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Infrared dynamics of massive scalars from the complementary series in de Sitter space

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We continue a previous study about the infrared loop effects in the D-dimensional de Sitter space for a real scalar ϕ^4 theory from the complementary series whose bare mass belongs to the interval $\frac{\sqrt{3}}{4}(D-1) < m \leq \frac{D-1}{2}$, in units of the Hubble scale. The lower bound comes from the appearance of discrete states in the mass spectrum of the theory when that bound is violated, causing large IR loop effects in the vertices. We derive an equation which allows us to perform a self-consistent resummation of the leading IR contributions from all loops to the two-point correlation functions in an expanding Poincaré patch of the de Sitter manifold. The resummation can be done for density perturbations of the Bunch-Davies state which violate the de Sitter isometry. There exist solutions having a singular (exploding) behavior, and therefore the backreaction can change the de Sitter geometry.

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I. INTRODUCTION

Quantum effects in de Sitter (dS) spacetime have received considerable attention in recent years [1–68]. These effects are rather different from those in flat or anti-de Sitter space [69]. As is shown in [58,59,64–66,68], the peculiar infrared (IR) behavior of interacting nonconformal fields in dS space is that there are large IR loop corrections or divergences even for very massive fields for any initial state (see [67] for a review). The quantum corrections eventually become of the same order or even dominate the tree-level contributions, and the situation is similar to the one encountered in nonstationary condensed matter theory [70,71] (see also [72,73] for the secular loop effects in strong electric fields in QED and [74] for the secular growth of loop corrections to the Hawking radiation).

In this paper we consider a real, massive, minimally coupled scalar field theory:

$$S = \int d^D x \sqrt{|g|}\left[\frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right]. \qquad (1)$$

The theory in question is restricted to the expanding Poincaré patch (EPP) of dS space:

$$ds^2 = \frac{1}{\eta^2}[-d\eta^2 + d\vec{x}^2], \qquad \eta = e^{-t}, \qquad (2)$$

where the conformal time ranges from $\eta = +\infty$ at past infinity ($t = -\infty$) to $\eta = 0$ at future infinity ($t = +\infty$). Throughout this paper we set the radius of the dS spacetime to one. Our goal is to check whether the assumption of negligible backreaction is self-consistent or not for various initial conditions.

The EPP coordinate system has a well-known peculiarity: Due to the presence of the conformal factor $1/\eta^2$ multiplying the spatial part of the metric, every wave experiences an IR shift towards future infinity; i.e., future infinity of the EPP corresponds to the IR limit of the physical momentum, while past infinity corresponds to the UV limit. Correspondingly, a generic free scalar mode in the EPP has the following properties [75,76]: First, due to spatial flatness of the EPP, a scalar mode may be factorized in terms of plane waves as follows: $\phi_p(\eta,\vec{x}) = \eta^{(D-1)/2} h(p\eta) e^{-i\vec{p}\vec{x}}$, where $h(p\eta)$ is a solution of the Bessel equation of order $\nu = \sqrt{(\frac{D-1}{2})^2 - m^2}$. Second, any Bessel function of this order behaves as follows:

$$h(p\eta) = \begin{cases} A\frac{e^{ip\eta}}{\sqrt{p\eta}} + B\frac{e^{-ip\eta}}{\sqrt{p\eta}} & p\eta \gg |\nu| \\ C(p\eta)^\nu + D(p\eta)^{-\nu} & p\eta \ll |\nu|. \end{cases} \qquad (3)$$

Here A, B, C, D are complex constants which are fixed by the canonical commutation relations and some other additional criterion. Due to the symmetries of the EPP, one cannot disentangle the comoving momentum p and conformal time η; all physical quantities depend on the combination $p\eta$ which is referred to as the physical momentum. Near past infinity of the chosen EPP, the physical momentum $p\eta$ tends to infinity and every mode

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behaves asymptotically as a plain wave in flat spacetime. This is because high energy modes are not sensitive to the comparatively small curvature of the background. One can actually introduce, in that region, a notion of particle with positive energy because the free Hamiltonian can be diagonalized there. Thus, at past infinity of the EPP, the background gravitational field is effectively switched off. Equivalently, for a given mode function, ϕ_p behaves as a plain wave in flat space when $p\eta \gg |\nu|$. On the other hand, for low physical momenta the behavior of the modes [see Eq. (3)] is very different from the one in flat space. It is exactly the latter region of physical momenta $p\eta \ll |\nu|$ that generates large IR corrections to correlation functions.

This is, roughly speaking, the origin of the strong IR effects in dS space that are discussed in the present paper.

As one can see from (3), the character of the behavior of the mode functions (and, hence, of the IR effects [66]) depends on whether m is greater or smaller than $(D-1)/2$ in units of the dS radius. Scalar fields with $m > (D-1)/2$ are associated with the principal series of representations of the dS group. In this case ν is purely imaginary and the mode functions (3) oscillate at small physical momenta. The IR physics in this case has been extensively studied in [58,59,64–68].

Here we want to explore the case of light scalars (the complementary series). Masses of such fields obey $0 < m \leq (D-1)/2$ in units of the dS radius. Now ν is real, and the mode functions (3) *do not* oscillate at future infinity of the EPP.

A. Schwinger-Keldysh formalism

The action (1) defines a field theory in a nonstationary background (2). Therefore, a perturbative expansion of the correlation functions should be constructed in terms of three propagators [70] (see also [67,71,77]). Two of them are the standard retarded and advanced propagators, which are purely algebraic, i.e., they do not depend on the chosen Fock space realization of the (free or tree-level) theory,

$$D_0^{\substack{R\\A}}(\eta_1,\vec{x}_1;\eta_2,\vec{x}_2) = \pm\theta(\mp\Delta\eta_{12})\langle[\phi(\eta_1,\vec{x}_1),\phi(\eta_2,\vec{x}_2)]\rangle,\qquad \Delta\eta_{12} = \eta_1 - \eta_2, \tag{4}$$

where $[\cdot,\cdot]$ is the commutator. The Keldysh propagator is the "vacuum" expectation value of the anticommutator:

$$D_0^K(\eta_1,\vec{x}_1;\eta_2,\vec{x}_2) = \frac{1}{2}\langle\{\phi(\eta_1,\vec{x}_1),\phi(\eta_2,\vec{x}_2)\}\rangle. \tag{5}$$

The Keldysh propagator does depend on the Fock space realization of the theory.

The EPP is invariant under space translations $\vec{x}\to\vec{x}+\vec{a}$. We will only consider quantizations where space translations are unbroken. This means, in particular, that we assume that all the propagators depend on the difference vectors $\vec{x}_2 - \vec{x}_1$. It is therefore advantageous to Fourier transform all the quantities w.r.t. the above difference vectors[1]:

$$D_0^{K,R,A}(p|\eta_1,\eta_2) \equiv \int d^{D-1}x e^{-i\vec{p}\,\vec{x}} D_0^{K,R,A}(\eta_1,\vec{x};\eta_2,0). \tag{6}$$

A partial Fourier transformation is also helpful to keep track of the behavior of each mode separately with a given physical momentum. Here we give the Fourier-transformed tree-level retarded and advanced propagators [77]:

$$D_0^{\substack{R\\A}}(p|\eta_1,\eta_2) = \pm\theta(\mp\Delta\eta_{12})2(\eta_1\eta_2)^{\frac{D-1}{2}}\mathrm{Im}[h(p\eta_1)h^*(p\eta_2)]. \tag{7}$$

If the initial state $|\Psi\rangle$ respects the spatial translational invariance, the (tree-level) Keldysh propagator can be written as follows:

$$D_0^K(p|\eta_1,\eta_2) = (\eta_1\eta_2)^{\frac{D-1}{2}}\left[\left(\frac{1}{2} + \langle\Psi, a_{\vec{p}}^+ a_{\vec{p}}\Psi\rangle\right)h(p\eta_1)h^*(p\eta_2) + \langle\Psi, a_{\vec{p}}a_{-\vec{p}}\Psi\rangle h(p\eta_1)h(p\eta_2) + \mathrm{H.c.}\right]. \tag{8}$$

In a ground state $a_{\vec{p}}|\Psi\rangle = 0$, the latter expression reduces to

$$D_0^K(p|\eta_1,\eta_2) = (\eta_1\eta_2)^{\frac{D-1}{2}}\mathrm{Re}[h(p\eta_1)h^*(p\eta_2)]. \tag{9}$$

When the mass is nonzero there is a one-parameter family of dS invariant quantizations known as the α-vacua (see [78–80]). In all these cases the Keldysh propagator $D_0^K(1,2)$ depends only on the invariant geodesic distance between the two points (while this happens for tree-level $D_0^{R,A}$ for any given state $|\Psi\rangle$).

II. DIFFERENT TYPES OF SECULAR EFFECTS. RESULTS OF THIS PAPER

There are different sorts of secularly growing contributions in nonstationary situations, in general, and in dS space, in particular. To begin, there is a secular growth which is specific to dS space and is already present at tree level (see e.g. [31,81–83]). It exists for all minimally coupled scalar tachyons, a family of fields whose squared mass is negative or zero—it includes the massless

[1]Note that due to the expansion of the EPP, every spatially inhomogeneous perturbation fades away at future infinity. Thus, the IR effects under study, appearing from the future infinity region of the EPP, are not very sensitive to such inhomogeneous perturbations. Hence, our methods are also applicable in the presence of such perturbations.

scalar field. In these cases canonical quantization gives rise to two-point functions that do not depend only on the geodesic distance, and there exists no Fock space representation where the representation of the dS group is unitary [see [82,83]; for masses equal to $m^2 = -n(n+d-1)$ with n a non-negative integer, there exists, however, a construction similar to the Gupta-Bleuler quantization of the free photon field].

By subtracting the UV divergence, one finds that the correlation functions grow with time; for instance, for the massless scalar field one has $D_0(t,\vec{x}) \propto |\log\eta| \sim t$ for the coincident points. Furthermore, taking into account loops e.g. for a $\lambda\phi^4$ self-interaction, the secular growth is $\Delta_n D(t,\vec{x}) \propto t(\lambda t^2)^n$ where n grows with the number of loops.

Such a secular growth results in a breakdown of perturbation theory. In fact, for every λ, however small, after a time t long enough, $\lambda t^2 \sim 1$ and quantum corrections become of the same order of the tree-level amplitude. A resummation of (at least) the leading contributions from loops is therefore mandatory for a meaningful perturbation calculation in dS space-time.

One possible scheme to perform the resummation has been proposed in [84]. It makes use of the stochastic approach to quantum field theory and allows, in a certain limit, the resummation of the secular corrections to the Bunch-Davies (BD) vacuum (or its analog for the massless scalar) in the EPP [31]. After the resummation the dS invariance of the correlation functions in the massless scalar field theory is restored. The approach of [84] allows us to control various sorts of secular contributions (without disentangling them) for scalar fields with any non-negative mass belonging to the complementary series. But it does this only in very special situations, and the determination of the exact limits of validity of this approach is an important, separate issue. In particular, for obvious reasons, this method cannot be applied to global dS space or to the contracting Poincaré patch. Also, it is not applicable in the EPP for strong enough initial density perturbations of the BD state—when there are strong nonlinear correlations, even if they are spatially homogeneous. The point is that the approach described in [84] exploits a stochastic differential equation with a *linear* random source in a *nonlinear* (self-interacting) theory. It is not applicable for the case when one has strong enough nonlinearities. We will come back to this point below.

Another well-known example of a secular IR loop correction in dS space is the following:

$$\Delta_n D(p|\eta_1,\eta_2) \propto (\eta_1\eta_2)^{(D-1)/2}[\lambda^2 \log(\eta_1/\eta_2)]^n = e^{-\frac{D-1}{2}(t_2+t_1)}\lambda^{2n}|t_1-t_2|^n \qquad (10)$$

in the scalar $\lambda\phi^4$ theory, when $|t_2-t_1| \to \infty$.

Such secular growth is quite universal and is present even for positive mass [47–49,85]. Usually, such growth is caused by some imaginary contributions to the self-energy; it is also present in Minkowski space-time, e.g. if one chooses an initial density matrix other than Planckian and describes the instability of quasiparticles (in the latter case, however, the dS volume factor $e^{-\frac{D-1}{2}(t_2+t_1)}$ is not present). This secular effect is present in all the propagators, including the retarded and advanced ones, and can be seen in their partially Fourier transforms. It may also be present in the vertices.

Some further comments are in order here for understanding the physical origin and the differences between various types of secular effects. In flat space-time, in the standard nonstationary situation, the Fourier transform of a propagator $D_0(p|t_1,t_2)$ is proportional to $e^{i\omega(p)(t_2-t_1)}$, where $\omega(p)$ is the dispersion relation of the model under consideration. The secular corrections to the self-energy are absorbed by the coefficient in front of the exponential at every loop order. They also depend on $t_2 - t_1$. After the resummation the growth in question at the leading order can be eliminated by a shift of the dispersion relation $\omega(p) \to \omega(p) + i\Gamma(p)$ or by a mass renormalization, when the contribution to the self-energy is real. This growth cannot be attributed just to the IR effects in a proper sense because it also appears in the UV domain.

Yet another secular effect at loop level is of the form

$$\Delta_n D^K(p|\eta_1,\eta_2) \propto (\eta_1\eta_2)^{(D-1)/2}\left[\lambda^2 \log\left(\frac{1}{\eta_1\eta_2}\right)\right]^n = e^{-\frac{D-1}{2}(t_2+t_1)}[\lambda^2(t_1+t_2)]^n \qquad (11)$$

which at leading order in λ is present *only* in the Keldysh propagator when $t_2+t_1 \to \infty$ and $t_2 - t_1 = \mathrm{const}$ (in the conformal coordinates of the EPP in dS space the above condition is as follows: $p\sqrt{\eta_1\eta_2} \to 0$, with $\eta_1/\eta_2 = \mathrm{const}$). In this limit both points of the propagator are sent to future infinity of the EPP while their coordinate time distance is held fixed. Such growth is also universal to practically any nonstationary situation, including the flat space nonstationary initial density matrix, strong electric fields in QED [72,73], and Hawking radiation [74]. Moreover, it is also present for massless scalars in dS space but is mixed with the other contributions described above.

The origin of the $\lambda^2(t_1+t_2)$ growth can be understood from the Keldysh propagator shown in Eq. (8). In the interaction picture, which we use here, $a_{\vec{p}}$ and $a^+_{\vec{p}}$ are time independent in the Gaussian approximation. In this approximation all the time dependence is in the modes ϕ_p. This means that in the Gaussian theory $\langle\Psi, a^+_{\vec{p}} a_{\vec{p}}\Psi\rangle$ and $\langle\Psi, a_{\vec{p}} a_{-\vec{p}}\Psi\rangle$ remain constant. In particular, if they were chosen to be zero, they always remain zero.

But if one turns on self-interactions, the above quantities, namely the population numbers and the anomalous quantum averages for the *exact modes*, start to depend on t_1 and t_2, and they receive the leading $\lambda^2(t_2+t_1)$ corrections in the limit under consideration.[2] This is generally true for any nonstationary situation. Because of the different functional dependence, this secular growth cannot be reabsorbed in a change of the dispersion relation $\omega(p) \to \omega(p) + i\Gamma(p)$ or by a mass renormalization: $\lambda^2(t_2 + t_1)e^{i\omega(p)(t_2-t_1)} \neq e^{i(\omega(p)+\delta\omega)(t_2-t_1)}$. In this paper we focus on secular effects of the latter type. These effects can strongly affect the commonly accepted picture of the strength of stress-energy fluxes. In fact, the common wisdom is that quantum loop corrections lead only to UV renormalization, while IR secular effects can just lead to mass renormalization or to a mode decay rate. Hence, according to common wisdom quantum, corrections do not contribute to the stress-energy tensor,[3] and one can safely apply the Gaussian approximation. In this approximation, one sees only the amplification of the zero-point fluctuations.

Indeed, if loop corrections are irrelevant, one can use tree-level correlation functions such as in (9) to calculate the stress-energy tensor or the electric current: namely, in this way, one can find Schwinger's current in strong electric fields, the Hawking energy flux in the gravitational collapse, and the Bunch-Davies expectation value of the stress-energy tensor in dS space. However, the time dependence of the level populations and the anomalous quantum averages for the exact modes, which we have just described, may drastically change the energy-momentum tensor (see [67,72–74] for a number of examples). In fact, in this case, to calculate the stress-energy tensor, one has to use an analog of (8), with time-dependent $\langle a_p^+ a_p\rangle$ and $\langle a_p a_{-p}\rangle$ which are attributed to the comoving volume.

A. Complementary series

For masses $m < (D-1)/2$ (the complementary series) generic modes behave as real powers of the conformal time [as opposed to oscillating imaginary powers, see Eq. (3)]. As a result, the dominant term in $h(p\eta_1)h^*(p\eta_2)$ is proportional to $\eta_2\eta_1 = e^{-|\nu|(t_1+t_2)}$. Hence, for light scalars the aforementioned universal secular effects, which are present even for massive fields, can be mixed up in the linear approximation to the Dyson-Schwinger equation. Moreover, because the modes do not oscillate, the imaginary contributions to the self-energy may lead to a mass renormalization rather than to a decay rate in the same approximation. This fact makes it difficult to disentangle the different secular effects from each other, while it is necessary to disentangle them to understand quantum field theory in dS space, as we have explained in the previous paragraph. But this distinction is out of reach of the methods of [31,84].

The situation regarding the fields of the complementary series has been explored in e.g. [52–57]. The resummation of the secularly growing loops in the BD state has been attempted there by using different methods, including the solution of the Dyson-Schwinger equation in the large N limit and the exact renormalization group equation for the IR cutoff. Their results agree with those of [84] and extend them to the case of noncoincident points of the two-point correlation functions. An interesting solution of the Dyson-Schwinger equations for the retarded, advanced and Keldysh propagators has been found. The approach adopted in [52,55,57] allows us to simultaneously resum all the aforementioned different sorts of IR effects in the large N limit without disentangling them.

However, in our opinion there remain certain unsatisfactory features which make the study of the dS light scalar field dynamics not yet complete. The point is that the resummation is done in [52–57] only for the BD state and for the cases when the dS isometry is respected at every step of the quantization. It is quite understandable that in this case, at the leading order quantum effects just lead to the renormalization of the cosmological constant and the mass of the field. But the following question remains: what happens if one considers density perturbations of the BD state which (necessarily) violate the dS isometry? Nevertheless, it is natural to consider perturbations of a highly symmetric state and trace their destiny: one should check whether they grow or fade away as the time goes by. That is what we propose to do in the present paper.

For massive scalars with an exact BD initial state at the lightlike boundary of the EPP, one can respect the dS isometry at every loop order [59] (see e.g. also [67]). Moreover, as we explain in Sec. VII, in this case the system of Dyson-Schwinger equations, which allows us to resum leading loop corrections, reduces to a single linear integro-differential equation. For the complementary series a similar equation was obtained in [52,55,57]. The difference of the situation in the latter references with respect to our case is that their equation was obtained in the large N limit and resums leading diagrams and IR effects that are a bit different, some of which are subleading in our approximation.

The situation with slight violations of the dS isometry is quite different. One cannot just put an initial comoving number density n_p^0 for the exact modes at past infinity (i.e. the lightlike boundary) of the EPP because then the physical density would become infinite. Due to the symmetries of the EPP, the appropriate way of approaching the problem is as follows. As we have explained above, every

[2] In general nonstationary situations, correlation functions depend on each of their arguments separately, $D(t_1, t_2) = \bar{D}(t_1 + t_2, t_1 - t_2)$, rather than the distance between them, $D(t_2 - t_1)$; e.g. there are contributions to the propagator of the form $\Delta D(t_1, t_2) \propto n_p(\frac{t_1+t_2}{2})e^{-i\omega(p)(t_1-t_2)}$ for some $n_p(t)$.

[3] Note that in the strong background fields, we calculate correlation functions and avoid using the notion of particles, unless it is appropriate for the interpretation of the energy-momentum flux [67]. The latter one can be found from the correlation functions. Namely, one can encounter situations in which there is a nontrivial energy flux, but there is no any suitable separation of it into particles—into something that obeys the energy composition principle.

physical quantity in the EPP is a function of the physical momentum, $P = p\eta$. Hence, one has to put an initial Cauchy condition for a given physical momentum, i.e. $n^0(P) = n(P_0)$, rather than just for initial conformal time η_0. A somewhat similar approach was adopted in [31,84]. The initial density perturbation violates the dS isometry. The main difference with respect to the standard approach [84] is that here, in general, we obtain a *nonlinear* integro-differential equation. For a very small initial density perturbation, when the initial comoving number density $n(P_0)$ is much smaller than 1, this equation can be reduced to a linear one, which is similar to the equation considered in [52,55,57]. However, if the initial perturbation is of order one, the full nonlinear equation has explosive solutions.

Finally, let us remark that in dS space there can be secular IR effects in the vertices. They are very important for the resummation. The point is that to do the resummation, one has to solve the system of Dyson-Schwinger equations for all propagators and vertices. As we explain in the main body of the text, for lower masses, higher and higher point functions become relevant in the IR limit.[4] This makes the problem under consideration practically impossible to solve, unless one can drop the equations for the vertices and invent some suitable ansatz for the two-point functions only. This is possible only if the leading secular growth is present in propagators and is absent in the vertices.

In our paper, we check the presence of the secular effects for the vertices and explain their physical origin. They appear due to the presence of bound states in the spectrum of the theory and signal that higher point correlations become relevant. We find the situation when these bound states and related secular effects in the vertices are absent. Then we perform the self-consistent resummation of the leading effects from all loops only when there are no secular effects in the vertices. We do that for arbitrary, not necessary isometry respecting, density perturbations of the BD state. We derive an equation which provides the resummation, find its solutions, and discuss their physical meaning.

III. GENERAL DISCUSSION OF THE LOOP CORRECTIONS TO THE PROPAGATORS

Let us start by discussing the loop corrections to the Keldysh propagator $D^K(p|\eta_1,\eta_2)$ having chosen an initial dS invariant state at past infinity of the EPP. It is not difficult to show that the massive $\lambda\phi^4$ theory does not possess any secularly growing contributions (of the last sort, which is described in the Introduction) to any propagator at the first-loop "bubble" diagram order ($\sim\lambda$). However, at the second-loop order ($\sim\lambda^2$) there is a large IR contribution to D^K which is of interest for us. Two-loop diagrams that contain large IR corrections are of the "sunset" type:

$$\begin{aligned}\Delta_2 D^K(p|\eta_1,\eta_2) = \frac{\lambda^2}{6}\int \frac{d^{D-1}\vec{q}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\vec{q}_2}{(2\pi)^{D-1}}\iint_{+\infty}^{0}\frac{d\eta_3 d\eta_4}{(\eta_3\eta_4)^D}\\ \times\Big[3D_0^K(p|\eta_1,\eta_3)D_0^K(q_1|\eta_3,\eta_4)D_0^K(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2)\\ -\frac{1}{4}D_0^K(p|\eta_1,\eta_3)D_0^A(q_1|\eta_3,\eta_4)D_0^A(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2)\\ -\frac{3}{4}D_0^R(p|\eta_1,\eta_3)D_0^K(q_1|\eta_3,\eta_4)D_0^A(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2)\\ +D_0^R(p|\eta_1,\eta_3)D_0^K(q_1|\eta_3,\eta_4)D_0^K(q_2|\eta_3,\eta_4)D_0^K(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2)\\ -\frac{3}{4}D_0^R(p|\eta_1,\eta_3)D_0^K(q_1|\eta_3,\eta_4)D_0^R(q_2|\eta_3,\eta_4)D_0^R(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2)\\ -\frac{1}{4}D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D_0^R(q_2|\eta_3,\eta_4)D_0^R(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^K(p|\eta_4,\eta_2)\\ +3D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D_0^K(q_2|\eta_3,\eta_4)D_0^K(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^K(p|\eta_4,\eta_2)\Big]. \qquad (12)\end{aligned}$$

Below we do not consider UV divergences but assume some kind of UV renormalization, i.e. that masses of the fields and coupling constants have been set equal to their physical renormalized values. It is probably worth stressing here that mixed expressions, where the partial Fourier transformation has been taken w.r.t. the spatial coordinates, are not sensitive to the UV divergences. In fact, to reveal the latter, one needs an extra integration in the vertices: namely, it is necessary to

[4] As we explain below, these secular effects in the vertices are present only for the fields whose mass is lower than a certain bound, $m < \frac{\sqrt{3}}{4}(D-1)$. And as one lowers the mass, higher and higher point correlation functions start to grow in the IR limit.

transform back to the spacetime variables $(\eta, \vec{x})$. The leading IR contributions to $\Delta_2 D^K(p|\eta_1,\eta_2)$ are hidden within the following expression [67]:

$$\Delta_2 D^K(p|\eta_1,\eta_2) \approx (\eta_1\eta_2)^{\frac{D-1}{2}}[h(p\eta_1)h^*(p\eta_2)n_2(p\eta) + h(p\eta_1)h(p\eta_2)\kappa_2(p\eta) + \text{c.c.}], \tag{13}$$

where $\eta = e^{-t} = \sqrt{\eta_1\eta_2} = e^{-\frac{t_1+t_2}{2}}$ is the average conformal time and

$$n_2(p\eta) = \frac{\lambda^2}{3}\int_\infty^\eta\int_\infty^\eta d\eta_3 d\eta_4 h(p\eta_3)h^*(p\eta_4)F(p,\eta_3,\eta_4),$$

$$\kappa_2(p\eta) = -\frac{2\lambda^2}{3}\int_\infty^\eta\int_\infty^{\eta_3} d\eta_3 d\eta_4 h^*(p\eta_3)h^*(p\eta_4)F(p,\eta_3,\eta_4),$$

$$F(p,\eta_3,\eta_4) = \int \frac{d^{D-1}q_1}{(2\pi)^{D-1}}\frac{d^{D-1}q_2}{(2\pi)^{D-1}}(\eta_3\eta_4)^{D-2} \times h(q_1\eta_4)h^*(q_1\eta_3)h(q_2\eta_4)h^*(q_2\eta_3)h(|\vec{p}-\vec{q}_1-\vec{q}_2|\eta_4)h^*(|\vec{p}-\vec{q}_1-\vec{q}_2|\eta_3), \tag{14}$$

where the subscript 2 in n_2 and κ_2 denotes the second loop contribution.

In deriving the above representation for $\Delta_2 D^K$ from (7), (9) and (12) in the limit $p\sqrt{\eta_1\eta_2} \to 0$ with $\eta_1/\eta_2 = \text{const}$, we neglected the difference between η_1 and η_2 and replaced both of them by the average conformal time η in the arguments of the Heaviside θ functions inside $D^{R,A}$ (see [67] for more details).

In the following we will estimate (13) and see that for generic modes h these quantities grow as $p\eta \to 0$ even when zero values of n_0 and κ_0 are chosen at past infinity (see the tree-level expressions for D^K).

The character of the two-loop corrections to the retarded and advanced propagators depends neither on the choice of the mode functions h nor on the mass of the field. It is the same as in the case of the principal series [67,85] (see also [71] for a more general discussion). In fact, the two-loop contribution to D^R is as follows:

$$\Delta_2 D^R(p|\eta_1,\eta_2) = \frac{\lambda^2}{6}\int\frac{d^{D-1}\vec{q}_1}{(2\pi)^{D-1}}\int\frac{d^{D-1}\vec{q}_2}{(2\pi)^{D-1}}\iint_{+\infty}^0 \frac{d\eta_3 d\eta_4}{(\eta_3\eta_4)^D} \times\Big[3D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D_0^K(q_2|\eta_3,\eta_4)D_0^K(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^R(p|\eta_4,\eta_2) - \frac{1}{4}D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D_0^R(q_2|\eta_3,\eta_4)D_0^R(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^R(p|\eta_4,\eta_2)\Big]. \tag{15}$$

Due to the presence of D^R inside the loop and in the external legs, the limits of integration over $\eta_{3,4}$ are such that $\eta_1 > \eta_3 > \eta_4 > \eta_2$. As a result, the integral (15) does not contain growing corrections when η_1/η_2 is held fixed. The situation with the advanced propagator is the same.

We will discuss loop corrections to the vertices below for specific choices of h separately.

IV. BUNCH-DAVIES FIELDS

In this section we examine the BD fields of the complementary series. The BD modes are proportional to the Hankel functions: $h(x) \propto H_\nu^{(1)}(x) = J_\nu(x) + iY_\nu(x)$; h^* is just the complex conjugate of h [76]. When $p\eta \to \infty$ they behave as $h(p\eta) \sim \frac{e^{ip\eta}}{\sqrt{p\eta}}$ and represent pure waves at past infinity of the EPP or in-modes. These modes diagonalize the free Hamiltonian at past infinity. On the other hand, when $p\eta \to 0$ they behave as $h(p\eta) \approx A_-(p\eta)^{-\nu} + iA_+(p\eta)^\nu + B(p\eta)^{-\nu+2}$ where $A_\pm$ and B are real constants. In this paper we consider fields from the complementary series; i.e. ν is real and $0 < \nu < (D-1)/2$. We keep the $Bx^{-\nu+2}$ term because this term will dominate over A_+x^ν, as $x \to 0$, if $\nu > 1$, and the presence of A_+ is important as we will see below. The results in our paper are valid for $\nu \leq 1$, but our discussion can be straightforwardly extended to the case of $1 < \nu < (D-1)/2$ (for $D > 3$), if we take into account the B term in h.

A. Corrections to the Keldysh propagator

Unlike the case of the principal series [67], the function $F(p,\eta_3,\eta_4)$ in (13) may contain large contributions as

$p \to 0$. In order to estimate this function let us divide the area of integration over $d^{D-1}q_{1,2}$ into four regions: a region where $|\vec{q}_{1,2}| \lesssim |\vec{p}|$, another where $|\vec{q}_{1,2}| \gtrsim |\vec{p}|$ and two other regions where $|\vec{q}_1| \lesssim |\vec{p}| \lesssim |\vec{q}_2|$ or $|\vec{q}_2| \lesssim |\vec{p}| \lesssim |\vec{q}_1|$. Here we present only vague estimates; in the next section we discuss the origin of these complications more rigorously and explain their physical meaning.

To estimate the contribution to $F(p, \eta_3, \eta_4)$ from the first region, we can approximate $|\vec{q}_{1,2} - \vec{p}| \approx p$ and take into account that, in the IR limit in question, we have that $p \to 0$. Hence, we can Taylor expand around zero all the mode functions in the expression for $F(p, \eta_3, \eta_4)$. Then, the contribution to $F(p, \eta_3, \eta_4)$ from the first region, $|\vec{q}_{1,2}| \lesssim |\vec{p}|$, is

$$F_{(1)}(p, \eta_3, \eta_4) = \int\limits_{|q_1|,|q_2|<|p|} \frac{d^{D-1}q_1}{(2\pi)^{D-1}} \frac{d^{D-1}q_2}{(2\pi)^{D-1}} (\eta_3\eta_4)^{D-2} h(q_1\eta_4) h^*(q_1\eta_3) h(q_2\eta_4) h^*(q_2\eta_3) h(p\eta_4) h^*(p\eta_3)$$
$$\sim \int\limits_{|q_1|,|q_2|<|p|} d^{D-1}q_1 d^{D-1}q_2 q_1^{-2\nu} q_2^{-2\nu} p^{-2\nu} (\eta_3\eta_4)^{D-2-3\nu} \sim (\eta_3\eta_4)^{D-2-3\nu} p^{2(D-1-3\nu)}. \tag{16}$$

In the second region (where $q_{1,2} \geq p$) the largest IR contribution to $F(p, \eta_3, \eta_4)$ comes from $q_{1,2} \gg p$. Again we can Taylor expand all mode functions to obtain

$$F_{(2)}(p, \eta_3, \eta_4) \sim \int\limits_{|q_1|,|q_2|>|p|} d^{D-1}q_1 d^{D-1}q_2 q_1^{-2\nu} q_2^{-2\nu} (|\vec{q}_1 + \vec{q}_2|)^{-2\nu} (\eta_3\eta_4)^{D-2-3\nu}.$$

Finally, the estimates in the remaining two regions give the same result:

$$F_{(3)}(p, \eta_3, \eta_4) \sim \int\limits_{|\vec{q}_1|<|\vec{p}|} d^{D-1}\vec{q}_1 \int\limits_{|\vec{p}|<|\vec{q}_2|} d^{D-1}\vec{q}_2 q_1^{-2\nu} q_2^{-4\nu} (\eta_3\eta_4)^{D-2-3\nu} \sim F_{(4)}(p, \eta_3, \eta_4). \tag{17}$$

Therefore, when $D - 1 - 3\nu < 0$ there is a large IR contribution to $F(p, \eta_3, \eta_4)$ coming from the regions where either q_1 or q_2 or both are smaller than p.

Consequently, in this case n and κ receive large IR contributions from the integral over $q_{1,2}$ in the region $p < q_{1,2}$ and the region $q_{1,2} < p$. This is one difference between the case of the complementary series and the principal series which gets large IR contributions only from the region $|\vec{q}_{1,2}| \gg |\vec{p}|$ [66,67].

Note however that when $D - 1 - 3\nu > 0$, $F(p, \eta_3, \eta_4)$ is well behaved and n_2 and κ_2 in (13) receive large IR contributions only from $|\vec{p}| < |\vec{q}_{1,2}|$. In other words, in this case the situation is similar to the principal series [67]. There is however a difference: in the principal series n_2, κ_2 receive logarithmic IR corrections [67] while here loop contributions are powerlike:

$$n_2(p\eta) \approx \frac{\lambda^2}{3} |A_-|^2 (p\eta)^{-2\nu} \int \frac{d^{D-1}q_1}{(2\pi)^{D-1}} \frac{d^{D-1}q_2}{(2\pi)^{D-1}} \int_\infty^1 \int_\infty^1 d\eta_3 d\eta_4 (\eta_3\eta_4)^{D-2-\nu}$$
$$\times h(q_1\eta_4) h^*(q_1\eta_3) h(q_2\eta_4) h^*(q_2\eta_3) h(|\vec{q}_1 + \vec{q}_2|\eta_4) h^*(|\vec{q}_1 + \vec{q}_2|\eta_3),$$
$$\kappa_2(p\eta) \approx -\frac{2\lambda^2}{3} \lambda^2 (A_-^*)^2 (p\eta)^{-2\nu} \int \frac{d^{D-1}q_1}{(2\pi)^{D-1}} \frac{d^{D-1}q_2}{(2\pi)^{D-1}} \int_\infty^1 \int_\infty^1 d\eta_3 d\eta_4 (\eta_3\eta_4)^{D-2-\nu}$$
$$\times h(q_1\eta_4) h^*(q_1\eta_3) h(q_2\eta_4) h^*(q_2\eta_3) h(|\vec{q}_1 + \vec{q}_2|\eta_4) h^*(|\vec{q}_1 + \vec{q}_2|\eta_3). \tag{18}$$

These expressions are obtained from (13) via the change of variables $q \to q\eta, \eta_{3,4} \to \eta_{3,4}/\eta$, neglecting p in comparison with $q_{1,2}$ and expanding $h(p\eta_{3,4})$ to the leading order in the limit $p\eta_{3,4} \to 0$. But, after substituting Eq. (18) into $\Delta_2 D^K$ in (13) and expanding $h(p\eta_{1,2})$ around zero, these leading expressions cancel out. Moreover, the large IR contributions coming from the term $Bx^{-\nu+2}$ in the expansion of $h(x)$ also disappear from the final expression for $\Delta_2 D^K$.

The largest IR correction to $\Delta_2 D^K$ comes from the subleading contributions to n and κ. To obtain this correction, one has to express all the modes in Eq. (13) using the Bessel functions J_ν and Y_ν. Then, in one of the four h's $[h(p\eta_{1,2})$ and $h(p\eta_{3,4})]$, we have to single out $J_\nu \sim x^\nu$, as $x \to 0$, while in the other three, $Y_\nu \sim x^{-\nu}$, as $x \to 0$. The corresponding expressions do not cancel out but provide the leading IR contribution to $\Delta_2 D^K$.

To calculate approximately the resulting leading contribution, we neglect p in comparison with $q_{1,2}$ under the integrals in (13). There is however a change in the lower limits of integration over η_3 and η_4 which are set to ν/p.

In doing this approximation we just neglect the contributions to $\Delta_2 D^K$ from the high physical momenta $p\eta_{3,4} \gg \nu$, where the physics is practically the same as in flat space.

After changing the integration variables $u = p\sqrt{\eta_3\eta_4}$ and $v = \sqrt{\frac{\eta_3}{\eta_4}}$, we can harmlessly extend the integration over v from infinity to zero. The integrals remain finite, and the prefactors of the expressions that we will find below are just slightly changed by the contributions from the high physical momenta.

Finally, we expand $h(p\eta_{3,4})$ around zero, perform the integration over u from ν to $p\eta$, and keep only the terms that are divergent or leading in the expression for $\Delta_2 D^K$, as $p\eta \to 0$:

$$\Delta_2 D^K(p|\eta_1,\eta_2) \approx \frac{8A_-^3 A_+}{3(2\pi)^{2(D-1)}} \frac{\lambda^2 \log(p\eta/\nu)\eta^{D-1}}{(p\eta)^{2\nu}} \times \left\{ \int_1^\infty dv v^{-D} G(v) \left(-\frac{1}{2\nu} v^{2\nu} + \frac{1}{v^{2\nu}} \right) - \int_0^1 dv v^{-D} G(v) \left(\frac{1}{2\nu} v^{-2\nu} + v^{2\nu} \right) \right\}, \tag{19}$$

where

$$G(v) = \iint \frac{d^{D-1}q_1}{(2\pi)^{D-1}} \frac{d^{D-1}q_2}{(2\pi)^{D-1}} h(q_1 v^2) h^*(q_1) h(q_2 v^2) h^* \times (q_2) h(|\vec{q}_1 + \vec{q}_2| v^2) h^*(|\vec{q}_1 + \vec{q}_2|). \tag{20}$$

Although λ is small, the loop correction becomes comparable to the tree-level contribution. In fact, as $p\eta \to 0$, the sum of the tree-level and the second loop correction to the Keldysh propagator is as follows:

$$D_0^K + \Delta_2 D^K \approx \eta^{D-1}/(p\eta)^{2\nu} \left[a + b\lambda^2 \log \frac{p\eta}{\nu} \right], \tag{21}$$

where the constants a and b are computed from the expressions above.

As a side remark, if $D - 1 - 3\nu < 0$, the IR contributions to n_2 and κ_2 have the following form:

$$n_2(p\eta) \propto \lambda^2 (p\eta)^{D-1-6\nu}, \quad \kappa_2(p\eta) \propto \lambda^2 (p\eta)^{D-1-6\nu}. \tag{22}$$

We do not present here the full expressions for n_2, κ_2 and $\Delta_2 D^K$ because in this case we will not do the resummation of the leading IR contributions from all loops. Again, after substituting the above leading expressions (22) into $\Delta_2 D^K$, they cancel out. What survives in $\Delta_2 D^K$ is coming from subleading contributions to n_2, κ_2 and κ_2^*. Corrections to $\Delta_2 D^K$ are always logarithmic as in (19) and (21), but the coefficients in front of them are different depending on whether $D - 1 - 3\nu$ is greater or lower than zero.

B. Correction to the vertices

For the vertices it is more convenient to use the nonstationary diagrammatic technique before the Keldysh rotation (see e.g. [67] for the explanation and notations). Then, the one-loop correction from the $(--)$ "fish" diagrams to the vertices is as follows:

$$\lambda^{--}(\eta_1,\eta_2,p_1,p_2,p_3,p_4) = (-i\lambda)^2(\eta_1\eta_2)^{D-1}\delta^{(D-1)}(\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4) \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \times \{\theta(\eta_1-\eta_2)h(|\vec{q}-\vec{p}_1|\eta_1)h^*(|\vec{q}-\vec{p}_1|\eta_2) + \theta(\eta_2-\eta_1)h(|\vec{q}-\vec{p}_1|\eta_2)h^*(|\vec{q}-\vec{p}_1|\eta_1)\} \times \{\theta(\eta_2-\eta_1)h(|\vec{p}_2+\vec{q}|\eta_2)h^*(|\vec{p}_2+\vec{q}|\eta_1) + \theta(\eta_1-\eta_2)h(|\vec{p}_2+\vec{q}|\eta_1)h^*(|\vec{p}_2+\vec{q}|\eta_2)\}. \tag{23}$$

Here the indices "+" and "−" are attributed to the two internal vertices in the one-loop diagram describing correction to the tree-level vertex. The situation with the other vertices, λ^{+-}, λ^{-+} and λ^{++}, is very similar. We would like to check if (23) contains large corrections in the limit $p_i\eta_{1,2} \to 0$, $i = 1, 2, 3, 4$.

As in the case of the Keldysh propagator, we divide the domain of integration over the internal momentum $\vec{q}$ into regions. One region is where $q \geq (p_1p_2p_3p_4)^{\frac{1}{4}}$. To estimate λ^{--} in this domain we observe that the largest contribution to the vertices comes from $q \sim (p_1p_2p_3p_4)^{1/4} \to 0$. (In the next section we present more rigorous observations.) Hence, Taylor expanding all mode functions around zero, we obtain the contribution to the vertex from the first region as follows:

$$\Delta_1\lambda^{--}(\eta_1,\eta_2,p_1,p_2,p_3,p_4) \sim (-i\lambda)^2(\eta_1\eta_2)^{D-1-2\nu}\delta^{(D-1)}(\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4) \times \int_{|q|>\sqrt[4]{p_1p_2p_3p_4}} d^{D-1}q q^{-4\nu}. \tag{24}$$

If $D - 1 - 4\nu < 0$, this expression is large. In the opposite case, when $D - 1 - 4\nu > 0$, the integral in (24) is convergent in the IR limit under consideration.

To estimate the vertex contribution in the domain of integration where $q \leq (p_1p_2p_3p_4)^{\frac{1}{4}}$, we can neglect q in comparison with p_i. Then

$$\Delta_2\lambda^{--}(\eta_1,\eta_2,p_1,p_2,p_3,p_4) \sim (-i\lambda)^2(\eta_1\eta_2)^{D-1-2\nu}(p_1p_2)^{-2\nu}\delta^{(D-1)}(\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4)\int_{|q|<\sqrt[4]{p_1p_2p_3p_4}} d^{D-1}q. \tag{25}$$

Again, if $D-1-4\nu>0$, this expression does not contain large contributions. If, however, $D-1-4\nu<0$, there are large IR corrections, which are similar to (24). In the next subsection we discuss more rigorously the physical origin and the meaning of these problems with the vertex corrections.

V. PHYSICAL ROOTS OF THE SECULAR EFFECTS IN THE VERTICES AND SELF-ENERGIES

When

$$\nu > \frac{D-1}{4} \tag{26}$$

[i.e. when $m<\frac{\sqrt{3}}{4}(D-1)$] there are potentially dangerous IR corrections to the vertices, as $p_i\eta_i\to 0$. Furthermore, when

$$\nu > \frac{D-1}{3} \tag{27}$$

[i.e. when $m<\frac{\sqrt{5}}{6}(D-1)$] there are also potentially dangerous IR contributions to the Keldysh propagator (self-energy). Such contributions can complicate the problem of the summation of the leading IR corrections in all loops because the IR limit of the entire system of the Dyson-Schwinger equations has to be solved for propagators and vertices simultaneously. This is to be compared to the case of the principal series, where the problem may be reduced to the solution of only one Dyson-Schwinger equation for the Keldysh propagator [67].

To better understand what is going on, recall that the spectrum of a theory of a neutral scalar meson in flat space is made of the mass shell $p^2=m^2$ plus a continuum including all two-particle states, the three-particle states, etc., which starts at $p^2=4m^2$. The very existence of the complementary series of fields in dS space makes the situation rather different. The situation becomes clearer when we express, say, the BD two-point functions by using the coordinate-independent plane-wave representation introduced in [86,87].

To this aim, in this section we look at either the real or the complex dS manifold as submanifolds of the complex Minkowski manifold with one spacelike dimension more:

$$dS_D=\{x\in M_{D+1}\colon x\cdot x=-R^2=-1\}\quad\text{and}$$
$$dS_D^{(c)}=\{z\in M_{D+1}^{(c)}\colon z\cdot z=-R^2=-1\}, \tag{28}$$

where $x\cdot y=x^0y^0-\vec{x}\cdot\vec{y}$. The dS metric is obtained by restriction of the ambient space-time interval to dS_D. In particular, for the EPP we get

$$x(t,\mathbf{x})=\begin{cases} x^0=\operatorname{sh}t+\frac{e^t}{2}|\mathbf{x}|^2\\ x^i=e^t\mathbf{x}^i\\ x^D=\operatorname{ch}t-\frac{e^t}{2}|\mathbf{x}|^2,\end{cases} \tag{29}$$

$$ds^2=(dx_0^2-dx_1^2-\dots dx_D^2)|_{dS_D}=dt^2-\exp(2t)d\mathbf{x}^2. \tag{30}$$

There exists a remarkable set of solutions of the dS Klein-Gordon equation which may be interpreted as dS plane waves [86–89]. For a complex dS event z, a given nonzero lightlike vector $\xi\in C_+$ and a complex number $\lambda\in\mathbf{C}$, the homogeneous function

$$z\mapsto(\xi\cdot z)^\lambda \tag{31}$$

satisfies the massive (complex) Klein-Gordon equation:

$$(\Box_z+m_\lambda^2)(\xi\cdot z)^\lambda=0. \tag{32}$$

The above plane waves are holomorphic in the future and past dS tuboids $\mathcal{T}_\pm$ (which are related to the spectral condition of dS QFT [86,87,90]) obtained as the intersections of the ambient tubes [91] with the complex dS manifold:

$$\mathcal{T}_\pm=\{x\pm iy\in dS_d^{(c)}\colon y^2=y\cdot y>0,y^0>0\}. \tag{33}$$

The parameter λ is unrestricted here; i.e. we may consider complex squared masses $m_\lambda^2=-\lambda(\lambda+D-1)$. The symmetry $\lambda\to(-\lambda-D+1)$ also implies that

$$(\Box_z+m_\lambda^2)(\xi\cdot z)^{-\lambda-D+1}=0. \tag{34}$$

The two boundary values of the complex waves (one from each tuboid) are homogeneous distributions of degree λ and are solutions of the dS Klein-Gordon equation on the real dS manifold:

$$(\Box_x+m_\lambda^2)(\xi\cdot x)_\pm^\lambda=0,\qquad(\Box_x+m_\lambda^2)(\xi\cdot x)_\pm^{1-D-\lambda}=0. \tag{35}$$

The waves depend, in a $\mathcal{C}^\infty$ way, on ξ and are entire functions of λ.

We may now introduce a class of maximally analytic vacua (the BD vacua—one for each complex squared mass) by specifying their two-point functions. In the following plane-wave expansion we take the first point z in the backward tuboid $\mathcal{T}_-$, the second point z' in the forward tuboid $\mathcal{T}_+$, and λ is not a pole of $\Gamma(-\lambda)\Gamma(\lambda+D-1)$:

$$W_\lambda(z,z') = w_\lambda(z\cdot z') = \frac{\Gamma(-\lambda)\Gamma(\lambda+D-1)e^{-i\pi(\lambda+\frac{D-1}{2})}}{2^{D+1}\pi^D} \times \int_{S_0} (\xi\cdot z)^{1-D-\lambda}(\xi\cdot z')^\lambda d\boldsymbol{\xi}. \tag{36}$$

The integral is performed on the spherical basis S_0 of the future cone: $S_0 = \{\xi : \xi^2 = 0, \xi_0 = 1\}$; $d\boldsymbol{\xi}$ denotes the spherical invariant measure. One gets that the above Fourier-like representation may be evaluated in terms of Legendre functions of the first kind:

$$w_\lambda(z\cdot z') = \frac{\Gamma(-\lambda)\Gamma(\lambda+D-1)}{2(2\pi)^{D/2}}(\zeta^2-1)^{-\frac{D-2}{4}}P^{-\frac{D-2}{2}}_{\lambda+\frac{D-2}{2}}(\zeta), \quad \zeta = z\cdot z'. \tag{37}$$

It is clear from (37) that $\zeta \mapsto w_\lambda(\zeta) = w_{-\lambda-D+1}(\zeta)$ is holomorphic in $\mathbf{C}\backslash(-\infty,-1]$, i.e. everywhere except on the locality cut (maximal analyticity property).

When we specialize the above construction to fields having real and positive squared masses, we immediately understand that there are two types of waves. The first is the principal series $\lambda = -\frac{D-1}{2} + i\nu$ with $\nu \in \mathbf{R}$; in this case, waves have an oscillatory character:

$$\phi(x) = (\xi\cdot x)_\pm^{-\frac{D-1}{2}+i\nu}, \qquad m^2 = \left(\frac{D-1}{2}\right)^2 + \nu^2. \tag{38}$$

The second type is the complementary series $\lambda = -\frac{D-1}{2} + \nu$ with $|\nu| < (D-1)/2$. Here waves do not oscillate but decay more slowly at infinity:

$$\phi(x) = (\xi\cdot x)_\pm^{-\frac{D-1}{2}+\nu}, \qquad m^2 = \left(\frac{D-1}{2}\right)^2 - \nu^2. \tag{39}$$

Although redundant, let us write explicitly the above Fourier-like representation of the BD vacua in the two cases of interest: for $\nu \in \mathbf{R}$, theories of the principal series have the following two-point functions:

$$W_{i\nu}(z,z') = w_{i\nu}(\zeta) = \frac{\Gamma(\frac{D-1}{2}+i\nu)\Gamma(\frac{D-1}{2}-i\nu)}{2^{D+1}\pi^D e^{-\pi\nu}} \times \int_{S_0}(\xi\cdot z)^{-\frac{D-1}{2}-i\nu}(\xi\cdot z')^{-\frac{D-1}{2}+i\nu}d\boldsymbol{\xi}. \tag{40}$$

For $\nu \in \mathbf{R}$ and $|\nu| < (D-1)/2$, theories of the complementary series have the following two-point functions:

$$W_\nu(z,z') = w_\nu(\zeta) = \frac{\Gamma(\frac{d-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{2^{D+1}\pi^D e^{i\pi\nu}} \times \int_{S_0}(\xi\cdot z)^{-\frac{D-1}{2}-\nu}(\xi\cdot z')^{-\frac{D-1}{2}+\nu}d\boldsymbol{\xi}. \tag{41}$$

Now let us come back to our main line of thought. When studying the corrections to the propagators and to the vertices, we are led to consider the distributions $W^2(x,x')$ and $W^3(x,x')$. Let us focus on the Wick-square $W^2(x,x')$ of the two-point function and consider a theory of the principal series. The asymptotic behavior can be crudely estimated by looking at the square of a plane wave: $(\xi\cdot x)_\pm^{-(D-1)+2i\nu}$. At infinity, this function behaves better than a wave of the principal series; since $W^2_{i\nu}(x,x')$ is a positive-definite distribution, it should be possible to write an expansion of it just in terms of two-point functions of the principal series as follows:

$$W^2_{i\nu}(\zeta) = \int \kappa\rho_{i\nu}(i\kappa)W_{i\kappa}(\zeta)d\kappa. \tag{42}$$

This property should also remain true for fields of the complementary series as long as

$$-(D-1) + 2|\nu| \le -\frac{D-1}{2} \tag{43}$$

i.e. as long as $|\nu| \le \frac{D-1}{4}$. To understand what happens when this bound is violated, we need to examine the above Källén-Lehmann representation more closely. The problem of finding the weight ρ has been solved in [92] in a more general case, namely, for the product of two distributions $w_{i\nu}(\zeta)$ and $w_{i\lambda}(\zeta)$ belonging to the principal series (40); the following integral representation holds:

$$w_{i\nu}(\zeta)w_{i\lambda}(\zeta) = \int_{\mathbf{R}} \kappa\rho_{i\nu,i\lambda}(i\kappa)w_{i\kappa}(\zeta)d\kappa, \tag{44}$$

where the Källén-Lehmann weight has the following remarkable explicit expression:

$$\kappa\rho_{i\nu,i\lambda}(i\kappa) = \frac{1}{2^5\pi^{\frac{D+5}{2}}\Gamma(\frac{D-1}{2})} \times \frac{\kappa\,\mathrm{sh}\,\pi\kappa\prod_{\epsilon,\epsilon',\epsilon''=\pm1}\Gamma(\frac{D-1}{4}+\frac{i\epsilon\kappa+i\epsilon'\nu+i\epsilon''\lambda}{2})}{\Gamma(\frac{D-1}{2}+i\kappa)\Gamma(\frac{D-1}{2}-i\kappa)}. \tag{45}$$

By using such an explicit result, we may perform analytic continuation in the mass parameters to obtain the Källén-Lehmann representation for the product $w_\alpha(\zeta)w_\beta(\zeta)$ belonging to the complementary series and violating the bound $\alpha+\beta < \frac{D-1}{2}$ (see [92] for more details). The result is as follows: if N is a non-negative integer such that

$$\frac{D-1}{4} + N < \frac{1}{2}(\alpha+\beta) < \frac{D-1}{4} + N + 1 \tag{46}$$

provided

$$N < \frac{D-1}{4}, \tag{47}$$

the Källén-Lehmann representation of the product $w_\alpha(\zeta)w_\beta(\zeta)$ incudes $N+1$ discrete terms:

$$w_\alpha(\zeta)w_\beta(\zeta) = \int_{\mathbf{R}} \kappa\rho_{\alpha,\beta}(i\kappa)w_{i\kappa}(\zeta)d\kappa + \sum_{n=0}^{N} A_n(\alpha,\beta)w_{\frac{D-1-2\alpha-2\beta}{2}+2n}(\zeta),$$

where

$$A_n(\alpha,\beta) = \frac{\Gamma(\alpha-n)\Gamma(\beta-n)\Gamma(\alpha+\beta-n)\Gamma(\frac{D-1}{2}-\alpha+n)\Gamma(\frac{D-1}{2}-\beta+n)\Gamma(\frac{D-1}{2}+n)}{4n!\pi^{\frac{D+1}{2}}\Gamma(\frac{D-1}{2})\Gamma(-\frac{D-1}{2}-2n+\alpha+\beta)\Gamma(-2n+\alpha+\beta)\Gamma(D-1+2n-\alpha-\beta)} \times \frac{(-1)^n\Gamma(n+\frac{D-1}{2}-\alpha-\beta)}{\Gamma(2n+\frac{D-1}{2}-\alpha-\beta)} \tag{48}$$

provided neither α nor β is an integer (if $\alpha+\beta<\frac{D-1}{2}$, the formula holds without the A_n terms as for the principal series). It is easy to check that $\kappa\rho_{\alpha,\beta}(\kappa)\geq 0$. Furthermore, all the factors in $A_n(\alpha,\beta)$—except the last fraction—are positive since the arguments of the Gamma functions are positive. The last fraction is of the form

$$\frac{(-1)^n\Gamma(n+x)}{\Gamma(2n+x)} = (-1)^n\prod_{q}^{2n-1}(q+x)^{-1}. \tag{49}$$

The last product contains n negative factors, and the result is positive.

The conclusion that can be drawn from the above analysis may be surprising. Let us consider fixing the idea of a dS space-time dimension $D=4$ and consider free fields of the complementary series violating the bound (43), i.e. fields such that $\nu>3/4$ for the complementary series. From the above analysis it follows that the two-particle subspace of the Fock space relative to such a field contains a discrete component of parameter $2\nu-3/2$. In other words, a free field theory of mass

$$m^2 = \frac{9}{4}-\nu^2 < \frac{27}{16} \tag{50}$$

contains, in the two-particle states, discrete terms ("bound states") of mass

$$M^2 = (3-2\nu)2\nu < \frac{9}{4}. \tag{51}$$

Similarly, when considering the three-particle subspace of a light free field, one sees that the behavior at infinity is not worse than that of the principal series provided

$$-\frac{3}{2}(D-1)+3\nu \leq -\frac{D-1}{2}. \tag{52}$$

On the other hand, a more laborious calculation shows that when $\nu>\frac{D-1}{3}$, as before, "bound states" appear in the three-particle subspace of the theory.

VI. REMARKS ON LOOP CORRECTIONS FOR THE OUT-MODES

To complete the discussion we also have to consider loop corrections for other α-modes. In fact, here we have an interacting theory, and we take it in the IR limit. Then, modes different from the BD modes may play an important role: for instance, for the principal series, due to interactions, the in-ground state is transformed in the out-ground state in future infinity [67]; hence, the loop resummation has to be done with the use of out-modes. On the other hand, it is not hard to see that the structure of the loop corrections for generic α-modes is very similar to the BD case. The only exceptions are the out-modes, in a sense that we explain now.

For the out-modes, $h(x)\propto J_\nu(x)$ while $h^*(x)\propto iY_\nu(x)$. These modes are related to the BD-modes via a Bogoliubov rotation. The main difference in comparison with the BD modes is that for $x\to 0$ the behavior of the modes is $h(x)\approx B_+x^\nu$ and $h^*(x)\approx B_-x^{-\nu}$, as $x\to 0$; here B_+ is some real constant, while B_- is purely imaginary; h^* is not the complex conjugate of h.[5] Thus, for the out-modes the IR behavior of h^* is different from that of h, contrary to what happens for BD modes and even for generic α-modes.

[5]Note that in the case of the principal series, when $m>(D-1)/2$, ν is imaginary. Hence, in our previous papers we were able to choose J_ν and its complex conjugate as the basis of mode functions. However, when ν is real, then J_ν is also real. As a result, we have to choose J_ν and iY_ν as the basis. This way we obtain the proper commutation relations for the creation and annihilation operators. One encounters a similar situation in flat space if the Bogoliubov rotation goes from $e^{\pm i\omega t}$ modes to $\cos(\omega t)$ and $i\sin(\omega t)$.

Note, however, that in the UV limit the out-modes behave as $h(p\eta) \sim \frac{\cos(p\eta - \nu\pi/2 - \pi/4)}{\sqrt{p\eta}}$ and $h^*(p\eta) \sim \frac{\sin(p\eta - \nu\pi/2 - \pi/4)}{\sqrt{p\eta}}$. These mixtures of the positive and negative energy modes $e^{\pm ip\eta}$ spoil the UV behavior of the corresponding propagators. This is the characteristic feature of a generic α-mode, the only exception being the BD modes, which exhibit the standard UV behavior. Note, however, that although out-modes have an incorrect UV behavior, they can be relevant in the IR limit in the presence of an interaction (the prototypical example is the Cooper pairing that is incorrect in the UV limit but provides a correct description of the IR physics).

For the out-modes the two-loop large IR correction to the Keldysh propagator is also contained in the expressions of Eq. (13). It is not hard to show that for any mass parameter ν the function $F(p, \eta_3, \eta_4)$ contains no large IR contribution. The calculation of n_2, κ_2 and κ_2^* proceeds along the same lines as for the principal series in the BD case [67]. The final answer for n_2, κ_2 and κ_2^* is the following:

$$
\begin{aligned}
n_2(p\eta) &\approx \frac{2\lambda^2 B_+ B_-}{3} \log\left(\frac{\nu}{p\eta}\right) \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_0^\infty dv v^{2D-2\nu-3} h(l_1 v^2) h^*(l_1) h(l_2 v^2) h^*(l_2) h(|\vec{l}_1 + \vec{l}_2| v^2) h^*(|\vec{l}_1 + \vec{l}_2|),\\
\kappa_2(p\eta) &\approx -\frac{2\lambda^2 B_-^2}{3\nu} (p\eta)^{-2\nu} \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_0^1 dv v^{2D-2\nu-3} h(l_1 v^2) h^*(l_1) h(l_2 v^2) h^*(l_2) h(|\vec{l}_1 + \vec{l}_2| v^2) h^*(|\vec{l}_1 + \vec{l}_2|),\\
\kappa_2^*(p\eta) &\approx -\frac{2\lambda^2 B_+^2}{3\nu} (p\eta)^{2\nu} \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_0^1 dv v^{2D-2\nu-3} h(l_1 v^2) h^*(l_1) h(l_2 v^2) h^*(l_2) h(|\vec{l}_1 + \vec{l}_2| v^2) h^*(|\vec{l}_1 + \vec{l}_2|).
\end{aligned}
\tag{53}
$$

Taking into account the behavior of $h(p\eta_{1,2})$ and $h^*(p\eta_{1,2})$ at future infinity, we find that the leading IR correction to the Keldysh propagator comes from n alone and is logarithmic:

$$
\Delta_2 D^K(p, \eta_1, \eta_2) = \eta^{D-1} B_+ B_- \left[\left(\frac{\eta_1}{\eta_2}\right)^\nu + \left(\frac{\eta_2}{\eta_1}\right)^\nu\right] n_2(p\eta). \tag{54}
$$

It seems that out-modes may allow the loop resummation for light fields $D - 1 - 4\nu < 0$ as is the case for the principal series [67]. The point is that the resummation can be done by solving the system of Dyson-Schwinger equations, but this system is covariant under the simultaneous Bogoliubov transformation of the modes and n, κ and κ^*. Moreover, it is straightforward to show that for the out-modes the vertices do not receive any large corrections in the limit $p_i \eta_{1,2} \to 0$, $i = 1, 2, 3, 4$. This is true because of the peculiar relation between the IR behavior of h and h^* for the out-modes.

Thus, it seems convenient to try and solve the system of Dyson-Schwinger equations with the use of out-modes. However, unlike the case of the principal series [67], any small excitation of κ and κ^* on top of the out-ground state leads to growing rather than damping effects. This can be seen in the Dyson-Schwinger system of equations at linear order in κ and κ^* (see [67] for the details and methods).

VII. RESUMMATION: PRELIMINARY DISCUSSION

Thus, for all the α-modes, loop effects [$\sim\lambda^2 \log(p\eta)$] can become large as $p\eta \to 0$ even when λ^2 is very small. The important point is that loop corrections are not suppressed in comparison with classical tree-level contributions to propagators and vertices. Hence, to understand the physics in dS space, one has to sum unsuppressed IR corrections from all loops.

In the EPP there is also the distinct problem of summing the dS-invariant IR corrections to the correlation functions of the exact BD state (see e.g. [52–57,68,93]). Only in this case do loop corrections respect the dS isometry (in the EPP) [59] (see also [10,67,68]). Here, however, we consider an initial nonsymmetric density perturbation on top of the BD state at past infinity of the EPP. We would like to trace the destiny of such a perturbation and to understand the effect of the large IR contributions, as the system progresses towards future infinity.

As we have explained in the Introduction, we cannot just put initial comoving density n_p^0 at past infinity of the EPP because then the physical density will be infinite. One has to put the initial value n_p^0 at an initial Cauchy surface $+\infty > \eta_0 > 0$. Moreover, due to the UV divergences the comoving momentum should also be cut off at the UV scale p_0. But, as was explained above, $n(p\eta)$ and $\kappa(p\eta)$ are attributed to the comoving volume and, hence, do not change before $p\eta \sim \nu$. Their behavior for $p\eta > \nu$ is not much different from that in flat space-time. Thus, cutting simultaneously comoving momentum and conformal time integrals effectively amounts to cutting the physical momentum integrals at ν. On the other hand, due to the symmetries of the EPP (or isometries of the dS space) we can put an initial comoving density at an initial value of the physical momentum $P_0 \equiv (p\eta)_0 \sim \nu$ and cut off all the integrals over the physical momentum at this value. This is what was proposed in the Introduction.

To sum loop contributions we should solve the Dyson-Schwinger system of equations for the propagators, self-energies and vertices. We would like to sum only powers of the leading contribution $\lambda^2 \log(p\eta)$ and neglect subdominant terms, such as powers of $\lambda^4 \log(p\eta)$ or $\lambda^2 \log(\eta_1/\eta_2)$ etc.

As we have seen above, the retarded and advanced propagators do not receive large IR contributions from the first and second loops, but only from higher loops which may be produced from the lower loop corrections to the Keldysh propagator. This means that such contributions to $D^{R,A}$ are suppressed by higher powers of λ. Since we want to sum only the leading corrections, we may use the tree-level expressions for $D^{R,A}$ everywhere (renormalized with the leading UV corrections), which is what we will do below.

However, at variance with the principal series case [67], we saw that in the complementary case, large IR contributions to the vertices may also arise. This happens for generic α-modes. To avoid that difficulty, here we restrict our consideration to fields with m sufficiently large $(D-1-4\nu > 0)$ $m > \frac{\sqrt{3}}{4}(D-1)$.

When the resummation is done for the exact BD state in the EPP, the leading contributions come from the summation of the sunset bubbles. In fact, if one puts the above two-loop logarithmic correction to the Keldysh propagator into the internal legs of the sunset diagram, the correction is suppressed as $\lambda^4 \log(p\eta)$ (because of its logarithmic behavior and of the integration over the complete range of the physical momentum inside the loops). The situation is very similar to the standard UV renormalization: if one again puts loop-corrected expressions inside the loops, they lead to subleading corrections, while the leading corrections come from the multiplication of the bubbles. This is exactly the reason why in the Dyson-Schwinger equation one can put the exact Keldysh propagator only into one of the external legs. This fact was used in part in [52,55,57]. As a result, in this case the Dyson-Schwinger equation reduces to a linear integro-differential equation. However, if we cut the integration over the physical momentum at P_0 and put an initial value $n(P_0)$, for the comoving density of the exact modes, the situation becomes very different. Now the internal legs also bring leading corrections of the type $|\lambda^2 \log(p\eta)|^n$. Then one also has to put the exact Keldysh propagators in the internal legs inside the loops, and a nonlinear integro-differential equation follows. The latter has a much farther-reaching realm of solutions.

VIII. IR SOLUTION OF THE DYSON-SCHWINGER EQUATIONS

In the previous section we justified why for BD modes and for $D-1-4\nu > 0$ the Dyson-Schwinger system of equations reduces to the equation for the Keldysh propagator alone:

$$\begin{aligned} D^K(p|\eta_1,\eta_2) \approx{}& D_0^K(p|\eta_1,\eta_2) + \frac{\lambda^2}{6}\int \frac{d^{D-1}\vec{q}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\vec{q}_2}{(2\pi)^{D-1}}\iint_{+\infty}^{0}\frac{d\eta_3 d\eta_4}{(\eta_3\eta_4)^D} \\ &\times\Big[3D_0^K(p|\eta_1,\eta_3)D^K(q_1|\eta_3,\eta_4)D^K(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2) \\ &-\frac{1}{4}D_0^K(p|\eta_1,\eta_3)D_0^A(q_1|\eta_3,\eta_4)D_0^A(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2) \\ &-\frac{3}{4}D_0^R(p|\eta_1,\eta_3)D^K(q_1|\eta_3,\eta_4)D_0^A(q_2|\eta_3,\eta_4)D_0^A(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2) \\ &+D_0^R(p|\eta_1,\eta_3)D^K(q_1|\eta_3,\eta_4)D^K(q_2|\eta_3,\eta_4)D^K(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2) \\ &-\frac{3}{4}D_0^R(p|\eta_1,\eta_3)D^K(q_1|\eta_3,\eta_4)D_0^R(q_2|\eta_3,\eta_4)D_0^R(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D_0^A(p|\eta_4,\eta_2) \\ &-\frac{1}{4}D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D_0^R(q_2|\eta_3,\eta_4)D_0^R(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D^K(p|\eta_4,\eta_2) \\ &+3D_0^R(p|\eta_1,\eta_3)D_0^R(q_1|\eta_3,\eta_4)D^K(q_2|\eta_3,\eta_4)D^K(|\vec{p}-\vec{q}_1-\vec{q}_2||\eta_3,\eta_4)D^K(p|\eta_4,\eta_2)\Big]. \end{aligned} \tag{55}$$

This equation is formally the same as Eq. (12). The difference is that the exact propagator D^K and tree-level propagators D_0^R and D_0^A appear on the rhs wherever appropriate. Again, Eq. (55) is invariant under simultaneous Bogoliubov rotations of the modes and consequently of n, κ and κ^*.

We want to solve this equation in the infrared limit $p\eta \lesssim \nu$ where modes behave as $h(p\eta) = A_-(p\eta)^{-\nu} + iA_+(p\eta)^{\nu}$. For the reasons mentioned at the beginning of Sec. III, we restrict our attention to the case $\nu \leq 1$. For $D \leq 5$ this includes the above restriction on ν. In the IR limit under consideration, we obtain

$$D_0^{\substack{R\\A}}(p|\eta_1,\eta_2) = \mp\,\theta(\mp\,\Delta\eta_{12})2A_-A_+(\eta_1\eta_2)^{\frac{D-1}{2}}\left[\left(\frac{\eta_1}{\eta_2}\right)^{\nu} - \left(\frac{\eta_2}{\eta_1}\right)^{\nu}\right], \qquad p\eta_{1,2} \to 0. \tag{56}$$

We use the same ansatz for the Keldysh propagator that was introduced in a previous paper for the principal series [67]:

$$D^K(p|\eta_1,\eta_2) \approx \eta^{D-1}\left\{h(p\eta_1)h^*(p\eta_2)\left[\frac{1}{2}+n_e(p\eta)\right]+h(p\eta_1)h(p\eta_2)\kappa_e(p\eta)\right\}+\text{c.c.},\qquad \eta=\sqrt{\eta_1\eta_2}, \tag{57}$$

where the subscript e of n, κ and κ^* designates the exact (resummed) contribution. Unlike the case of the principal series, this ansatz solves (55) only in the regions when all physical momenta in the above expressions are less than ν. Hence, we substitute the asymptotic approximate expressions for all mode functions for the small values of their arguments. In fact, this region brings the leading IR contributions to the integrals in (55).

Thus, keeping only the leading contributions, we obtain from (57) the following expression for the Keldysh propagator:

$$D^K(p|\eta_1,\eta_2) \approx A_-^2\eta^{D-1}\frac{N(p\eta)}{(p\eta)^{2\nu}}, \tag{58}$$

where $N(p\eta)=1+2n_e(p\eta)+\kappa_e(p\eta)+\kappa_e^*(p\eta)$ in terms of the original n_e, κ_e and κ_e^*. We assume that the initial condition for this quantity does not contain κ and κ^*, but it can contain n. As we have explained in the Introduction, we put this condition at some initial value of the physical momentum, $n_0(p\eta)=n(P_0)$. This is the initial density perturbation on top of the BD state.

There are two points which are worth stressing at this moment. First, for generic values of $N(p\eta)$, the propagator $D^K(p|\eta_1,\eta_2)$ is not a function of the geodesic distance. This is true although the combination $p\eta$ respects part of the dS isometry—e.g. the simultaneous rescalings $p\to\sigma p$ and $\eta\to\eta/\sigma$. Thus, any initial value for $N(p\eta)$ different from 1 violates the dS isometry. Second, substituting the tree-level value $N(p\eta)=1$ on the rhs of (55) would reproduce the two-loop contribution to D^K obtained above in the case when all modes are approximated by their values at $q_{1,2}\eta_{3,4}\ll\nu$.

Now we can substitute expressions (56) and (58) on the rhs of (55). The leading contributions will be given by the first two and last two terms of (55). The other terms give rise to expressions which are suppressed by higher powers of $p\eta\to 0$. Also, it is convenient to make a change of variables $u=p\sqrt{\eta_1\eta_2}$, $v=\sqrt{\frac{\eta_3}{\eta_4}}$, and $\vec{l}_i=\vec{p}_i\sqrt{\eta_3\eta_4}$. Finally, we get

$$N(p\eta)\approx N(P_0)-\frac{\lambda^2}{3}A_-^6A_+^2\int_\nu^{p\eta}\frac{du}{u}[N(u)+N(P_0)]\int_{\frac{1}{\nu}}^{\nu}\frac{dv}{v}\int^{p\eta<|\vec{l}_{1,2}|<\nu}\frac{d^{D-1}\vec{l}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\vec{l}_2}{(2\pi)^{D-1}}$$
$$\times\left[\theta(v-1)\frac{1}{v^{2\nu}}-\theta(1-v)v^{2\nu}\right]\left\{3A_-^2\frac{N(l_1)}{l_1^{2\nu}}\frac{N(l_2)}{l_2^{2\nu}}\left[v^{2\nu}-\frac{1}{v^{2\nu}}\right]-A_+^2\left[v^{2\nu}-\frac{1}{v^{2\nu}}\right]^3\right\}, \tag{59}$$

where $N(P_0)$ is the initial value of $N(p\eta)$ and $P_0\sim\nu\gg p\eta$. In (59) we neglected p in comparison with q_i; this is made possible by the condition $D-1-4\nu>0$. This equation can be recast in the following form:

$$N(p\eta)-N(P_0)\approx-\int_\nu^{p\eta}\frac{du}{u}[N(u)+N(P_0)]\left[\Gamma_1\left(\int_{p\eta}^{\nu}dll^{D-2-2\nu}N(l)\right)^2-\Gamma_2\right], \tag{60}$$

where

$$\Gamma_1=\frac{\lambda^2A_-^8A_+^2S_{D-2}^2}{(2\pi)^{2(D-1)}}\int_{\frac{1}{\nu}}^{\nu}\frac{dv}{v}\left[\theta(v-1)\frac{1}{v^{2\nu}}-\theta(1-v)v^{2\nu}\right]\left[v^{2\nu}-\frac{1}{v^{2\nu}}\right]>0,$$

$$\text{and}\quad\Gamma_2=\frac{\lambda^2A_-^6A_+^4S_{D-2}^2}{3(2\pi)^{2(D-1)}}\int_{\frac{1}{\nu}}^{\nu}\frac{dv}{v}[\theta(v-1)v^{-2\nu}-\theta(1-v)v^{2\nu}][v^{2\nu}-v^{-2\nu}]^3>0. \tag{61}$$

Here S_{D-2} is the area of the $(D-2)$-dimensional sphere.[6] The above equation can be transformed into an integro-differential equation by differentiating both sides w.r.t. $\log(p\eta)$:

[6]First, the same equation is obtained for $\kappa^*(p\eta)$ instead of $N(p\eta)$, by using the out-modes instead of the BD modes in (55).

$$\frac{\partial N(p\eta)}{\partial \log(\frac{p\eta}{\nu})} \approx -[N(p\eta) + N(P_0)]\left[\Gamma_1\left(\int_{p\eta}^{\nu} dll^{D-2-2\nu}N(l)\right)^2 - \Gamma_2\right] + 2\Gamma_1 N(p\eta)(p\eta)^{D-1-2\nu}\int_{\nu}^{p\eta}\frac{du}{u}[N(u)+N(P_0)]\int_{p\eta}^{\nu} dll^{D-2-2\nu}N(l). \tag{62}$$

Unlike the case of principal series [67], this equation has no clear kinetic or particle interpretation. This is because, in this case, the modes do not oscillate at future infinity. Hence, we cannot neglect the time dependence of N in comparison with that of the modes: here $N(p\eta)$ is not a slow function. However, solving the equation for the given initial conditions provides the resummation of the leading corrections from all loops (for the considered initial conditions).

In the limit $p\eta \ll \nu$ let us consider

$$N(p\eta) = C(p\eta)^{\alpha} \tag{63}$$

where C is a constant of integration which depends on the initial conditions. Since $p\eta \ll P_0 \sim \nu$ for $\alpha > 0$ we have $N(p\eta) \ll N(P_0)$, and it can be easily seen that (63) cannot solve (60). But when $\alpha < 0$ then $N(p\eta) \gg N(P_0)$. Hence, substituting into (60) and neglecting $N(P_0)$ in comparison with N, we obtain

$$\alpha \approx -\Gamma_1 C^2\left[\frac{\nu^{(D-1-2\nu+\alpha)} - (p\eta)^{(D-1-2\nu+\alpha)}}{D-1-2\nu+\alpha}\right]^2 + \Gamma_2. \tag{64}$$

If $D-1-2\nu+\alpha > 0$ and $p\eta \ll \nu$, on the rhs of this equation we can neglect the $p\eta$ dependence, and α is a constant, as it should be. This solution is valid only for

$$\frac{C\nu^{(D-1-2\nu+\alpha)}}{D-1-2\nu+\alpha} > \sqrt{\frac{\Gamma_2}{\Gamma_1}} \tag{65}$$

(because otherwise $\alpha > 0$).

For the solution under consideration we have that $D^K(p|\eta_1,\eta_2) \approx \frac{A^2C^2}{p^{D-1}}(p\eta)^{D-1-2\nu+\alpha}$. The Keldysh propagator blows up only if $D-1-2\nu+\alpha < 0$, but this cannot happen for the solution in question. Thus, all solutions of the type under consideration describe smooth behavior of the correlation functions, even if they violate dS isometry. Such solutions are realized by a mild initial perturbation over the BD state. Note that such a solution is very similar to the one obtained in [52–57], if one keeps in the latter only the leading term as $p\eta \to 0$ (and we do keep only the leading contributions in the limit in question). Furthermore, here we have an obvious stationary solution, $\alpha = 0$, $N(p\eta) = N(P_0) = C = \sqrt{\frac{\Gamma_2}{\Gamma_1}}\frac{D-1-2\nu}{\nu^{D-1-2\nu}}$.

It is tempting to compare the solutions (63) and (64) to the one considered in [52,57]. There are certain differences. Namely, our solution is valid for generic dS-violating initial conditions. As a result, the parameter α in (63) depends on the parameter C, which defines the initial value of $N(p\eta)$. The situation should reduce to the one considered in [52,57], when one takes the BD state exactly. In terms of (63) this corresponds to $N(\nu) = 1$. In this case, as follows from (64), $\alpha \sim \frac{\lambda^2}{m^4}$, if $m^2 \ll 1$ (this is the approximation in which one can compare the two results under discussion). This answer coincides parametrically with the one found in [52,57]. The coefficients, however, are different. The reason for this is due to the difference between the sort of approximations made and the sort of leading diagrams that are resummed in [52,57] and in our paper.

However, apart from the stable solutions, Eq. (60) also has a singular (exploding) one. Consider indeed

$$N(p\eta) = \frac{C}{(p\eta - p\eta_*)^{\alpha}}, \tag{66}$$

where C, $0 < \eta_* < \eta$ and $\alpha > 0$ are some real constants, which may depend on the initial conditions. We assume that such a behavior of $N(p\eta)$ is valid in the limit when η is very close to η_*. After the substitution of this solution into (60) and neglecting the suppressed terms, we obtain the following relation between the constants C, η_* and α:

$$\frac{1}{(p\eta - p\eta_*)^{\alpha}} \approx \frac{\Gamma_1 C^2 (p\eta_*)^{D-1-2\nu}}{(\alpha-1)(p\eta - p\eta_*)^{3(\alpha-1)}}. \tag{67}$$

This equation establishes $\alpha = 3/2$ and a relation between C and η_*. In this case the Keldysh propagator blows up at a finite proper time.[7] Then, the expectation value of the stress-energy tensor also blows up (which would appear on the rhs of the Einstein equations due to the quantum fluctuations). This means that the backreaction is not negligible. One possibility is that the cosmological constant is secularly screened because the expectation value of the stress-energy tensor under discussion does not respect the dS isometry. This is the subject of a separate study. Here we do not consider the backreaction issue.

[7]It is probably worth stressing at this point that here we are talking about superhorizon modes; hence, even for very short periods of time, the intuition gained from the flat space-time physics is not applicable here.

IX. CONCLUSIONS

Equation (19) shows that there are secularly growing corrections to the Keldysh propagator starting from two-loop order—i.e. from the sunset diagrams. As explained in Sec. IV A this equation is valid for $\frac{\sqrt{5}}{6}(D-1) < m < \frac{D-1}{2}$. Moreover, as explained in Sec. IV B, for $m < \frac{\sqrt{3}}{4}(D-1)$ there are large IR contributions to the vertices. The physical origin of these complications is explained in Sec. V.

Since the quantum corrections to the propagator become of the same order as tree-level contributions, when $|\lambda^2 \log(p\eta)| \sim 1$ they have to be resummed. We perform a self-consistent resummation, by resumming only the leading corrections in $\lambda^2 \log(p\eta)$ and dropping all the subleading ones—those which are suppressed by higher powers of λ or do not contain $\log(p\eta)$, as $p\eta \to 0$.

The resummation amounts to solving the relevant Dyson-Schwinger equation. For the Keldysh propagator we make the ansatz (58) with unknown $N(p\eta)$, and we take tree-level (perhaps UV renormalized) expressions for the retarded propagators, the advanced propagators and the vertices because they do not receive large IR contributions at leading order for the mass range under consideration.

Equation (60) follows from the Dyson-Schwinger equation. Its solution allows us to perform the resummation of leading secular IR corrections from all loops. Note that $N(p\eta)$ is a quantity attributed to the comoving volume; its physical meaning is, however, less clear than in the principal series case. However, we can solve (60) and find the behavior of the Keldysh propagator at future infinity.

The solutions (63) and (64) describe the smooth behavior of the Keldysh propagator and correspond to a mass renormalization. This situation is very similar to the one encountered in [52–57].

However, the self-consistent resummation (60) produces a nonlinear equation. As a result, it also has the exploding solution (66). Which one of the solutions is realized depends on the initial conditions $N(P_0)$. The blow up happens at finite proper time and cannot be washed away by the EPP expansion because $N(p\eta)$ is attributed to the comoving volume.

In conclusion, in dS space the backreaction on quantum effects can also be strong for massive fields (see also [67]).

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