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Study of the DKK and $DK\bar{K}$ systems

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Using the fixed center approximation to Faddeev equations, we investigate the DKK and $DK\bar{K}$ three-body systems, considering that the DK dynamically generates, through its $I=0$ component, the $D_{s0}^*(2317)$ molecule. According to our findings, for the $DK\bar{K}$ interaction we find evidence of a state $I(J^P)=1/2(0^-)$ just above the $D_{s0}^*(2317)\bar{K}$ threshold and around the $Df_0(980)$ threshold, with mass of about 2833–2858 MeV, made mostly of $Df_0(980)$. On the other hand, no evidence related to a state from the DKK interaction is found. The state found could be seen in the $\pi\pi D$ invariant mass.

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I. INTRODUCTION

The study of three-body systems is one of the starting points in the study of nuclei and nuclear dynamics. The traditional quantum mechanical approach to this problem is based on the Faddeev equations [1], and the main application was done for three-nucleon systems. The simplicity of the Faddeev equations is deceiving since, in practice, its evaluation is very involved, and one approximation or another is done to solve them. One popular choice is the use of separable potentials to construct the two-body scattering amplitudes via the Alt-Grassberger-Sandhas (AGS) form of the Faddeev equations [2]. Incorporation of chiral symmetry into the scheme has led to interesting developments [3]. Another way to tackle these three-body systems is by using a variational method [4–6]. Gradually, other systems involving not only nucleons or hyperons but also mesons were tackled. The interaction of K^-d at threshold was thoroughly investigated using Faddeev equations [7,8] or approximations to them, basically, the fixed center approximation (FCA) [9]. The investigation of a possible state of K^-pp nature has also received much attention [10–16] and, according to the calculations done in Ref. [17], the recent J-PARC experiment [18] has found support for this state.

Another step in this direction was the investigation of systems with two mesons and one baryon. Surprisingly, it was found in Refs. [19–21] that with such systems, one could obtain the low energy baryon states of $J^P=1/2^+$. Work in this direction with different methods was also done in Ref. [6] for the $\bar{K}\bar{K}N$ system and in Ref. [22] for the $K\bar{K}N$ system. In this latter case a bound system developed, giving rise to an N^* state around 1920 MeV, mostly made of an $Na_0(980)$, which was also predicted in Ref. [21].

Systems of three mesons also followed, and in Ref. [23] the $\phi K\bar{K}$ system was studied and shown to reproduce the properties of the $\phi(2170)$. Similarly, in Ref. [24] the $KK\bar{K}$ system is studied, and the bound cluster found is associated with the $K(1460)$. Another similar system, $\pi K\bar{K}$, is studied in Ref. [25], and the state found is associated with the $\pi(1300)$. The $\eta K\bar{K}$ and $\eta' K\bar{K}$ systems are also studied in Refs. [25–27], and they are revised in Ref. [28] with the full Faddeev equations and more solid results. Along the same lines, the $\pi\bar{K}K^*$ system is studied in Ref. [29] and found to generate a state that is identified with the $\pi_1(1600)$.

An important result was found in Refs. [19–21,23]. In the Faddeev equations one uses input from the two-body amplitudes of the different components, and the off-shell part of the amplitudes appears in the calculations. This off-shell part is unphysical, and observables cannot depend on it. The finding in those works was that the use of chiral Lagrangians provides three-body contact terms that cancel the off-shell two-body contributions. In other calculations empirical three-body forces are introduced which might have some genuine part, but an important part of them will serve the purpose of effectively canceling these unphysical off-shell contributions. Rather than introducing these terms empirically and fitting them to some data, the message of those works is that to make predictions it is safer to use as input only on-shell two-body amplitudes, without extra three-body terms, and an example of this is given in Ref. [21].

An extension to the charm sector was also performed. The DNN system, analogous to the $\bar{K}NN$ system, is studied in Ref. [30], and the NDK, $\bar{K}DN$, and $ND\bar{D}$ molecules are studied in Ref. [31]. The $\rho D^*\bar{D}^*$ system is studied in Ref. [32] and the $\rho D\bar{D}$ in Ref. [33].

In the hidden charm sector a resonance is found for the $J/\psi K\bar{K}$ system which is associated with the $Y(4260)$ in Ref. [34]. Closer to our work is the one of Ref. [35], where the $DK\bar{K}$ is studied using QCD sum rules and Faddeev equations and in both methods a state coupling strongly to

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$Df_0(980)$ is found. We study this system with a different method and, in addition, the DKK system.

The former works show a constant feature, which is that systems that add $K\bar{K}$ to another particle generate states in which the $K\bar{K}$ clusters around the $f_0(980)$ or the $a_0(980)$. The DKK system benefits from the DK attraction that forms the $D_{s0}^*(2317)$ according to works using chiral Lagrangians and the unitary approach [36–41]. It is also supported by analysis of lattice QCD data [42,43]. However, the KK interaction is repulsive, and the system might not bind. On the other hand, the $DK\bar{K}$ system has repulsion for $D\bar{K}$ in $I = 1$, and attraction for $I = 0$, and the DK interaction is attractive, as it is also the $K\bar{K}$. Altogether this latter system could have more chances to bind than the DKK system, a detailed calculation is called for to find the answer, and this is the purpose of the present work.

The starting point of our approach is to use the FCA with a preexisting molecule, which is the $D_{s0}^*(2317)$, formed by the DK interaction. In addition, another K (or $\bar{K}$) is introduced which is allowed to undergo multiple scattering with the D and K components of the molecule. The result, as we shall see, is that in the DKK system we do not see the signal of a three-body bound state; however, in the $DK\bar{K}$ system we find a peak that we interpret as the $K\bar{K}$ fusing to produce the $f_0(980)$ which then gets bound to the D meson, and a narrow peak appears at an energy below the $Df_0(980)$ threshold. Such a state could be seen in the $\pi\pi D$ invariant mass.

II. FORMALISM

The FCA to Faddeev equations is useful when a light hadron H_3 interacts with a cluster H composed of two other hadrons H_1 and H_2, $H[H_1H_2]$, which are heavier than the first one, i.e., $M_{(H[H_1H_2])} > M_{H_3}$. This cluster comes from the two-body interaction between the hadrons H_1 and H_2 that can be described using a chiral unitary approach in coupled channels. Hence, the Faddeev equations in this approximation have as an input the two-body t matrices for the different pairs of mesons which form the system, and in this way, the generated bound states and resonances are encoded. In our case, we have $H_1 = D$ and $H_2 = K$, while $H_3 = \bar{K}$ if we consider the $DK\bar{K}$ interaction or $H_3 = K$ for the DKK system. Both three-body interactions involve the $D_{s0}^*(2317)$ and $f_0(980)/a_0(980)$ molecules that, according to Refs. [39,44], are dynamically generated through DK and $K\bar{K}$ interactions, respectively, taking into account their associated coupled channels. Therefore, the following channels contribute to the three-body interaction systems with which we are concerned: (1) $K^-[D^+K^0]$, (2) $K^-[D^0K^+]$, (3) $\bar{K}^0[D^0K^0]$, (4) $[D^+K^0]K^-$, (5) $[D^0K^+]K^-$, and (6) $[D^0K^0]\bar{K}^0$ for the $DK\bar{K}$ interaction, and (1) $K^+[D^+K^0]$, (2) $K^+[D^0K^+]$, (3) $K^0[D^+K^+]$, (4) $[D^+K^0]K^+$, (5) $[D^0K^+]K^+$, and (6) $[D^+K^+]K^0$ for the DKK system. Note that the states (1), (2), and (3) are the same as (4), (5), and (6), respectively. Their distinction signifies that the interaction in the FCA formalism occurs with the particle outside the cluster, which is represented by the brackets $[\ldots]$, and the particle of the cluster next to it. This allows for a compact formulation that describes all the charge exchange steps and distinguishes the interaction with the right or left component of the cluster [17]. These channels will contribute to the $T_{DK\bar{K}}$ and T_{DKK} three-body scattering matrices, and if those interactions generate bound states or resonances, they will manifest as a pole in the solutions of the Faddeev equations. In what follows, we discuss how to construct these three-body scattering matrices and their solutions for both the $DK\bar{K}$ and DKK systems.

A. $DK\bar{K}$ and DKK three-body systems

In order to write the contributions to Faddeev equations of all the channels mentioned previously, we adopt the following procedure to construct the relevant amplitudes: For each channel the anti-kaon (kaon) meson on the left side in (1), (2), and (3) interacts with the hadron on its right side. Similarly, for (4), (5), and (6) the K or $\bar{K}$ to the right interacts with the particle to its left. In doing so, we can distinguish the order of the anti-kaon (kaon) and two other mesons with which the anti-kaon (kaon) interacts first and last. This procedure is similar to that used in Ref. [17] to study the $\bar{K}NN$ interaction. For instance, in the $DK\bar{K}$ system, the channel (1) $K^-[D^+K^0]$ in the initial state means that the K^- interacts with the D^+ meson to its right. The channel (4) $[D^+K^0]K^-$ indicates that the K^- interacts with the K^0 to its left. This procedure allows us to divide the multiple anti-kaon (kaon) scattering process in such a way that the formulation of the multiple scattering becomes easier.

In order to illustrate the structure of the multiple scattering in the fixed center approximation, we define the partition functions T_{ij}^{FCA}, which contain all possible intermediate multiple steps, where the first index refers to the initial $\bar{K}[DK]$, (1), (2), and (3) or $[DK]\bar{K}$ (4), (5), and (6) states and the second index to the final state. If we consider the $K^-[D^+K^0] \to K^-[D^+K^0]$ amplitude denoted by T_{11}^{FCA}, which is diagramatically represented in Fig. 1, we obtain the following expression [17,45]:

$$T_{11}^{\mathrm{FCA}}(s) = t_1 + t_1 G_0 T_{41}^{\mathrm{FCA}} + t_2 G_0 T_{61}^{\mathrm{FCA}}, \qquad (1)$$

which tells us that the transition from the $K^-[D^+K^0]$ to itself is given in terms of a single and double scattering, coupled to the amplitudes T_{ij}^{FCA} related to the other channels. As a result, the three-body problem is given in terms of the T_{ij}^{FCA} partitions, where the i, j indices run from 1 to 6 and stand for the initial and final channels, respectively; as we discuss later, they can be displayed in a matrix form.

In Eq. (1), s is the Mandelstam variable that is equal to the square of the three-body energy system, while t_1 and t_2 are, respectively, the $D^+K^- \to D^+K^-$ and $D^+K^- \to D^0\bar{K}^0$

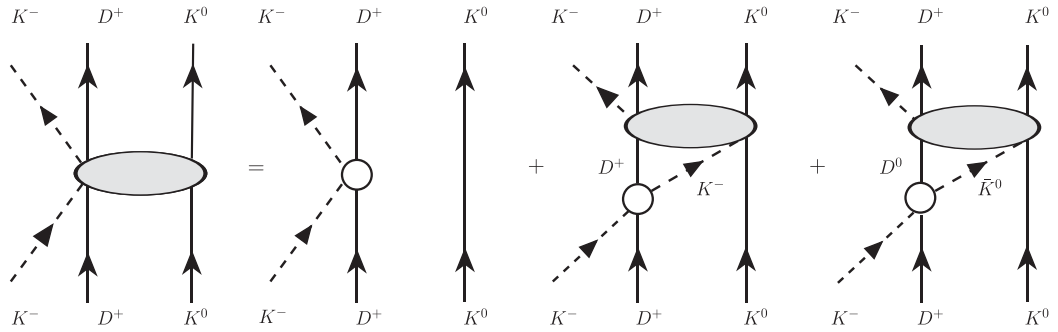


FIG. 1. Feynman diagrams for the K^- multiple scattering of the process $K^-D^+K^0$. The white circle indicates the $D\bar{K} \to D\bar{K}$ scattering amplitude, while the gray bubble is associated with the one for $DK\bar{K}$.

two-body scattering amplitudes studied in Ref. [39], in which the authors have applied the chiral unitary approach in coupled channels to investigate the $D\bar{K}$ and DK two-body interaction. Here, G_0 is the kaon propagator [46] between the particles of the cluster, which is evaluated using the equation below:

$$G_0(s) = \frac{1}{2M_{D_{s0}^*}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{F_R(\mathbf{q})}{(q^0)^2 - \omega_K^2(\mathbf{q}) + i\epsilon}, \quad (2)$$

with $\omega_K^2(\mathbf{q}) = \mathbf{q}^2 + m_K^2$, and q^0 is the energy carried by the kaon meson in the cluster rest frame where $F_R(\mathbf{q})$ is calculated, which corresponds to the following expression:

$$q^0(s) = \frac{s - m_K^2 - M_{D_{s0}^*}^2}{2M_{D_{s0}^*}}. \quad (3)$$

In this work, we use the isospin symmetric masses such that m_D and m_K are the D and K meson average masses, respectively, while $M_{D_{s0}^*}$ is the D_{s0}^* molecule mass. This molecule dynamics does not come into play explicitly in our formalism. The information on the molecule is encoded in the function $F_R(\mathbf{q})$ appearing in Eq. (2), the form factor, which is related to the cluster wave function by a Fourier transform, as discussed in Refs. [45,47]. According to these works, for the form factor to be used consistently, the theory that generates the bound states and resonances (clusters) -the chiral unitary approach, which is developed for scattering amplitudes- has to be extended to wave functions. This was done in those references for s-wave bound states and s-wave resonant states as well as in states with arbitrary angular momentum [48]. In our work we need the form factor expression only for s-wave bound states, which is given by [45]

$$F_R(\mathbf{q}) = \frac{1}{N} \int_{|\mathbf{p}|,|\mathbf{p}-\mathbf{q}|<\Lambda} d^3\mathbf{p} \frac{1}{M_{D_{s0}^*} - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \times \frac{1}{M_{D_{s0}^*} - \omega_D(\mathbf{p}-\mathbf{q}) - \omega_K(\mathbf{p}-\mathbf{q})}, \quad (4)$$

where $\omega_D(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_D^2}$ and the normalization factor N is

$$N = \int_{|\mathbf{p}|<\Lambda} d^3\mathbf{p} \left(\frac{1}{M_{D_{s0}^*} - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \right)^2. \quad (5)$$

The upper integration limit Λ has the same value of the cutoff used to regularize the loop DK, adjusted in order to get the $D_{s0}^*(2317)$ molecule from the DK interaction.

Analogously to T_{11}^{FCA} expressed in Eq. (1), we can calculate all the relevant multiple scattering amplitudes, the partitions T_{ij}^{FCA}, using diagrams similar to the one in Fig. 1. As a result, they can be written as

$$T_{ij}^{\mathrm{FCA}}(s) = V_{ij}^{\mathrm{FCA}}(s) + \sum_{l=1}^{6} \tilde{V}_{il}^{\mathrm{FCA}}(s) G_0(s) T_{lj}^{\mathrm{FCA}}(s), \quad (6)$$

where V_{ij} and $\tilde{V}_{il}$ are the elements of the following matrices:

$$V^{\mathrm{FCA}} = \begin{pmatrix} t_1 & 0 & t_2 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 & 0 & 0 \\ t_2 & 0 & t_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_6 & t_7 \\ 0 & 0 & 0 & 0 & t_7 & t_8 \end{pmatrix},$$

$$\tilde{V}^{\mathrm{FCA}} = \begin{pmatrix} 0 & 0 & 0 & t_1 & 0 & t_2 \\ 0 & 0 & 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_2 & 0 & t_4 \\ t_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & t_6 & t_7 & 0 & 0 & 0 \\ 0 & t_7 & t_8 & 0 & 0 & 0 \end{pmatrix}. \quad (7)$$

Therefore, according to Eq. (6), in our case we can solve the three-body problem in terms of the multiple scattering amplitudes given by partitions T_{ij}^{FCA}, which contain only

the $D\bar{K}$ and $K\bar{K}$ two-body amplitudes. Thus, for the $DK\bar{K}$ system the solution to the scattering equation, Eq. (6), will be

$$T_{ij}^{\mathrm{FCA}}(s) = \sum_{l=1}^{6}[1 - \tilde{V}^{\mathrm{FCA}}(s)G_0(s)]_{il}^{-1}V_{lj}^{\mathrm{FCA}}(s). \quad (8)$$

Analogously, for the DKK system, we have the same solution as in Eq. (8). However, in this case, the $\tilde{V}^{\mathrm{FCA}}$ and V^{FCA} matrices, in terms of the DK and KK two-body amplitudes, are now given by

$$V^{\mathrm{FCA}} = \begin{pmatrix} \bar{t}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{t}_2 & \bar{t}_3 & 0 & 0 & 0 \\ 0 & \bar{t}_3 & \bar{t}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \\ 0 & 0 & 0 & 0 & \bar{t}_6 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \end{pmatrix},$$

$$\tilde{V}^{\mathrm{FCA}} = \begin{pmatrix} 0 & 0 & 0 & \bar{t}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{t}_2 & \bar{t}_3 \\ 0 & 0 & 0 & 0 & \bar{t}_3 & \bar{t}_4 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \\ 0 & \bar{t}_6 & 0 & 0 & 0 & 0 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

The elements of the matrices in Eqs. (7) and (9), i.e., $t_1, t_2, \ldots, t_8$ and $\bar{t}_1, \ldots, \bar{t}_6$, related to the three-body interaction $DK\bar{K}$ and DKK systems are the two-body scattering matrix elements, respectively, given by

$$\begin{aligned} t_1 &= t_{D^+K^-\to D^+K^-}, \quad t_4 = t_{D^0\bar{K}^0\to D^0\bar{K}^0}, \quad t_7 = t_{K^+K^-\to K^0\bar{K}^0}, \\ t_2 &= t_{D^+K^-\to D^0\bar{K}^0}, \quad t_5 = t_{K^0K^-\to K^0K^-}, \quad t_8 = t_{K^0\bar{K}^0\to K^0\bar{K}^0}, \\ t_3 &= t_{D^0K^-\to D^0K^-}, \quad t_6 = t_{K^+K^-\to K^+K^-}, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \bar{t}_1 &= t_{D^+K^+\to D^+K^+}, \quad \bar{t}_4 = t_{D^+K^0\to D^+K^0}, \\ \bar{t}_2 &= t_{D^0K^+\to D^0K^+}, \quad \bar{t}_5 = t_{K^+K^0\to K^+K^0}, \\ \bar{t}_3 &= t_{D^0K^+\to D^+K^0}, \quad \bar{t}_6 = t_{K^+K^+\to K^+K^+}, \end{aligned} \quad (11)$$

which we discuss in the next subsection.

It is important to mention that, in this work, we are using the Mandl and Shaw normalization, which has different weight factors for the particle fields. In order to use these factors in a consistent manner in our problem, we should take into account how they appear in the single-scattering and double-scattering as well as in the full amplitude. The detailed calculation on how to do this can be found in Refs. [45–47]. According to these works, this is done by multiplying the two-body amplitudes by the factor $M_c/M_{1(2)}$, where M_c is the cluster mass while $M_{1(2)}$ is associated with the mass of the hadrons H_1 and H_2. In our case, we have M_c/M_D for the two-body amplitudes related to the $D\bar{K}(DK)$ and M_c/M_K for the one related to the $K\bar{K}(KK)$ appearing in Eqs. (10) and (11).

Once we solve the Faddeev equations for the systems we are concerned with, we have to write this solution in such a way that it represents the amplitude of a $\bar{K}(K)$ meson interacting with the D_{s0}^* molecule, which is the DK cluster written into an $I = 0$ combination. Taking into account that $|DK(I=0)\rangle = (1/\sqrt{2})|D^+K^0 + D^0K^+\rangle$ [recall $(D^+, -D^0)$ is the isospin doublet] and summing the cases where the odd $\bar{K}(K)$ interacts first to the left (right) of the cluster and finishes interacting at the left (right), we obtain the following combination for both $DK\bar{K}$ and DKK systems,

$$\begin{aligned} T_{X-D_{s0}^*} = \frac{1}{2}(&T_{11}^{\mathrm{FCA}} + T_{12}^{\mathrm{FCA}} + T_{14}^{\mathrm{FCA}} + T_{15}^{\mathrm{FCA}} + T_{21}^{\mathrm{FCA}} + T_{22}^{\mathrm{FCA}} \\ &+ T_{24}^{\mathrm{FCA}} + T_{25}^{\mathrm{FCA}} + T_{41}^{\mathrm{FCA}} + T_{42}^{\mathrm{FCA}} + T_{44}^{\mathrm{FCA}} + T_{45}^{\mathrm{FCA}} \\ &+ T_{51}^{\mathrm{FCA}} + T_{52}^{\mathrm{FCA}} + T_{54}^{\mathrm{FCA}} + T_{55}^{\mathrm{FCA}}), \end{aligned} \quad (12)$$

where X denotes a $\bar{K}$ in the $DK\bar{K}$ case and a K meson for the DKK interaction.

B. Two-body amplitudes

In order to solve the Faddeev equations using the FCA for the systems we are concerned with, we need to know the two-body scattering amplitudes appearing in Eqs. (10) and (11). They were studied in Refs. [39,44]. These amplitudes are calculated using the chiral unitary approach (for a review see [49]). In this model, the transition amplitudes between the different pairs of mesons are extracted from Lagrangians based on symmetries such as chiral and heavy quark symmetries. Then, they are unitarized using them as the kernels of the Bethe-Salpeter equation, which, in its on-shell factorization form, is given by

$$t = (1 - vG)^{-1}v, \quad (13)$$

where G is the two-meson loop function and its expression in the dimensional regularization method is

$$\begin{aligned} G(s_i) = \frac{1}{16\pi^2}\Bigg\{&\alpha_i(\mu) + \log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s_i}{2s_i}\log\frac{m_2^2}{m_1^2} \\ &+ \frac{p}{\sqrt{s_i}}[\log(s_i - m_2^2 + m_1^2 + 2p\sqrt{s_i}) \\ &- \log(-s_i + m_2^2 - m_1^2 + 2p\sqrt{s_i}) \\ &+ \log(s_i + m_2^2 - m_1^2 + 2p\sqrt{s_i}) \\ &- \log(-s_i - m_2^2 + m_1^2 + 2p\sqrt{s_i})]\Bigg\}, \end{aligned} \quad (14)$$

with m_1 and m_2 standing for the i-channel meson masses in the loop and p the three-momentum in the two-meson center-of-mass energy, $\sqrt{s_i}$. In Eq. (14), μ is a scale fixed *a priori*, and the subtraction constant $\alpha(\mu)$ is a free parameter. In Ref. [39], μ is considered to be equal to 1500 MeV for the $D\bar{K}$ system, corresponding to $\alpha_{D\bar{K}} = -1.15$. On the other hand, since the amount of DK content in $D_{s0}^*(2317)$ is about 70% [42], we consider just one channel, with $\alpha_{DK} = -0.925$, adjusted to provide the $D_{s0}^*(2317)$ peak, corresponding to a cutoff value equal to 650 MeV. This value also has to be used as the upper limit in the integrals given by Eqs. (4) and (5). For the $f_0(980)/a_0(980)$ we consider the same channels as Refs. [50,51], where a cutoff equal to 600 MeV was used to regularize the loops, given by

$$G(s_l) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{\omega_1(\mathbf{q}) + \omega_2(\mathbf{q})}{2\omega_1(\mathbf{q})\omega_2(\mathbf{q})} \times \frac{1}{(P^0)^2 - [\omega_1(\mathbf{q}) + \omega_2(\mathbf{q})]^2 + i\epsilon}, \quad (15)$$

where $(P^0)^2 = s_l$, the two-body center-of-mass energy squared. The index l stands for the following channels: (1) $\pi^+\pi^-$, (2) $\pi^0\pi^0$, (3) K^+K^-, (4) $K^0\bar{K}^0$, (5) $\eta\eta$, and (6) $\pi^0\eta$. In each channel $\omega_{1(2)}(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_{1(2)}^2}$, where $m_{1(2)}$ is the mass of the mesons inside the loop.

In order to get the scattering amplitude for the KK interaction, we follow Ref. [44]. First, we have to find the kernel v to be used in Eq. (13). This kernel is the lowest order amplitude describing the KK interaction, and it is calculated using the chiral Lagrangian

$$\mathcal{L}_2 = \frac{1}{12 f_\pi^2} \langle (\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M \Phi^4 \rangle, \quad (16)$$

where $\langle \ldots \rangle$ means the trace in the flavor space of the $SU(3)$ matrices appearing in Φ and M, while f_π is the pion decay constant. The matrices Φ and M are given by

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix},$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}, \quad (17)$$

where in M we have taken the isospin limit ($m_u = m_d$), and in Φ we take $\eta_8 = \eta$. Hence, from Eqs. (16) and (17) we can calculate the tree-level amplitudes for K^+K^0 and K^+K^+, which, after projection in the s-wave, read as

$$v_{K^+K^0 \to K^+K^0} = \frac{1}{2f_\pi^2}(s_{KK} - 2m_K^2),$$
$$v_{K^+K^+ \to K^+K^+} = \frac{1}{f_\pi^2}(s_{KK} - 2m_K^2), \quad (18)$$

where s_{KK} is the Mandelstam variable s in the KK center-of-mass frame. From these equations one finds that $v_{KK}^{I=0} = 0$ (and $t_{KK}^{I=0} = 0$), and taking the unitary normalization appropriate for identical particles $|K^+K^+, I = 1\rangle = |K^+K^+\rangle/\sqrt{2}$, we find $v_{KK}^{I=1} = \frac{1}{2} v_{K^+K^+ \to K^+K^+}$. The $t_{KK}^{I=1}$ amplitude will be $t_{KK}^{I=1} = (1 - v_{KK}^{I=1} G_{KK})^{-1} v_{KK}^{I=1}$, and then $t_{KK}^{I=1}$ has to be multiplied by two to restore the good normalization. Therefore, using these expressions we obtain the KK scattering amplitudes $\bar{t}_5$ and $\bar{t}_6$ present in Eq. (11) ($\bar{t}_6 = t_{KK}^{I=1}, \bar{t}_5 = \frac{1}{2} t_{KK}^{I=1}$, with $t_{KK}^{I=1}$ the good normalization), where we have used a cutoff of 600 MeV to regularize the KK loops, the same cutoff that was used in the $K\bar{K}$ and coupled channels system. After these considerations we are able to determine all the two-body amplitudes in Eqs. (10) and (11).

It is worth mentioning that the arguments of the partitions $T_{ij}^{\mathrm{FCA}}(s)$ and the $t_i(s_i)$ two-body amplitudes are different. While the former is written into the three-body center-of-mass energy $\sqrt{s}$, the latter is given in the two-body one. In order to write the $\sqrt{s_i}$'s in terms of $\sqrt{s}$, we use the same transformations as in Refs. [46,52], which are

$$s_{DK(D\bar{K})} = m_K^2 + m_D^2 + \frac{1}{2M_{D_{s0}^*}^2}(s - m_K^2 - M_{D_{s0}^*}^2) \times (M_{D_{s0}^*}^2 + m_D^2 - m_K^2), \quad (19)$$

where the subscript $DK(D\bar{K})$ stands for the two-body channels associated with the energy in the center-of-mass frame of $DK(D\bar{K})$. Analogously, for the energy in the $KK(K\bar{K})$ center-of-mass frame, we have

$$s_{KK(K\bar{K})} = 2m_K^2 + \frac{1}{2M_{D_{s0}^*}^2}(s - m_K^2 - M_{D_{s0}^*}^2) \times (M_{D_{s0}^*}^2 + m_K^2 - m_D^2). \quad (20)$$

In this work, we call this set of transformations "prescription I." In order to estimate the uncertainties in our calculations, we use another set of transformations, which we call "prescription II," given by

$$s_{DK(D\bar{K})} = \left(\frac{\sqrt{s}}{M_{D_{s0}^*} + m_K}\right)^2 \left(m_K + \frac{m_D M_{D_{s0}^*}}{m_D + m_K}\right)^2 - \mathbf{P}_2^2 \quad (21)$$

and

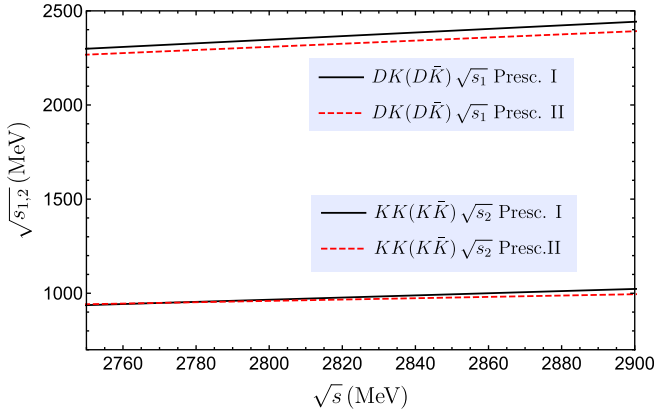


FIG. 2. Energy distribution in the center-of-mass frame of each two-body system as a function of the total energy of the three-body system, using prescriptions I and II. Here $s_1 = s_{DK(D\bar{K})}$ and $s_2 = s_{KK(K\bar{K})}$. The lower curves are for KK or $K\bar{K}$, and the upper curves are for DK or $D\bar{K}$.

$$s_{KK(K\bar{K})} = \left(\frac{\sqrt{s}}{M_{D_{s0}^*} + m_K}\right)^2 \left(m_K + \frac{m_K M_{D_{s0}^*}}{m_D + m_K}\right)^2 - \mathbf{P}_1^2, \tag{22}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ stand for the momenta of the D and K mesons in the cluster, which we take to be equal such that the kinetic energy in the DK cluster is of the order of the binding energy; hence, $\mathbf{P}_1^2 = \mathbf{P}_2^2 = 2\tilde{\mu}B_{D_{s0}^*} = 2\tilde{\mu}(m_D + m_K - M_{D_{s0}^*})$, with $\tilde{\mu}$ the reduced mass of DK. This prescription is based on another one discussed in Refs. [32,52], which shares the binding energy among the three particles proportionally to their respective masses.

III. RESULTS

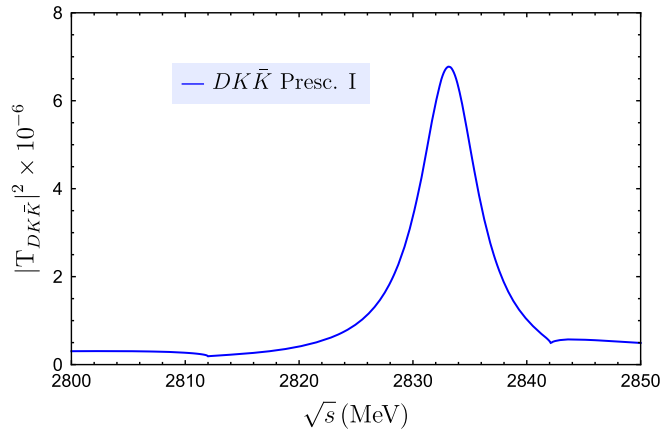
In all our calculations we use $m_K = 495$ MeV, $m_D = 1865$ MeV, $m_{D_{s0}^*(2317)} = 2317$ MeV, $m_\pi = 138$ MeV, $m_\eta = 548$ MeV, and $f_\pi = 93$ MeV. In Fig. 2 we plot the energies in the center-of-mass frame of each of the two-body systems as a function of the energy of the center-of-mass frame of the three-body system, according to Eqs. (19)–(22). Both prescriptions map the energy range around 2812 MeV, which corresponds to the threshold of $D_{s0}^*(2317)K$ [or $D_{s0}^*(2317)\bar{K}$], to an energy range around each of the thresholds of the two-body interactions, i.e., the KK system (or $K\bar{K}$) interacts in an energy range around 990 MeV in its center-of-mass frame, which corresponds to $2m_K$, and the DK system (or $D\bar{K}$) interacts in an energy range around 2360 MeV, which corresponds to $m_K + m_D$.

The main uncertainty in our calculation is the difference between these two ways of mapping the total energy into the center-of-mass frame of each two-body system. This feature was also found in other works using FCA, for instance in Ref. [52].

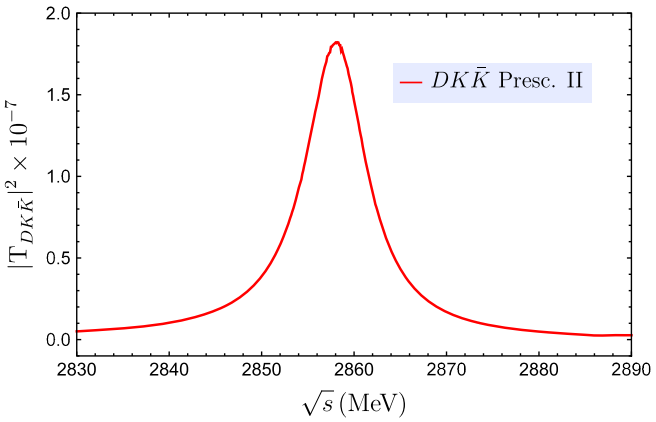
A. The $DK\bar{K}$ system

In Fig. 3(a) we show the result of the total Faddeev amplitude squared from Eq. (12) using prescription I. We see a strong peak around 2833 MeV, which could be interpreted as a $D[f_0(980)/a_0(980)]$ bound state since it is below the $D[f_0(980)/a_0(980)]$ threshold of 2855 MeV. On the other hand, using prescription II we observe a peak around 2858 MeV, as can seen in Fig. 3(b), and now could be interpreted as a $D[f_0(980)/a_0(980)]$ resonance since it is above its threshold.

In order to investigate if this strong peak in the $DK\bar{K}$ system comes mostly from $K\bar{K}$ merging into $a_0(980)$ or $f_0(980)$, we have separated the $K\bar{K}$ amplitudes (that enter in the Faddeev equations) in the isospin basis and selected only one contribution at a time. In Fig. 4 we show the results where the $I = 0$ component of $K\bar{K}$ was removed; therefore, there is no $f_0(980)$ contribution. In this figure we can clearly see the shape of the $a_0(980)$ in the three-body amplitude, which peaks around 2842 MeV in prescription I



(a)



(b)

FIG. 3. Results for the total $DK\bar{K}$ amplitude squared using prescriptions I (left) and II (right).

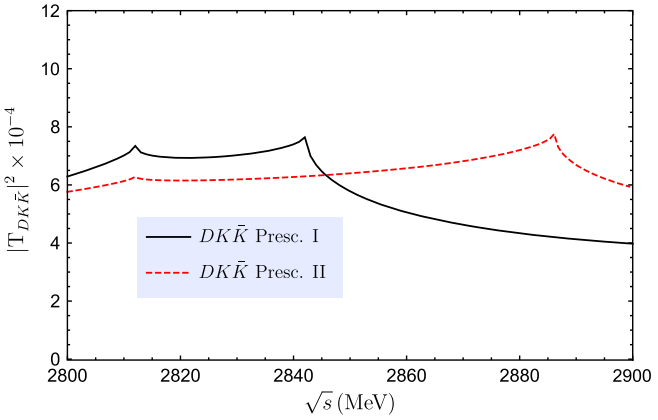


FIG. 4. Results for the $DK\bar{K}$ amplitude squared after removing the $f_0(980)$ contribution, using prescriptions I and II.

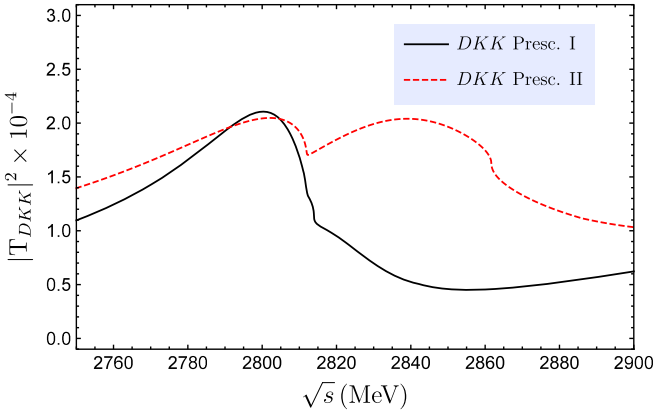


FIG. 5. Results for the total DKK amplitude squared using prescriptions I and II.

(and 2886 MeV in prescription II); according to Fig. 2, this corresponds to 990 MeV in the $K\bar{K}$ center-of-mass frame, exactly where the $a_0(980)$ peak results from the $I = 1$ $K\bar{K}$ two-body amplitude. Notice that when we removed the $I = 0$ isospin component from the $K\bar{K}$ amplitude, the strength of the peaks in $|T_{DK\bar{K}}|^2$ have decreased by about 2 orders of magnitude in both prescriptions, pointing out that the $f_0(980)$ is indeed the most important contribution coming from $K\bar{K}$. It is interesting to recall that the same conclusion was obtained in [35], where no apparent signal for $Da_0(980)$ was found. Furthermore, the small cusps seen in both prescriptions at 2812 MeV in Fig. 4 correspond to the $D_{s0}^*(2317)\bar{K}$ threshold. In Table I we compile the results of both prescriptions.

The results for the $DK\bar{K}$ system point to the formation of a three-body state: the $D[f_0(980)/a_0(980)]$, in which the $Df_0(980)$ is the strongest contribution in both prescriptions. Specifically, in prescription I the $Df_0(980)$ state would be bound by about 20 MeV, while in prescription II it would correspond to a resonance. This latter result would be similar to the findings of Ref. [35], where a peak is seen at higher energy, forming a $Df_0(980)$ resonant state at 2890 MeV.

As mentioned previously, the difference between the results of prescriptions I and II should be interpreted as the main uncertainty in our approach, but what emerges from both pictures is that a $Df_0(980)$ state is formed, slightly bound or unbound.

TABLE I. Comparison between position and intensity of the peaks found in the $DK\bar{K}$ amplitude.

	Prescription I		Prescription II	
	$\sqrt{s}$	$\lvert T\rvert^2$	$\sqrt{s}$	$\lvert T\rvert^2$
Total	2833	6.8×10^6	2858	1.8×10^7
$I = 1$ only	2842	7.7×10^4	2886	7.8×10^4

We note that the theoretical uncertainty of the present method is of the order of 25 MeV. To put this number in a proper context, we recall that the uncertainty in the QCD sum rules method in Ref. [35] is far larger, with a mass given by $m_{Df_0} = (2926 \pm 237)$ MeV (the uncertainty for the mass in the Faddeev method of Ref. [35] is not given).

B. The *DKK* system

In Fig. 5 we show the DKK total amplitude squared from Eq. (12) using prescriptions I and II. We can see that in both prescriptions, the amplitude decreases around 2812 MeV, which corresponds to the $D_{s0}^*(2317)K$ threshold, and both have a maximum below this threshold; however, prescription II also develops a broad structure above threshold, but no clear peak that could indicate the formation of a bound state or a resonance is found.

As a physical interpretation we could say that, even though the interaction of the external K with the D inside the cluster is attractive (the same responsible for the strong binding that generates the $D_{s0}^*(2317)$ cluster), the repulsion of the external K with the K inside the cluster seems to be of the same magnitude and prevents the DKK system from forming a bound state.

One might be tempted to associate the peak below threshold with a physical state, but this is not the case. Indeed, one should note that the strength of $|T_{DKK}|^2$ in Fig. 5 is about 3 orders of magnitude smaller than for $|T_{DK\bar{K}}|^2$ in Fig. 3, which simply indicates that no special hadron structure has been formed in this case.

IV. CONCLUSIONS

In this work, we have used the FCA to Faddeev equations in order to look for bound states or resonances generated from $DK\bar{K}$ and DKK three-body interactions. The cluster DK in the $I = 0$ component is the well-known $D_{s0}^*(2317)$ bound state studied by means of the chiral unitary approach. From the $DK\bar{K}$ interaction we found an

$I(J^P) = 1/2(0^-)$ state with mass about 2833–2858 MeV, where the uncertainties were estimated by taking into account two different prescriptions to obtain $\sqrt{s_{D\bar{K}}}$ and $\sqrt{s_{K\bar{K}}}$ from the total energy of the system $\sqrt{s}$. Our findings corroborated those of Ref. [35], where the authors studied the $DK\bar{K}$ interaction using two different nonperturbative calculation tools, the QCD sum rules and the Faddeev equations without FCA. They found a state around 2890 MeV, which is above the $Df_0(980)$ threshold. As we have pointed out before, this state could be seen in the $\pi\pi D$ invariant mass distribution. Therefore, as in Ref. [35], we also suggest the search for such a state in future experiments. On the other hand, for the DKK system we found an enhancement effect, but with a very small strength compared to the $DK\bar{K}$ system, and it should not be related to a physical bound state. In this case, the repulsion between KK seems to be of the same magnitude as the attraction on the DK interaction, preventing the formation of the three-body molecular state.

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