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Quark mixing in an S_3 symmetric model with two Higgs doublets

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We construct a model where the smallness of the masses of first quark generations implies the near block diagonal nature of the Cabibbo-Kobayashi-Maskawa matrix and vice versa. For this setup, we rely on a two Higgs-doublet model structure with an S_3 symmetry. We show that an SM-like Higgs emerges naturally from such a construction. Moreover, the ratio of two VEVs, $\tan\beta$, can be precisely determined from the requirement of the near masslessness of the up- and down-quarks. The flavor changing neutral current structure that arises from our model is also very predictive.

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The standard model (SM) does not provide any connection between quark masses and mixings: they are independent parameters to be fixed by the experimental observations. There have been many attempts where, by imposing intergenerational symmetries, relations between the masses and mixings have been obtained (see [1,2] for review).

In this article, we present an attempt to relate two features of quark masses and mixings. The first of these two is the fact that the first generation quarks are very light compared to the other ones whereas the second concerns the near block-diagonal structure of the quark mixing matrix, or the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This second feature comes out very clearly in the Wolfenstein parametrization [3] of the CKM matrix, where each element is written in a power series of a small parameter λ. If we keep only terms up to the linear order in λ, the CKM matrix is indeed block-diagonal. We propose a connection between these two features by invoking an S_3 symmetry.

Many works on flavor model building using S_3 symmetry have been done in the past [4–26]. In these constructions one usually employs, for the scalar sector, a three Higgs doublet structure which goes well with the aesthetic idea of having three replicas of Higgs doublets in conformity with three generations of fermions [27–41]. Even more complicated scalar structures are not uncommon [42–48]. In this paper, we rely on a two Higgs-doublet model (2HDM) scalar structure [49,50] which is much more economical in terms of independent parameters. Although the idea of a 2HDM with S_3 symmetry has been conceived lately [24], some distinct implications have not been emphasised earlier. For example, we will show that an S_3 symmetric 2HDM potential naturally delivers an SM-like Higgs boson which can be identified with the scalar resonance observed at the LHC with signal strengths in close agreement with the SM predictions [51].

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We will also demonstrate how, in our scenario, the requirement of near-masslessness for the first generation of quarks dictates a particular value of $\tan\beta$, which will simultaneously render the CKM matrix block-diagonal. For intuitive understanding of the model Lagrangian and the conclusions that follow from it, a brief overview of the S_3 symmetry is in order.

The discrete symmetry group S_3 has three irreducible representations: **1**, **1′** and **2**. We pick a basis such that the generators in the **2** representation are given by

$$a = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, \qquad b = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}. \tag{1}$$

Note that a is of order 3, whereas b is of order 2. The rest of the elements can be obtained by taking products of powers of these two elements. In this basis the quark fields transform under S_3 in the following way:

$$\mathbf{2}\colon \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \qquad \begin{bmatrix} u_{1R} \\ u_{2R} \end{bmatrix}, \qquad \begin{bmatrix} d_{1R} \\ d_{2R} \end{bmatrix}, \tag{2a}$$

$$\mathbf{1}\colon Q_3, \qquad u_{3R}, \qquad d_{3R}, \tag{2b}$$

where the Q_A's ($A = 1, 2, 3$) are the usual left-handed SU(2) quark doublets, whereas the u_{AR}'s and d_{AR}'s are the right-handed up-type and down-type quark fields respectively, which are singlets of the SU(2) part of the gauge symmetry. Note that the square brackets, in Eqs. (1) and (2) as well as in the subsequent text, denote the doublet representation of S_3, and has nothing to do with the representation of the enclosed fields under SU(2). Similarly, in the Higgs sector, there are two SU(2) doublets $\phi_i (i = 1, 2)$, and their transformation under the S_3 symmetry is as follows:

$$\mathbf{2}\colon \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \equiv \Phi. \tag{3}$$

We write the potential of the theory as follows:

$$\begin{aligned} V(\Phi) &= \mu_1^2(\phi_1^\dagger\phi_1) + \mu_2^2(\phi_2^\dagger\phi_2) - (\mu_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}) \\ &\quad + \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 \\ &\quad + \lambda_3\{(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2\}. \end{aligned} \tag{4}$$

Note that the most general form of the quadratic part, written in the first line of Eq. (4), is not S_3-symmetric unless the coefficients satisfy some special conditions. If these conditions are not met, the quadratic terms can softly break the S_3 symmetry, and we allow for such terms. We will consider various scenarios with the quadratic terms in a short while. The quartic part is, however, the most general S_3-symmetric.

The parameters in the quartic part of the potential must be real because of hermiticity of the Lagrangian. In the quadratic part, the parameters μ_1^2 and μ_2^2 are also real. The parameter μ_{12}^2 can be complex, but its phase can be absorbed by redefining either ϕ_1 or ϕ_2. Thus, all parameters in $V(\Phi)$ can be taken to be real without any loss of generality. It has been argued [32] that in this case the vacuum expectation values (VEVs) can also be taken to be real. Denoting the VEV of ϕ_i by v_i we write the doublets after symmetry breaking in the form

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + i\zeta_i) \end{pmatrix}, \tag{5}$$

and use the standard notation

$$v_1 = v\cos\beta, \qquad v_2 = v\sin\beta, \tag{6}$$

where the W- and Z-boson masses are proportional to $v \approx 246$ GeV. Assuming both v_1 and v_2 to be nonzero, minimization of the potential $V(\Phi)$ gives

$$\mu_1^2 = \mu_{12}^2 \tan\beta - (\lambda_1 + \lambda_3)v^2, \tag{7a}$$

$$\mu_2^2 = \mu_{12}^2 \cot\beta - (\lambda_1 + \lambda_3)v^2. \tag{7b}$$

Let us discuss the physical scalar spectrum of the model. In the charged boson sector, one combination of $\phi_1^\pm$ and $\phi_2^\pm$, to be denoted by $w^\pm$, is an unphysical field that does not appear in the physical spectrum. The orthogonal combination, $H^\pm$, is a physical charged scalar. The combinations are given by

$$\begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}. \tag{8}$$

The mass of the physical charged scalar is

$$M_{H^\pm}^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2\lambda_3 v^2. \tag{9}$$

In the pseudoscalar sector, one combination z becomes unphysical after symmetry breaking, and there is one physical pseudoscalar field A. They are related to the fields ζ_1 and ζ_2 by exactly the same matrix that appears in Eq. (8). The mass of A is given by

$$M_A^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2(\lambda_2 + \lambda_3)v^2. \tag{10}$$

The mass matrix for the scalar part can be written as

$$V_{\text{mass}}^{\text{S}} = (\,h_1 \quad h_2\,)\frac{1}{2}\mathbb{M}_S^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \tag{11}$$

with

$$\mathbb{M}_S^2 = \mu_{12}^2 \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} + 2(\lambda_1 + \lambda_3) \begin{pmatrix} v_1^2 & v_1 v_2 \\ v_1 v_2 & v_2^2 \end{pmatrix}. \tag{12}$$

The diagonalization of $\mathbb{M}_S^2$ will lead to two neutral physical scalars h and H, related to h_1 and h_2 by the same orthogonal matrix that appears in Eq. (8), and whose masses are given by

$$m_H^2 = \frac{2\mu_{12}^2}{\sin 2\beta}, \qquad m_h^2 = 2(\lambda_1 + \lambda_3)v^2. \tag{13}$$

At this point, one should note that in the case of 2HDMs, the combination $H^0 = (v_1 h_1 + v_2 h_2)/v$ has SM-like couplings at the tree level. But, in general, H^0 is not a physical eigenstate. The limit where H^0 is aligned with one of the physical CP-even scalars, is known as the "alignment limit" for 2HDMs. This is indeed the case in the present model, viz., that the eigenstate h is the same as H^0. Thus the alignment limit emerges naturally [52] in our scenario. Hence, by identifying h with the 125 GeV scalar observed at the LHC, our model becomes consistent, by design, with the LHC Higgs data [51].

Looking at the spectrum, we can identify the following different scenarios.

(1) If $\mu_1^2 = \mu_2^2$ and $\mu_{12}^2 = 0$, the potential is completely S_3-symmetric, and is in fact invariant under a much bigger symmetry: an SO(2) symmetry under which

$$\begin{aligned} \phi_1 &\to \phi_1\cos\alpha - \phi_2\sin\alpha, \\ \phi_2 &\to \phi_1\sin\alpha + \phi_2\cos\alpha. \end{aligned} \tag{14}$$

Thus, after gauge symmetry breaking when the ϕ_i's develop vacuum expectation values (VEVs), we will have a massless scalar, a Goldstone boson as seen clearly from Eq. (13). This is not the scenario that we advocate.

(2) If $\mu_1^2 \neq \mu_2^2$ and $\mu_{12}^2 = 0$, the potential is not S_3 symmetric, but Eq. (13) shows that we will still have a massless boson. Thus, this is not our desired scenario either.

(3) If $\mu_1^2 = \mu_2^2$ and $\mu_{12}^2 \neq 0$, there exists no massless scalar, but Eq. (13) shows that we will now have $\tan\beta = 1$ or $v_1 = v_2$ because the potential has an

exchange symmetry $\phi_1 \leftrightarrow \phi_2$. As we discuss later, this scenario will be detrimental to our aim.

(4) If $\mu_1^2 \neq \mu_2^2$ and $\mu_{12}^2 \neq 0$, there is no massless scalar and also $\tan\beta$ can be arbitrary. This is the scenario that will be useful for us, implying that the soft-breaking terms are absolutely necessary.

We now present the most general Yukawa couplings involving the u_R quarks that is consistent with the gauge and S_3 symmetries. The S_3 symmetry cuts down on the number of Yukawa couplings drastically, and we obtain only the following couplings involving right-chiral u-type quarks:

$$\begin{aligned}\mathcal{L}_Y^{(u)} &= -A_u(\bar{Q}_1\tilde{\phi}_1 + \bar{Q}_2\tilde{\phi}_2)u_{3R} - B_u\{(\bar{Q}_1\tilde{\phi}_2 + \bar{Q}_2\tilde{\phi}_1)u_{1R} \\ &\quad + (\bar{Q}_1\tilde{\phi}_1 - \bar{Q}_2\tilde{\phi}_2)u_{2R}\} - C_u\bar{Q}_3(\tilde{\phi}_1 u_{1R} + \tilde{\phi}_2 u_{2R}) \\ &\quad + \text{H.c.}\end{aligned} \tag{15}$$

We have used the standard abbreviation $\tilde{\phi}_i = i\sigma_2\phi_i^*$. The Yukawa couplings of the d_R quarks can be obtained by replacing u_{AR} by d_{AR}, $\{A, B, C\}_u$ by $\{A, B, C\}_d$, and $\tilde{\phi}_i$ by ϕ_i in Eq. (15). Although the Yukawa couplings, in general, may be complex, we will discuss later that all but one phase can be absorbed in the field redefinitions.

After symmetry breaking, the mass matrices that arise in the quark sector have the following form:

$$\mathcal{M}_q = \frac{v}{\sqrt{2}}\begin{pmatrix} B_q\sin\beta & B_q\cos\beta & A_q\cos\beta \\ B_q\cos\beta & -B_q\sin\beta & A_q\sin\beta \\ C_q\cos\beta & C_q\sin\beta & 0\end{pmatrix}, \tag{16}$$

where the subscripted index q can take the value u for the up-type quarks, and d for the down-type quarks. It is well-known that these matrices can be diagonalized through bi-unitary transformations, e.g., one can find two unitary matrices U_u and V_u, for the up-sector, such that $U_u\mathcal{M}_u V_u^\dagger$ is diagonal. The CKM matrix is then given by $U_u U_d^\dagger$.

The matrices U_u and U_d are the unitary matrices which diagonalize, through similarity transformations, the hermitian matrices $\mathcal{M}_u\mathcal{M}_u^\dagger$ and $\mathcal{M}_d\mathcal{M}_d^\dagger$ respectively. From Eq. (16), we obtain

$$\begin{aligned}&\mathcal{M}_q\mathcal{M}_q^\dagger \\ &= \frac{1}{2}v^2\begin{pmatrix} a_q^2\cos^2\beta + b_q^2 & \frac{1}{2}a_q^2\sin 2\beta & B_qC_q^*\sin 2\beta \\ \frac{1}{2}a_q^2\sin 2\beta & a_q^2\sin^2\beta + b_q^2 & B_qC_q^*\cos 2\beta \\ B_q^*C_q\sin 2\beta & B_q^*C_q\cos 2\beta & c_q^2\end{pmatrix},\end{aligned} \tag{17}$$

where $a_q = |A_q|$ etc. Clearly, the three eigenvalues of $\mathcal{M}_u\mathcal{M}_u^\dagger$ would be the mass squared of the three up sector quarks, namely m_u^2, m_c^2 and m_t^2, and the three eigenvalues of $\mathcal{M}_d\mathcal{M}_d^\dagger$ would be m_d^2, m_s^2 and m_b^2. Introducing the shorthand notation

$$x = \frac{2m^2}{v^2} \tag{18}$$

for any fermion with mass m, we can find the eigenvalues by solving the characteristic equation,

$$\begin{aligned}&x^3 - (a^2 + 2b^2 + c^2)x^2 + (a^2 + b^2)(b^2 + c^2)x \\ &\quad - a^2b^2c^2\sin^2 3\beta = 0,\end{aligned} \tag{19}$$

with subscripts u or d attached to the Yukawa couplings, as the case may be. Note that this equation is free from the phase of BC^*, which is the only phase that is present in Eq. (17).

Looking at the Lagrangian of Eq. (15) and the corresponding Lagrangian involving d_{iR}, we see why only one phase is present in Eq. (17). Any phase of A_u and A_d can be absorbed by redefining the fields u_{3R} and d_{3R}. After this, both B_u and B_d can be made real by redefining the fields u_{1R}, u_{2R} and d_{1R}, d_{2R}. Finally, either C_u or C_d can be made real by choosing the phase of Q_3, but one of them remains complex. Alternatively, one can make both C_u or C_d real first, by redefining the right-chiral quark fields, and then either B_u or B_d can be made real by choosing the phases of Q_1 and Q_2. Either way, one of the C_q's or one of the B_q's can be complex in the most general case. In what follows, we will assume that all Yukawa couplings are real, and use the lower-case symbols for them.

Before entering into a discussion of the eigenvalues obtained as solutions of Eq. (19), let us have some idea of the form of the diagonalizing matrix. As a first step, we can diagonalize only the terms in Eq. (17) that are proportional to a_q^2. This is done, e.g., by a matrix

$$\mathcal{U} = \begin{pmatrix} 0 & 0 & 1 \\ \sin\beta & -\cos\beta & 0 \\ \cos\beta & \sin\beta & 0\end{pmatrix}. \tag{20}$$

Note that this matrix does not depend on the Yukawa couplings, and is therefore the same for the up-type and down-type mass matrices. Applying a similarity transformation with this matrix on $\mathcal{M}\mathcal{M}^\dagger$, we obtain

$$\begin{aligned}M^2 &= \mathcal{U}\mathcal{M}\mathcal{M}^\dagger\mathcal{U}^\dagger \\ &= \frac{1}{2}v^2\begin{pmatrix} c^2 & -bc\cos 3\beta & bc\sin 3\beta \\ -bc\cos 3\beta & b^2 & 0 \\ bc\sin 3\beta & 0 & a^2 + b^2\end{pmatrix},\end{aligned} \tag{21}$$

with subscripts u and d attached for quarks of positive and negative charges respectively.

In the preamble of the article, we said that we want to relate the almost-masslessness of first generation quarks with the almost-block-diagonal form of the CKM matrix. We now narrow down the scenario in which we can have one zero eigenvalue in both up-type and down-type quark sector, as well as a block-diagonal CKM matrix.

First we note that if one solution of Eq. (19) is zero, then the x-independent term should vanish in that equation. In this case, the eigenvalues of $\mathcal{M}\mathcal{M}^\dagger$ are

$$0, \qquad \frac{1}{2}v^2(b^2+c^2), \qquad \frac{1}{2}v^2(a^2+b^2). \tag{22}$$

For the diagonalizing matrix, we now consider two different cases.

Case 1: Some Yukawa couplings vanish. Surely, the x-independent term in Eq. (19) can vanish if at least one of the Yukawa couplings is zero. Looking at Eq. (21), we see that $a=0$ does not make M^2 block-diagonal, so we reject this possibility. If either b or c vanishes, the matrix M^2 becomes completely diagonal. This means that for b or $c=0$, the same matrix $\mathcal{U}$ will diagonalize both $\mathcal{M}_u\mathcal{M}_u^\dagger$ and $\mathcal{M}_d\mathcal{M}_d^\dagger$ making the CKM matrix a unit matrix. Therefore making some Yukawa coupling vanish to obtain one zero mass does not produce the desirable block-diagonal structure of the CKM matrix.

One should recall that making $\mu_1^2=\mu_2^2$ leads to $\tan\beta=1$, which in view of Eq. (19) demands that one of the Yukawa couplings must be zero in order to obtain zero mass eigenvalue. For this reason we discard this option about the parameters of $V(\Phi)$.

Case 2: A specific ratio of the two VEVs. This is a more attractive possibility. From the characteristic equation, Eq. (19), one can see that zero eigenvalue can also be ensured if

$$\sin 3\beta = 0. \tag{23}$$

Discarding the trivial solution $\beta=0$, we obtain the solution $\beta=\frac{1}{3}\pi$ which implies that $\tan\beta=\sqrt{3}$, i.e., $v_2=\sqrt{3}v_1=\sqrt{3}v/2$.[1] Looking at Eq. (21) now, we see that this value of β also makes the matrix M^2 block-diagonal, and one obtains

$$M^2=\mathcal{U}\mathcal{M}\mathcal{M}^\dagger\mathcal{U}^\dagger=\frac{1}{2}v^2\begin{pmatrix} c^2 & bc & 0\\ bc & b^2 & 0\\ 0 & 0 & a^2+b^2\end{pmatrix}. \tag{24}$$

Notice that the third generation has been singled out, and therefore $v\sqrt{(a^2+b^2)/2}$ can be readily identified with the mass of the third generation quark. In order that it be much heavier than the quarks in the first two generations, we need

$$a^2 \gg b^2, c^2 \tag{25}$$

in both up and down sectors.

Complete diagonalization would require a further similarity transformation affecting the upper 2×2 block. This will involve the values of the Yukawa couplings. Thus, we obtain

$$U_u=\mathcal{O}_u\mathcal{U}, \qquad U_d=\mathcal{O}_d\mathcal{U}, \tag{26}$$

where

$$\mathcal{O}_q=\begin{pmatrix}\cos\theta_q & -\sin\theta_q & 0\\ \sin\theta_q & \cos\theta_q & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{27}$$

with

$$\tan\theta_q=\frac{c_q}{b_q}. \tag{28}$$

From Eq. (26) the CKM matrix can now be written as,

$$V_{\rm CKM}=U_uU_d^\dagger=\mathcal{O}_u\mathcal{O}_d^\dagger = \begin{pmatrix}\cos(\theta_u-\theta_d) & -\sin(\theta_u-\theta_d) & 0\\ \sin(\theta_u-\theta_d) & \cos(\theta_u-\theta_d) & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{29}$$

Thus the difference $\theta_u-\theta_d$, which can be identified with the Cabibbo angle, θ_C.

In passing, we make a point about the VEV alignment, i.e., the value of β, dictated by Eq. (23). It reflects our choice of the representation for S_3. Had we chosen a different representation, the value of β would in general be different. But the physical implications should be independent of the representation, and so the block-diagonal form of the CKM matrix would still result.

Having reproduced the leading order effects of the mixing matrix in the Wolfenstein parametrization as a consequence of the masslessness of the first generation quarks, we now explore whether one can do better. So far, the conclusions that we derived came from Eq. (23), which is a statement about the relative magnitude of the VEVs of the two Higgs doublets. Note that this relation is not protected by any symmetry. Suppose we deviate from Eq. (23) by a small amount such that

$$\sin 3\beta = \delta. \tag{30}$$

Since δ is expected to be small, we do not expect the heavier quark masses to be altered very much by this change. The sums of eigenvalues etc. will also not change appreciably. The only thing that will change dramatically is the product of all eigenvalues, which should be the

[1]In case of a three Higgs doublet model with S_3 symmetry, the minimization of potential leads to a vev alignment $v_1=\sqrt{3}v_2$ which in turn implies a residual $\mathbb{Z}_2$ symmetry [38]. In the present case, no such implication is possible. Moreover, note that if we decide to choose a different representation of the generator b, using $\cos(\frac{1}{3}\pi+2\gamma)$ in place of $\frac{1}{2}$ and $\sin(\frac{1}{3}\pi+2\gamma)$ in place of $\frac{\sqrt{3}}{2}$ in Eq. (1), the components of the S_3 doublets will be linear combinations of the original ones, but Eq. (23) adjusts itself to $\sin 3(\beta'-\gamma)=0$ or $\tan(\beta'-\gamma)=\sqrt{3}$, which is the same since β', the ratio of VEVs of the two Higgs doublets in this new representation, is given by $\beta'=\beta-\gamma$.

x-independent term in Eq. (19). Therefore the first generation quark masses will be given, in the notation of Eq. (18), by

$$x_u = \frac{2m_u^2}{v^2} \approx \frac{a_u^2 b_u^2 c_u^2 \delta^2}{(a_u^2 + b_u^2)(b_u^2 + c_u^2)} \approx \frac{b_u^2 c_u^2 \delta^2}{b_u^2 + c_u^2} \tag{31a}$$

$$x_d = \frac{2m_d^2}{v^2} \approx \frac{a_d^2 b_d^2 c_d^2 \delta^2}{(a_d^2 + b_d^2)(b_d^2 + c_d^2)} \approx \frac{b_d^2 c_d^2 \delta^2}{b_d^2 + c_d^2}, \tag{31b}$$

where in the last step we have used the hierarchy mentioned in Eq. (25). These equations, along with Eq. (28), to a good approximation, implies $m_u^2 \approx \frac{1}{4} m_c^2 \delta^2 \sin^2 2\theta_u$ and $m_d^2 \approx \frac{1}{4} m_s^2 \delta^2 \sin^2 2\theta_d$. Thus using the fact $\theta_u - \theta_d = \theta_C$ and taking all the experimental uncertainties into account we have found $\delta > 0.2$ which is inconsistent with our assumption of small δ in Eq. (30). Therefore, this minimal framework is not sufficient to reproduce the observed masses of the first generation quarks.

Now, for the sake of completeness, we comment on the flavor changing neutral currents (FCNC) in our model. To set up the notations we first lay out the Yukawa Lagrangian for 2HDM in the following form [49]:

$$\mathcal{L}_Y = -\sum_{k=1}^{2} [\bar{\mathbf{Q}}_L \Gamma_k \phi_k \mathbf{d}_R + \bar{\mathbf{Q}}_L \Delta_k \tilde{\phi}_k \mathbf{u}_R] + \text{H.c.}, \tag{32}$$

where we have kept the notation for the field the same as in Eq. (15) but put them in boldface font to remind ourselves that the generation indices have been suppressed. Unlike Eq. (15), here we also take into account the Yukawa Lagrangian for the down sector too. Here $\Delta_{1,2}$ and $\Gamma_{1,2}$ represent the Yukawa matrices in the up and down sectors respectively. By comparing Eqs. (32) and (15) we can write,

$$\Delta_1 = \begin{pmatrix} 0 & b_u & a_u \\ b_u & 0 & 0 \\ c_u & 0 & 0 \end{pmatrix}, \qquad \Delta_2 = \begin{pmatrix} b_u & 0 & 0 \\ 0 & -b_u & a_u \\ 0 & c_u & 0 \end{pmatrix}, \tag{33}$$

and the Γ_k's can be obtained by replacing the subscript u by the subscript d in the matrices. Now, the Yukawa Lagrangian in terms of physical fields can be written as

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = &-\frac{h}{v}(\bar{\mathbf{d}} D_d \mathbf{d} + \bar{\mathbf{u}} D_u \mathbf{u}) + \frac{H}{v}[\bar{\mathbf{d}}(N_d P_R + N_d^\dagger P_L)\mathbf{d} \\ &+ \bar{\mathbf{u}}(N_u P_R + N_u^\dagger P_L)\mathbf{u}] - \frac{iA}{v}[\bar{\mathbf{u}}(N_u P_R - N_u^\dagger P_L)\mathbf{u} \\ &- \bar{\mathbf{d}}(N_d P_R - N_d^\dagger P_L)\mathbf{d}] + \frac{\sqrt{2}H^+}{v}\bar{\mathbf{u}}[V_{\text{CKM}} N_d P_R \\ &- N_u^\dagger V_{\text{CKM}} P_L]\mathbf{d} + \text{H.c.},\end{aligned} \tag{34}$$

where D_u and D_d are the diagonal mass matrices in the up and down sectors respectively. Note that the SM-like scalar, h, does not have any FCNC couplings. This is a direct consequence of the natural alignment that we have talked about earlier. The matrices N_u and N_d, in Eq. (34), carry the information of FCNC in the up and down sectors respectively and are given by,

$$N_u = \frac{1}{\sqrt{2}} U_u (\Delta_1 v_2 - \Delta_2 v_1) V_u^\dagger, \tag{35a}$$

$$N_d = \frac{1}{\sqrt{2}} U_d (\Gamma_1 v_2 - \Gamma_2 v_1) V_d^\dagger. \tag{35b}$$

Note that the expressions for V_u and V_d can be obtained from diagonalizing $\mathcal{M}^\dagger \mathcal{M}$ for both up and down sectors. The matrices $\mathcal{M}^\dagger \mathcal{M}$ can be obtained from $\mathcal{M}\mathcal{M}^\dagger$ by making the interchange $a \leftrightarrow c$ in the Yukawa couplings. Because of this interchange, the matrix V is different from U in two respects. First, the matrix corresponding to $\mathcal{U}$ should have the last two rows interchanged so that the eigenvalues can occur in the same order. Second, the angle θ_q should be replaced by θ'_q, which will be given by $\tan \theta'_q = a_q / b_q$. In view of the hierarchy mentioned in Eq. (25), we can use these to write $\sin \theta'_q \approx 1$, $\cos \theta'_q \approx b_q / a_q$, neglecting higher order terms in b_q / a_q. Replacing the Yukawa couplings by the mass eigenvalues and the angles θ_q, we obtain

$$N_d \approx \begin{pmatrix} -\frac{3}{2} m_s \sin 2\theta_d & 0 & -m_b \sin \theta_d \\ \frac{1}{2} m_s (3\cos^2 \theta_d - 1) & 0 & m_b \cos \theta_d \\ 0 & m_s \cos \theta_d & 0 \end{pmatrix}, \tag{36}$$

neglecting corrections of order m_s / m_b. A similar expression for N_u can be obtained from Eq. (36) by replacing θ_d, m_s, m_b with θ_u, m_c, m_t respectively. Thus the FCNCs are uniquely determined by θ_u or θ_d. One should keep in mind that this represents the FCNC couplings at the leading order, i.e., when the CKM matrix is block-diagonal. In a more complete framework where the CKM matrix can be reproduced exactly these FCNC matrices are expected to get small corrections. The important thing to notice is that, already at the leading order, the FCNC couplings are suppressed at least by m_b / v, so they are naturally small in this model. Because of this, the lower bound on FCNC-mediating bosons are low, about 3 TeV as opposed to about 100 TeV that is obtained for $O(1)$ couplings. Moreover, the bounds from the electroweak T-parameter can also be evaded if the nonstandard scalars, H, A and $H^\pm$ are nearly degenerate [53,54].

In summary, we connect two apparently disjoint experimental observations namely, the tiny masses of first generation of quarks and the near block-diagonal structure of the CKM matrix in a simple setup of 2HDM with an S_3 symmetry. We attribute these two features of the quark sector to a particular value of $\tan \beta$. An added bonus of our model is the existence of a light scalar, which can be identified with the 125 GeV Higgs observed at the LHC, in view of a naturally emerging alignment limit. Admittedly, the exact CKM matrix and correct nonzero masses for the first generation of quarks could not be reproduced in this minimalistic scenario. Perhaps our setup can be taken as a constituent in a more elaborate framework which can address the full quark structure.

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