial_\mu K^*_\nu]\\ &\quad + [\partial_\mu K^{*\nu}\phi(\rho)_\nu - K^{*\nu}\partial_\mu\phi(\rho)_\nu]K^{*\mu}\}. \end{aligned} \tag{1}$$

In the three-point QCDSR approach, phenomenological and theoretical sides are carried out separately, and then are equated. The theoretical side involves perturbation and nonperturbation contributions. First, the perturbative contribution of each vertex is calculated; then the quark-quark and quark-gluon condensations of nonperturbative parts are found and added to the perturbative contribution. The phenomenological side is expanded, based upon hadronic parameters such as meson and quark mass, decay constant, and strong coupling constant. The theoretical side representing the Feynman diagrams is real, whereas the phenomenological side is imaginary. These two sides are related by the dispersion relation. In order to avoid divergences, double Borel transformation is used and continuum and higher terms are ignored.

The present study is organized in four sections. After the introduction, in Sec. II we introduce three-point QCD sum rules and calculate the strong coupling constants of the $K^*K\pi$, ϕKK, ϕK^*K^*, and ρK^*K^*. In Sec. III, the numerical results of the coupling constants are given and in this section, the decay widths of the decay modes $K^* \to K\pi$ and $\phi \to KK$ are determined. The paper ends with a conclusion in Sec. IV.

II. THREE-POINT QCD SUM RULES FORMALISM FOR THE COUPLING CONSTANT DETERMINATION

In three-point QCDSR, the standard method for obtaining a vertex coupling constant is by introducing

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the correlation function that is given by the following formula:

(1) corresponding to the $K - K^*(\phi) - \pi(K)$ vertex:
(a) for $K^*(\phi)$ off shell,

$$\Pi_\mu^{K^*(\phi)}(p,p') = i^2 \int d^4x d^4y e^{i(p'\cdot x - p\cdot y)} \times \langle 0|\mathcal{T}(j^K(x) j_\mu^{K^*(\phi)}(0) j^{\pi(K)}(y))|0\rangle, \tag{2}$$

(b) for $\pi(K)$ off shell,

$$\Pi_\mu^{\pi(K)}(p,p') = i^2 \int d^4x d^4y e^{i(p'\cdot x - p\cdot y)} \times \langle 0|\mathcal{T}(j^K(x) j^{\pi(K)}(0) j_\mu^{K^*(\phi)}(y))|0\rangle; \tag{3}$$

(2) corresponding to the $\phi(\rho) - K^* - K^*$ vertex:
(a) for K^* off shell,

$$\Pi_{\mu\nu\alpha}^{K^*}(p,p') = i^2 \int d^4x d^4y e^{i(p'\cdot x - p\cdot y)} \times \langle 0|\mathcal{T}(j_\alpha^{K^*}(x) j_\nu^{K^*}(0) j_\mu^{\phi(\rho)}(y))|0\rangle, \tag{4}$$

(b) for $\phi(\rho)$ off shell,

$$\Pi_{\mu\nu\alpha}^{\phi(\rho)}(p,p') = i^2 \int d^4x d^4y e^{i(p'\cdot x - p\cdot y)} \times \langle 0|\mathcal{T}(j_\alpha^{K^*}(x) j_\nu^{\phi(\rho)}(0) j_\mu^{K^*}(y))|0\rangle. \tag{5}$$

Each [(2)–(5)] equation can be written in theoretical [operator product expansion (OPE)] and in phenomenological (physical) representations. In the OPE part of each interpolating current can be expanded in terms of quark fields in the following form:

(i) for the pseudoscalar ($\mathcal{P}$) meson,

$$j^{\mathcal{P}}(x) = q(x)\gamma_5 q'(x), \tag{6}$$

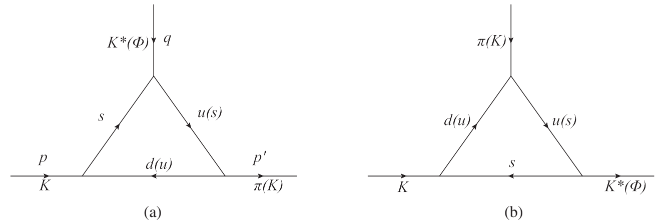

FIG. 1. Perturbative diagrams for (a) off-shell $K^*(\phi)$ and (b) off-shell $\pi(K)$.

(ii) for the vector($\mathcal{V}$) meson,

$$j_\mu^{\mathcal{V}}(x) = q(x)\gamma_\mu q'(x), \tag{7}$$

where $q(x)$ and $q'(x)$ are up, down, or strange quark fields. In general, the correlation function is written, using operator product expansion, as

$$\Pi^{\mathcal{P}(\mathcal{V})} = \Pi_{\text{per}}^{\mathcal{P}(\mathcal{V})} + \Pi_{\text{nonper}}^{\mathcal{P}(\mathcal{V})}, \tag{8}$$

which contain several Lorentz structures. Theoretically all the structures are equal and give the same results, and we have the option to choose any structure. But in order to reduce the effects of approximations, we prefer to select the structure that is more stable. To calculate the perturbation contribution (Fig. 1), the product of interpolating currents in Eqs. (6) and (7) is expanded by using Wick's theorem and is substituted in "perturbation vacuum" brackets in Eqs. (2)–(5). If the dispersion relation is taken into account, the general form of the correlation function can be written as

$$\Pi_{\text{per}}^{\mathcal{P}(\mathcal{V})} = -\frac{1}{(2\pi)^2}\int ds \int ds' \frac{\rho^{\mathcal{P}(\mathcal{V})}(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}, \tag{9}$$

where ρ is the spectral density and can be calculated by Cutkosky rules.

As a result, the spectral densities corresponding to each vertex can be written as

$$\rho_\mu^{\mathcal{P}(\mathcal{V})} = 4N_c I_0 \left[B_2\left(km_1m_2 + m_1m_3 - km_2m_3 - m_3^2 + (k+1)\frac{\Delta}{2} - k\frac{u}{2}\right) + \left(m_1m_3 - m_3^2 - \frac{\Delta}{2}\right)\right]p'_\mu + \cdots, \tag{10}$$

$$\rho_{\mu\nu\alpha}^{\mathcal{V}} = -N_c[(B_2 - B_1)(2m_1m_2 + 2m_1m_3 - 2m_2m_3 - 2m_3^2 + 2\Delta - u) + 4A_1 + 8(C_6 - C_3) + I_0(2m_2m_3 - 4m_3^2 + \Delta' + 2m_1m_3 + \Delta)]g_{\mu\nu}q_\alpha + \cdots, \tag{11}$$

where … denotes other Lorentz structures and $k = 1$ for off-shell $\mathcal{P}$ and $k = -1$ for off-shell $\mathcal{V}$. Also, the parameters used in the spectral densities are given by

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{\frac{1}{2}}(s,s',q^2)},\\
\lambda(a,b,c) &= a^2+b^2+c^2-2ac-2bc-2ac,\\
\Delta &= (s+m_3^2-m_1^2),\\
\Delta' &= (s'+m_3^2-m_2^2),\\
u &= s+s'-q^2,\\
B_1 &= \frac{1}{\lambda(s,s',q^2)}[2s'\Delta-\Delta' u],\\
B_2 &= \frac{1}{\lambda(s,s',q^2)}[2s\Delta'-\Delta u],\\
A1 &= -\frac{1}{2\lambda(s,s',q^2)}[4ss'm_3^2-s\Delta'^2-s'\Delta^2-u^2m_3^2+u\Delta\Delta'],\\
C_3 &= \frac{1}{2\lambda^2(s,s',q^2)}[8s'^2m_3^2\Delta s-2s'm_3^2\Delta u^2-4um_3^2\Delta' ss'+u^3m_3^2\Delta'-2s'^2\Delta^3\\
&\quad +3s'u\Delta^2\Delta'-2\Delta'^2\Delta ss'-\Delta'^2\Delta u^2+us\Delta^3],\\
C_6 &= \frac{1}{2\lambda^2(s,s',q^2)}[8s^2m_3^2\Delta' s'-2s^2\Delta'^3-4um_3^2\Delta ss'-2\Delta^2\Delta' ss'+3us\Delta'^2\Delta\\
&\quad -2sm_3^2\Delta u^2+s'u\Delta^3+u^3m_3^2\Delta-\Delta^2\Delta' u^2],
\end{aligned}
\tag{12}
$$

where the color factor of the quark is $N_c=3$ and the masses of quarks are given as

(1) corresponding to the $K^*(\phi)K\pi(K)$ vertex,
   (a) for $K^*(\phi)$ off shell: $m_1=m_s$ $m_2=m_u(m_s)$ and $m_3=m_d(m_u)$,
   (b) for $\pi(K)$ off shell: $m_1=m_d(m_u)m_2=m_u(m_s)$ and $m_3=m_s$,

(2) corresponding to the $\phi(\rho)K^*K^*$ vertex,
   (a) for $\phi(\rho)$ off shell: $m_1=m_s(m_u)$, $m_2=m_s(m_d)$ and $m_3=m_u(m_s)$,
   (b) for K^* off shell: $m_1=m_s(m_u)m_2=m_u(m_s)$ and $m_3=m_s(m_d)$.

Next, the nonperturbation contribution of the theoretical side that contains the quark-quark and quark-gluon condensations has been discussed and finally both of them lead to the same condensation operator. Here the gluon-gluon contribution can be ignored because all the participating quarks are light [29,30]. The nonperturbative contribution that plays a significant role in quark-quark and quark-gluon condensations is given in Fig. 2 and can be presented as the function of $r=p^2-m_q^2$ and $r'=p'^2-m_{q'}^2$.

The physical part of the correlation function corresponding to the phenomenology can be calculated, by the unitary relation, obtained by inserting three complete sets of hadronic intermediate states in Eqs. (2)–(5) with the same quantum number as the interpolating currents. By separating the higher and continuum states, and isolating the ground states, we have

$$
\begin{aligned}
\Pi_\mu^{K^*(\phi)} &= \frac{\langle 0|j^{\pi(K)}|\pi(K)(p')\rangle\langle 0|j^K|K(p)\rangle\langle\pi(K)(p')K(p)|K^*(\phi)(q,\epsilon)\rangle\langle K^*(\phi)(q,\epsilon)|j_\mu^{K^*(\phi)}|0\rangle}{(q^2-m_{K^*(\phi)}^2)(p^2-m_K^2)(p'^2-m_{\pi(K)}^2)}\\
&\quad + \text{higher and continuum states},
\end{aligned}
\tag{13}
$$

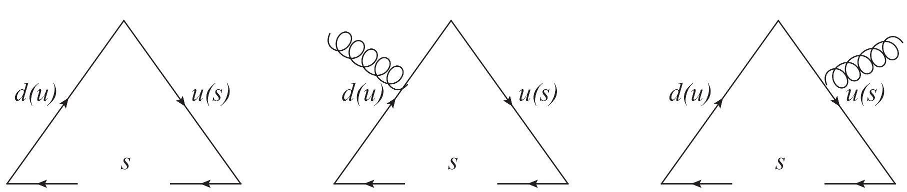

FIG. 2. Nonperturbative diagrams for $\pi(K)$ off shell.

$$\Pi_{\mu}^{\pi(K)} = \frac{\langle 0|j_{\mu}^{K^*(\phi)}|K^*(\phi)(p',\epsilon')\rangle\langle 0|j^K|K(p)\rangle\langle K^*(\phi)(p',\epsilon')K(p)|\pi(K)(q)\rangle\langle\pi(K)(q)|j^{\pi(K)}|0\rangle}{(q^2-m_{\pi(K)}^2)(p^2-m_K^2)(p'^2-m_{K^*(\phi)}^2)} + \text{higher and continuum states}, \tag{14}$$

$$\Pi_{\mu\nu\alpha}^{\phi(\rho)} = \frac{\langle 0|j_{\alpha}^{K^*}|K^*(p',\epsilon')\rangle\langle 0|j_{\mu}^{K^*}|K^*(p,\epsilon)\rangle\langle K^*(p',\epsilon')K^*(p,\epsilon)|\phi(\rho)(q,\epsilon'')\rangle\langle\phi(\rho)(q,\epsilon'')|j_{\nu}^{\phi(\rho)}|0\rangle}{(q^2-m_{\phi(\rho)}^2)(p^2-m_{K^*}^2)(p'^2-m_{K^*}^2)} + \text{higher and continuum states}, \tag{15}$$

$$\Pi_{\mu\nu\alpha}^{K^*} = \frac{\langle 0|j_{\alpha}^{K^*}|K^*(p',\epsilon')\rangle\langle 0|j_{\mu}^{\phi(\rho)}|\phi(\rho)(p,\epsilon)\rangle\langle K^*(p',\epsilon')\phi(\rho)(p,\epsilon)|K^*(q,\epsilon'')\rangle\langle K^*(q,\epsilon'')|j_{\nu}^{K^*}|0\rangle}{(q^2-m_{K^*}^2)(p^2-m_{\phi(\rho)}^2)(p'^2-m_{K^*}^2)} + \text{higher and continuum states}. \tag{16}$$

Generally the matrix elements are defined on the basis of leptonic decay constants or strong coupling constants such as

$$\begin{aligned}
\langle\mathcal{V}(p,\epsilon)\mathcal{V}(p',\epsilon')|\mathcal{V}(q,\epsilon'')\rangle &= ig_{\mathcal{V}\mathcal{V}\mathcal{V}}(q^2)[-q_\nu g_{\mu\alpha}+(q+p')_\mu g_{\alpha\nu}-(q+p)_\alpha g_{\nu\mu}]\times\epsilon^\mu(p)\epsilon'^\alpha(p')\epsilon''^\nu(q),\\
\langle\mathcal{P}(p)\mathcal{P}(p')|\mathcal{V}(q,\epsilon)\rangle &= -g_{\mathcal{P}\mathcal{P}\mathcal{V}}^{\mathcal{V}}(q^2)(p_\nu+p'_\nu)\epsilon^\nu(q),\\
\langle 0|j^{\mathcal{P}}|\mathcal{P}(p)\rangle &= \frac{m_{\mathcal{P}}^2 f_{\mathcal{P}}}{m_q+m_{q'}},\\
\langle 0|j_\mu^{\mathcal{V}}|\mathcal{V}(q,\epsilon)\rangle &= m_{\mathcal{V}}f_{\mathcal{V}}\epsilon_\mu(q),
\end{aligned} \tag{17}$$

in which $f_{\mathcal{P}}$ and $f_{\mathcal{V}}$ are the leptonic decay constants of the pseudoscalar and vector meson respectively, $m_{\mathcal{P}}$ and $m_{\mathcal{V}}$ are meson masses, and ϵ_μ, ϵ'_μ, and ϵ''_μ are the polarization vectors that satisfy the following completeness relation:

$$\epsilon_\mu^*\epsilon_{\mu'} = -g_{\mu\mu'} + \frac{P_\mu P_{\mu'}}{m_{\mathcal{V}}^2}.$$

Using Eqs. (17) and after some calculations, one can rewrite Eqs. (13)–(16) as

$$\begin{aligned}
\Pi_{\mu}^{K^*(\phi)} &= -g_{K^*(\phi)K\pi(K)}^{K^*(\phi)}(q^2)\frac{m_{K^*(\phi)}m_K^2 m_{\pi(K)}^2 f_{K^*(\phi)}f_K f_{\pi(K)}}{(m_s+m_d)(m_d+m_u)(p^2-m_K^2)(p'^2-m_{\pi(K)}^2)(q^2-m_{K^*(\phi)}^2)}\\
&\quad\times\frac{(m_{\pi(K)}^2-m_K^2-m_{K^*(\phi)}^2)}{m_{K^*(\phi)}^2}(p'_\mu+\cdots)+\text{higher and continuum states},\\
\Pi_{\mu}^{\pi(K)} &= -g_{K^*(\phi)K\pi(K)}^{\pi(K)}(q^2)\frac{m_{K^*(\phi)}m_K^2 m_{\pi(K)}^2 f_{K^*(\phi)}f_K f_{\pi(K)}(m_K^2+m_{K^*(\phi)}^2-q^2)}{(m_s+m_d)(m_d+m_u)(p^2-m_K^2)(p'^2-m_{K^*(\phi)}^2)(q^2-m_{\pi(K)}^2)}\\
&\quad\times\frac{1}{m_{K^*}^2}(p'_\mu+\cdots)+\text{higher and continuum states},\\
\Pi_{\mu\nu\alpha}^{\phi(\rho)} &= -g_{\phi(\rho)K^*K^*}^{\phi(\rho)}(q^2)\frac{m_{K^*}^2 m_{\phi(\rho)}f_{\phi(\rho)}f_{K^*}^2(3m_{K^*}^2+m_{K^*}^2-q^2)}{2m_{K^*}^2(p^2-m_{K^*}^2)(p'^2-m_{K^*}^2)(q^2-m_{\phi(\rho)}^2)}(g_{\mu\nu}q_\alpha+\cdots)\\
&\quad+\text{higher and continuum states},\\
\Pi_{\mu\nu\alpha}^{K^*} &= -g_{\phi(\rho)K^*K^*}^{K^*}(q^2)\frac{m_{K^*}^2 m_{\phi(\rho)}f_{\phi(\rho)}f_{K^*}^2(3m_{K^*}^2+m_{\phi(\rho)}^2-q^2)}{2m_{K^*}^2(p^2-m_{\phi(\rho)}^2)(p'^2-m_{K^*}^2)(q^2-m_{K^*}^2)}(g_{\mu\nu}q_\alpha+\cdots)\\
&\quad+\text{higher and continuum states}.
\end{aligned} \tag{18}$$

In order to calculate the strong coupling constants by QCD sum rules, the theoretical side and the phenomenological side of each vertex are equated. The higher and continuum states are almost unknown and can lead to divergences. The situation can be improved and divergences are suppressed if we apply the double Borel transformation to both sides of the correlation function with respect to the p^2 and p'^2 as follows:

$$B_{p^2}(M_1^2)\left(\frac{1}{p^2-m^2}\right)^n=\frac{(-1)^n}{\Gamma(n)}\frac{e^{-\frac{m^2}{M_1^2}}}{(M_1^2)^{(n-1)}},$$
$$B_{p'^2}(M_2^2)\left(\frac{1}{p'^2-m^2}\right)^n=\frac{(-1)^n}{\Gamma(n)}\frac{e^{-\frac{m^2}{M_2^2}}}{(M_2^2)^{(n-1)}},\tag{19}$$

where M_1^2 and M_2^2 are the Borel parameters.

The final result for coupling constants is obtained as follows:

$$g^{\pi(K)}_{K^*(\phi)K\pi(K)}(q^2)=-\frac{(m_d+m_u)(m_d+m_s)m_{K^*(\phi)}(q^2-m^2_{\pi(K)})}{m^2_{\pi(K)}m_K^2f_{K^*(\phi)}f_{\pi(K)}f_K(m^2_{\pi(K)}+m^2_{K^*(\phi)}-q^2)}e^{\frac{m^2_{K^*(\phi)}}{M_1^2}}e^{\frac{m_K^2}{M_2^2}}$$
$$\times\left\{-\frac{1}{4\pi^2}\int_{(m_d+m_s)^2}^{s_0^{K^*(\phi)}}ds'\int_{s_1}^{s_0^{K^*(\phi)}}ds\rho^{\pi(K)}_{K^*(\phi)K\pi(K)}(s,s',q^2)e^{-\frac{s}{M_1^2}}e^{-\frac{s'}{M_2^2}}+\frac{C^{\pi(K)}_{K^*(\phi)K\pi(K)}}{12M_1^2M_2^2}\langle s\bar{s}\rangle\right\},$$
$$g^{K^*(\phi)}_{K^*(\phi)K\pi(K)}(q^2)=-\frac{(m_d+m_s)(m_d+m_u)(q^2-m^2_{K^*})}{m_{K^*}m^2_{\pi(K)}m_K^2f_{K^*}f_{\pi(K)}f_K(m^2_{\pi(K)}-m_K^2-m^2_{K^*(\phi)})}e^{\frac{m^2_{\pi(K)}}{M_1^2}}e^{\frac{m_K^2}{M_2^2}}$$
$$\times\left\{-\frac{1}{4\pi^2}\int_{(m_d+m_s)^2}^{s_0^K}ds'\int_{s'_1}^{s_0^{\pi}(K)}ds\rho^{K^*}_{K^*(\phi)K\pi(K)}(s,s',q^2)e^{-\frac{s}{M_1^2}}e^{-\frac{s'}{M_2^2}}+\frac{C^{K^*(\phi)}_{K^*(\phi)K\pi(K)}}{12M_1^2M_2^2}\langle d\bar{d}\rangle(\langle u\bar{u}\rangle)\right\},$$
$$g^{K^*}_{\phi(\rho)K^*K^*}(q^2)=-\frac{2(q^2-m^2_{K^*})}{m_\phi f_{\phi(\rho)}f^2_{K^*}(3m^2_{K^*}+m^2_{\phi(\rho)}-q^2)}e^{\frac{m^2_{\phi(\rho)}}{M_1^2}}e^{\frac{m^2_{K^*}}{M_2^2}}$$
$$\times\left\{-\frac{1}{4\pi^2}\int_{(m_u+m_s)^2}^{s_0^{K^*}}ds'\int_{s_2}^{s_0^{\phi(\rho)}}ds\rho^{K^*}_{\phi(\rho)K^*K^*}(s,s',q^2)e^{-\frac{s}{M_1^2}}e^{-\frac{s'}{M_2^2}}+\frac{C^{K^*}_{\phi(\rho)K^*K^*}}{12M_1^2M_2^2}\langle s\bar{s}\rangle(\langle d\bar{d}\rangle)\right\},$$
$$g^{\phi(\rho)}_{\phi(\rho)K^*K^*}(q^2)=-\frac{2(q^2-m^2_{\phi(\rho)})}{m_{\phi(\rho)}f_{\phi(\rho)}f^2_{K^*}(4m^2_{K^*}-q^2)}e^{\frac{m^2_{K^*}}{M_1^2}}e^{\frac{m^2_{K^*}}{M_2^2}}$$
$$\times\left\{-\frac{1}{4\pi^2}\int_{(m_u+m_s)^2}^{s_0^{K^*}}ds'\int_{s'_2}^{s_0^{K^*}}ds\rho^{\phi(\rho)}_{\phi(\rho)K^*K^*}(s,s',q^2)e^{-\frac{s}{M_1^2}}e^{-\frac{s'}{M_2^2}}+\frac{C^{\phi(\rho)}_{\phi(\rho)K^*K^*}}{12M_1^2M_2^2}\langle u\bar{u}\rangle(\langle s\bar{s}\rangle)\right\},\tag{20}$$

where s_1, s'_1, s_2, and s'_2 are the lower limits of the integrals over s as

$$s_i=\frac{(m_3^2+q^2-m_1^2-s')(m_1^2s'-q^2m_3^2)}{(m_1^2-q^2)(m_3^2-s')},\tag{21}$$

and we presented the contributions of the quark-quark and quark-gluon condensations as

$$\Pi^{\mathrm{off-shell}}_{\mathrm{nonper}}=\langle q\bar{q}\rangle\frac{C^{\mathrm{off-shell}}}{12M_1^2M_2^2},\tag{22}$$

where the details are given in the Appendix.

III. NUMERICAL ANALYSIS

At this point, numerical analysis is presented to calculate the strong coupling constants. The QCDSR expression of vertex strong coupling constants has some inputs including quark and meson masses, decay constants, continuum thresholds s_0 and s'_0, and Borel parameters M_1^2 and M_2^2.

The values of masses are presented in Table I. Table II show the leptonic decay constants used in the present analysis. The condensation values are given by $\langle s\bar{s}\rangle=(0.8\pm0.2)\langle u\bar{u}\rangle$, $\langle u\bar{u}\rangle=\langle d\bar{d}\rangle=-(0.240\pm0.010\ \mathrm{GeV})^3$ [31].

The Borel parameters are not physical so the physical results should be independent of their variations and for this reason the ranges of Borel parameters are selected so that strong coupling constants remain almost unchanged and stable. Another condition that should be considered in

TABLE I. The values of quark and meson masses in GeV [32].

m_s	m_ϕ	m_{K^*}	m_ρ	m_K	m_π
0.14	1.02	0.89	0.78	0.49	0.14

TABLE II. The leptonic decay constants in MeV.

f_ϕ	f_{K^*}	f_ρ	f_K	f_π
234 ± 10[33]	217 ± 7[32,34]	216 ± 5[32,34]	156.1 ± 8[32,34]	130.4 ± 2[32,34]

range selection is that, with the large increase in Borel parameters, the continuum and higher state contributions cannot be ignored.

The continuum thresholds (s_0 and s'_0) are the upper limit of the integrals in Eqs. (20) and are not physical too. So as mentioned previously, our results should have independency to this parameters. The values of the continuum thresholds are $s_0 = (m + \Delta)^2$ and $s'_0 = (m' + \Delta')^2$, where m and m' are the masses of initial and final particles, respectively, and $0.6 \leq \Delta(\Delta') \leq 0.9$. In Fig. 3, we plot the strong coupling constants $g^{\pi}_{K^*K\pi}$ versus Borel parameters M_1^2 and M_2^2 at $Q^2 = -q^2 = 1$ GeV. So by using the conditions mentioned above, our calculations lead to suitable regions as $7\ \mathrm{GeV}^2 \leq M^2 \leq 14\ \mathrm{GeV}^2$ for all vertices.

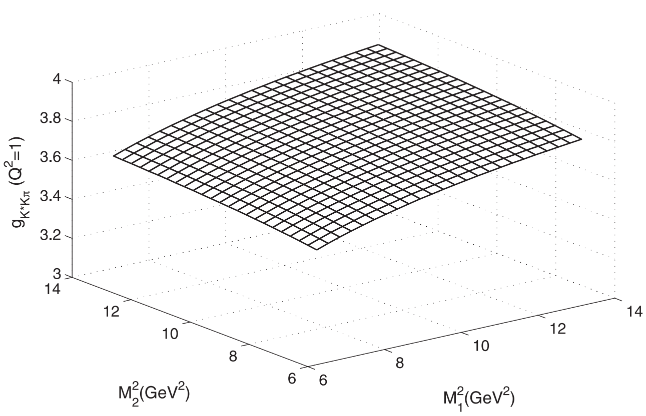

FIG. 3. $g^{\pi}_{K^*K\pi}$ dependencies on Borel parameters M_1 and M_2 at $Q^2 = 1$ GeV.

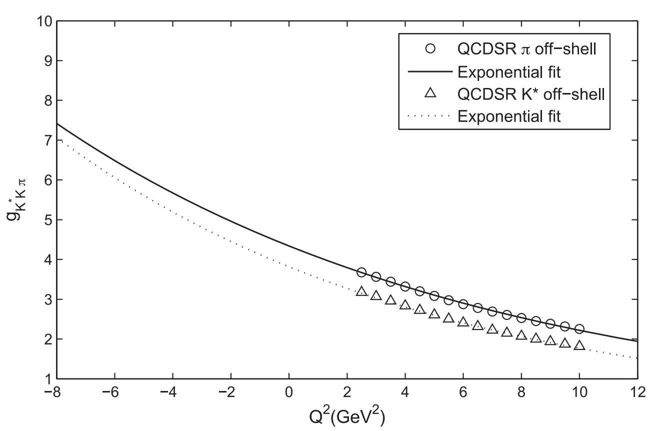

FIG. 4. The strong coupling constant $g^{\pi}_{K^*K\pi}$ dependencies on Q^2.

By acquiring the suitable Borel parameters and using Table I and Table II data, we are able to present the numerical solution of Eqs. (20). In order to reach the strong coupling constant values, defined at $Q^2 = -m^2_{\mathrm{off-shell}}$, a function of $g(Q^2)$ must be found that fits the numerical solution, because in this region, QCDSR leads to invalid results. Our numerical solutions can have a good fit by exponential fit function, i.e.,

$$g(Q^2) = Ae^{-Q^2/B}. \tag{23}$$

As an example, we draw the strong coupling constant $g^{\pi}_{K*K\pi}$ versus Q^2 in Fig. 4. The A and B values of two different off-shell mesons for all vertices are given in Table III.

To reduce the error in extrapolating the strong coupling constant quantities, three sets are considered. If the initial and final particles are similar, then $\Delta = \Delta'$ and $\Delta_{\mathrm{set\,I}} = 0.7$ GeV, $\Delta_{\mathrm{set\,II}} = 0.8$ GeV and $\Delta_{\mathrm{set\,III}} = 0.9$ GeV. Otherwise the values of Δ and Δ' parameters are chosen to be (0.7,0.6) GeV for set I, (0.8,0.7) GeV for set II, and (0.9,0.8) GeV for set III, so that the larger value corresponds to the heavier particle.

Table IV shows the coupling constant values of the $K^*K\pi$, ϕKK, ϕK^*K^*, and ρK^*K^*. The error estimation in

TABLE III. Parameters appearing in the fit functions for all the vertices.

	Set I		Set II		Set III	
Form factor	$A(\Delta_1)$	$B(\Delta_1)$	$A(\Delta_2)$	$B(\Delta_2)$	$A(\Delta_3)$	$B(\Delta_3)$
$g^{\pi}_{K^*K\pi}$	3.16	14.43	4.33	14.92	5.78	15.51
$g^{K^*}_{K^*K\pi}$	2.55	11.65	3.82	13.00	6.24	16.69
$g^{K}_{\phi KK}$	4.24	15.59	5.00	10.09	5.78	16.38
$g^{\phi}_{\phi KK}$	3.61	16.70	4.25	16.23	4.89	16.80
$g^{K^*}_{\phi K^*K^*}$	5.28	17.42	6.75	18.27	8.43	19.27
$g^{\phi}_{\phi KK}$	5.57	20.70	7.13	20.45	8.93	20.50
$g^{K^*}_{\rho K^*K^*}$	4.83	13.18	6.56	14.48	8.65	15.63
$g^{\rho}_{\rho K^*K^*}$	6.57	17.48	8.49	18.46	10.71	19.38

TABLE IV. The strong coupling constants of all the vertices.

Coupling constant		Coupling constant	
$g^{\pi}_{K^*K\pi}$	5.42 ± 1.60	$g^{K^*}_{K^*K\pi}$	5.36 ± 1.65
$g^{K}_{\phi KK}$	5.08 ± 1.09	$g^{\phi}_{\phi KK}$	4.56 ± 1.52
$g^{K^*}_{\phi K^*K^*}$	6.86 ± 1.45	$g^{\phi}_{\phi K^*K^*}$	7.59 ± 1.64
$g^{K^*}_{\rho K^*K^*}$	7.06 ± 1.55	$g^{\rho}_{\rho K^*K^*}$	8.88 ± 1.73

TABLE V. Values of the strong coupling constants using different approaches.

	Our approach	Ref. [39]	Ref. [40]	Ref. [41]	Ref. [35]	Ref. [42]	Ref. [43]	Ref. [44]	Ref. [45]	Ref. [38]
$g_{K^*K\pi}$	5.39 ± 1.63	5.7	4.6	3.76	3.43	8.7	5.5	6.38	4.6	5.43 5.85
		Ref. [41]	Ref. [35]	Ref. [46]	Ref. [47]	Ref. [48]	Ref. [49]			
$g_{\phi KK}$	4.82 ± 1.3	5.55	5.93 4.51	5.77	4.7	4.9	4.56			
		Ref. [35]	Ref. [35]							
$g_{\phi K^*K^*}$	7.22 ± 1.55	5.93	4.51							
		Ref. [36]	Ref. [41]							
$g_{\rho K^*K^*}$	7.97 ± 1.64	3.025	5.55							

TABLE VI. Values of the full widths using different approaches in MeV.

Decay	Our approach	Ref. [50]	Ref. [51]	Ref. [52]	Ref. [53]	Experiment [32]
$K^* \to K\pi$	46.8	46	21	52	51	47.4 ± 0.6
$\phi \to KK$	5.58	3.3	2.5		4.2	4.266 ± 0.031

the coupling constant results is obtained by considering the uncertainties in the physical and nonphysical input parameters. Our calculations show that the most effective factors in the error results are the decay constants and the continuum thresholds. To estimate the theoretical error corresponding to the Borel masses, the behavior of the coupling constants is investigated within the Borel windows ($7\ \mathrm{GeV}^2 \leq M^2 \leq 14\ \mathrm{GeV}^2$) by varying the Borel mass values while fixing all other variables. It is found that such variations lead to less than ~5% of deviations in the values of coupling constants. For example, in the case of $g^{\pi}_{K^*K\pi}$ we obtain 3.05% of error. But outside this selected window for different vertices, the values of coupling constants are not even close to each other.

Table V presents a comparison between our results and the predictions of other calculations. In the SU(3) invariant Lagrangian [35–37], the coupling constants of ϕK^*K^* and ρK^*K^* can be related to each other by $g_{\phi K^*K^*} = g_{\phi KK}$ and $g_{\rho K^*K^*} = g_{\rho KK}$, and according to [38], for different charge states we have $g_{K^*K\pi} = \sqrt{3} g_{K^{*+}K^+\pi^0} = \sqrt{3/2} g_{K^{*+}K^0\pi^+}$ that are used in the comparison of Table V. The strong coupling constant values of $g_{\phi KK}$ and $g_{\phi K^*K^*}$ in Ref. [35] have two results corresponding to $\sin\theta = 0.761$ and $\sin\theta = 1$.

Finally, we are interested in determining the decay width of the $K^{*+}K^0\pi^+$ and ϕK^+K^- decay mode via

$$\Gamma = \frac{g^2_{\mathcal{V}\mathcal{P}\mathcal{P}'} P^3}{6\pi m^2_{\mathcal{V}}},$$

$$P^2 = \frac{[m^2_{\mathcal{V}} - (m_{\mathcal{P}} + m_{\mathcal{P}'})^2][m^2_{\mathcal{V}} - (m_{\mathcal{P}} - m_{\mathcal{P}'})^2]}{4m^2_{\mathcal{V}}}, \qquad (24)$$

so that by using the fraction (Γ_i/Γ) from [32], the full width can be achieved. The obtained decay width is compared to the results from other methods and experiments in Table VI.

IV. CONCLUSIONS

In this research, we used the three-point QCD sum rules to study the $K^*K\pi$, ϕKK, ϕK^*K^*, and ρK^*K^* vertices, leading to the calculation of the strong coupling constants of these vertices. The results are compared to the findings of other models, and predictions. Except for ρK^*K^*, all the rest of the coupling constants have good agreement with the models summarized in Table V. Evaluation of the strong coupling constants could give useful information about the nature of strong interactions. For example, we calculated the decay widths of $K^* \to K\pi$ and $\phi \to KK$, and their comparison to the other models shows good agreement with the experimental results, especially in the $K^* \to K\pi$ decay that is very close to values from experiment.

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APPENDIX: EXPLICIT EXPRESSION FOR C-COEFFICIENTS IN EQ. (22)

In this Appendix, the explicit expressions of $C^{\pi(K)}_{K^*(\phi)K\pi(K)}$, $C^{K^*(\phi)}_{K^*(\phi)K\pi(K)}$, $C^{\phi(\rho)}_{\phi(\rho)K^*K^*}$, and $C^{K^*}_{\phi(\rho)K^*K^*}$ are given by

$$
\begin{aligned}
C^{\pi(K)}_{K^*(\phi)K\pi(K)} = \Big(& 12m_1M_1^2M_2^2 + 12m_3M_1^2M_2^2 + 12m_3m_1^2M_2^2 + 6m_1m_3^2M_2^2 - 6m_2m_3^2M_2^2 \\
& + 3m_2m_0^2M_2^2 - 9m_1m_0^2M_2^2 + 6m_1m_2m_3M_1^2 - 6m_3m_1^2M_1^2 - 6m_3m_2^2M_1^2 \\
& + 6m_3q^2M_1^2 + 12m_1m_3^2M_1^2 + 6m_2m_3^2M_1^2 - 3m_2m_0^2M_1^2 - 9m_1m_0^2M_1^2 \\
& - 6m_1^3m_3^2 - 6m_1m_2^2m_3^2 + 6m_1m_3^2q^2 + 3m_0^2m_1^3 + 3m_1m_2^2m_0^2 \\
& - 3m_1m_0^2q^2 + 3m_1m_2^2m_3^2\frac{M_1^2}{M_2^2} - \frac{3}{2}m_1m_2^2m_0^2\frac{M_1^2}{M_2^2} + 3m_1^2m_2m_3^2\frac{M_2^2}{M_1^2} \\
& + 6m_1^3m_3^2\frac{M_2^2}{M_1^2} - \frac{3}{2}m_1^2m_2m_0^2\frac{M_2^2}{M_1^2} - 3m_1^3m_0^2\frac{M_2^2}{M_1^2}\Big) \times e^{-\frac{m_1^2}{M_1^2}}e^{-\frac{m_2^2}{M_2^2}},
\end{aligned}
$$

$$
\begin{aligned}
C^{K^*(\phi)}_{K^*(\phi)K\pi(K)} = \Big(& 12m_1M_1^2M_2^2 + 6m_3M_1^2M_2^2 + 12m_3m_1^2M_2^2 + 6m_1m_3^2M_2^2 + 6m_2m_3^2M_2^2 \\
& - 3m_2m_0^2M_2^2 - 9m_1m_0^2M_2^2 - 6m_1m_2m_3M_1^2 + 6m_3m_1^2M_1^2 + 12m_1m_3^2M_1^2 \\
& - 6m_2m_3^2M_1^2 + 3m_2m_0^2M_1^2 - 9m_1m_0^2M_1^2 - 6m_1^3m_3^2 - 6m_1m_2^2m_3^2 \\
& + 6m_1m_3^2q^2 + 3m_0^2m_1^3 + 3m_1m_2^2m_0^2 - 3m_1m_0^2q^2 + 3m_1m_2^2m_3^2\frac{M_1^2}{M_2^2} \\
& - \frac{3}{2}m_1m_2^2m_0^2\frac{M_1^2}{M_2^2} - 3m_1^2m_2m_3^2\frac{M_2^2}{M_1^2} + 6m_1^3m_3^2\frac{M_2^2}{M_1^2} + \frac{3}{2}m_1^2m_2m_0^2\frac{M_2^2}{M_1^2} - 3m_1^3m_0^2\frac{M_2^2}{M_1^2}\Big) \times e^{-\frac{m_1^2}{M_1^2}}e^{-\frac{m_2^2}{M_2^2}},
\end{aligned}
$$

$$
\begin{aligned}
C^{\phi(\rho)}_{\phi(\rho)K^*K^*} = \Big(& 6m_1M_1^2M_2^2 + 6m_2M_1^2M_2^2 + 3m_3M_1^2M_2^2 + 6m_3m_1^2M_2^2 + 6m_1m_2m_3M_2^2 \\
& + 6m_2m_3^2M_2^2 - 3m_1m_0^2M_2^2 - 3m_2m_0^2M_2^2 - 3m_1m_0^2M_2^2 + 6m_1m_3^2M_2^2 \\
& + 3m_1m_2m_3M_1^2 - 3m_3m_1^2M_1^2 - 6m_3m_2^2M_1^2 + 3m_3q^2M_1^2 + 6m_2m_3^2M_1^2 \\
& + 6m_1m_3^2M_1^2 - 3m_2m_0^2M_1^2 - 3m_1m_0^2M_1^2 - 3m_1^3m_3^2 - 3m_2^3m_3^2 - 3m_1m_2^2m_3^2 \\
& - 3m_2m_1^2m_3^2 + 3m_1m_3^2q^2 + \frac{3}{2}m_2m_3^2q^2 + \frac{3}{2}m_0^2m_1^3 + \frac{3}{2}m_0^2m_2^3 \\
& + \frac{3}{2}m_1m_2^2m_0^2 + \frac{3}{2}m_2m_1^2m_0^2 - \frac{3}{2}m_1m_0^2q^2 - \frac{3}{2}m_2m_0^2q^2 + 3m_1m_2^2m_3^2\frac{M_1^2}{M_2^2} \\
& + 3m_2^3m_3^2\frac{M_1^2}{M_2^2} - \frac{3}{2}m_1m_2^2m_0^2\frac{M_1^2}{M_2^2} - \frac{3}{2}m_2^3m_0^2\frac{M_1^2}{M_2^2} + 3m_2m_1^2m_3^2\frac{M_2^2}{M_1^2} + 3m_1^3m_3^2\frac{M_2^2}{M_1^2} \\
& - \frac{3}{2}m_2m_1^2m_0^2\frac{M_2^2}{M_1^2} - \frac{3}{2}m_1^3m_0^2\frac{M_2^2}{M_1^2}\Big) \times e^{-\frac{m_1^2}{M_1^2}}e^{-\frac{m_2^2}{M_2^2}} = C^{K^*}_{\phi(\rho)K^*K^*}.
\end{aligned}
$$

[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).
[2] S. S. Agaev, K. Azizi, and H. Sundu, Phys. Rev. D 95, 034008 (2017).
[3] Z. G. Wang, Eur. Phys. J. C 77, 432 (2017).
[4] W. Chen, H. X. Chen, T. G. Steele, and S. L. Zhu, Phys. Rev. D 95, 114005 (2017).
[5] K. Azizi, Y. Sarac, and H. Sundu, Phys. Rev. D 95, 094016 (2017).
[6] W. Chen, H. X. Chen, T. G. Steele, and S. L. Zhu, Phys. Rev. D 92, 054002 (2015).
[7] M. E. Bracco and M. Nielsen, Phys. Rev. D 82, 034012 (2010).
[8] N. Ghahramany, R. Khosravi, and K. Azizi, Phys. Rev. D 78, 116009 (2008).

[9] N. Ghahramany and R. Khosravi, Phys. Rev. D **80**, 016009 (2009).
[10] N. Ghahramany, R. Khosravi, and Z. Naseri, Phys. Rev. D **81**, 036005 (2010).
[11] N. Ghahramany and R. Khosravi, Phys. Rev. D **81**, 016012 (2010).
[12] F. S. Navarra, M. Nielsen, M. E. Bracco, M. Chiapparini, and C. L. Schat, Phys. Lett. B **489**, 319 (2000).
[13] F. S. Navarra, M. Nielsen, and M. E. Bracco, Phys. Rev. D **65**, 037502 (2002).
[14] M. E. Bracco, M. Chiapparini, A. Lozea, F. S. Navarra, and M. Nielsen, Phys. Lett. B **521**, 1 (2001).
[15] M. E. Bracco, M. Chiapparini, F. S. Navarra, and M. Nielsen, Phys. Lett. B **659**, 559 (2008).
[16] R. D. Matheus, F. S. Navarra, M. Nielsen, and R. R. da Silva, Phys. Lett. B **541**, 265 (2002).
[17] R. R. da Silva, R. D. Matheus, F. S. Navarra, and M. Nielsen, Braz. J. Phys. **34**, 236 (2004).
[18] Z. G. Wang and S. L. Wan, Phys. Rev. D **74**, 014017 (2006).
[19] Z. G. Wang, Nucl. Phys. **A796**, 61 (2007).
[20] F. Carvalho, F. O. Durães, F. S. Navarra, and M. Nielsen, Phys. Rev. C **72**, 024902 (2005).
[21] M. E. Bracco, M. Chiapparini, F. S. Navarra, and M. Nielsen, Phys. Lett. B **605**, 326 (2005).
[22] M. E. Bracco, A. J. Cerqueira, M. Chiapparini, A. Lozea, and M. Nielsen, Phys. Lett. B **641**, 286 (2006).
[23] L. B. Holanda, R. S. Marques de Carvalho, and A. Mihara, Phys. Lett. B **644**, 232 (2007).
[24] R. Khosravi and M. Janbazi, Phys. Rev. D **87**, 016003 (2013).
[25] M. E. Bracco and M. Nielsen, Phys. Rev. D **82**, 034012 (2010).
[26] A. Cerqueira, Jr., B. Osrio Rodrigues, and M. E. Bracco, Nucl. Phys. **A875**, 11004 (2011); **A874**, 130 (2012).
[27] G. L. Yu, Z. Y. Li, and Z. G. Wang, Eur. Phys. J. C **75**, 243 (2015).
[28] H. Sundu, J. Y. Sungu, S. Sahin, N. Yinelek, and K. Azizi, Phys. Rev. D **83**, 114009 (2011).
[29] H. Sundu, J. Y. Sungu, S. Sahin, N. Yinelek, and K. Azizi, Phys. Rev. D **83**, 114009 (2011).
[30] M. Janbazi, N. Ghahramany, and E. Pourjafarabadi, Eur. Phys. J. C **74**, 2718 (2014).
[31] B. L. Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006).
[32] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[33] I. Bediaga and M. Nielsen, Phys. Rev. D **68**, 036001 (2003).
[34] H. M. Choi, C. R. Ji, Z. Li, and H. Y. Ryu, Phys. Rev. C **92**, 055203 (2015).
[35] D. Y. Chen, X. Liu, and T. Matsuki, Eur. Phys. J. C **72**, 2008 (2012).
[36] M. R. Talabtash and H. Mehraban, Phys. Scr. **89**, 125301 (2014).
[37] A. M. Torres, K. P. Khemchandani, F. S. Navarra, and M. Nilsen, Phys. Rev. D **90**, 114023 (2014).
[38] N. Jaus, Phys. Rev. D **67**, 094010 (2003).
[39] S. Prelovsek, L. Leskovec, C. B. Lang, and D. Mohler, Phys. Rev. D **88**, 054508 (2013).
[40] C. D. Lu, Y. L. Shen, and W. Wang, Phys. Rev. D **73**, 034005 (2006).
[41] X. Liu, B. Zhang, L. L. Shen, and S. L. Zhu, Phys. Rev. D **75**, 074017 (2007).
[42] S. L. Zhu, Eur. Phys. J. A **4**, 277 (1999).
[43] J. Nebred and J. R. Pelaez, Phys. Rev. D **81**, 054035 (2010).
[44] Z. Fu and K. Fu, Phys. Rev. D **86**, 094507 (2012).
[45] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[46] B. Mohammadi and H. Mehraban, Prog. Theor. Phys. **128**, 477 (2012).
[47] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D **57**, 4334 (1998).
[48] T. M. Aliev, I. Kanik, and M. Savci, Eur. Phys. J. C **31**, 197 (2003).
[49] G. Li, Q. Zhao, and B. S. Zou, Phys. Rev. D **77**, 014010 (2008).
[50] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B **744**, 1 (2015).
[51] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996); T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997); T. Barnes, N. Black, and P. R. Page, Phys. Rev. D **68**, 054014 (2003); F. E. Close and E. S. Swanson, Phys. Rev. D **72**, 094004 (2005); T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[52] M. A. Ivanov, Y. L. Kalinovsky, and C. D. Roberts, Phys. Rev. D **60**, 034018 (1999).
[53] M. P. Khanna and M. K. Volkov, Theor. Math. Phys. **102**, 158 (1995); M. K. Volkov, D. Ebert, and M. Nagy, Int. J. Mod. Phys. A **13**, 5443 (1998); M. K. Volkov, V. L. Yudichev, and D. Ebert, J. Phys. G **25**, 2025 (1999).