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Leptonic current structure and azimuthal asymmetry in deeply inelastic scattering

Hong-Fei Zhang*

Department of Physics, School of Biomedical Engineering, Third Military Medical University, Chongqing 400038, China

Zhan Sun†

School of Science, Guizhou Minzu University, Guiyang 500025, People's Republic of China

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We present a compact form of the leptonic currents for the computation of the processes involving an initial virtual boson (photon, $W^{\pm}$, or Z_0). For deeply inelastic scattering, once the azimuthal angle of the plane expanded by the initial- and final-state leptons is integrated over in the boson-proton center-of-mass frame, the azimuthal-asymmetric terms vanish, which, however, is *not* true when some physical quantities (such as the transverse momentum of the observed particle) are specified in the laboratory frame. The misuse of the symmetry may lead to wrong results.

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I. INTRODUCTION

Deeply inelastic scattering (DIS) provides a useful probe to the partonic structure of hadrons and photons. It uses leptons to collide with the species of particles we study and observes the kinematics of the scattered leptons, which can provide information for the structure of the targets. Taking electron-proton scattering as an example, the kinematics of the scattered leptons can be described by any two of the following four variables:

$$Q^2 = -q^2 \equiv -(k-k')^2, \qquad W^2 = (P+q)^2,$$
$$x = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot k}, \tag{1}$$

where P, k, k', and q are, as illustrated in Fig. 1, the momenta of the initial proton, the initial and final lepton, and the virtual boson, respectively. The differential cross section can be written, in the limit $P^2/Q^2 \ll 1$, as

$$\mathrm{d}\sigma = \frac{1}{4P\cdot k}\frac{1}{N_c N_s} L_{\mu\nu}\frac{1}{(Q^2)^2} H^{\mu\nu}\mathrm{d}\Phi'\mathrm{d}\Phi_H, \tag{2}$$

where $1/(N_c N_s)$ is the color and spin average factor, $L_{\mu\nu}$ and $H^{\mu\nu}$ are the leptonic and hadronic tensors, respectively, and

$$\mathrm{d}\Phi' = \frac{\mathrm{d}^3k'}{(2\pi)^3 2k'_0},$$
$$\mathrm{d}\Phi_h = \frac{\mathrm{d}^3p}{(2\pi)^3 2p_0},$$
$$\mathrm{d}\Phi_X = (2\pi)^4\delta^4\left(P+q-p-\sum_i p_i\right)\prod_i \frac{\mathrm{d}^3p_i}{(2\pi)^3 2p_{i0}},$$
$$\mathrm{d}\Phi_H = \mathrm{d}\Phi_h \mathrm{d}\Phi_X. \tag{3}$$

*hfzhang@ihep.ac.cn
†zhansun@cqu.edu.cn

Here, i runs over all the final states other than the scattered lepton and the tagged hadron (p). The leptonic tensor can be directly calculated and obtained as

$$L_{\mu\nu} = 4\pi\alpha \mathrm{Tr}(\not{k}\gamma_\mu \not{k}'\gamma_\mu)$$
$$= 8\pi\alpha Q^2\left[\left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}\right) + \frac{(2k-q)_\mu(2k-q)_\nu}{Q^2}\right]$$
$$\equiv 8\pi\alpha Q^2 l_{\mu\nu}, \tag{4}$$

where α is the electromagnetic coupling and the lepton mass and the contributions from the Z_0 propagator are neglected for the moment and will be considered in the sequel. Having $d\Phi_H$ integrated over, the structure of the hadronic tensor,

$$W^{\mu\nu}(P,q) \equiv \int H^{\mu\nu}(P,q,h,p_1,\ldots,p_n)\mathrm{d}\Phi_H, \tag{5}$$

is restricted by the Lorentz covariance and, thus, can be decomposed into the linear combination of the current conserving dimensionless basic tensors as

$$W^{\mu\nu} = \left(-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right)F_1(x,Q^2)$$
$$+\frac{1}{Q^2}\left(q+\frac{Q^2}{P\cdot q}P\right)^\mu\left(q+\frac{Q^2}{P\cdot q}P\right)^\nu\frac{1}{2x}F_2(x,Q^2)$$
$$-\frac{i}{P\cdot q}\epsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta F_3(x,Q^2), \tag{6}$$

where F_1, F_2, and F_3 are the structure functions of the proton and $\epsilon^{0123} = 1$ in our convention. In the Bjorken limit, Q^2, $P\cdot q \to \infty$, the structure functions obey an approximate scaling law; i.e., they depend only on the dimensionless variable x [1–3]. This scaling law indicates that, in the high-energy limit, the hadrons interact via

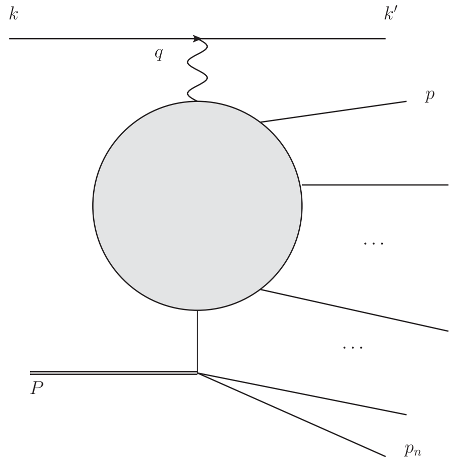

FIG. 1. The illustrative diagram for the DIS processes.

pointlike partons inside them [4,5]. The structure functions describe the parton distributions in high-energy hadrons (see, e.g., [6,7]). The Q^2 scaling of the structure functions can be well described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations [8–10].

Through a short calculation, one can obtain

$$l_{\mu\nu}W^{\mu\nu} = 2F_1(x,Q^2) + \frac{2-2y}{xy^2}F_2(x,Q^2). \tag{7}$$

It is easy to verify that, by setting

$$l_{\mu\nu} = \frac{2-2y+y^2}{y^2}\epsilon_{\mu\nu} - \frac{6-6y+y^2}{y^2}\epsilon_{L\mu\nu}, \tag{8}$$

or equivalently

$$l_{\mu\nu} = \frac{2-2y+y^2}{y^2}\epsilon_{T\mu\nu} - \frac{4(1-y)}{y^2}\epsilon_{L\mu\nu}, \tag{9}$$

where

$$\epsilon_{\mu\nu} = -g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2},$$
$$\epsilon_{L\mu\nu} = -\frac{1}{Q^2}\left(q_\mu + \frac{Q^2}{P\cdot q}P_\mu\right)\left(q_\nu + \frac{Q^2}{P\cdot q}P_\nu\right),$$
$$\epsilon_{T\mu\nu} = \epsilon_{\mu\nu} - \epsilon_{L\mu\nu}, \tag{10}$$

one can reproduce the contraction $L_{\mu\nu}W^{\mu\nu}$ for the real form of the leptonic tensor presented in Eq. (4). Note that the leptonic momenta k and k' do not appear in the reduced leptonic tensor presented in Eq. (8) or Eq. (9), the employment of which, thus, can greatly simplify the calculation of the cross sections.

For semi-inclusive DIS (SIDIS), an additional final state other than the scattered lepton is measured. The transverse-spin and azimuthal asymmetry will emerge [11–24], which,

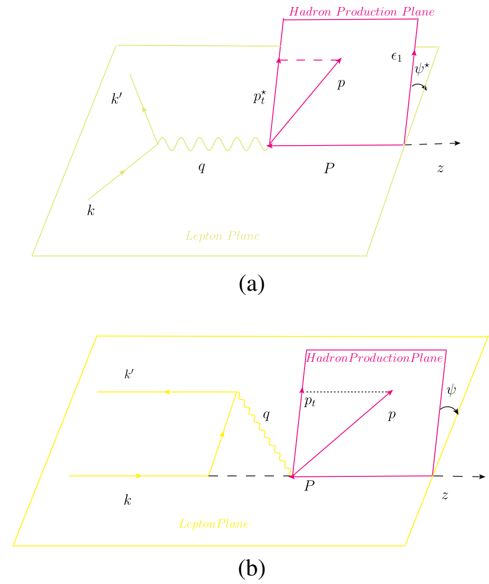

FIG. 2. Illustrations of the process in z frames (a) and the laboratory frame (b).

in different kinematic regions, can provide crucial information for the transverse-momentum-dependent parton distribution and the higher twist transverse-spin-dependent multiparton correlation functions. On the experiment side, the HERMES [25] and COMPASS [26] Collaborations measured the azimuthal asymmetries in SIDIS off unpolarized targets and observed nonvanishing cosine modulations and their strong dependence on the kinematical variables. All the discussions in the literature on the azimuthal angle and transverse spin are carried out in such frames in which the vector boson and the target travel along the opposite direction. Without the loss of generality, we assume the spatial momentum of the vector boson is along the z direction and name such frames as z frames [see Fig. 2(a)]. In any frame other than z frames, the azimuthal angle (ψ) dependence of the plane expanded by the initial and final leptons around the z axis is not a simple trigonometric function and, with ψ integrated over, does not vanish. To our astonishment, numerous works employed the form of the leptonic tensor in Eq. (8) presenting results for the processes in which the transverse momentum (p_t) or the rapidity (Y) of the observed particle in the laboratory frame is specified or a cut is applied on these parameters. As a matter of fact, in the cases stated above, this form of the leptonic tensor will lead to wrong results. Among these works[1] (see, e.g., [27]) are some highly cited articles and calculations adopted by Monte Carlo generators.

In this paper, we will provide a compact form of the leptonic tensor, which, on the one hand, is valid in all kinds

[1]There are many papers adopting reference frames not belonging to the class of z frames and, at the same time, the reduced leptonic tensor; however, we list only one of them here.

of processes, on the other hand, involves only momenta in a hadronic process, and, thus, helps with the simplification of the computation. In Sec. II, we present the azimuthal-dependent form of the leptonic tensor, including a short note on the application of our approach in the e^+e^- annihilation processes in Sec. II C. In Sec. III, we discuss in detail the difference between the real form of the leptonic tensor and the reduced ones, following which is the concluding remark in Sec. IV.

II. THE FORM OF THE LEPTONIC TENSOR

We will provide a compact form of the leptonic tensor for SIDIS and then a comprehensive form of the leptonic tensor for the most generalized DIS. Having ψ integrated over in z frames, Eq. (8) will be automatically reproduced. However, when some physical quantities, e.g., p_t or Y, in the laboratory frame are specified, the integration over ψ (in any frame) does not lead to the conventional leptonic tensor in Eq. (8). In the next section, we will see that the difference between the correct and the wrong results can be huge.

A. The leptonic tensor for SIDIS

Let us first investigate the processes in which only one final-state hadron (p) is observed. Integrating over all the other hadronic final states, one can define the hadronic tensor

$$W_h^{\mu\nu}(P,q,p) \equiv \int H^{\mu\nu}(P,q,p,p_1,\ldots,p_n)\mathrm{d}\Phi_X. \tag{11}$$

Since W_h is a Lorentz covariant tensor and dependent only on the momenta P, q, and p, it can thus be decomposed as the linear combination of the tensors constituted of $-g^{\mu\nu}$, P, q, and p. Note that W_h satisfies the following equation:

$$q_\mu W_h^{\mu\nu} = 0. \tag{12}$$

Thus, with the above elements, only four independent tensors can be constructed. One can define the normalized longitudinal and transverse vectors, respectively, as

$$\begin{aligned} \epsilon_L &= \frac{1}{Q}\left(q + \frac{Q^2}{P\cdot q}P\right), \\ \epsilon_1 &= \frac{1}{p_t^\star}(p - \rho P - zq), \end{aligned} \tag{13}$$

where $p_t^\star$ is the transverse momentum of p in z frames, M is the mass of p, $z = P\cdot p/P\cdot q$ is the elasticity coefficient, and

$$\rho = \frac{p\cdot q + zQ^2}{P\cdot q} = \frac{p_t^{\star 2} + M^2 + z^2Q^2}{2zP\cdot q}. \tag{14}$$

Apparently, we have

$$\epsilon_L^{\mu\nu} = -\epsilon_L^\mu \epsilon_L^\nu. \tag{15}$$

We label all the physical quantities calculated in z frames by the superscript $\star$ hereinafter. It is easy to check the following relations:

$$\begin{aligned} &q\cdot\epsilon_L = q\cdot\epsilon_1 = P\cdot\epsilon_1 = 0, \\ &\epsilon_L^2 = 1, \epsilon_1^2 = -1. \end{aligned} \tag{16}$$

As a matter of fact, ϵ_1 is a normalized vector perpendicular to both P and q and parallel to the transverse component of the momentum p in z frames, as illustrated in Fig. 2(a). Accordingly, W_h can be decomposed as

$$\begin{aligned} W_h^{\mu\nu} &= W_g\epsilon^{\mu\nu} + W_L\epsilon_L^\mu\epsilon_L^\nu \\ &\quad + W_{LT}(\epsilon_L^\mu\epsilon_1^\nu + \epsilon_1^\mu\epsilon_L^\nu) + W_T\epsilon_1^\mu\epsilon_1^\nu, \end{aligned} \tag{17}$$

where W_g, W_L, W_{LT}, and W_T are the coefficients of the four independent normalized tensors.

With a short calculation, one can obtain

$$\begin{aligned} l_{\mu\nu}W_h^{\mu\nu} &= 2W_g + \frac{4(1-y)}{y^2}W_L \\ &\quad - \frac{4(2-y)}{y^2}\sqrt{1-y}\cos\psi^\star W_{LT} \\ &\quad + \left[1 + \frac{2-2y}{y^2} + \frac{2-2y}{y^2}\cos(2\psi^\star)\right]W_T, \end{aligned} \tag{18}$$

where $\psi^\star$ is the azimuthal angle of the lepton plane around the z axis relative to the hadron production plane in z frames, as is illustrated in Fig. 2(a). It is easy to verify that, with

$$\begin{aligned} l^{\mu\nu} &= A_g\epsilon^{\mu\nu} + A_L\epsilon_L^\mu\epsilon_L^\nu \\ &\quad + A_{LT}(\epsilon_L^\mu\epsilon_1^\nu + \epsilon_1^\mu\epsilon_L^\nu) + A_T\epsilon_1^\mu\epsilon_1^\nu, \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_g &= 1 + \frac{2(1-y)}{y^2} - \frac{2(1-y)}{y^2}\cos(2\psi^\star), \\ A_L &= 1 + \frac{6(1-y)}{y^2} - \frac{2(1-y)}{y^2}\cos(2\psi^\star), \\ A_{LT} &= \frac{2(2-y)}{y^2}\sqrt{1-y}\cos\psi^\star, \\ A_T &= \frac{4(1-y)}{y^2}\cos(2\psi^\star), \end{aligned} \tag{20}$$

one can reproduce the results in Eq. (18). It seems as if the expression of the reduced leptonic tensor in Eq. (19) is more complicated than the real one in Eq. (4). However, the reduced one involves only the momenta in the hadronic process; in other words, the leptonic momenta do not appear. Correspondingly, the number of independent Lorentz invariants in the calculation, while employing Eq. (19), is reduced by 3, which can greatly simplify the computation, especially when the calculation is carried out at the loop level.

Taking into account the current conservation, the leptonic tensor in Eq. (19) can be rewritten as

$$l^{\mu\nu} = C_1(-g^{\mu\nu}) + C_2 P^\mu P^\nu + C_3 \frac{P^\mu p^\nu + p^\mu P^\nu}{2} + C_4 p^\mu p^\nu, \tag{21}$$

where

$$\begin{aligned} C_1 &= A_g, \\ C_2 &= \frac{4x}{yS}(A_L - 2\beta A_{LT} + \beta^2 A_T), \\ C_3 &= \frac{4x}{Q p_t^\star}(A_{LT} - \beta A_T), \\ C_4 &= \frac{1}{p_t^{\star 2}} A_T, \end{aligned} \tag{22}$$

with

$$\beta = \frac{p_t^{\star 2} + M^2 + z^2 Q^2}{2zQ p_t^\star}. \tag{23}$$

The leptonic tensor expressed in Eq. (21) has replaced the normalized vectors by the momenta of the interacting particles, which is more suitable for using in calculations.

The leptonic tensor is more complicated for charged and weak neutral current DIS, when antisymmetric tensors also participate. This is true for the cases in which the beams and targets are polarized as well. For most of the cases, the antisymmetric part of the leptonic tensor is proportional to the following normalized structure:

$$l^a_{\mu\nu} = \frac{2}{Q^2}\epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta, \tag{24}$$

and that of the hadronic tensor $W_h^{\mu\nu}$ can be decomposed as

$$W_h^{a\mu\nu}(P,q,p) = W_L^a \frac{1}{Q}\epsilon^{\mu\nu\alpha\beta} q_\alpha \epsilon_{L\beta} + W_T^a \frac{1}{Q}\epsilon^{\mu\nu\alpha\beta} q_\alpha \epsilon_{1\beta}. \tag{25}$$

One can obtain

$$l^a_{\mu\nu} W_h^{a\mu\nu} = \frac{2(2-y)}{y} W_L^a - \frac{4}{y}\sqrt{1-y}\cos\psi^\star W_T^a. \tag{26}$$

To reproduce the results in the above equation, the leptonic tensor can be written in terms of the hadron momenta as

$$l^a_{\mu\nu} = \frac{2-y}{y}\frac{1}{Q}\epsilon_{\mu\nu\alpha\beta} q^\alpha \epsilon_L^\beta + \frac{2}{y}\sqrt{1-y}\cos\psi^\star \frac{1}{Q}\epsilon_{\mu\nu\alpha\beta} q^\alpha \epsilon_1^\beta. \tag{27}$$

For inclusive DIS when the target is transversely polarized, one can directly employ Eqs. (19) and (27), with ϵ_1 replaced by the polarization vector of the target.

B. The leptonic tensor for the most generalized DIS

When several final states are observed and/or the polarization of the leptonic beam or the target is specified, the hadron momenta and the polarization vectors are not constrained in a plane; accordingly, the hadronic tensor is not only related to P, q, and p. We need to introduce another vector, ϵ_2, which, in association with ϵ_L and ϵ_1, consists of a complete set of normalized, mutually orthogonal vectors perpendicular to q. Here and in the following, we say a 4-vector a is perpendicular to q when $a \cdot q = 0$. We define ϵ_2 as

$$\epsilon_2^\mu = \frac{1}{Q}\epsilon^{\mu\nu\alpha\beta} q_\nu \epsilon_{L\alpha} \epsilon_{1\beta}. \tag{28}$$

Apparently, we have

$$q\cdot\epsilon_2 = P\cdot\epsilon_2 = p\cdot\epsilon_2 = 0, \qquad \epsilon_2^2 = -1. \tag{29}$$

Namely, ϵ_1 and ϵ_2 are the two normalized, mutually orthogonal transverse vectors in z frames. Any vector perpendicular to q can be decomposed into the linear combination of ϵ_L, ϵ_1, and ϵ_2. Note that $2k - q$ is perpendicular to q. To express $2k - q$ in terms of these three vectors, we need to calculate $2k \cdot \epsilon_i$ $(i = L, 1, 2)$, which are obtained as

$$\begin{aligned} 2k\cdot\epsilon_L &= Q\left(\frac{2}{y} - 1\right), \\ 2k\cdot\epsilon_1 &= -\frac{2Q}{y}\sqrt{1-y}\cos\psi^\star, \\ 2k\cdot\epsilon_2 &= -\frac{2Q}{y}\sqrt{1-y}\sin\psi^\star. \end{aligned} \tag{30}$$

Then, $2k - q$ can be expressed as

$$2k - q = Q\left(\frac{2}{y} - 1\right)\epsilon_L + \frac{2Q}{y}\sqrt{1-y}\cos\psi^\star\epsilon_1 + \frac{2Q}{y}\sqrt{1-y}\sin\psi^\star\epsilon_2, \tag{31}$$

employing which we can obtain the expression of $l^{\mu\nu}$ as

$$l^{\mu\nu} = A_1\epsilon_L^\mu\epsilon_L^\nu + A_2(\epsilon_L^\mu\epsilon_1^\nu + \epsilon_1^\mu\epsilon_L^\nu) + A_3(\epsilon_L^\mu\epsilon_2^\nu + \epsilon_2^\mu\epsilon_L^\nu) + A_4\epsilon_1^\mu\epsilon_1^\nu + A_5(\epsilon_1^\mu\epsilon_2^\nu + \epsilon_2^\mu\epsilon_1^\nu) + A_6\epsilon_2^\mu\epsilon_2^\nu, \tag{32}$$

where

$$A_1 = \frac{4(1-y)}{y^2},$$
$$A_2 = \frac{2(2-y)}{y^2}\sqrt{1-y}\cos\psi^\star,$$
$$A_3 = \frac{2(2-y)}{y^2}\sqrt{1-y}\sin\psi^\star,$$
$$A_4 = 1+\frac{2(1-y)}{y^2}+\frac{2(1-y)}{y^2}\cos(2\psi^\star),$$
$$A_5 = \frac{2(1-y)}{y^2}\sin(2\psi^\star),$$
$$A_6 = 1+\frac{2(1-y)}{y^2}-\frac{2(1-y)}{y^2}\cos(2\psi^\star), \tag{33}$$

and the relation

$$\epsilon^{\mu\nu} = -\epsilon_L^\mu\epsilon_L^\nu + \epsilon_1^\mu\epsilon_1^\nu + \epsilon_2^\mu\epsilon_2^\nu \tag{34}$$

has been employed. Then one can find out the relations among ϵ_1, ϵ_2, and $\epsilon_T^{\mu\nu}$ defined in Eq. (10) as

$$\epsilon_T^{\mu\nu} = \epsilon_1^\mu\epsilon_1^\nu + \epsilon_2^\mu\epsilon_2^\nu. \tag{35}$$

With $\psi^\star$ integrated from 0 to 2π, the form of the leptonic tensor presented in Eq. (8) or Eq. (9) can be reproduced. However, note that the above statement is true only when the hadronic part of the cross section is independent of $\psi^\star$.

With Eq. (31), the antisymmetric tensor $l^a_{\mu\nu}$ can be obtained as

$$l^a_{\mu\nu} = \frac{2-y}{y}\frac{1}{Q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\epsilon_L^\beta + \frac{2}{y}\sqrt{1-y}\cos\psi^\star\frac{1}{Q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\epsilon_1^\beta + \frac{2}{y}\sqrt{1-y}\sin\psi^\star\frac{1}{Q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\epsilon_2^\beta. \tag{36}$$

The integration over $\psi^\star$ leaves only the first term in Eq. (36), which is explicitly written

$$l^a_{\mu\nu} = \frac{2-y}{y}\frac{1}{Q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\epsilon_L^\beta. \tag{37}$$

Equations (19), (27), (32), and (36) can reduce the number of the Lorentz invariants involved and, consequently, help to simplify the calculation. Furthermore, they can provide important information for the structure of the cross sections.

To complete our discussion, we note that, in collinear factorization, one can simultaneously replace P by the parton momentum in Eqs. (13) and (14). Another issue to address is that Eq. (19) can be reproduced by setting $A_3 = A_5 = 0$ in Eq. (32), while Eq. (27) can be reproduced by dropping the last term on the right-hand side of Eq. (36). Actually, with Eq. (34), one can obtain the following relations:

$$A_1 = A_L - A_g, \qquad A_2 = A_{LT},$$
$$A_3 = 0, \qquad A_4 = A_g + A_T,$$
$$A_5 = 0, \qquad A_6 = A_g. \tag{38}$$

C. Notes on the e^+e^- annihilation

The approach discussed above can also be applied to single inclusive particle production in e^+e^- annihilation, the squared amplitude (without averaging the initial spin) for which can be expressed as

$$|\mathcal{A}|^2 = \frac{1}{s^2}L_{\mu\nu}H^{\mu\nu}. \tag{39}$$

The hadronic tensor, after the integration of all the phase space other than the observed particle and neglecting the contributions from the Z-boson propagator, will be reduced into the combination of the following two tensors:

$$-g^{\mu\nu}+\frac{q^\mu q^\nu}{s}, \qquad \left(p-\frac{p\cdot q}{s}q\right)^\mu\left(p-\frac{p\cdot q}{s}q\right)^\nu, \tag{40}$$

where q is the sum of the initial momenta and p is the momentum of the observed particle. Taking the current conservation into account, one can easily obtain

$$L_{\mu\nu} = 4\pi\alpha s\left[-g_{\mu\nu}(1+\cos^2\theta)+\frac{p_\mu p_\nu}{\boldsymbol{p}^2}(1-3\cos^2\theta)\right], \tag{41}$$

where θ is the angle between $\boldsymbol{p}$ and the spatial momentum of e^- (or e^+) in the e^+e^- center-of-mass frame. Integrating over $\cos\theta$, the second term on the right-hand side of Eq. (41) will vanish, and the first term will reduce to $8\pi\alpha s/3$. We can rewrite Eq. (41) in the Lorentz invariant form by substituting the following equations:

$$\boldsymbol{p}^2 = \frac{(p\cdot q)^2}{s} - M^2, \qquad \cos^2\theta = \frac{(p\cdot k - p\cdot k')^2}{\boldsymbol{p}^2 s}, \tag{42}$$

where k and k' are the momenta of e^- and e^+, respectively, and M is the mass of p.

III. COMPARISON OF THE REDUCED LEPTONIC TENSOR TO THE REAL ONE

Equations (8), (9), and (37) are the generally used formulas in many papers in the computation in DIS; thus, we name them as the conventional leptonic tensors. However, these formulas are not valid when some quantities, e.g., p_t or Y, in the laboratory frame are specified. This is generally because, as long as these quantities are specified, the hadronic part of the cross section is *not* independent of $\psi^\star$, which is actually manifest regarding the following relations:

$$p_t^2 = p_t^{\star 2} + z^2 Q^2 (1-y) - 2zQp_t^\star \sqrt{1-y}\cos\psi^\star,$$
$$Y = \frac{1}{2}\ln\{[p_t^{\star 2} + M^2 + z^2(1-y)Q^2 - 2z\sqrt{1-y}Qp_t^\star \cos\psi^\star]/(4y^2 z^2 E_l^2)\}, \tag{43}$$

where E_l is the energy of the incident lepton in the laboratory frame. The derivation of Eq. (43) can be found in the Appendix. Once p_t^2 or Y is specified, the sine and cosine terms in Eqs. (20) and (33) do not vanish after the integration over $\psi^\star$.

To make our point clearer, we present here a more explicit form of the cross section for the case in which the p_t in the laboratory frame is specified. Here we constrain our discussions to only the symmetric leptonic tensor in SIDIS. The differential cross section for one final-state hadron production can be expressed as [according to Eq. (2)]

$$\mathrm{d}\sigma = \frac{1}{N_c N_s}\frac{4\pi\alpha}{SQ^2} l_{\mu\nu} W_h^{\mu\nu} \mathrm{d}\Phi' \mathrm{d}\Phi_h, \tag{44}$$

where

$$S = 2P\cdot k \tag{45}$$

is the squared colliding energy. The phase space can be obtained as

$$\mathrm{d}\Phi' = \frac{1}{32\pi^3}\mathrm{d}Q^2 \mathrm{d}y \mathrm{d}\psi^\star,$$
$$\mathrm{d}\Phi_h = \frac{1}{32\pi^3 z}\mathrm{d}p_t^{\star 2}\mathrm{d}z. \tag{46}$$

If we define

$$w_g = -g_{\mu\nu}W_h^{\mu\nu},$$
$$w_L = \epsilon_{L\mu}\epsilon_{L\nu}W_h^{\mu\nu},$$
$$w_{LT} = (\epsilon_{L\mu}\epsilon_{1\nu} + \epsilon_{1\mu}\epsilon_{L\nu})W_h^{\mu\nu},$$
$$w_T = \epsilon_{1\mu}\epsilon_{1\nu}W_h^{\mu\nu}, \tag{47}$$

the differential cross section can then be expressed as

$$\mathrm{d}\sigma = \frac{\alpha}{256\pi^5 N_s N_c S Q^2 z}\sum_i A_i w_i \mathrm{d}Q^2 \mathrm{d}y \mathrm{d}p_t^{\star 2}\mathrm{d}z\mathrm{d}\psi^\star, \tag{48}$$

where i runs over g, L, LT, and T.

Apparently, A_i are functions of y and $\psi^\star$, while w_i are functions of Q^2, y, $p_t^\star$, and z. If one measures only the quantities in z frames, e.g., $p_t^\star$ is fixed, w_i do not depend on $\psi^\star$; accordingly, the integration over $\psi^\star$ makes the cosine terms in A_i vanish. However, if, e.g., p_t in the laboratory frame is fixed, the values of $p_t^\star$ and $\psi^\star$ are constrained in a curved surface. When $\psi^\star$ varies, w_i changes accordingly. In this case, the cosine terms in A_i will not vanish after the integration over $\psi^\star$.

To be more explicit, we can replace $\mathrm{d}p_t^{\star 2}$ by $\mathrm{d}p_t^2$ with the Jacobian multiplied. The Jacobian can be easily obtained regarding Eq. (43) as

$$J \equiv \left|\frac{\partial p_t^{\star 2}}{\partial p_t^2}\right| = \frac{p_t^\star}{\sqrt{p_t^2 - (1-y)z^2 Q^2 \sin^2\psi^\star}}. \tag{49}$$

Apparently, we have the following inequalities:

$$\int_0^{2\pi} d\psi^\star \cos\psi^\star J \neq 0,$$
$$\int_0^{2\pi} d\psi^\star \cos(2\psi^\star) J \neq 0. \tag{50}$$

Correspondingly, the conventional leptonic tensors cannot be reproduced with the integration over $\psi^\star$.

We can conclude that the structure functions F_1 and F_2 are not sufficient to describe the cross sections when p_t or Y are not taken to cover all their possible values. Even for the processes in which $p_\mu P_\nu W_h^{\mu\nu} = p_\mu p_\nu W_h^{\mu\nu} = 0$, the conventional leptonic tensor in Eq. (8) or Eq. (9) will also lead to wrong results.

We use R to denote the ratio of the differential cross section calculated with the employment of the leptonic tensor presented in Eqs. (8), (9), and (37) to the correct one, say, that obtained using Eq. (4). As an example, we investigate the J/ψ inclusive production in photonic current DIS, which has been studied in Ref. [28]. However, it adopted the conventional leptonic tensor to present results for p_t and rapidity distributions in the laboratory frame. It is worth noting that, with the same form of the leptonic tensor as given in Ref. [28], we can reproduce their results. R as functions of p_t^2 and the rapidity of the J/ψ (y_ψ) in the laboratory frame are presented in Fig. 3. As Q^2 increases, the difference between the value of R and 1 becomes larger in the high-p_t region. For $Q^2 = 400\ \mathrm{GeV}^2$, the value of R can be as large as 3.1 in the high-p_t region and as small as 0.6 in the low-p_t region. For specified values of z and integrated Q^2 in the range $4\ \mathrm{GeV}^2 < Q^2 < 100\ \mathrm{GeV}^2$, the p_t^2 dependence of R is also studied. R peaks at around $p_t^2 \approx 70\ \mathrm{GeV}^2$ and $z \approx 0.7$, where $R = 1.8$ is obtained. The difference between the wrong and correct results for specified values of y_ψ, however, is not so large as that for specified values of p_t^2. In midrapidity regions, especially $-0.5 < y_\psi < 0.5$, where most of the J/ψ events are produced, R is almost equal to 1, which means in this region, even with the wrong form of the leptonic tensor, one can generally reproduce the correct results. However, this might be true only for specific processes. For a different

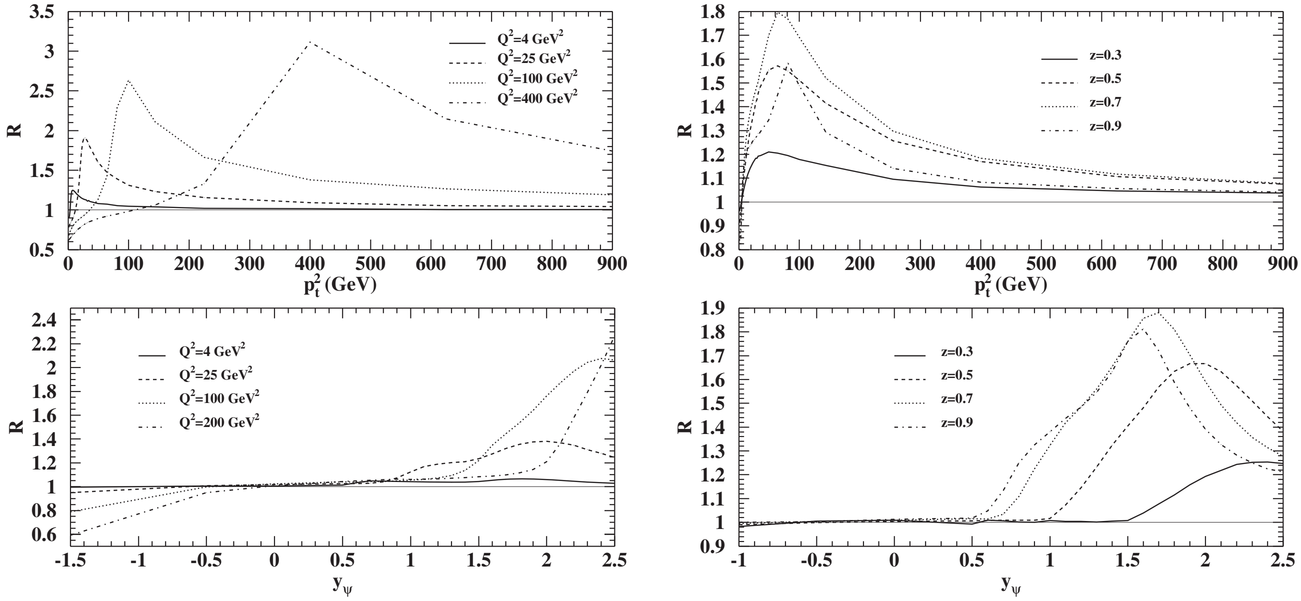

FIG. 3. The ratio R as a function of p_t and y_ψ for different values of Q^2 and z. The invariant mass of the virtual photon and the proton lies in the region $60\ \mathrm{GeV} < W < 240\ \mathrm{GeV}$. For the left-hand-side plots, $0.3 < z < 0.9$, while for the right-hand-side plots, $4\ \mathrm{GeV}^2 < Q^2 < 100\ \mathrm{GeV}^2$.

process, R can be different from 1 even in midrapidity regions.

IV. SUMMARY

In summary, we investigated the structure of the leptonic tensors in DIS and SIDIS. The most general forms of the leptonic tensor are presented in Eqs. (32), (33), and (36), which can greatly simplify the computation by reducing the number of independent Lorentz invariants. Moreover, they explicitly prove the azimuthal structure of the cross sections consists, in addition to the $\psi^\star$-independent terms, of those proportional to $\cos\psi^\star$, $\sin\psi^\star$, $\cos(2\psi^\star)$, and $\sin(2\psi^\star)$, respectively, which after the integration over $\psi^\star$ will vanish. For SIDIS, the symmetric leptonic tensor reduces to a more compact form, which involves only four independent normalized tensors, while the antisymmetric one consists of only two independent normalized tensors. Taking the J/ψ inclusive production in DIS as an example, we demonstrate that the reduced formalism of the leptonic tensor presented in Eq. (8) or Eq. (9) [as well as in Eq. (37)] cannot give correct predictions when some physical observables, such as p_t and the rapidity, in the laboratory frame are measured. However, many works are still using these reduced leptonic currents in computations. Our work can provide a reference for the future phenomenological studies in DIS, including the physics at the future Electron-Ion Collider.

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APPENDIX: RELATION BETWEEN p_t AND $p_t^\star$

Since p_t is invariant under the boost along the z axis, we thus parameterize all the momenta in the virtual-photon-proton ($\gamma^\star p$) center-of-mass frame and the laboratory frame.

The invariants $k\cdot p$ and $P\cdot p$ can be expressed in the laboratory frame as

$$k\cdot p = E_l\sqrt{p_t^2+M^2}e^{Y},$$
$$P\cdot p = E_p\sqrt{p_t^2+M^2}e^{-Y}, \tag{A1}$$

respectively, where E_l and E_p are the energies of the incident lepton and proton, respectively, and the forward z direction is defined as that of the incident proton. Then we have

$$4(k\cdot p)(P\cdot p) = 4E_lE_p(p_t^2+M^2) = S(p_t^2+M^2). \tag{A2}$$

$2P\cdot p$ can also be obtained in terms of the hadronic variables as

$$2P\cdot p = yzS, \tag{A3}$$

and, thus, we have

$$p_t^2+M^2 = 2yzk\cdot p. \tag{A4}$$

Now, we calculate $2k\cdot p$. Apparently, it depends on $\psi^\star$. To obtain its explicit expression, we need to parameterize the momenta in the $\gamma^\star p$ center-of-mass frame. In the $\gamma^\star p$ center-of-mass frame, the forward z direction is defined as that of the incident virtual photon, which is consistent with the HERA experiment conventions. Since the invariant energy of the $\gamma^\star p$ system is W, we can obtain their energies and longitudinal momenta as

$$P_0^\star = \frac{W^2+Q^2}{2W},$$
$$P_l^\star = -\frac{W^2+Q^2}{2W},$$
$$q_0^\star = \frac{W^2-Q^2}{2W},$$
$$q_l^\star = \frac{W^2+Q^2}{2W}. \quad \text{(A5)}$$

If we define k^μ as

$$k^\mu = (k_0^\star, \boldsymbol{k}_t^\star, k_l^\star), \quad \text{(A6)}$$

we can calculate $2k\cdot P$ and $2k\cdot q$ as

$$2k\cdot P = S = \frac{W^2+Q^2}{W}(k_0^\star + k_l^\star),$$
$$2k\cdot q = -Q^2 = \frac{W^2-Q^2}{W}k_0^\star - \frac{W^2+Q^2}{W}k_l^\star. \quad \text{(A7)}$$

Then we can obtain $k_0^\star$ and $k_l^\star$ as

$$k_0^\star = \frac{S-Q^2}{2W},$$
$$k_l^\star = \frac{1}{2W}\left(Q^2 + \frac{W^2-Q^2}{W^2+Q^2}S\right). \quad \text{(A8)}$$

$k_t^{\star 2}$ can be calculated via

$$k_t^{\star 2} = k_0^{\star 2} - k_l^{\star 2} = Q^2\frac{1-y}{y^2}. \quad \text{(A9)}$$

p can be expressed in the $\gamma^\star p$ center-of-mass frame as

$$p^\mu = (m_t^\star(e^{Y^\star} + e^{-Y^\star}), \boldsymbol{p}_t^\star, m_t^\star(e^{Y^\star} - e^{-Y^\star})), \quad \text{(A10)}$$

where

$$m_t^\star = \sqrt{p_t^{\star 2} + M^2}. \quad \text{(A11)}$$

Then we can obtain $2k\cdot p$ as

$$2k\cdot p = \frac{1}{yz}[p_t^{\star 2} + M^2 + (1-y)z^2Q^2 - 2z\sqrt{1-y}Qp_t^\star\cos\psi^\star]. \quad \text{(A12)}$$

With Eq. (A4), we obtain

$$p_t^2 = p_t^{\star 2} + (1-y)z^2Q^2 - 2z\sqrt{1-y}Qp_t^\star\cos\psi^\star. \quad \text{(A13)}$$

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