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Vacuum radiation pressure fluctuations and barrier penetration

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We apply recent results on the probability distribution for quantum stress tensor fluctuations to the problem of barrier penetration by quantum particles. The probability for large stress tensor fluctuations decreases relatively slowly with increasing magnitude of the fluctuation, especially when the quantum stress tensor operator has been averaged over a finite time interval. This can lead to large vacuum radiation pressure fluctuations on charged or polarizable particles, which can in turn push the particle over a potential barrier. The rate for this effect depends sensitively upon the details of the time averaging of the stress tensor operator, which might be determined by factors such as the shape of the potential. We make some estimates for the rate of barrier penetration by this mechanism and argue that in some cases this rate can exceed the rate for quantum tunneling through the barrier. The possibility of observation of this effect is discussed.

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I. INTRODUCTION

In a recent paper [1], we showed how the one loop radiative correction to potential scattering and to quantum tunneling may be obtained from simple arguments involving the vacuum fluctuations of the time-averaged quantized electric field. In particular, the one loop enhancement of the quantum tunneling rate obtained by Flambaum and Zelevinsky [2] may be understood as the vacuum electric field giving the particle an extra boost to get over the barrier. The effects of vacuum electric field fluctuations on light propagation in nonlinear materials were discussed in Refs. [3,4].

In the present paper, we will discuss the effect of vacuum radiation pressure fluctuations in enhancing tunneling rates. Here we are dealing with fluctuations of the electromagnetic stress tensor, rather than of the fields themselves. The role of classical radiation pressure on electrons and atoms in astrophysics has long been studied [5]. The variance of the radiation pressure fluctuations in a coherent state, which plays a role in laser interferometer detectors of gravity waves, was calculated in Refs. [6–9]. The variance of the time averaged radiation pressure fluctuations in the vacuum state has been treated by several authors in the context of Casimir force fluctuations [8,10,11]. Time averaging will play a crucial role in our analysis as well. The fluctuations of a quantum stress tensor operator at a single spacetime point are not defined in the sense that all of the moments, beyond the first moment, of such an operator diverge. In general, time averaging of the quantum stress tensor is needed to yield finite results for the moments. It is also true that the correlation and n-point functions of a stress tensor operator are finite provided that none of the spacetime points involved are at null separations. The Fourier transform of a correlation function yields a power spectrum, which can be useful for the study of the variance of the fluctuations. This approach was used in Refs. [12,13] to study fluctuations of a mirror in the vacuum.

In the present paper, we will consider the effects of large radiation pressure fluctuations in the vacuum state. By "large," we mean fluctuations which are much larger than the root-mean-square value found in calculations of the variance. The probability distributions for quantum stress tensor vacuum fluctuations have been discussed in Refs. [14–16]. These distributions contain the information needed to go beyond calculations of the variance of the fluctuations, a fact which was acknowledged by Barton [10]. The part of the probability distribution which describes large fluctuations is determined by the higher moments ($n \gg 2$) of the time averaged operator. Thus approaches which focus upon the variance or the power spectrum of the fluctuations, such as were used in Refs. [6–13], are not particularly useful for the study of large fluctuations. A key result is that the distributions for stress tensor fluctuations fall relatively slowly as the magnitude of the fluctuation increases, much more slowly than does the Gaussian distribution which describes time averaged electric field fluctuations. This means that large radiation pressure fluctuations are not so rare as one might have expected. This is especially the case when the relevant stress tensor has been averaged over a finite time interval [16], that is, with an averaging function which is strictly zero outside of a finite interval. Such an averaging function may be viewed as describing a measurement made over a finite time. Here we will explore the possible role of large vacuum radiation pressure fluctuations in pushing a particle over a barrier more quickly than it would tunnel through the barrier.

It is well known that at finite temperature it is possible for particles to acquire enough energy to fly over a barrier

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without tunneling, a process known as thermal activation. The effect we will consider bears some similarities to thermal activation, but can occur at zero temperature. Our effect is also related to the noise-induced activation studied by Antunes *et al.* in Ref. [17]. These authors treat a model of a quantum particle in a double well potential which is linearly coupled to a bath of quantum oscillators. They find a form of activation at zero temperature which can be ascribed to the quantum fluctuations of the oscillator bath. A key difference between the model of Ref. [17] and that in the present paper is that we assume the particle to be coupled quadratically to the quantized electromagnetic field through the stress tensor. This leads to the possibility of large, non-Gaussian fluctuations.

The outline of this paper is as follows: The results of Ref. [16] on probability distributions will be summarized in Sec. II and extended to the specific case of electromagnetic radiation pressure fluctuations. The effects of vacuum radiation pressure fluctuations on barrier penetration by charged particles will be examined in Sec. III. Estimates of the magnitude of this effect will be given, and the conditions under which it can dominate quantum tunneling will be discussed. The possible role of radiation pressure fluctuations in nuclear fusion will be treated in Sec. IV. The effect of radiation pressure fluctuations on polarizable, uncharged, particles will be discussed in Sec. V. Section VI summarizes and discusses the main results of the paper.

Units in which $\hbar = c = 1$ and Lorentz-Heaviside units for electromagnetic quantities will be used unless otherwise noted.

II. PROBABILITY OF LARGE STRESS TENSOR FLUCTUATIONS

In this section, we first review previous results on the probability distribution function for quantum stress tensor fluctuations, and then apply these results to the specific case of vacuum pressure fluctuations of the quantized electromagnetic field.

A. Finite duration measurements and the probability distribution

Here we summarize the key results of Ref. [16] which will be needed in the present paper. Let $Q(t)$ be an operator which is a quadratic function of a free field operator and define its time average with respect to $f(t)$ by

$$T = \int_{-\infty}^{\infty} Q(t) f(t) dt, \tag{1}$$

where

$$\int_{-\infty}^{\infty} f(t) dt = 1. \tag{2}$$

In general, it is the time average, T, rather than the local operator, Q, which is observable in the sense that one may assign a well-defined probability distribution to T, but not to Q. The key idea is that measurements of a quantum stress tensor which occur in a finite time interval should be described by a sampling function of time, $f(t)$, which is smooth and has compact support. Thus $f(t)$ is taken to be a C^∞, but nonanalytic, function which is strictly zero outside of a finite time interval whose width is approximately τ. The Fourier transform of such a function will have an asymptotic form for large argument which falls faster than any power but more slowly than an exponential function. Define the Fourier transform by

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega t} f(t). \tag{3}$$

A useful set of compactly supported sampling functions is defined by

$$\hat{f}(\omega) = e^{-|\omega|^\alpha}, \tag{4}$$

where $0 < \alpha < 1$. (Units in which $\tau = 1$, following the notation in Ref. [16], are adopted temporarily. Later, we return to general units for τ when needed for clarity.) The corresponding functions of time, $f(t)$, are expressible in terms of Fox H-functions [18,19]. For our purposes, we only require that Eq. (4) hold asymptotically for $\omega \gg 1$. This will be sufficient to give the switching behavior which we now discuss. We will also require that $\hat{f}(\omega) \geq 0$. We can arrange for the initial switch-on of $f(t)$, to occur at $t = 0$. In this case, the functional form of $f(t)$ as $t \to 0^+$ is

$$f(t) \sim t^{-\mu} e^{-w t^{-\nu}}, \tag{5}$$

where

$$\nu = \frac{\alpha}{1-\alpha}, \tag{6}$$

$$\mu = \frac{2-\alpha}{2(1-\alpha)}, \tag{7}$$

and

$$w = (1-\alpha)\alpha^{\alpha/(1-\alpha)}. \tag{8}$$

The switch-off at the end of the finite interval will have the same functional form. The parameter α describes both the rate of decrease of $\hat{f}(\omega)$, and the behavior of $f(t)$ at the switch-on and switch-off. A simple electrical circuit which has a switch-on corresponding to $\alpha = 1/2$ was described in Ref. [16]. In this case, $f(t) \propto t^{-3/2} e^{-1/(4t)}$ as $t \to 0^+$.

The asymptotic form of the Fourier transform, $\hat{f}(\omega)$, determines the rate of growth of the moments of the sampled stress tensor and in turn, the probability for large fluctuations. Let T be a normal-ordered quadratic operator which has been averaged with the sampling function $f(t)$ and define its moments by

$$\mu_n = \langle 0|T^n|0\rangle. \tag{9}$$

We express T in a mode sum of creation and annihilation operators as

$$T = \sum_{ij}(A_{ij}a_i^\dagger a_j + B_{ij}a_i a_j + B_{ij}^* a_i^\dagger a_j^\dagger), \tag{10}$$

where the coordinate space mode functions are assumed to be plane waves proportional to $e^{-i\omega t}$. Now μ_n may be expressed as a sum of nth degree polynomials in the coefficients A_{ij} and B_{ij}. These coefficients have the functional forms

$$A_{ij} \propto (\omega_i\omega_j)^{(p-2)/2}\hat{f}(\omega_i - \omega_j) \tag{11}$$

and

$$B_{ij} \propto (\omega_i\omega_j)^{(p-2)/2}\hat{f}(\omega_i + \omega_j), \tag{12}$$

where p is an integer determined by the dimensions of the operator T. In the case of stress tensor operators, which will be our primary concern, $p = 3$. However, we will consider the possibility of larger values of p in Sec. V.

It was argued in Ref. [16] that there is one term in the expression for μ_n which dominates for $n \gg 1$. This term is

$$M_n = 4\sum_{j_1\cdots j_n} B_{j_1j_2}A_{j_2j_3}A_{j_3j_4}\cdots A_{j_{n-1}j_n}B^*_{j_nj_1}. \tag{13}$$

The dominance of this term can be understood as arising from the relative minus sign in the argument of the $\hat{f}$ factor in A_{ij}, as compared to that in B_{ij}. The dominant term contains the maximum number of factors of A_{ij}, which fall more slowly with increasing ω_i. In any case, $M_n < \mu_n$ as all of the terms neglected in M_n are positive, because $\hat{f}(\omega) \geq 0$. Thus M_n gives a lower bound on the exact moments. This will in turn give a lower bound on the probability of large fluctuations. In the case where T is a time average of $:\dot{\varphi}^2:$, where φ is the massless scalar field,

$$M_n = k_n\int_0^\infty d\omega_1\cdots d\omega_n(\omega_1\cdots\omega_n)^p\hat{f}(\omega_1+\omega_2) \times\hat{f}(\omega_2-\omega_3)\cdots\hat{f}(\omega_{n-1}-\omega_n)\hat{f}(\omega_n+\omega_1), \tag{14}$$

where

$$k_n = \frac{1}{(2\pi^2)^n} \tag{15}$$

and $p = 3$. For $n \gg 1$, the asymptotic form of M_n becomes

$$M_n \simeq k_n[2\pi f(0)]^{n-2}\frac{p![(n-1)p]!}{(np+1)!}\int_0^\infty du\hat{f}^2(u)u^{np+1}, \tag{16}$$

and if $\hat{f}$ has the form given in Eq. (4), we have

$$M_n \simeq k_n[2\pi f(0)]^{n-2}\frac{p![(n-1)p]!}{\alpha(np+1)!2^{(np+2)/\alpha}}\Gamma\left[\frac{(np+2)}{\alpha}\right]. \tag{17}$$

The last factor in this expression reveals that for large n, the moments grow as $(pn/\alpha)!$.

This rapid rate of growth of the moments leads to a slow decrease in the tail of the probability distribution. Now return to arbitrary units for the sampling time τ and define the dimensionless variable $x = T\tau^{p+1}$. Let $P(x)$ be the probability distribution describing the probability of finding various values of T in a measurement. As explained in Refs. [14,15], this probability distribution has a lower bound at the quantum inequality bound on expectation values of T in an arbitrary state, $x = -x_0 < 0$, but no upper bound, so

$$\int_{-x_0}^\infty P(x)dx = 1. \tag{18}$$

The asymptotic form for $P(x)$ for large x may be written as

$$P(x) \sim c_0x^b e^{-ax^c}. \tag{19}$$

The constants c_0, a, b, and c may be determined from Eq. (17) to be [16]

$$c = \frac{\alpha}{p}, \tag{20}$$

$$b = c\left(\frac{2}{\alpha} - p - 1\right) - 1 = \frac{2-\alpha}{p} - (\alpha+1), \tag{21}$$

$$a = 2[2\pi f(0)B]^{-\alpha/p}, \tag{22}$$

and

$$c_0 = ca^{(b+1)/c}B_0p!\alpha^{-(p+2)}2^{-(2/\alpha)}[2\pi f(0)]^{-2}. \tag{23}$$

Here the constants B_0 and B are defined by

$$k_n = B_0B^n. \tag{24}$$

Thus for the case of $:\dot{\varphi}^2:$, we have $B_0 = 1$ and $B = 1/(2\pi^2)$.

Because the moments μ_n grow faster than $n!$ as $n \to \infty$, the probability distribution $P(x)$ cannot be uniquely determined by its moments. However, the average behavior of the asymptotic form in Eq. (19) can be inferred from the rate of growth of the moments, as was discussed in Refs. [15,16]. It is of interest to seek alternative derivations of the vacuum stress tensor probability distribution, $P(x)$. One possibility is numerical diagonalization in a modified theory with a finite number of degrees of freedom. This possibility is under investigation. It may also be possible to apply functional approaches, such as the Schwinger-Keldysh or closed time path method. However, so far this type of approach has been used primarily in perturbative treatments and would need to be extended to apply to nonperturbative problems such as that of the probability distribution.

B. Radiation pressure fluctuations

Now we wish to apply the results summarized in the previous subsection to the case of vacuum electromagnetic radiation pressure fluctuations. These are fluctuations of the time averaged energy or momentum flux components of the electromagnetic stress tensor. Consider the momentum flux in the z direction

$$T^{tz} = (\mathbf{E}\times\mathbf{B})^z = E^x B^y - E^y B^x, \tag{25}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the quantized electric and magnetic field operators, respectively. Let S^z be the momentum flux sampled with $f(t)$

$$S^z = \int_{-\infty}^{\infty} T^{tz}(t,\mathbf{x}) f(t) dt, \tag{26}$$

where the sampling is in time at a fixed spatial location. Note that T^{tz}, and hence S^z are automatically normal ordered, as $\langle 0|T^{tz}|0\rangle = 0$. The nth moment of S^z is

$$\mu_n = \langle 0|(S^z)^n|0\rangle = \int_{-\infty}^{\infty} dt_1 f(t_1) \int_{-\infty}^{\infty} dt_2 f(t_2)\cdots \times \int_{-\infty}^{\infty} dt_n f(t_n) \langle 0|T_1^{tz} T_2^{tz}\cdots T_n^{tz}|0\rangle, \tag{27}$$

where $T_j^{tz} = T^{tz}(t_j,\mathbf{x})$. When $n \gg 1$, we expect $\mu_n \sim M_n \sim C_n$, where C_n is the nth connected moment.

We expect the high moments of the time averages of both T^{tz} and of $:\dot\varphi^2:$ to be of the form of Eq. (17) with $p = 3$, but with different values for the constants k_n. We may relate $k_n(T^{tz})$ to $k_n(:\dot\varphi^2:)$, the latter of which are given by Eq. (15), using a variation of the argument in Sec. III B of Ref. [15]. The connected moments of $:\dot\varphi^2:$ may be expressed as a sum of the possible connected contractions of the form

$$\dot\varphi_1\dot\varphi_1\dot\varphi_2\dot\varphi_2\dot\varphi_3\dot\varphi_3\cdots\dot\varphi_n\dot\varphi_n\,, \tag{28}$$

where the subscripts label operators at different spacetime points. Here the contraction of the form

$$\dot\varphi_i\cdots\dot\varphi_j \tag{29}$$

contributes a factor of $\langle\dot\varphi_i\,\dot\varphi_j\rangle$ in the expression for $C_n(\dot\varphi^2)$. The number of terms in $C_n(\dot\varphi^2)$ may be counted as follows: The first operator to contact has $2(n-1)$ possible partners with which it may be contracted. After this is done, the next operator has $2(n-2)$ possible partners. Thus the total number of terms will be

$$[2(n-1)][2(n-2)]\cdots 2 = 2^{n-1}(n-1)!. \tag{30}$$

The corresponding calculation for the nth connected moment of S^z, $C_n(S_z)$ will involve

$$\langle (E^x B^y - E^y B^x)_1 (E^x B^y - E^y B^x)_2 \cdots (E^x B^y - E^y B^x)_n \rangle. \tag{31}$$

The contractions of the electric and magnetic field operators are related to those for $\dot\varphi$ by the relations

$$\langle E_i(t)E_j(t')\rangle = \langle B_i(t)B_j(t')\rangle = \frac{2}{3}\delta_{ij}\langle\dot\varphi(t)\dot\varphi(t')\rangle, \tag{32}$$

and

$$\langle E_i(t)B_j(t')\rangle = 0, \tag{33}$$

where all operators are at the same spatial point. This means that E_1^x can only contract with other E^x operators, etc. Thus E_1^x has $n-1$ possible contractions, and B_1^y can only contract with other B^y operators whose associated E^x operator is still uncontracted, as otherwise a disconnected moment would result. This leads to $n-2$ possibilities. The next E^x operator has $n-3$ possibilities, etc. Thus a total of $(n-1)!$ terms arise from $E^x B^y$, and an equal number from $E^y B^x$, leading to a total of $2(n-1)!$ terms in $C_n(S_z)$. Equation (32) tells us that each contraction of electromagnetic field operators contributes a factor of $2/3$ to $C_n(S_z)$, compared to the contribution of a $\dot\varphi$ contraction to $C_n(\dot\varphi^2)$. Thus, we may write

$$k_n(S_z) = \left(\frac{2}{3}\right)^n \frac{2(n-1)!}{2^{n-1}(n-1)!} k_n(\dot\varphi^2) = \frac{4}{(6\pi^2)^n}, \tag{34}$$

where $k_n(\dot\varphi^2)$ is given by Eq. (15). This leads to

$$B_0 = 4 \quad \text{and} \quad B = \frac{1}{6\pi^2} \tag{35}$$

for S^z. This result may also be derived by an alternative argument which involves direct evaluation of the vacuum expectation value of a product of S^z operators.

As $p = 3$ for T^{tz}, and hence for S^z, the probability distribution $P(x)$ is a function of $x = \tau^4 S^z$. However, unlike the case of operators such as $\dot\varphi^2$ or the energy density, there is no lower bound, and the distribution is symmetric $P(-x) = P(x)$. The normalization becomes

$$\int_{-\infty}^{\infty} P(x)dx = 1. \tag{36}$$

The asymptotic form for $|x| \gg 1$ is still given by Eq. (19), and the constants c, b, and a are given by Eqs. (20), (21), and (22), respectively, with $p = 3$ and B as in Eq. (35). However, the constant c_0 is now one-half of that given by Eq. (23). The values of the parameters in the tail of the radiation pressure probability distribution become

$$c = \frac{\alpha}{3}, \tag{37}$$

$$b = -\frac{4\alpha + 1}{3}, \tag{38}$$

$$a = 2\left[\frac{f(0)}{3\pi}\right]^{-\alpha/3}, \tag{39}$$

and

$$c_0 = \frac{1}{4\alpha^4}\left[\frac{f(0)}{3\pi}\right]^{2(2\alpha-1)/3}[2\pi f(0)]^{-2}. \tag{40}$$

C. Cumulative probability distribution

Often we are more interested in a cumulative probability distribution, rather than $P(x)$ itself. Define

$$P_>(x) = \int_x^{\infty} P(y)dy, \tag{41}$$

which is the probability to find any value of y with $y \geq x$ in a given measurement. If $x \gg 1$, we may use the asymptotic form for $P(x)$ given in Eq. (19) to find

$$P_>(x) \approx \frac{c_0}{a^{2/c}c}\Gamma\left(\frac{2}{c}, ax^c\right) \approx \frac{c_0}{ac}x^{1+b-c}\mathrm{e}^{-ax^c} = \mathrm{e}^{-F(x)}, \tag{42}$$

where $\Gamma(\frac{2}{c}, ax^c)$ is an incomplete gamma function, and

$$F(x) = ax^c - (1 + b - c)\ln x - \ln\left(\frac{c_0}{ac}\right). \tag{43}$$

The constants a and c_0 depend upon $f(0)$, the value of the sampling function at $t = 0$ in $\tau = 1$ units. Given that $f(t)$ has unit area and characteristic width τ, we expect $f(0)$ to be of order one. Simple choices, such as that illustrated in Fig. 4 of Ref. [16], give a slightly larger value. For the purposes of our estimates, we will set $f(0) = \pi/2$. Then the coefficients which appear in Eqs. (19) and (42) for S^z depend only upon the parameter α and are listed in Table I for selected values of α.

TABLE I. Coefficients for the radiation pressure probability distribution.

α	c	b	a	c_0	$1+b-c$	$\ln(\frac{c_0}{ac})$
$\frac{1}{2}$	$\frac{1}{6}$	-1	2.70	0.0411	$-\frac{1}{6}$	-2.39
$\frac{1}{3}$	$\frac{1}{9}$	$-\frac{7}{9}$	2.44	0.310	$\frac{1}{9}$	0.132
$\frac{1}{4}$	$\frac{1}{12}$	$-\frac{2}{3}$	2.32	1.19	$\frac{1}{4}$	1.82

D. Validity of the worldline approximation

The probability distributions treated in Ref. [16] and reviewed earlier in this section involve only time averaging, that is, averaging along the worldline of a point particle in inertial motion. However, in realistic physical situations, such as those to be discussed in the next section, some averaging in space as well may occur. A systematic treatment of the effects of both space and time averaging will appear in Ref. [20], including a discussion of the range of validity of the worldline approximation. This discussion will be briefly summarized here. The effect of spatial averaging can be described by a spatial sampling function $g(\mathbf{x})$, with three-dimensional Fourier transform $\hat{g}(\mathbf{k})$. Now the expressions for the moments, such as Eq. (14), will contain factors of $\hat{g}$ in addition to those of $\hat{f}$, and integrations over d^3k_j. Let $s = \ell/\tau$ denote the ratio the characteristic scale of the spatial sampling, ℓ, to the temporal scale, τ, and assume $s \ll 1$. In this case, we expect the worldline approximation to hold for the lower moments, and hence the inner part of the probability distribution.

This statement can be made more quantitative as follows: For $\omega \lesssim 1/s$, we have $\hat{g} \approx 1$. (Recall that ω is dimensionless in $\tau = 1$ units.) The dominant contribution in ω to the nth moment comes near the maximum of the integrand in Eq. (16), which is

$$\omega_n \approx \left(\frac{n}{2c}\right)^{1/\alpha} \tag{44}$$

if $\hat{f}$ has the form in Eq. (4). Thus the worldline approximation gives an accurate estimate for the nth moment if

$$n \lesssim 2cs^{-\alpha}. \tag{45}$$

For $n \gg 1$, we have

$$\mu_n = \int_{-\infty}^{\infty} x^n f(x)dx \approx 2c_0\int_0^{\infty} x^{n+b}\mathrm{e}^{-ax^c}dx, \tag{46}$$

for the case of the momentum flux S^z. The dominant contribution to this integral comes near the maximum of its integrand,

$$x_n \approx \left(\frac{n}{ac}\right)^{1/c}. \quad (47)$$

We may now combine these results to infer that the worldline result should give a good approximation to $P(x)$ for

$$x \lesssim \left(\frac{2}{a}\right) s^{-p}. \quad (48)$$

For the case of stress tensors such as S^z, where $p = 3$ and $a \approx 2$, as illustrated in Table I, we find that the worldline approximation gives an accurate estimate for $P(x)$ when

$$x \lesssim s^{-3}. \quad (49)$$

In addition, we need to have $x \gg 1$, so that the asymptotic probability distribution, Eq. (19), is valid. We will see below that there is a large region where both conditions may be satisfied.

E. Dependence upon the switching parameter α

A crucial feature of the asymptotic probability distributions given in Eqs. (19) and (42) is the sensitive dependence upon the parameter α. A small decrease in the value of α can cause a significant increase in the probability of a large stress tensor fluctuation. Recall that this parameter was defined in Eq. (4), which gives the asymptotic behavior of the Fourier transform, $\hat{f}(\omega)$ of a wide class of compactly supported C^∞ sampling functions. The Fourier transform of such a function must fall faster than any power, but slower than an exponential, and Eq. (4) describes the simplest class of functions with this behavior. The rate of decrease of $\hat{f}(\omega)$ for large ω is linked to the switch-on behavior of the sampling function $f(t)$ through Eqs. (5), (6), (7), and (8). Recall that if $\hat{f}(\omega)$ is exactly given by Eq. (4), then $f(t)$ is a Fox H-function, but we are considering a broader class of functions for which Eq. (4) need only hold asymptotically. Our view is that the specific form of the sampling function should be determined by the details of the physical system. Note that the variance of the vacuum radiation pressure fluctuations. which was addressed in Refs. [8,10–13], is much less sensitive to the details of the sampling function than is the probability of a large fluctuation, which is the topic addressed here. Note that Eq. (47) implies that the probability distribution for a large value of $x \gg 1$ is determined by moments of order

$$n \approx acx^c \gg 1. \quad (50)$$

This reiterates the point made earlier that studies of the variance or the power spectrum are not adequate for understanding large fluctuations.

III. BARRIER HOPPING

In this section, we will discuss the possible effects of quantum radiation pressure fluctuations on barrier penetration by quantum particles. Consider the situation illustrated in Fig. 1, where a particle of mass m and energy E_0 is incident upon a potential barrier $V(z)$, with classical turning points at $z = z_1$ and $z = z_2$, where $E_0 = V(z_1) = V(z_2)$. The probability of quantum tunneling through the barrier is given in the WKB approximation by

$$P_{\mathrm{WKB}} = \mathrm{e}^{-G}, \quad (51)$$

where

$$G = 2\int_{z_1}^{z_2} \sqrt{2m[V(z) - E_0]}dz. \quad (52)$$

The mean value theorem implies the existence of z_m, such that $z_1 \leq z_m \leq z_2$ and

$$G = 2\sqrt{2m[V(z_m) - E_0]}d, \quad (53)$$

where $d = z_2 - z_1$ is a measure of the spatial width of the barrier. Define a speed v_1 by

$$v_1 = \sqrt{2[V(z_m) - E_0]/m}, \quad (54)$$

which is the speed of a nonrelativistic particle with kinetic energy $V(z_m) - E_0$. Now we can express G as

$$G = 2v_1\left(\frac{d}{\lambda_C}\right), \quad (55)$$

where $\lambda_C = 1/m$ is the reduced Compton wavelength of the particle. Thus, the WKB tunneling probability decreases as an exponential of the product of speed v_1 as a fraction of the speed of light, and of the width of the barrier as a multiple of the Compton wavelength.

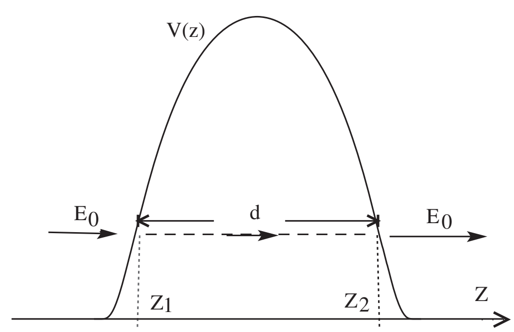

FIG. 1. A quantum particle with energy E_0 tunnels through a potential barrier $V(z)$. The classical turning points are at $z = z_1$ and $z = z_2$. The characteristic spatial width of the barrier is $d = z_2 - z_1$.

A. The effect of large vacuum radiation pressure fluctuations

Now consider the possibility that the particle, while still to the left of the barrier in Fig. 2, is subjected to a radiation pressure fluctuation in the $+z$ direction. If the magnitude and duration of this fluctuation are sufficiently large, it could push the particle over the barrier. Let σ be the scattering cross section for radiation by the particle, such as the Thompson cross section for a nonrelativistic charged particle. The average force exerted on the particle by the pressure fluctuation is σS^z, and the work done if the particle moves a distance d to the right during the fluctuation will be

$$\Delta E = \sigma S^z d. \tag{56}$$

If $\Delta E > V_{\rm max} - E_0$, where $V_{\rm max}$ is the maximum value of the potential, then the particle will fly over the barrier, if the duration of the fluctuation is sufficiently long. Let v_0 be the average speed of the particle as it goes over the barrier, and let

$$\tau = \frac{d}{v_0} \tag{57}$$

be the required duration (in arbitrary units). Here we assume that the motion of the particle is nonrelativistic so that the radiation pressure in the rest frame of the particle is approximately equal to that in the rest frame of the potential barrier. For the purpose of a rough estimate, assume that the fluctuation is sufficiently large that ΔE is at least a few times larger than $V_z - E_0$ everywhere and take $\Delta E \approx \frac{1}{2} m v_0^2$. Now we may combine the above relations to write the dimensionless x as

$$x = \tau^4 S^z \approx \frac{m d^3}{2\sigma v_0^2}. \tag{58}$$

Let the particle have an electric charge of q, so σ is the Thompson cross section

$$\sigma = \sigma_T = \frac{q^4}{6\pi m^2}. \tag{59}$$

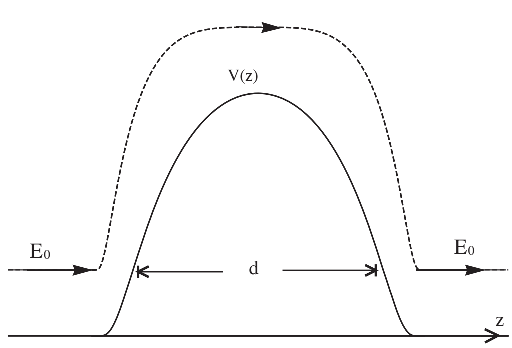

FIG. 2. Here the particle temporarily receives extra energy from a quantum radiation pressure fluctuation, which allows it to fly over the barrier.

Now we can write

$$x \approx \frac{3\pi m^3 d^3}{q^4 v_0^2}. \tag{60}$$

Note that if we hold all other variables fixed and increase v_0, and hence ΔE, then x decreases, so $P_>(x)$ typically increases, and the fluctuation becomes more probable. This arises because the factor of $1/v_0^4$ coming from τ^4 dominates over the factor of v_0^2 in ΔE.

If the cumulative probability $P_>(x)$ is greater than $P_{\rm WKB}$, or

$$F(x) < G, \tag{61}$$

then the radiation pressure fluctuations will dominate over quantum tunneling. This can occur if d is sufficiently large, as $G \propto d$ but F grows more slowly than linearly in d. For example, if $\alpha = 1/2$, then $F \propto \sqrt{d}$ for large d. For smaller values of α, the growth of F with increasing d becomes even slower.

Recall that in Sec. II D, we argued that the validity of the worldline approximation for stress tensor fluctuations requires

$$x s^3 \lesssim 1, \tag{62}$$

where s is the ratio of the spatial to the temporal averaging scales. In the case of a particle with a scattering cross section σ, we will take the spatial scale to be of order $\sqrt{\sigma}$, and set

$$s = \frac{\sqrt{\sigma}}{\tau} = \frac{q^2 \lambda_C}{\sqrt{6\pi} d} v_0. \tag{63}$$

Now Eq. (62) becomes

$$x s^3 = \frac{q^2}{2\sqrt{6\pi}} v_0 \lesssim 1, \tag{64}$$

where the factors of λ_C and d have canceled. Let $q = Ze$, and recall that $e^2/4\pi \approx 1/137$ is the fine structure constant to write Eq. (64) as

$$\left(\frac{Z}{10}\right)^2 \lesssim \frac{1}{v_0}. \tag{65}$$

This condition for the validity of the worldline approximation is generally satisfied for nonrelativistic ($v_0 \ll 1$) elementary particles and smaller nuclei.

Consider the case of radiation pressure fluctuations on a particle whose charge has a magnitude e such as an electron or proton, so $Z = 1$. For the purposes of an estimate, assume that $v_1 \approx v_0$. For given values of α and v_0, we may use Eqs. (43), (55), and (60), combined with the data in

TABLE II. Dominance of radiation pressure fluctuations. For given α and v_0, this table lists the value of the width d at which radiation pressure fluctuations begin to dominate over quantum tunneling.

α	v_0	G	d/λ_C	x	s^{-3}
$\frac{1}{2}$	0.5	132	132	1.0×10^{10}	1.9×10^{12}
$\frac{1}{2}$	0.1	1770	8880	7.8×10^{16}	7.3×10^{19}
$\frac{1}{3}$	0.5	12.5	12.5	8.8×10^{6}	1.6×10^{9}
$\frac{1}{3}$	0.1	54.1	271	2.2×10^{12}	2.1×10^{15}
$\frac{1}{4}$	0.5	0.64	0.64	1.2×10^{3}	2.2×10^{5}
$\frac{1}{4}$	0.1	3.8	19	7.6×10^{8}	7.0×10^{11}

Table I, to find the value of x and hence of d at which $F(x) = G$. A few examples are listed in Table II. As before, we have estimated the spatial dimension of the worldtube of the particle to be of order $\sqrt{\sigma} \approx 0.021\lambda_C$, so the ratio of the spatial to the temporal sampling lengths is

$$s = \frac{\sqrt{\sigma}}{\tau} \approx \frac{v_0\lambda_C}{47d}. \tag{66}$$

We can draw several inferences from the data in Table II. First, as the characteristic speed v_0 increases, the relative effect of radiation pressure fluctuations increases. This comes from the decrease in the sampling time τ and the corresponding decrease in the parameter x. The value $v_0 = 0.5$ is at the upper limit of validity of a nonrelativistic treatment but gives a reasonable order of magnitude estimate of the maximum effect attainable in this treatment. For $\alpha = 1/2$, radiation pressure fluctuations only dominate over quantum tunneling in a regime where both effects are very small. For example, for $\alpha = 1/2$ and $v_0 = 0.5$, the probability of both effects at the crossover point is of the order of e^{-132}. However, as α decreases, the relative effect of radiation pressure fluctuations increases rapidly. For $\alpha = 1/4$ and $v_0 = 0.1$, at the point that $F = G$, the probability of a particle being kicked over the barrier by a vacuum fluctuation is $e^{-3.8} \approx 0.02$, and for barriers with width $d > 19\lambda_C$, radiation pressure fluctuations will dominate. In all of the cases illustrated, $xs^3 \ll 1$, so the worldline approximation seems to be valid. At the same time, $x \gg 1$, so the asymptotic form, Eq. (19), of the probability distribution holds.

B. Sources of the switching

In this subsection, we will discuss possible physical origins of the switching function, $f(t)$, which averages the T^{tz} component of the electromagnetic stress tensor to produce the averaged momentum flux on the particle. We are working within the hypothesis that this function must be determined by the details of the physical situation or measurement. In the case of a quantum particle impinging upon a potential barrier, one possibility is an interplay between the shape of the particle's wave packet, and the geometry of the barrier. Consider a particle moving in one space dimension with wave function $\psi(z,t)$, and hence probability density $|\psi(z,t)|^2$. It is reasonable to require this to be a compactly supported function of t at fixed z, or at least be strictly zero before some specified time. This will always be the case if the source of the particle was switched on at a finite time in the past. Although it is often convenient to use Gaussian wave packets, or other functions with infinite tails in both directions, these are idealizations which imply a source in the infinite past.

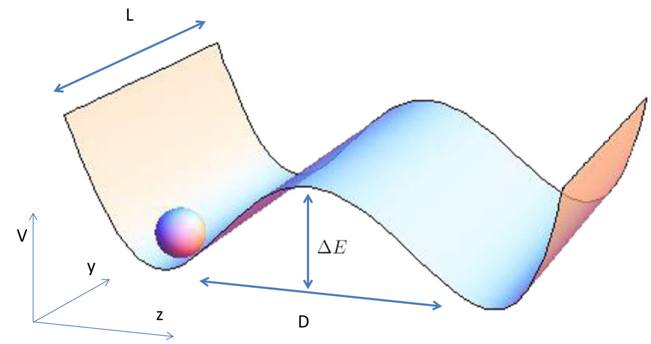

FIG. 3. A particle moves along a potential trough in the y direction, which modulates the radiation pressure fluctuations in the z direction. These fluctuations may in turn push the particle over the barrier.

Whether the potential $V(z)$ needs to be a compactly supported function of z is less clear. However, it seems reasonable to consider such potentials, which describe systems with a finite spatial extent. In this case, we might suppose that the sampling of the quantum stress tensor by the particle occurs while the probability density $|\psi(z,t)|^2$ and the potential $V(z)$ overlap in space. In this case, $f(t)$ would be zero before the leading edge of the wave packet reaches the potential, and drops again to zero after the wave packet has split into transmitted and reflected components which have left the region where $V(z) \neq 0$. It is also possible to consider potentials of the form $V(t,z)$, with explicit time dependence. Recall that a simple electrical circuit with switch-on corresponding to $\alpha = 1/2$ was discussed in Ref. [16].

Other possibilities can involve motion in more than one space dimension, as illustrated in Fig. 3. Here the particle is initially moving in the y direction in the local minimum of a potential trough on the left. The detailed shape of the potential as a function of y, as well as the shape of the particle wave packet, define a switching function for the components of the electromagnetic stress tensor, including T^{tz}. This in turn creates an averaged force in the $+z$ direction, which can cause the particle to jump over the local maximum of the potential to the trough on the right of the barrier. The temporal switch-on might be modulated by the shape of the potential in the y direction.

IV. APPLICATIONS TO NUCLEAR FUSION

An example of barrier penetration by a charged particle arises in nuclear fusion, where a smaller projectile nucleus must penetrate the Coulomb barrier of a larger target nucleus. For small projectile nuclei, a simple quantum tunneling calculation gives reasonable agreement with experiment. However, for larger projectile nuclei, such as ^{16}O or ^{40}A, the simple calculation underestimates the fusion cross section, often by many orders of magnitude [21,22]. This is usually ascribed to effects such as deformation of the target nucleus. However, we will explore the possibility that large vacuum radiation pressure fluctuations could be large enough to explain the observed cross sections.

We will consider as an example the fusion of ^{40}A with ^{154}Sm. At a center-of-mass energy of $E_{\rm cm} = 113.7$ MeV, the experimentally measured cross section is [23]

$$\sigma_{\rm exp} = 0.51 \pm 0.10 \text{ mb}. \tag{67}$$

First, we review the theoretical calculation of the cross section using quantum tunneling in a simple model [24]. Let μ be the reduced mass of the system and $k = \sqrt{2E/\mu}$ be the wave number. The cross section may be expressed in a partial wave expansion as

$$\sigma(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l, \tag{68}$$

where P_l is the transmission probability through the barrier for the lth wave. The potential for this wave can be modeled by an inverted harmonic oscillator potential

$$V_l(r) = -\frac{1}{2}\omega_0^2 \mu (r - R_0)^2 + E_l, \tag{69}$$

where

$$E_l = E_0 + \frac{l(l+1)}{2\mu R_0^2}. \tag{70}$$

Here ω_0, E_0, and R_0 are parameters which are determined semiempirically. A fit to the proximity function given in Ref. [25] leads to the values $E_0 = 123.4$ MeV, $R_0 = 12.26$ fm, and $\omega = 4.16$ MeV. This potential models Coulomb repulsion at larger distances and nuclear attractive forces at shorter distances and is illustrated in Fig. 4. The quantum tunneling probability, P_l, for this potential is given by the Hill-Wheeler formula [26]

$$P_l(E) = \frac{1}{1 + \exp[2\pi(E_l - E)/\omega_0]}. \tag{71}$$

If we evaluate the predicted cross section using Eqs. (68) and (71), with the above choices for the parameters, the result is

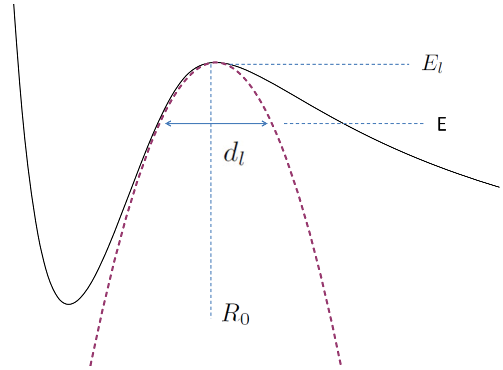

FIG. 4. Sketch of Coulomb barrier for nuclear fusion. The solid curve is the actual potential, which combines Coulomb repulsion at large separation and attractive nuclear force at short separation. The dashed curve is the inverted quadratic potential which is tangent to the actual one at the maximum point. Here d_l is the effective width of the barrier at energy E.

$$\sigma_{HW} \approx 6 \times 10^{-6} \text{ mb} \approx 10^{-5} \sigma_{\rm exp}. \tag{72}$$

Clearly, the model described above fails badly for below-barrier energies, $E < E_0$. However, it does give reasonable results for the above-barrier case.

We now explore the hypothesis that the observed cross section in the below-barrier case can be explained by large vacuum radiation pressure fluctuations, described by the tail of the cumulative probability distribution given in Eq. (42). Let

$$P_l = P_>(x_l) \approx \frac{c_0}{ac} x_l^{1+b-c} e^{-a x_l^c}, \tag{73}$$

where

$$x_l = \frac{\mu d_l^3}{2\sigma_T v_0^2}. \tag{74}$$

Here σ_T is the Thompson cross section, Eq. (59), and d_l is the width of barrier for the lth partial wave, defined by

$$V_l\left(R_0 \pm \frac{1}{2} d_l\right) = E. \tag{75}$$

The solutions of this equation are

$$d_l = d_0[1 + \xi l(l+1)]^{1/2}, \tag{76}$$

where

$$d_0 = \frac{2}{\omega_0}\sqrt{\frac{2(E_0 - E)}{\mu}} \tag{77}$$

and

$$\xi = \frac{4}{(\mu\omega_0 R_0 d_0)^2}. \tag{78}$$

Define

$$S = \frac{k^2}{\pi}\sigma, \tag{79}$$

so we have

$$S = \frac{c_0}{ac}\sum_{l=0}^{\infty}(2l+1)\{x_0[(1+l(l+1)\xi]^{3/2}\}^{1+b-c} \times \mathrm{e}^{-a\{x_0[1+l(l+1)\xi\}^{3/2}]^c}. \tag{80}$$

For the cases of interest here, this series converges well when about 10^3 terms are included.

We take the parameters c, b, a, and c_0 to be those given by Eqs. (37)–(40), with $f(0) = \pi/2$, and hence functions of α alone. The quantities x_0 and ξ are determined by the parameters specific to the $^{40}\mathrm{A} + {}^{154}\mathrm{Sm}$ system, and may be expressed as

$$\xi = 4.8 \times 10^{-4} \tag{81}$$

and

$$x_0 = 6.0 \times 10^7. \tag{82}$$

In addition, we have $d_0 \approx 2.3$ fm in this case. More generally, we can write

$$\xi = 7.4 \times 10^{-4}\left(\frac{4\ \mathrm{MeV}}{\omega_0}\right)^2\left(\frac{32\mathrm{u}}{\mu}\right)^2\left(\frac{2\ \mathrm{fm}}{d_0}\right)^2\left(\frac{12\ \mathrm{fm}}{R_0}\right)^2 \tag{83}$$

$$x_0 = 3.0 \times 10^7\left(\frac{\mu}{\mathrm{u}}\right)^3\left(\frac{Z}{18}\right)^2\left(\frac{d_0}{2\ \mathrm{fm}}\right)^3\left(\frac{0.1}{v_0}\right)^2 \tag{84}$$

for any nuclear fusion case, where Z is the atomic number of the incoming nucleus.

In the case of the $^{40}\mathrm{A} + {}^{154}\mathrm{Sm}$ system, $Z = 18$ and $\mu \approx 32\mathrm{u}$. At a center-of-mass energy of $E_{\mathrm{cm}} \approx \frac{1}{2}\mu v_0^2 \approx 114$ MeV, we have $v_0 \approx 0.085$. This leads to $(Z/10)^2 v_0 \approx 0.3$. Thus the criterion for the validity of the worldline approximation, Eq. (65), is satisfied to fair accuracy. This should be adequate for the order-of-magnitude estimates which we make.

If we replace the sum in Eq. (80) by an integral, $\sum_{l=0}^{\infty} \to \int_0^\infty dl$ then $S \to S_I$, where S_I may be expressed in terms of an incomplete gamma function:

$$S_I = \frac{2c_0}{3c^2\xi x_0^{2/3}}a^{-(5+3b)/(3c)}\Gamma\left(\frac{5+3b-3c}{3c}, a x_0^c\right). \tag{85}$$

If $a x_0^c \gg 1$, we have the asymptotic form

$$S_I \sim S_{IA} = \frac{2c_0}{3a^2c^2\xi}x_0^{1+b-2c}\mathrm{e}^{-a x_0^c}. \tag{86}$$

Now we wish to find the value of α which produces a value of σ which agrees with the experimental value, Eq. (67). This requires $S \approx 2.8$ at $E = 113.7$ MeV. The choices which arise from our best estimates of the nuclear parameters $\xi = 4.8 \times 10^{-4}$ and $x_0 = 6.0 \times 10^7$ lead to $\alpha \approx 0.27$. The result for α is only weakly sensitive to the values of ξ and x_0, and tend to lie in the range $0.25 \lesssim \alpha \lesssim 0.30$, with increases in either ξ or x_0 leading to smaller values for α. For example, $\xi = 10^{-4}$ and $x_0 = 10^7$ lead to $\alpha \approx 0.30$, while $\xi = 10^{-2}$ and $x_0 = 10^8$ lead to $\alpha \approx 0.25$. These results may be obtained from either the sum S or the integral form S_I, which agree very well with each other. Thus vacuum radiation pressure fluctuations with $\alpha \lesssim 0.3$ seem to be large enough to explain the observed cross section.

V. RADIATION PRESSURE FLUCTUATIONS ON A POLARIZABLE PARTICLE

In this section, we will consider the effects of vacuum radiation pressure fluctuations on an uncharged but electrically polarizable particle, such as an atom or a neutron. We will assume that the polarizability, α_0, is approximately independent of frequency. The Rayleigh scattering cross section for scattering of a monochromatic electromagnetic wave of angular frequency ω by such a particle is

$$\sigma_R = \frac{\alpha_0^2}{6\pi}\omega^4. \tag{87}$$

Thus we can write the force in the z direction on the particle as

$$f^z = \sigma_R(\mathbf{E}\times\mathbf{B})^z = \frac{\alpha_0^2}{6\pi}(\ddot{\mathbf{E}}\times\ddot{\mathbf{B}})^z. \tag{88}$$

We will assume that the vacuum fluctuations of this force arise from the fluctuations of the operator $(\ddot{\mathbf{E}}\times\ddot{\mathbf{B}})^z$. More precisely, they arise from the fluctuations of the time averaged operator

$$R^z = \int_{-\infty}^{\infty}(\ddot{\mathbf{E}}\times\ddot{\mathbf{B}})^z f(t)dt, \tag{89}$$

where the integrand is evaluated along the worldline of the particle. This operator is very similar to the operator S^z treated in Sec. II B, except for the additional time derivatives, which lead to $p = 7$ for R^z.

The dimensionless variable, x, in the probability distribution $P(x)$ for R^z is now $x = R^z\tau^8$. The asymptotic forms for $P(x)$ and for the cumulative distribution $P_>(x)$ have the forms in Eqs. (19) and (42), respectively.

TABLE III. Coefficients for the probability distribution of R^z.

α	c	b	a	c_0	$1+b-c$	$\ln(\frac{c_0}{ac})$
$\frac{1}{2}$	$\frac{1}{14}$	$-\frac{9}{7}$	2.27	8.86	$-\frac{5}{14}$	4.00
$\frac{1}{3}$	$\frac{1}{21}$	$-\frac{23}{21}$	2.18	319.	$-\frac{1}{7}$	8.03
$\frac{1}{4}$	$\frac{1}{28}$	-1	2.13	3784	$-\frac{1}{28}$	10.8

The numerical constants are determined as before, using $B_0 = 4$ and $B = 1/(6\pi^2)$, as for S^z, but now using $p = 7$. The results are displayed in Table III. Note that here $c = \alpha/7$, so $P(x)$ and $P_>(x)$ decrease very slowly with increasing x and hence increasing averaged force.

The criterion for the validity of the worldline approximation, Eq. (48), now becomes

$$xs^7 \lesssim 1, \tag{90}$$

where

$$s = \frac{r_0}{\tau}, \tag{91}$$

and $r_0 = \alpha_0^{\frac{1}{3}}$ is the characteristic size of the particle. Consider the situation treated in Sec. III A, where the particle can be pushed over a potential barrier by a vacuum force fluctuation. Here we find

$$x = \frac{3\pi m d^7}{\alpha_0^2 v_0^6} \approx \frac{10 m d^7}{r_0^6 v_0^6}, \tag{92}$$

and $s = v_0 r_0/d$. Hence $xs^7 = 10 m v_0 r_0$, and the worldline approximation is valid when

$$v_0 \lesssim \frac{1}{10 m r_0}. \tag{93}$$

This condition is difficult to satisfy for atoms. For the case of a hydrogen atom, for example, we would need $v_0 \lesssim 4 \times 10^{-7}$, or $E_0 \lesssim 8 \times 10^{-8}$ eV, which corresponds to a temperature below $0.1K$.

The case of the neutron seems more promising, which has a static electric polarizability of $\alpha_0 \approx 10^{-3}$ fm^3 [27–29], or a spatial size of $r_0 \approx 0.1$ fm. The validity of the worldline approximation requires $v_0 \lesssim 0.2$. Here we will give some estimates for the limiting case when $v_0 \approx 0.2$ and

$$x \approx 7.8 \times 10^{11} \left(\frac{d}{1\ \mathrm{fm}}\right)^7. \tag{94}$$

Here

$$G \approx 2\left(\frac{d}{1\ \mathrm{fm}}\right) \tag{95}$$

and F has the form in Eq. (43), with the coefficients given in Table III. As before, vacuum radiation pressure fluctuations dominate over quantum tunneling when $F < G$. For the case $\alpha = 1/2$, this begins to occur when $d \approx 80$ fm, so $F = G \approx 160$, so the rates due to both effects are very small. When $\alpha = 1/3$, we have $F = G$ at $d \approx 12.5$ fm, corresponding to $P_> = \mathrm{e}^{-12.5} \approx 3.7 \times 10^{-6}$. In the case $\alpha = 1/4$, we find that $F < G$ for all values of d, so the radiation pressure fluctuation effect dominates. For all values of $\alpha < 1$, for sufficiently large d, we have $F \propto d^\alpha$, and hence growing more slowly than G.

VI. SUMMARY AND DISCUSSION

In this paper, we have explored the hypothesis that large vacuum radiation pressure fluctuations can sometimes contribute noticeably to barrier penetration by quantum particles with energies below the maximum of the barrier. This barrier penetration is usually assumed to occur by quantum tunneling, the rate for which decreases exponentially with increasing barrier height or width. Our analysis is based upon recent results on the vacuum probability distributions for quantum stress tensor components averaged in time with a class of sampling function with compact support [16]. We argue that such functions, which vanish outside of a finite time interval, are more realistic descriptions of physical processes than are functions with tails extending into the infinite past and future. We also suggest that the choice of the sampling function should be determined by the details of the physical situation. Large vacuum radiation pressure fluctuations of the quantized electromagnetic field are described by a probability distribution which falls more slowly than exponentially, as an exponential of a fractional power of the sampled pressure. The relatively high probability of large vacuum radiation pressure fluctuations leads to the possibility that these fluctuations can temporarily give a particle enough energy to fly over the barrier classically. The probability of a large fluctuation increases with decreasing time duration of the sampling function, which measures the time required for the particle to traverse the barrier. Here we have studied the class of sampling functions reviewed in Sec. II A, which are described by the parameter α, which lies in the range $0 < \alpha < 1$. Smaller values of α are associated with a greater probability of large fluctuations. For nonrelativistic charged particles, the force exerted by radiation pressure is proportional to the Thompson cross section.

Some estimates for the rate of this process were given in Sec. III A. It was found that for sufficiently wide barriers, the vacuum radiation pressure effect can always dominate over usual quantum tunneling. Furthermore, for sufficiently large incident energies, and hence short sampling times, and for smaller values of α, the barrier penetration rate due to vacuum fluctuation may be large enough to be observable. In Sec. IV, we examined the possible role of vacuum radiation pressure fluctuations in nuclear fusion, especially heavy ion projectiles, where the observed fusion cross sections are much larger than predicted by simple barrier tunneling models. We find that radiation pressure fluctuations with $\alpha \lesssim 0.3$ could explain the observed cross sections.

In Sec. V, we turned to force fluctuations on electrically neutral, but polarizable, particles. Here the classical force is proportional to the Rayleigh scattering cross section and is proportional to the fourth power of the incident wave frequency. We argued that the quantum force fluctuations can be analyzed using the probability distribution for the time average of the operator $\ddot{\mathbf{E}} \times \ddot{\mathbf{B}}$, where $\mathbf{E}$ and $\mathbf{B}$ are the quantized electric and magnetic field operators, respectively. We find the asymptotic form of the probability distribution for this operator averaged with the same class of compactly supported sampling functions, and find that it falls even more slowly than does the distribution for averaged stress tensor components. We applied the result to barrier penetration by polarizable particles, using the neutron as an example. As in the case of charged particles, it is possible for vacuum force fluctuation effects to dominate over quantum tunneling.

In all cases, the effect is very sensitive to the details of the switching function, particularly to the value of the parameter α. This strong dependence is a new feature of the large vacuum fluctuations being treated in this paper, and does not appear when only the variance is considered, as was the case in earlier work [8,10,11]. Our view is that the functional form of the switching function should be determined by the details of the physical system being studied. Some progress in this direction has been made in the context of nonlinear optical models for lightcone fluctuations [3,4], where it was shown that the density profile of a slab of nonlinear material defines the relevant sampling function for electric field and squared electric field fluctuations. In the context of barrier penetration, we have conjectured in Sec. III B that a combination of the shape of the wave packet of the incident particle and the spatial dependence of the barrier potential may also define the relevant sampling function. However, it is not yet clear how to use this information to explicitly determine a value for α. This is a topic for future work. In the meantime, we may regard α as an undetermined phenomenological parameter which might be possible to determine by experiment.

ACKNOWLEDGMENTS

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