Axion production from primordial magnetic fields

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Production of axionlike particles (ALPs) by primordial magnetic fields may have significant impacts on cosmology. We discuss the production of ALPs in the presence of the primordial magnetic fields. We find a region of the ALP mass and photon coupling which realizes the observed properties of the dark matter with appropriate initial conditions for the magnetic fields. This region may be interesting in light of recent indications for the 3.5 keV lines from galaxy clusters. For a small axion mass, a region of previously allowed parameter spaces is excluded by overproduction of ALPs as a hot/warm dark matter component. Since the abundance of ALPs strongly depends on the initial conditions of primordial magnetic fields, our results provide implications for scenarios of magnetogenesis.

I. INTRODUCTION

Physics of the dark matter (DM) beyond the standard paradigm of the weakly interacting massive particle (WIMP) has recently enhanced its presence. Despite intensive searches, any indication of the existence of the WIMP has not been found so far. Moreover, in light of the naturalness of the electroweak symmetry breaking, null results of the LHC experiments may suggest nonstandard signatures of supersymmetry (SUSY) such as R-parity violation [1–3] or stealth SUSY [4–7] where the lightest supersymmetric particle cannot be the DM. If this is the case, another DM candidate is required.

Axionlike particles (ALPs) with very weak interactions and a tiny mass are promising alternatives to the WIMP idea (for reviews, see [8,9]). They may appear as pseudo-Nambu-Goldstone bosons (PNGBs) of some spontaneously broken global symmetries or by-products of string theory compactifications. Various production mechanisms of ALPs have been studied so far. Thermal production of ALPs is likely to give a large free-streaming length and prevent structure formation [9,10]. On the other hand, nonthermal production via the misalignment mechanism in the early Universe, as discussed in [11–13] for QCD axions, [14–17] for string axions and [18] for more general setups, can give rise to the observed cold dark matter (CDM). However, one possible problem in this production mechanism is a tight constraint from DM isocurvature perturbations (see, e.g., Ref. [19]). This constraint gives an upper bound on the inflationary scale and excludes high-scale inflation models which can be tested in near-future observations.[1]

Recently, Fermi has observed a deficit of secondary GeV gamma rays from TeV blazars [25–32]. This observation can be explained by intergalactic magnetic fields (IGMFs) that broaden the secondary cascade photons, with a characteristic field strength $B_0 \gtrsim 10^{-19}$ G at Mpc scales (smaller scale

IGMFs need a stronger strength to explain the deficit). Such IGMFs, if any, may have a primordial origin [primordial magnetic fields (PMFs)].[2] That is, we could imagine a scenario where there exist strong magnetic fields in the very early Universe. A natural question is then what the existence of the PMFs implies for the ALP DM paradigm.

In this article, we consider the production of ALPs via photon-axion conversion in the presence of PMFs. Photon-axion conversion is a process that has been well studied theoretically [41,42] and discussed in different contexts [43–46]. We show that a sufficiently large number of ALPs could be produced in the early Universe, with a relatively long free-streaming length, via this conversion process. We find a viable region of the ALP mass and photon coupling which predicts the appropriate properties for the DM such as its abundance and free-streaming length with suitable initial conditions for the PMFs. Moreover, for a small axion mass, a region of the previously allowed parameter space can be excluded by the upper limit on the hot/warm component of the DM if the strength of PMFs is relatively large but consistent with the present constraints on IGMFs.

II. EVOLUTION OF MAGNETIC FIELDS

Before discussing the ALP production from PMFs, we first summarize the setup and assumptions on the cosmological evolution of PMFs.[3] We here consider the case

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[1]For possible solutions to this problem, see, e.g., Refs. [20–24].

[2]There are several proposals of magnetogenesis in the early Universe such as inflationary magnetogenesis [33–35] (see also [36,37]) and magnetogenesis from a strong first order phase transition [38–40].

[3]Magnetic fields are generated in the early Universe as hypermagnetic fields and turn into (electro)magnetic fields at the electroweak phase transition/crossover. Assuming that the transition proceeds smoothly without a substantial change of the field strength, we do not distinguish the hyper gauge field from the electromagnetic field throughout the paper.

where nonhelical[4] PMFs are produced in the radiation dominated era at a temperature T_i (or a time $t = t_i$) with a causal process such as a strong first order phase transition in a hidden sector. The evolution of PMFs is described by magnetohydrodynamics (MHD) equations and is hard to evaluate in principle. However, it has been found that PMFs evolve according to a scaling law [50,51] until recombination and after that evolve adiabatically. Here we assume that PMFs have a spectrum (blue at large scales) described by the characteristic field strength B_p at the peak scale λ_B which is identified as the correlation length and that they enter the scaling regime quickly after their generation. In the absence of late time entropy production, the strength of PMFs at a conformal time τ before recombination can be roughly written in terms of that of the present IGMFs,[5]

$$B_p(\tau) = \left(\frac{a(\tau)}{a_0}\right)^{-2} \left(\frac{\tau}{\tau_{\rm rec}}\right)^{-n_B} B_0, \qquad (1)$$

where $a(\tau)$ and a_0 are the scale factor at τ and today, respectively, and $\tau_{\rm rec}$ is the conformal time at recombination. B_0 is the characteristic strength of the present IGMFs. In terms of the initial magnetic field strength $|\boldsymbol{B}| = B_i$, it is also written as

$$B_p(T) = B_i(T/T_i)^{2+n_B} \qquad (2)$$

in the radiation dominated era.

The exponent n_B in the scaling relations is subject to a controversy and differs by MHD simulations and analytical estimations. For example, in the direct cascade process, the exponent of the scaling law is obtained analytically [50–52] and also numerically [50] as

$$\text{(i)} \quad n_B = n/(2+n), \qquad (3)$$

where n is determined by the spectrum index of magnetic fields and fluid velocity fields.[6] On the other hand, it has been recently claimed that an "inverse transfer" process would occur [53,54] and the exponent of the scaling relation is identified as

$$\text{(ii)} \quad n_B = 1/2. \qquad (4)$$

To be fair, we have both possibilities for the evolution of nonhelical magnetic fields in mind. Note that the resultant ALP abundance strongly depends on the strength of magnetic fields at their generation but is not very sensitive to the exponents of the scaling laws of the evolution, as we will see.

In the above discussion, we have implicitly assumed that the correlation length is comparable to the largest processed eddy scale, $\lambda_B \sim v_A t$, with v_A being the Alfvén velocity that depends on the magnetic field strength. This feature is observed in the MHD simulation [50,55]. Thus the comoving correlation length is fixed at recombination, which gives the linear relation between the present strength and correlation length of the IGMFs as $\lambda_0 \simeq 1\ \mathrm{pc} \times (B_0/10^{-14}\ \mathrm{G})$ [50].[7] In other words, if the MFs are causally generated in the early Universe, the correct property of the IGMFs is given on the line $\lambda_0 \simeq 1\ \mathrm{pc} \times (B_0/10^{-14}\ \mathrm{G})$ in the B_0-λ_0 plane. The latest analysis of the TeV blazars by Fermi [32] gives a constraint to explain the deficit of the GeV cascade photons. Since the $\lambda_0 \simeq 1\ \mathrm{pc} \times (B_0/10^{-14}\ \mathrm{G})$ line is in the "ruled-out" region of the B_0-λ_0 plane in Ref. [32] for $B_0 \lesssim 10^{-16}\ \mathrm{G}$ and $\lambda_0 \lesssim 10^{-2}\ \mathrm{pc}$, the constraint reads $B_0 \gtrsim 10^{-16}\ \mathrm{G}$ and $\lambda_0 \gtrsim 10^{-2}\ \mathrm{pc}$ if the IGMFs are generated in the early Universe by a causal mechanism so that they satisfy the linear relation between B_0 and λ_0. On the other hand, the upper bound on the IGMF strength at Mpc scales is given by the cosmic microwave background (CMB) as $B_0 < 10^{-9}\ \mathrm{G}$ [56].

Since the energy density of PMFs decreases faster than that of radiation, the ratio between the energy density of PMFs $\rho_B = B_p^2/2$ and that of radiation is larger for higher temperatures. This, in turn, gives an upper bound on the initial strength of PMFs by requiring $\rho_B(t_i) < \rho_\gamma(t_i) \sim T_i^4$, depending on the scaling laws of the magnetic field evolution. Since we here do not specify the magnetogenesis mechanism, this energy consideration uniquely gives the upper bound on the initial field strength. Figure 1 shows constraints on the strength of magnetic fields B_i at the initial temperature T_i for each case of the exponents of the scaling laws. Note that the initial strength of PMFs can be expressed in terms of the present IGMF strength and the initial temperature through Eq. (1). We can see that upper bounds on the initial strength of PMFs and temperature that can explain the deficit of the GeV cascade photons from blazars are given as $B_i^{1/2} < 10^4(10^{10})\ \mathrm{GeV}$ and $T_i < 10^4(10^{10})\ \mathrm{GeV}$ for case (i) with $n = 5$ [52,57] [for case (ii)]. Hereafter we use these values as references.

III. AXION PRODUCTION

We here discuss how ALPs are produced by PMFs. ALPs are PNGBs of some global symmetries and couple to

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[4]In the maximally helical case, we would suffer from baryon overproduction [47–49]. For simplicity, we also do not consider the partially helical case.

[5]To be precise, the scaling relation with a constant n_B applies only to the radiation dominated era with turbulent plasma and does not hold in all of cosmic history due to the neutrino or photon streaming effect or matter domination. However, it turns out that the relation between the IGMFs and PMFs can be roughly evaluated as if the scaling relation holds until recombination. See the discussion in Refs. [50–52].

[6]$n \geq 3$ is required from the causality [51,52].

[7]Although one must be careful for the treatment of Alfvén velocity at recombination that is in the matter dominated era, it turns out that the evaluation with the "radiation domination approximation" gives roughly correct relations [50–52].
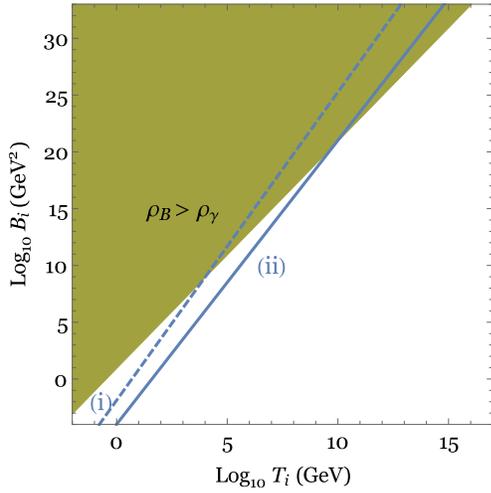
FIG. 1. Constraints on the initial strength of PMFs B_i and temperature T_i. The (green) shaded region is excluded by $\rho_B(t_i) > \rho_\gamma(t_i)$. The (blue) dashed and solid lines represent the initial conditions that predict $B_0 = 10^{-16}$ G for case (i) with $n = 5$ [52,57] and case (ii) respectively. In the regions below the lines, the deficit of the GeV cascade photons from TeV blazars cannot be explained by the PMFs. In that region, the PMFs are free from CMB constraints. Note that the analytic investigation discussed in the text as well as the MHD simulation suggests that the present IGMF strength and correlation length must satisfy the linear relation, $\lambda_0 \simeq 1$ pc $\times (B_0/10^{-14}$ G) [50]. By using this relation, we reinterpret the condition in Ref. [32] in which the IGMFs can explain the blazar observation as $B_0 \gtrsim 10^{-16}$ G and $\lambda_0 \gtrsim 10^{-2}$ pc.

matter and gauge bosons only derivatively. The coupling of an ALP ϕ to the electromagnetic field is given by

$$\mathcal{L}_\phi \supset -\frac{1}{4}g\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = g\phi \boldsymbol{E}\cdot\boldsymbol{B}, \qquad g \simeq \frac{\alpha}{2\pi}\frac{1}{f_\phi}, \quad (5)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual, and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, respectively. $\alpha \equiv e^2/4\pi$ is the fine structure constant and f_ϕ is the decay constant. We can see that in the presence of a background magnetic field this coupling induces a mixing between an ALP and the electric field whose polarization is parallel to the magnetic field.

The probability of photon-axion conversion, $\gamma \to \phi$, in the presence of plasma has been studied in Ref. [45] where the linearized field equations of the photon-axion system with an energy ω are given by

$$(\omega - i\partial_y + \mathcal{M})\begin{pmatrix}|\gamma\rangle \\ |\phi\rangle\end{pmatrix} = 0, \quad \mathcal{M} = \begin{pmatrix}\Delta_\gamma & \Delta_M \\ \Delta_M & \Delta_\phi\end{pmatrix}. \quad (6)$$

Here, the coordinate y is along the direction of propagation and we denote the state vectors of the photon and the ALP as $|\gamma\rangle$, $|\phi\rangle$. For the matrix $\mathcal{M}$, $\Delta_\phi \equiv -m_\phi^2/2\omega$ comes from

the ALP mass m_ϕ and $\Delta_M \equiv g\langle \boldsymbol{e}\cdot\boldsymbol{B}\rangle/2 \simeq gB_p/2$ originates from the photon-axion mixing in the presence of the magnetic field. Here, $\boldsymbol{e}$ denotes the photon polarization vector and the bracket represents the ensemble average. The upper left component comes from the nonzero effective photon mass in magnetized plasma. In the present case, the effective mass is dominated by the Debye mass, $m_D^2 \sim e^2T^2$, and we obtain $\Delta_\gamma \simeq -m_D^2/2\omega$. From the field equations (6), the photon-axion conversion probability after a distance of propagation $y = L$ is given by [45]

$$P(\gamma \to \phi) = (\Delta_M L)^2 \frac{\sin^2(\Delta_{\mathrm{osc}}L/2)}{(\Delta_{\mathrm{osc}}L/2)^2},$$

$$\Delta_{\mathrm{osc}}^2 \equiv (\Delta_\gamma - \Delta_\phi)^2 + 4\Delta_M^2. \quad (7)$$

For the parameter range of interest, $T \gtrsim B \gg m_\phi$, we approximately find $\Delta_{\mathrm{osc}} \simeq \Delta_\gamma$.

We now derive the kinetic equation for ALPs from the photon-axion conversion probability obtained above. Here we can use the same procedure in the case of production of sterile neutrinos via oscillations [58–66]. Following the discussion of [65,66] (see also Refs. [58–63]), the photon-axion conversion rate in a unit time is evaluated in terms of the probability averaged over photons in the ensemble,

$$\Gamma(\gamma \to \phi) = \frac{\Gamma_\gamma}{2}\langle P(\gamma \to \phi)\rangle, \quad (8)$$

where $\Gamma_\gamma \sim \alpha^2 T$ is the thermally averaged collision rate of photons. This expression of the conversion rate can be understood as follows. The collision of a photon leads to collapse of the photon wave function into either a pure photon eigenstate or a pure ALP eigenstate. Then, the collision is a measurement. The rate of the measurements is given by Γ_γ and the origin of the factor $1/2$ has been discussed in Ref. [64]. In the ensemble average of the probability, we take $\langle \omega \rangle \sim T$ and $\langle \sin^2(\Delta_{\mathrm{osc}}L/2)\rangle \to 1/2$ since $\Delta_{\mathrm{osc}}L \simeq \Delta_{\mathrm{osc}}\Gamma_\gamma^{-1} \gg 1$.

In the radiation dominated Universe, the evolution of the number-to-entropy ratio of ALPs, $\eta_\phi \equiv n_\phi/s$ (n_ϕ is the number density of ALPs and $s = 2\pi^2 g_{*s}T^3/45$ is the entropy density), obeys the following kinetic equation:

$$\frac{d\eta_\phi}{dt} = \Gamma(\gamma \to \phi)\frac{n_\gamma - n_\phi}{s} = cg^2\frac{B_p^2}{T}\left(1 - \frac{n_\phi}{n_\gamma}\right), \quad (9)$$

where n_γ is the number density of photons and we have introduced a numerical factor $c = \mathcal{O}(0.1)$.

If $H \gg \Gamma(\gamma \to \phi)$ is satisfied throughout the cosmic history, we obtain the ALP number-to-entropy ratio by integrating the kinetic equation with neglecting n_ϕ/n_γ as

$$\eta_\phi(t) = \int_{t_i}^{t} dt' cg^2 \frac{B_i^2 (T(t')/T_i)^{2(2+n_B)}}{T(t')} \simeq c'g^2 \frac{B_i^2 M_{\rm pl}}{T_i^3}, \quad (10)$$

where c' is a numerical factor with $\mathcal{O}(0.01)$. Here we have used Eq. (2) and $t = (2H)^{-1} \propto T^{-2}$ in the radiation dominated Universe. Since the photon-axion conversion rate decreases quickly, η_ϕ is fixed just after the magnetic field generation. The present ALP energy-to-entropy ratio is then given by

$$\frac{\rho_{\phi,B}}{s} \simeq 2 \times 10^{-10} \text{ GeV} \times \left(\frac{c'}{0.01}\right)\left(\frac{m_\phi}{10 \text{ keV}}\right)$$
$$\times \left(\frac{B_i}{(10^{11} \text{ GeV})^2}\right)^2 \left(\frac{T_i}{10^{11} \text{ GeV}}\right)^{-3} \left(\frac{g}{10^{-16} \text{ GeV}^{-1}}\right)^2. \quad (11)$$

If once $H \ll \Gamma(\gamma \to \phi)$ is satisfied in the cosmic history, ALPs are thermalized through the photon-axion conversion process and the present ALP energy-to-entropy ratio is evaluated as

$$\frac{\rho_{\phi,B}}{s} = \frac{m_\phi n_\phi}{s} \simeq 2.6 \times 10^{-9} \text{ GeV} \times \left(\frac{m_\phi}{1 \text{ keV}}\right). \quad (12)$$

Note that the thermalized axion number density to entropy ratio is fixed as $n_\phi/s = (\zeta(3)/\pi^2)T^3/((2\pi^2/45)g_{*s}T^3) \simeq 2.6 \times 10^{-3}$, where we take g_{*s} to be the value for all the Standard Model particles, $g_{*s} = 106.75$.

Since the present DM abundance in the Universe is given by $\rho_{\rm DM}/s = \Omega_{\rm DM}\rho_c/s \simeq 4.0 \times 10^{-10}$ GeV [67,68], the fiducial values of parameters in Eq. (11) can explain the abundance of the DM. Note that it is difficult to explain both the ALP DM abundance and the deficit of the GeV cascade photons from TeV blazars simultaneously, see Fig. 1. One possible solution is to consider the case where the PMFs evolve adiabatically first and start the direct cascade or the inverse transfer at a later time similar to the situation discussed in Ref. [47]. For example, consider the case where the magnetic fields are produced at $T = 10^{11}$ GeV with $B_i = 10^{22}$ GeV^2 and evolve adiabatically ($n_B = 0$) for a while. The ALP abundance fixed at the $T = 10^{11}$ GeV and it coincides with the present DM abundance. If the eddy scale of turbulent plasma gets comparable to the coherent length of the magnetic fields at $T \simeq 10^9$ GeV with $B_p \simeq 10^{18}$ GeV^2 and they evolve with the scaling law [(ii); $n_B = 1/2$] after that, the present strength and coherent length of IGMFs are $B_0 \sim 10^{-16}$ G and $\lambda_B \sim 0.01$ pc. Thus, the axion DM abundance and the blazar observation can be explained simultaneously. However, this scenario needs a new parameter (the initial correlation length or the time when the scaling law changes) and hence we have less predictive power. Moreover, unfortunately, we do not know

any concrete magnetogenesis mechanisms that lead to this scenario.

For ALPs produced by primordial magnetic fields to explain the present Universe, we need to know if the produced ALPs are stable and cold enough. ALPs can decay into two photons, $\phi \to \gamma\gamma$, through the photon coupling (5). The lifetime is given by [18]

$$\tau_\phi \simeq 10^{28} \text{ s} \times \left(\frac{g}{10^{-16} \text{ GeV}^{-1}}\right)^{-2} \left(\frac{m_\phi}{1 \text{ keV}}\right)^{-3}. \quad (13)$$

In the parameter range where ALPs produced by photon-axion conversion can explain the observed DM abundance, the ALP lifetime is easily much longer than the age of the Universe, t $\sim 10^{17}$ s.

The "temperature" of ALPs produced by this mechanism, T_ϕ, is the same to the photon at production and is just redshifted afterwards. The comoving free-streaming horizon at matter radiation equality is estimated as [69]

$$\lambda_{\rm FS} \simeq 1 \text{ Mpc} \times \left(\frac{m_\phi}{1 \text{ keV}}\right)^{-1} \left(\frac{T_\phi/T}{0.33}\right)$$
$$\times \left(1 + 0.03 \log\left[\left(\frac{m_\phi}{1 \text{ keV}}\right)\left(\frac{0.1}{\Omega_{\rm DM}h^2}\right)\left(\frac{0.33}{T_\phi/T}\right)\right]\right), \quad (14)$$

where $\Omega_{\rm DM}h^2$ is the present density parameter of the DM.[8] We here take into account the change in the effective numbers of relativistic degrees of freedom of the Standard Model sector, $T_\phi/T = (\frac{3.91}{106.75})^{1/3} \simeq 0.33$. A large free-streaming length prevents structure formation and is constrained by Lyman-α forest observations [69–71]. The upper bound of the length is around 1 Mpc and hence ALPs with $\lambda_{\rm FS} > 1$ Mpc cannot be the main component of the DM. For $m_\phi \lesssim 1$ keV, the constraint on the ratio of the energy density of ALPs in the total DM density $\rho_{\rm DM}$ is given by $\rho_{\rm ALP}/\rho_{\rm DM} < 0.6$ [69]. For $m_\phi \ll 1$ keV, the Planck 2015 temperature and polarization data give a stronger bound, $\rho_{\rm ALP}/\rho_{\rm DM} < 0.3 \times 10^{-2}$, interpreting the constraint of Ref. [72] in terms of the ALP energy density. We will see that these constraints exclude a region of the parameter space of ALPs.

Here we comment on other ALP production mechanisms. If the ALP ϕ stays at a different field point from the potential minimum during inflation, it begins to oscillate around the minimum of the potential when the Hubble parameter becomes comparable to the ALP mass. The oscillation behaves as a matter which survives until today, which is called the misalignment mechanism. The present energy-to-entropy ratio of ALPs produced by this mechanism is given by [18]

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[8]The estimate is the same as that in the case where ALPs are once thermalized.

$$\frac{\rho_{\phi,\phi_1}}{s} \sim 10 \text{ GeV} \times \left(\frac{m_\phi}{1 \text{ keV}}\right)^{1/2} \left(\frac{\phi_1}{10^{16} \text{ GeV}}\right)^2, \quad (15)$$

where ϕ_1 is the initial value of the ALP field and we have assumed that the ALPs start oscillation in the radiation dominated era and the ALP mass at the time when the oscillation starts is the same as the present mass m_ϕ. Natural values of ϕ_1 are around f_ϕ and, for a large decay constant, we often suffer from the ALP overproduction problem. The correct DM abundance (or much less ALP abundance) can be also obtained, however, by tuning the initial condition. The degree of tuning is estimated by $\Delta_\phi \equiv \phi_1/f_\phi$. The produced ALPs can give the CDM with a small free-streaming length. Note that they often generate too large DM isocurvature perturbation in the case of high-scale inflation.

ALPs are also produced via scattering of quarks and gluons in thermal equilibrium such as $gg \to g\phi$. If the temperature of the Universe is higher than the decoupling temperature of the scattering,

$$T_D \sim 10^6 \text{ GeV} \times \left(\frac{g}{10^{-10} \text{ GeV}^{-1}}\right)^{-2}, \quad (16)$$

ALPs are thermalized, which may cause an ALP overproduction problem. Even if the temperature is below the decoupling temperature, ALPs are still produced like gravitinos or freeze-in DM scenarios. The abundance of relic ALPs is evaluated as [8,73,74]

$$\frac{\rho_{\phi,th}}{s} \sim 10^{-16} \text{ GeV} \times \left(\frac{m_\phi}{1 \text{ keV}}\right)$$
$$\times \left(\frac{g}{10^{-16} \text{ GeV}}\right)^2 \left(\frac{T_R}{10^{11} \text{ GeV}}\right), \quad (17)$$

where T_R is the reheating temperature. We find that the number of thermally produced ALPs is smaller than that of ALPs produced from photon-axion conversion in the parameter range of interest.

IV. ALP DARK MATTER

We now investigate the parameter space of the ALP mass and photon-ALP coupling and identify a region where ALPs produced by photon-axion conversion can be responsible for the present DM. We also give constraints from overproduction of the ALP hot/warm DM.

Let us first summarize the known constraints on the ALP parameters. The first constraint on the ALP dark matter comes from its stability. As we have mentioned, the lifetime of ALPs τ_ϕ [Eq. (13)] must be larger than the age of the Universe, $t \sim 10^{17}$ s. Even if this is satisfied, partial decays of ALPs might cause phenomena inconsistent with astrophysical observations, such as the extragalactic background light (EBL) and extragalactic x-rays. These observations
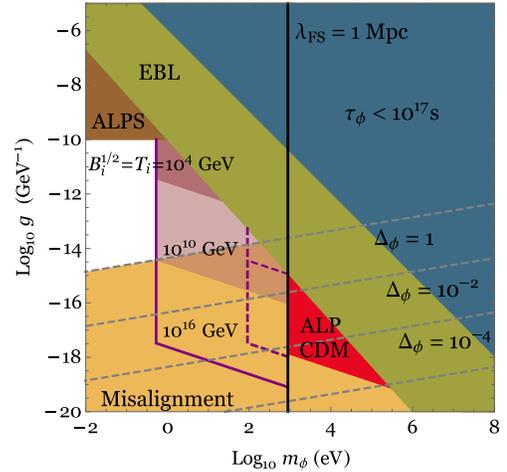

FIG. 2. The constraints on the parameter space of the ALP mass m_ϕ and the photon coupling g are shown. The blue region ($\tau_\phi < 10^{17}$ s), green region (EBL) and brown region (ALPS) are constrained by the ALP stability, astrophysical observations such as the EBL, and direct experimental and observational bounds on the photon coupling, respectively. The yellow region (Misalignment) can realize the correct abundance of the ALP DM produced by the misalignment mechanism with an appropriate tuning of the initial amplitude ϕ_1. The degree of tuning is shown by the gray dashed lines. The black solid line gives $\lambda_{FS} = 1$ Mpc. For $m_\phi \gtrsim 1$ keV, ALPs can be the CDM. The red region (ALP CDM) gives the correct abundance of the ALP DM produced by primordial magnetic fields for $T_i = B_i^{1/2} < 10^{16}$ GeV. For $m_\phi \lesssim 1$ keV, the parameter space can be constrained by the hot/warm DM, $\rho_{ALP}/\rho_{DM} < 0.3 \times 10^{-2}$. The light (thick) purple region is ruled out for $T_i = B_i^{1/2} = 10^{10}(10^4)$ GeV. The region above the solid purple line is ruled out for $T_i = B_i^{1/2} = 10^{16}$ GeV. The regions surrounded by the dashed purple lines are ruled out by a conservative constraint $\rho_{ALP}/\rho_{DM} < 0.6$.

lead to a constraint on the ALP mass and the photon coupling, which is roughly given as [18]

$$g < 10^{-10} \text{ GeV}^{-1} \times \left(\frac{m_\phi}{1 \text{ eV}}\right)^{-5/3}. \quad (18)$$

Furthermore, there are direct experimental and observational bounds on the photon coupling. The constraints come from the light-shining-through-walls experiment ALPS and the helioscopes CAST and SUMICO. Combining this with the constraint from the short lifetime of ALPs, it is required to satisfy $g < 10^{-10} \text{ GeV}^{-1}$ [18].

Now we explore the possibility of the ALP dark matter. From Eq. (11), we can see if ALPs produced by photon-axion conversion through the PMFs can be the CDM for $m_\phi \gtrsim 1$ keV. Since the ALP abundance is proportional to B_i^2 and T_i^{-3} and energy condition gives a constraint $B_i^{1/2} \lesssim T_i$, the large ALP abundance is obtained when $B_i^{1/2} \simeq T_i$ with large temperature at production. The red region in Fig. 2

represents the parameter space of the ALP mass m_ϕ and the photon coupling g where ALPs can be the CDM. Here we take the highest value of the initial temperature and the square root of the magnetic field strength as the possible highest temperature of the Universe, $T_i = B_i^{1/2} < 10^{16}$ GeV. The lowest initial temperature and the square root of the initial magnetic field strength that can explain the present DM are $T_i = B_i^{1/2} \simeq 10^{10-11}$ GeV. Therefore, unfortunately, it is difficult to explain the CDM and the deficit of the GeV cascade photons from blazars simultaneously (See Fig. 1).[9] However, it should be emphasized that IGMFs can be explained by other mechanisms that occur at some later time and hence weaker PMFs are not worrisome. Note that, in the region where ALPs produced by PMFs can explain the CDM, those produced by the misalignment mechanism must be suppressed somehow strangely. But it has a benefit that we do not suffer from too large DM isocurvature perturbation. Interestingly, it has been known that the collisionless CDM addresses some disagreements between the observations and numerical simulations of the galactic halos such as the "core-cusp" problem or "too big to fail" problem. The ALP DM produced by photon-axion conversion via PMFs with free-streaming length of $\mathcal{O}(0.1\text{--}1)$ Mpc might be the candidate to resolve the issues.

For $m_\phi \lesssim 1$ keV, we can constrain the ALP parameter space from the hot/warm DM argument, $\rho_{ALP}/\rho_{DM} < 0.3 \times 10^{-2}$. The light (thick) purple regions in Fig. 2 are excluded due to the overproduction of hot/warm ALPs for $T_i = B_i^{1/2} \simeq 10^{10}(10^4)$ GeV. These choices of the parameters $T_i = 10^4$ GeV and 10^{10} GeV predict IGMFs with $B_0 = 10^{-16}$ G today that can explain the deficit of the GeV cascade photons from blazars for the scaling laws in case (i) with $n = 5$ and case (ii), respectively (see Fig. 1). The region above the thick purple line is ruled out for $T_i = B_i^{1/2} \simeq 10^{16}$ GeV, the possible highest temperature of the Universe. We also show the parameter space excluded by the constraint $\rho_{ALP}/\rho_{DM} < 0.6$, a more conservative one, with the dashed purple lines. Note that for $g > 10^{-17}, 10^{-14}, 10^{-11}$ GeV^{-1} ALPs are thermalized for $T_i = 10^{16}, 10^{10}, 10^4$ GeV respectively and the region $m_\phi < 0.5(10^2)$ eV is not constrained by $\rho_{ALP}/\rho_{DM} < 0.3 \times 10^{-2}(0.6)$. Magnetogenesis models can be also constrained if future experiments identify ALPs within that parameter region.

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[9]As we have mentioned in Sec. III, if we consider the case where the PMFs evolve adiabatically first and start the direct cascade or the inverse transfer at a later time, one can construct a scenario where the blazar observation and the DM abundance are explained simultaneously. However, this scenario needs another phenomenological parameter, which loses the one-to-one correspondence between the DM abundance and the present IGMF strength and hence weakens the predictive power.

V. DISCUSSIONS

We have discussed ALP production in the presence of PMFs without specifying their origin and explored several features of the produced ALP. We have found a region of the ALP mass and photon coupling where ALPs produced via photon-axion conversion provide the correct abundance of the DM. In particular, the mass region for the ALP CDM via PMFs covers the point in Ref. [75] that may explain the recently indicated emission lines at 3.5 keV from galaxy clusters and the Andromeda galaxy [76,77]. These ALPs are different from those produced via the misalignment mechanism in that the free-streaming length is relatively long. If ALPs are detected in future observations, it might be possible to identify the dominant production mechanism by this feature.

It should be emphasized that the origin and the evolution of PMFs are still under discussion. A strong first order phase transition [38–40] can be a possible origin of PMFs, but at present we do not have a candidate for such phase transition. Inflationary magnetogenesis [33–35] is another option, but there are not satisfactory models (except for Ref. [78]). Although we have taken simple scaling laws of the evolution of PMFs for simplicity, their nature is not fully understood yet [50–54]. Note that we did not consider the possibility of backreaction to PMFs from photon-axion conversion. This might also change the evolution of PMFs and make it possible for the PMFs to be the origin of the IGMFs responsible for the deficit of GeV cascade photons from blazars and at the same time provide the ALP DM. The thorough treatment of this effect is left for a future investigation.

Our results give new implications for the nature of PMFs as well as ALPs. As a future investigation, it is worth exploring to construct a magnetogenesis mechanism with $T_i \simeq B_i^{1/2} > 10^{11}$ GeV for the consistent "ALP DM via PMF" scenario. Once we determine the strength and correlation length of the IGMFs by future gamma-ray observations, it will be interesting to construct a magneto-genesis model, to perform careful studies on evolution of PMFs and to find if the ALP DM and the IGMFs can be explained simultaneously. This will also identify the ALP mass and photon-axion coupling strength. For a small ALP mass, we have also pointed out the possible problem that hot/warm relic ALPs can constrain the properties of both PMFs and ALPs. If small mass ALPs will be detected, the strength of the PMFs is constrained.

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