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$K \to \pi\nu\bar{\nu}$ in the MSSM in light of the ϵ'_K/ϵ_K anomaly

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The standard model (SM) prediction for the CP-violating quantity ϵ'_K/ϵ_K deviates from its measured value by 2.8σ. It has been shown that this tension can be resolved within the minimal supersymmetric standard model (MSSM) through gluino-squark box diagrams, even if squarks and gluinos are much heavier than 1 TeV. The rare decays $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ are similarly sensitive to very high mass scales and the first one also measures CP violation. In this article, we analyze the correlations between ϵ'_K/ϵ_K and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ within the MSSM aiming at an explanation of ϵ'_K/ϵ_K via gluino-squark box diagrams. The dominant MSSM contribution to the $K \to \pi\nu\bar{\nu}$ branching fractions stems from box diagrams with squarks, sleptons, charginos, and neutralinos, and the pattern of the correlations is different from the widely studied Z-penguin scenarios. This is interesting in light of future precision measurements by KOTO and NA62 at J-PARC and CERN, respectively. We find $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 2(1.2)$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 1.4(1.1)$, if all squark masses are above 1.5 TeV, gaugino masses obey GUT relations, and if one allows for a fine-tuning at the 1%(10%) level for the gluino mass. Larger values are possible for a tuned CP violating phase. Furthermore, the sign of the MSSM contribution to ϵ'_K imposes a strict correlation between $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and the hierarchy between the masses $m_{\bar{U}}$, $m_{\bar{D}}$ of the right-handed up-squark and down-squark: $\mathrm{sgn}[\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu})] = \mathrm{sgn}(m_{\bar{U}} - m_{\bar{D}})$.

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I. INTRODUCTION

Flavor-changing neutral current (FCNC) decays of K mesons are extremely sensitive to new physics (NP) and probe virtual effects of particles with masses far above the reach of future colliders, especially if the corresponding observable is CP violating. Prime examples of such observables are ϵ_K and ϵ'_K measuring indirect and direct CP violation in $K \to \pi\pi$ decays and also $K_L \to \pi^0\nu\bar{\nu}$. While indirect CP violation was already found in 1964 [1], it took 35 more years to establish a nonzero value of ϵ'_K in 1999 by the NA48 and KTeV Collaborations [2,3]:

$$\mathrm{Re}\frac{\epsilon'_K}{\epsilon_K}\bigg|_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4} \quad (\mathrm{PDG}). \tag{1}$$

Until recently, large theoretical uncertainties precluded reliable predictions for $\mathrm{Re}(\epsilon'_K/\epsilon_K)$. Calculating the hadronic matrix elements with the large-N_c (dual QCD) method one finds a standard model (SM) value well below the experimental range given in Eq. (1) [4]. A major breakthrough has been the recent lattice-QCD calculation of Ref. [5], which gives support to the large-N_c result. The current status is [6]

$$\frac{\epsilon'_K}{\epsilon_K}\bigg|_{\rm SM} = (1.06 \pm 5.07) \times 10^{-4}, \tag{2}$$

which is consistent with $(\epsilon'_K/\epsilon_K)_{\rm SM} = (1.9 \pm 4.5) \times 10^{-4}$ [7]. Both results are based on the lattice numbers in

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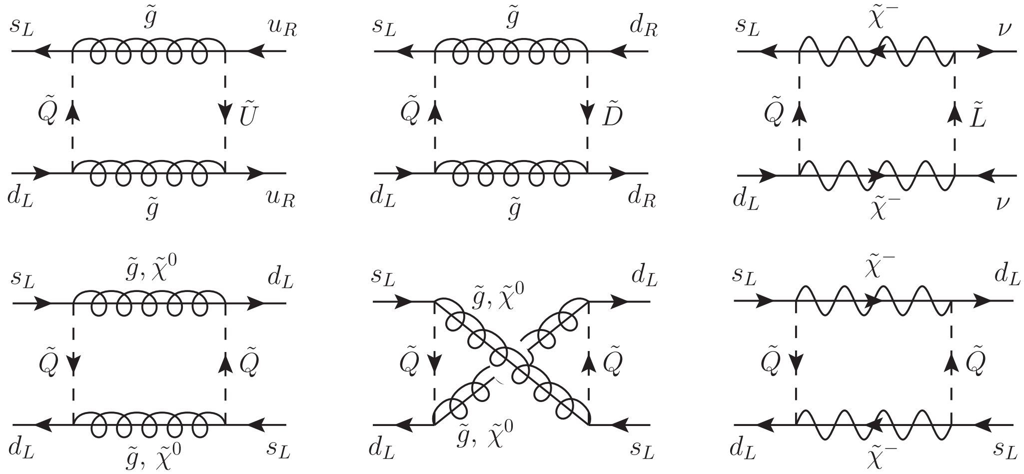

FIG. 1. Feynman diagrams of the dominant MSSM contributions to ϵ'_K, $K \to \pi\nu\bar{\nu}$, and ϵ_K in our scenario. $\tilde{Q}$ denotes a left-handed squark which is a down-strange mixture in our setup. $\tilde{U}$ ($\tilde{D}$) represents the right-handed up (down) squark. $\tilde{g}, \tilde{\chi}^0$, and $\tilde{\chi}^\pm$ stand for gluino, neutralino, and chargino, respectively, and $\tilde{L}$ denotes a charged slepton. *First row:* The first two box diagrams feed ϵ'_K through A_2 in Eq. (6) if $m_U \neq m_D$. The last diagram gives the ballpark of the MSSM contribution to $\mathcal{B}(K \to \pi\nu\bar{\nu})$. *Second row:* MSSM contributions to ϵ_K.

Refs. [5,8] and further use CP-conserving $K \to \pi\pi$ data to constrain some of the hadronic matrix elements involved. The SM prediction in Eq. (2) lies below the experimental value in Eq. (1) by 2.8σ.[1]

This tension can be explained by NP effects like Z' gauge bosons [10–14], models with modified Z-couplings [10,12,15,16], by a right-handed coupling of quarks to the W [17], within the littlest Higgs model [18], but also within the minimal supersymmetric standard model (MSSM) [19,20].

When pursuing such NP interpretations of the tension in ϵ'_K, it is natural to look for signatures in other $s \to d$ transitions which are, in general, correlated in UV complete models. To this end, the rare decays $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ play an important role. Within the SM, the branching ratios are predicted to be [21–23]

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (2.9 \pm 0.2 \pm 0.0) \times 10^{-11},$$
$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.3 \pm 0.3 \pm 0.3) \times 10^{-11}. \quad (3)$$

The first error summarizes the uncertainty from CKM parameters, the second one denotes the remaining theoretical uncertainties (in $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM}$, 0.04 is rounded to 0.0). The numbers in Eq. (3) are based on the best-fit result for the CKM parameters in Ref. [24]. Experimentally, we have [25]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm exp} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}, \quad (4)$$

and the 90% C.L. upper bound [26],

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm exp} \leq 2.6 \times 10^{-8}. \quad (5)$$

In the future, these measurements will be significantly improved. The NA62 experiment at CERN [27,28] is aiming to reach a precision of 10% compared to the SM already in 2018. In order to achieve 5% accuracy, more time is needed. Concerning $K_L \to \pi^0\nu\bar{\nu}$, the KOTO experiment at J-PARC aims in a first step at measuring $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ around the SM sensitivity [29,30]. Furthermore, the KOTO-step2 experiment will aim at 100 events for the SM branching ratio, implying a precision of 10% of this measurement [31].

In our MSSM scenario—in which the desired effect in ϵ'_K is generated via gluino-squark boxes [19]—correlations with $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ are not unexpected, since sizable box contributions also occur in these rare decays [32] (see Fig. 1). Reference [19] achieves sizable effects in ϵ'_K [33] together with a simultaneous efficient suppression of the supersymmetric QCD contributions to ϵ_K [34]. The suppression occurs because crossed and uncrossed gluino box diagrams cancel if the gluino mass is roughly 1.5 times the squark masses. With appropriately large left-left squark mixing angle and a CP phase, one can reconcile the measurements of ϵ_K and ΔM_K with the large value in Eq. (1) and squark and gluino masses in the multi-TeV range, so that there is no conflict with collider searches.

However, there is no such cancellation in the (dominant) chargino box contribution to $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ which permits potentially large effects.

[1]Calculations using chiral perturbation theory instead are consistent with both the measurement and Eq. (2), because they have larger errors [9].

This article is organized as follows: In the next section, we will review ϵ'_K/ϵ_K and $K \to \pi\nu\bar{\nu}$ within the MSSM. In Sec. III, we then perform the phenomenological analysis highlighting the correlations before we conclude in Sec. IV.

II. PRELIMINARIES

ϵ'_K/ϵ_K is given by [7]

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\rm exp}|{\rm Re}A_0^{\rm exp}}\left\{\frac{{\rm Im}A_2}{\omega_+} - (1-\hat{\Omega}_{\rm eff}){\rm Im}A_0\right\} \quad (6)$$

$\omega_+ = (4.53 \pm 0.02) \times 10^{-2}$, $|\epsilon_K^{\rm exp}| = (2.228 \pm 0.011) \times 10^{-3}$, $\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0) \times 10^{-2}$, and the amplitudes $A_I = \langle(\pi\pi)_I|\mathcal{H}^{|\Delta S|=1}|K^0\rangle$ involving the effective $|\Delta S| = 1$ Hamiltonian $\mathcal{H}^{|\Delta S|=1}$. Short-distance physics enters ${\rm Im}A_0$ and ${\rm Im}A_2$ through the Wilson coefficients in $\mathcal{H}^{|\Delta S|=1}$. The SM prediction of the renormalization-group (RG) improved Wilson coefficients is known to the next-to-leading order (NLO) of QCD and QED corrections [35] and the next-to-next-to-leading-order QCD calculation is underway [36]. Equation (2) is based on a novel analytic formula for the NLO RG evolution.

The Wilson coefficients multiply the four-quark operators Q_j whose hadronic matrix elements $\langle(\pi\pi)_I|Q_j|K^0\rangle$ must be calculated by nonperturbative methods. For some time these calculations for the matrix elements entering ${\rm Im}A_2$ are in good shape, thanks to precise results from lattice QCD [8]. However, ${\rm Im}A_0$ has become tractable with lattice QCD only recently [5].

CP-conserving data determine ${\rm Re}A_0$ and ω_+ in Eq. (6). ω_+ is essentially equal to the ratio ${\rm Re}A_2/{\rm Re}A_0$, except that it is calculated from charged rather than neutral kaon decays. The smallness of ω_+ encodes the famous "$\Delta I = 1/2$" rule ${\rm Re}A_0 \gg {\rm Re}A_2$. It leverages the ${\rm Im}A_2$ term in Eq. (6) and leads to the above-mentioned high sensitivity of ϵ'_K to new physics in this amplitude.

Following the approach of Ref. [19], we aim at explaining the discrepancy in ϵ'_K/ϵ_K with contributions to the Wilson coefficients $c'^{q}_{1,2}$. Therefore, we need the flavor (and CP) violation in the left-handed squark sector while the mass difference between the right-handed up- and down-squarks accounts for the necessary isospin violation.

The small errors in Eq. (3) show that the $K \to \pi\nu\bar{\nu}$ branching ratios are theoretically very clean. While $K_L \to \pi^0\nu\bar{\nu}$ is only sensitive to the CP violating part of the amplitude, $K^+ \to \pi^+\nu\bar{\nu}$ is dominated by the CP conserving part. In principle, many diagrams contribute to $K \to \pi\nu\bar{\nu}$ in the MSSM with generic sources of flavor violation [32]. However, since we are interested in a scenario with $s-d$ flavor violation in the left-handed squark sector, chargino-box contributions are numerically most important.

III. PHENOMENOLOGICAL ANALYSIS

Although the correlations between ϵ'_K/ϵ_K and $K \to \pi\nu\bar{\nu}$ in the MSSM have already been discussed in detail in Refs. [20,32,37,38], our study has several novelties. First of all, Refs. [32,37] were written before the appearance of the ϵ'_K anomaly, while we take into account the implication of the current deviation from the SM prediction. With the progress on the SM prediction, ϵ'_K implies a much sharper constraint on the MSSM parameters, resulting in tighter bounds on the deviations of $\mathcal{B}(K \to \pi\nu\bar{\nu})$ from the SM prediction. In addition, in our analysis we employ $m_{\bar{U}} \neq m_{\bar{D}}$ to generate large gluino box (*Trojan* penguin) [33] contributions to ϵ'_K, while Refs. [20,38] enhance ϵ'_K through Z penguins. Furthermore, we consider the latest LHC limits on the supersymmetric (SUSY) masses [39–42].

Defining the bilinear terms for the squarks as $M^2_{X,ij} = m^2_X(\delta_{ij} + \Delta_{X,ij})$ for $X = Q, \bar{U}, \bar{D}$, the numerically relevant parameters entering ϵ'_K, ϵ_K and $K \to \pi\nu\bar{\nu}$ in our analysis are

$$m_Q,\ |\Delta_{Q,12}|,\ \theta,\ M_3,\ M_2,\ M_1,\ m_{\bar{U}}/m_{\bar{D}},\ m_L. \quad (7)$$

Here m_Q is the universal mass parameter for the bilinear terms of the left-handed squarks which we define in the down-quark basis (i.e. the up-squark mass matrix is obtained via a CKM rotation from M^2_Q) .$\theta \equiv \arg(\Delta_{Q,12})$, M_3 is the gluino mass, M_2 (M_1) the wino (bino) mass, and m_L is the (universal) mass for the left-handed sleptons, respectively. The trilinear A-terms as well as the off-diagonal elements of the bilinear terms $\Delta_{X,ij}$ are set to 0 except for $\Delta_{Q,12}$ which generates the required flavor and CP violation in our setup. The values of the other (SUSY) parameters barely affect our results.[2]

The SUSY contribution to ϵ_K ($\epsilon_K^{\rm SUSY}$) and ΔM_K, originates from one-loop boxes with all possible combinations of gluinos, winos, and binos. For $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ we take into account all MSSM one-loop contributions [32]. However, numerically the chargino boxes turn out to be by far dominant in our setup. In ϵ'_K/ϵ_K, we include all SUSY QCD (SQCD) contributions as well as Z-penguin contributions originating from zchargino diagrams to the $I = 0$, 2 amplitudes with hadronic matrix elements evaluated at 1.3 GeV [6,19]. In the calculation of all contributions, we perform an exact diagonalization of the squark mass matrices.

In the SM contributions, we fix the relevant CKM elements to their best-fit values [24], in particular we set $V^*_{td}V_{ts} = (-3.22 + 1.41i) \times 10^{-4}$. In this way, we assume that the MSSM contributions to the standard unitarity-triangle analysis are small, so that the change in $V^*_{td}V_{ts}$ is

[2]We use the fixed values $\tan\beta = 10$, $\mu = M_A = m_Q$, $A_{ij} = 0$. We also fix $B_G = 1$, which parameterizes the matrix element of the chromomagnetic penguin operator Q_{8g}.

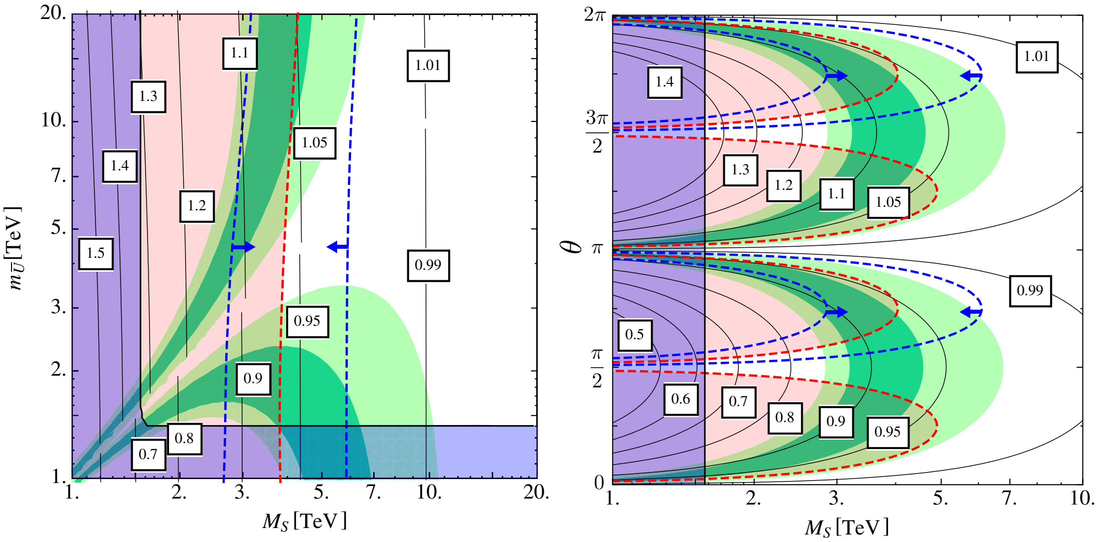

FIG. 2. Contours of $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0 \nu\bar{\nu})$. The ϵ'_K/ϵ_K discrepancy is resolved at the 1σ (2σ) level within the dark (light) green region. The red shaded region is excluded by ϵ_K at 95% C.L. using the inclusive value $|V_{cb}|$, while the region between the blue-dashed lines can explain the ϵ_K discrepancy which is present if the exclusive determination of V_{cb} is used [43]. The blue shaded region is excluded by the current LHC results from CMS and ATLAS [40–42]. $M_3/M_S = 1.5$, $m_L = 300$ GeV and GUT relations among gaugino masses are used. In the *left* plot, $\Delta_{Q,12} = 0.1\exp(-i\pi/4)$ for $m_{\bar U} > m_{\bar D} = m_Q = M_S$ (upper branch) and $\Delta_{Q,12} = 0.1\exp(i3\pi/4)$ for $m_{\bar U} < m_{\bar D} = m_Q = M_S$ (lower branch). In the *right* plot, $|\Delta_{Q,12}| = 0.1$ is used, $m_{\bar D} = 2m_{\bar U} = 2m_Q = 2M_S$ (for $0 < \theta < \pi$) and $m_{\bar U} = 2m_{\bar D} = 2m_Q = 2M_S$ (for $\pi < \theta < 2\pi$).

unimportant compared to the explicit MSSM contributions to ϵ'_K and $\mathcal{B}(K \to \pi\nu\bar{\nu})$. This is justified in typical MSSM scenarios with generic flavor violation.

First, we show a typical prediction for $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu})$ as a function of the squark masses in the left panel of Fig. 2. Here we assume universal diagonal elements for the left-handed and right-handed down squark mass matrices $M_S = m_Q = m_{\bar D}$ and use $m_L = 300$ GeV. We also choose $\Delta_{Q,12} = 0.1\exp(-i\pi/4)$ ($0.1\exp(i3\pi/4)$) for $m_{\bar U} > M_S$ ($m_{\bar U} < M_S$) regions to obtain a positive contribution to ϵ'_K. We impose $M_3/M_S = 1.5$ in order to obtain an efficient suppression of $\epsilon_K^{\rm SUSY}$ [19,34]. In addition, the GUT relation for M_2 and M_1 are imposed. The ϵ'_K/ϵ_K discrepancy between Eq. (1) and the second prediction in Eq. (2) is resolved at 1σ (2σ) within the dark (light) green region. The red shaded region is excluded by ϵ_K at 95% C.L. if the inclusive value of $|V_{cb}|$ is used, while the region between the blue-dashed lines can explain the ϵ_K discrepancy present if the exclusive determination of V_{cb} is used [43].[3] Note that $\theta = \pm\pi/4$ maximizes the effect in $\epsilon_K^{\rm SUSY}$, while the SUSY contributions to ϵ'_K/ϵ_K is maximized at $\theta = \pm\pi/2$ resulting instead in a vanishing effect in $\epsilon_K^{\rm SUSY}$. The blue shaded region is excluded by the current LHC results [40–42]. Here, in order to be conservative, we use the most stringent one, i.e. we maximize the bound which is a function of the neutralino mass. In this setup, we find that $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \simeq 1.05$–$1.1$ is predicted in light of the ϵ'_K/ϵ_K discrepancy (and the potential ϵ_K discrepancy) if $m_{\bar U} > m_{\bar D}$.

In the right panel of Fig. 2, the dependence on the *CP*-violating phase (θ) is shown. Here, we chose $|\Delta_{Q,12}| = 0.1$, and $m_{\bar D} = 2m_{\bar U} = 2m_Q = 2M_S$ ($m_{\bar U} = 2m_{\bar D} = 2m_Q = 2M_S$) for $0 < \theta < \pi$ ($\pi < \theta < 2\pi$). It can be seen that if θ is close $\pm\pi/2$, the constraint from ϵ_K is weakened while ϵ'_K as well as $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is enhanced.

Next, let us investigate upper and lower limits on $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$. In the following analysis, we fix the slepton mass close to the experimental limit ($m_L = 300$ GeV) [39] and use GUT relations among all three gaugino masses. Therefore, when one fixes the lightest squark mass, the relevant free parameters are only

$$|\Delta_{Q,12}|, \quad M_3, \quad m_{\bar U}/m_{\bar D}, \tag{8}$$

with $0 < |\Delta_{Q,12}| < 1$ and $0 < \theta < 2\pi$. In Fig. 3, the blue solid line encloses the maximally allowed region in the $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$–$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ plane (normalized by their SM values). The maximal values are obtained whenever the SUSY contributions to the $\Delta S = 2$ amplitude exactly cancel. The contour lines in the figures show the required value of M_3/M_S (imposing again GUT relations) for this cancellation. The maximal and minimal values for $\mathcal{B}(K \to \pi\nu\bar{\nu})$ are obtained by the decoupling one of the left-handed mixed down-strange squark while simultaneously

[3] The difference compared to Fig. 4 of Ref. [19] comes from $\Delta_{Q,13,23}$.

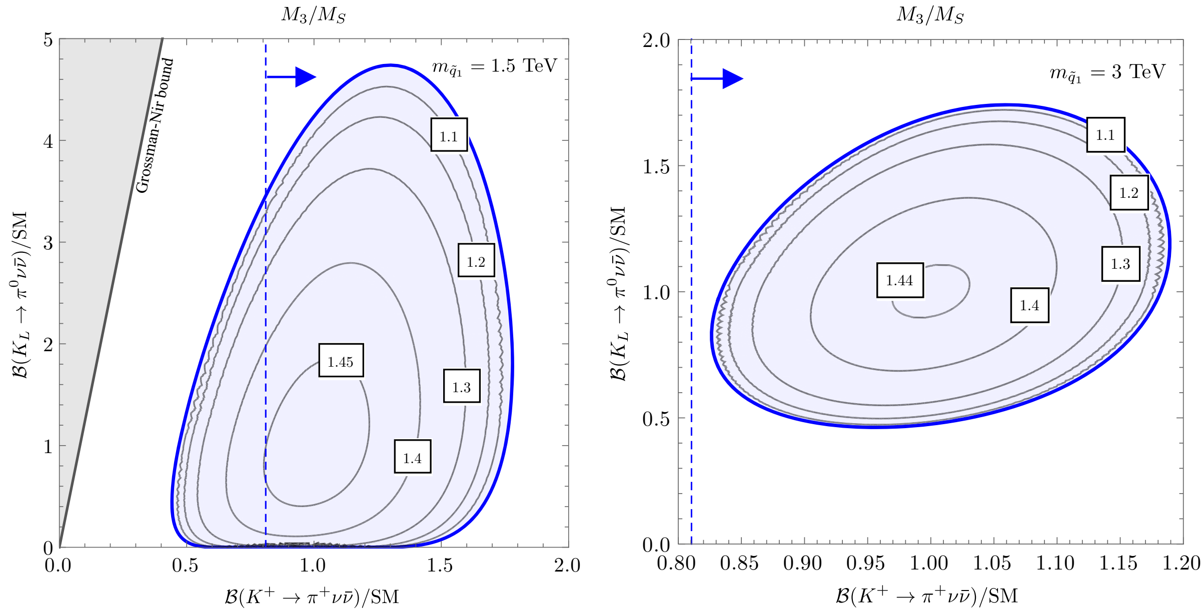

FIG. 3. Allowed region in the $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ plane. "SM" in the axis labels represents the corresponding value of the branching ratio within the SM. The contours show the values of M_3/M_S which is needed to cancel the SUSY contributions to ϵ_K. In the *left (right)* panel, the lightest squark mass is fixed at 1.5 (3) TeV. The gray shaded region is the Grossman-Nir bound [44]. The right sides of the blue dashed lines are the experimental result for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ given in Eq. (4).

maximizing their mixing. Since we assume equal diagonal entries of the bilinear terms this corresponds to the limit $m_Q \to \infty$ and $|\Delta_{Q,12}| \to 1$ which implies one light squark which is an equal admixture of the first and second generation of interaction eigenstates. Note that these results are independent of $m_{\bar{U}}/m_{\bar{D}}$, but $m_{\bar{U}}/m_{\bar{D}}$ is important when considering the correlation with ϵ'_K/ϵ_K. In the left and right panels, the lightest squark mass is fixed to 1.5 TeV and 3 TeV, respectively. The latest searches for first-generation squarks at the LHC imply $m_{\tilde{q}_1} \gtrsim 1.4$ TeV if the gluino is heavy and the neutralino is light [41,42]. We find that the upper allowed values for the branching ratios differ significantly from the SM predictions. However, in order to achieve these maximal values, severe fine-tuning of the gluino mass (with respect to the squark masses) or tuning of the CP violating phase is necessary: e.g. around $\theta = 3\pi/2$, ϵ_K^{SUSY} is much suppressed while $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is enhanced.

Let us now investigate the degree of fine-tuning of the gluino mass needed to suppress ϵ_K^{SUSY}. In Fig. 4, the necessary amount of the fine-tuning in the gluino mass with respect to the value for the exact cancellation is shown, again in the $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ plane like Fig. 3. In the light (dark) blue regions, the amount of fine-tuning is milder than 1% (10%), respectively while in the regions outside more sever fine-tuning is required in order to satisfy constraints from ϵ_K (using the inclusive $|V_{cb}|$ [43]) and ΔM_K at the 2σ level. This means that the gluino mass can be shifted from its value necessary for an exact cancellation in ϵ_K and ΔM_K (given by the contours in Fig. 3) by 1% (~20 GeV) and 10% (~200 GeV) without violating the constraints. Here we have scanned over all values of the CP violating phase θ. Alternatively, one can satisfy ϵ_K by tuning θ to values different from $\pm 90°$ by at most 0.9° (9°). In our plot, we have discarded such a tuned CP phase, which explains the white dents in the blue regions around $\theta = \pm 90°$. However, in the case of a tuned θ the correlation between the two $K \to \pi\nu\bar{\nu}$ branching ratios is stronger. The red contour show the SUSY contributions to ϵ'_K/ϵ_K and the current ϵ'_K/ϵ_K discrepancy is resolved at 1σ (2σ) within the dark (light) green region. The black dashed lines indicate the shifts of the boundaries of the green regions when the gluino is taken to be 10% heavier than in Fig. 3. The lightest squark mass is fixed to 1.5 TeV. In the left (right) panel, we used $m_{\bar{D}}/m_{\bar{U}} = 1.1\,(2)$ with $m_{\bar{U}} = m_Q$ for $0 < \theta < \pi$, and $m_{\bar{U}}/m_{\bar{D}} = 1.1\,(2)$ with $m_{\bar{D}} = m_Q$ for $\pi < \theta < 2\pi$. The same results are depicted in Fig. 5 but for a lightest squark mass of 3 TeV, and $m_{\bar{D}}/m_{\bar{U}} = 1.5\,(2)$ with $m_{\bar{U}} = m_Q$ is used for $0 < \theta < \pi$, or $m_{\bar{U}}/m_{\bar{D}} = 1.5\,(2)$ with $m_{\bar{D}} = m_Q$, in the left (right) panel.

Comparing Fig. 4 to Fig. 5 we can see that if $m_{\bar{U}}/m_{\bar{D}}$ (or $m_{\bar{D}}/m_{\bar{U}}$) differs more strongly from 1, $|\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\text{SM}}(K_L \to \pi^0\nu\bar{\nu})|$ is predicted to be smaller in light of the ϵ'_K/ϵ_K discrepancy. Figs. 4 and 5 also illustrate an important finding: There is a strict correlation between $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $m_{\bar{U}}/m_{\bar{D}}$: $\text{sgn}(\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\text{SM}}(K_L \to \pi^0\nu\bar{\nu})) = \text{sgn}(m_{\bar{U}} - m_{\bar{D}})$. This finding is easily understood by recalling that $\text{sgn}(m_{\bar{U}} - m_{\bar{D}})$ determines whether we must choose the CP phase θ between 0 and π or instead between π and 2π to generate the desired positive

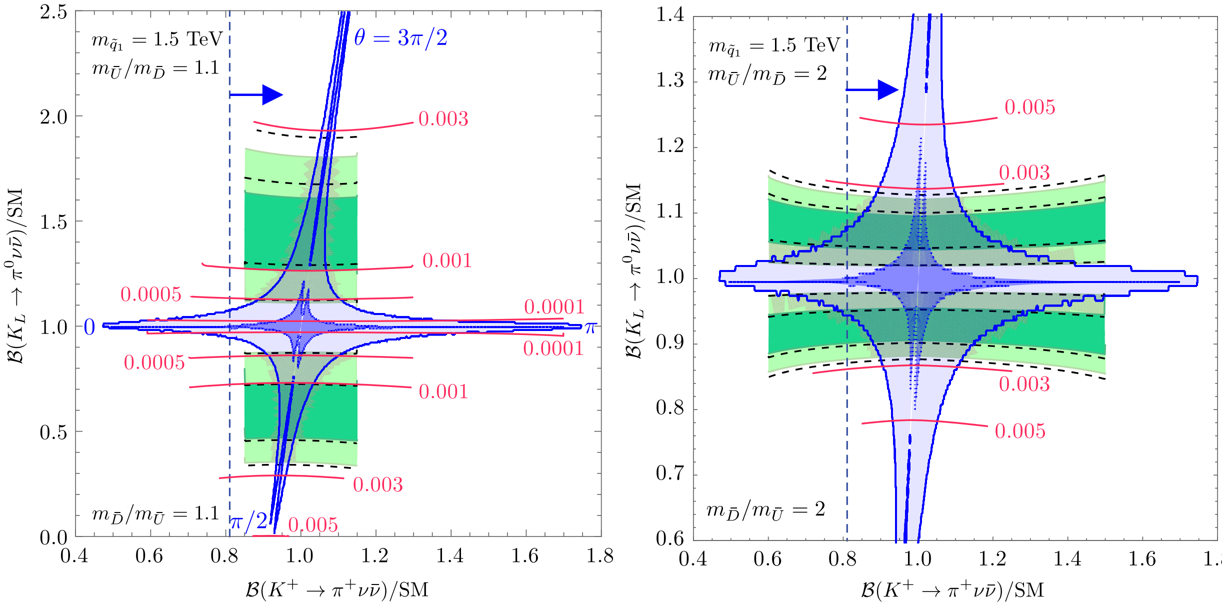

FIG. 4. The light (dark) blue region requires a milder parameter fine-tuning than 1% (10%) of the gluino mass compared to the value of Fig. 3 and a milder parameter tuning than 1% (10%) of the deviation of the CP violating phase from $\pm\pi/2$. The plot suggests that by tuning the CP violating phase θ close to $\pm\pi/2$ one can relax the fine-tuning of the gluino mass and still amplify the branching ratios. The red contour represents the SUSY contributions to ϵ'_K/ϵ_K, and the ϵ'_K/ϵ_K discrepancy is resolved at 1σ (2σ) within the dark (light) green region. The black dashed lines show the projected shifts of the boundaries of the green regions when the gluino is assumed to be 10% heavier. The lightest squark mass is fixed to 1.5 TeV. In the *left* panel, $m_{\bar{D}}/m_{\bar{U}} = 1.1$ ($m_{\bar{U}}/m_{\bar{D}} = 1.1$) is used for $0 < \theta < \pi$ ($\pi < \theta < 2\pi$) to obtain a positive SUSY contribution to ϵ'_K/ϵ_K. While, $m_{\bar{D}}/m_{\bar{U}} = 2$ ($m_{\bar{U}}/m_{\bar{D}} = 2$) is used for $0 < \theta < \pi$ ($\pi < \theta < 2\pi$) in the *right* panel. The region on the right side of the blue dashed lines are allowed by the current experimental measurements [given in Eq. (4)].

contribution to ϵ'_K. Now the sign of the MSSM contribution to $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ depends on the CP phase in the same way, but there is no explicit dependence of $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ on $m_{\bar{U},\bar{D}}$. The shape of the blue regions in Figs. 4 and 5 is a generic feature of NP models with FCNC transitions only among left-handed quarks and stems from the constraint of ϵ_K on the new CP phases [45].

Numerically, we observed $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 2(1.2)$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 1.4(1.1)$ in light of ϵ'_K/ϵ_K discrepancy, if all squark are

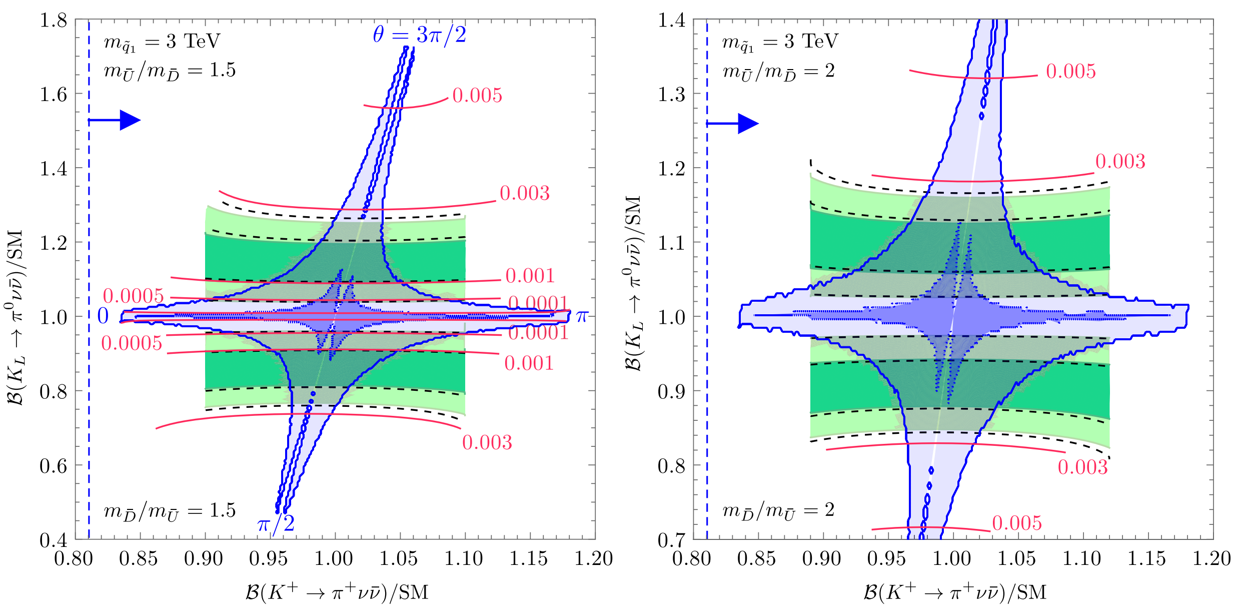

FIG. 5. Same as Fig. 4 but for $m_{\tilde{q}_1} = 3$ TeV, $m_{\bar{D}}/m_{\bar{U}}$ (or $m_{\bar{U}}/m_{\bar{D}}$) = 1.5 (*left* panel) and $m_{\bar{D}}/m_{\bar{U}}$ (or $m_{\bar{U}}/m_{\bar{D}}$) = 2 (*right*).

heavier than 1.5 TeV and if a 1% (10%) fine-tuning is permitted. Here and hereafter, the quoted fine-tuning corresponds to the fine-tuning of the gluino mass or, alternatively, the tuning of the *CP* violating phase. Similarly, $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 1.1$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 1.02$ are predicted, if all squark masses are above 3 TeV with a 10% fine-tuning.

Note that if $m_{\bar{U}}/m_{\bar{D}}$ is close to 1, the Trojan penguin contribution from the SUSY QCD box diagrams are suppressed and the gluino contribution to the chromomagnetic operator entering ϵ'_K/ϵ_K becomes dominant: for $m_{\bar{U}}/m_{\bar{D}} = 1.05$ (1.02), 25% (50%) of the SUSY contribution comes from the chromomagnetic operator for $m_{\tilde{q}_1} = 1.5$ TeV and larger values of $|\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu})|$ are predicted. However, it is shown that such a case always requires fine-tuning at the 1% level.

IV. DISCUSSION AND CONCLUSIONS

In this article, we have studied the correlations between ϵ_K, ϵ'_K, $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ in detail within the MSSM. In order to accommodate the ϵ'_K/ϵ_K anomaly, we generate isospin violation by a mass splitting between right-handed up and down-squark and flavor as well as *CP* violating by off-diagonal elements in the left-handed bilinear squark mass terms.

We find strong correlations between these observables depending (to a very good approximation) only on m_Q, $|\Delta_{Q,12}|, \theta, M_3, M_2, m_{\bar{U}}/m_{\bar{D}}, m_L$. In particular, we find the following prediction: $\mathrm{sgn}(\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu})) = \mathrm{sgn}(m_{\bar{U}} - m_{\bar{D}})$. This is in contrast to generic Z' models where couplings to leptons are in general free parameters, decoupling ϵ'_K/ϵ_K from $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$.

We show that $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is expected to be shifted with respect to the SM value by 5%–10% within the typical parameter region of our scenario. Even a larger shift is possible if one allows for fine tuning: $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 2(1.2)$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 1.4\,(1.1)$ for a fine-tuning at the 1% (10%) level.

It is also clearly shown that our scenario can be distinguished from those with dominant *Z*-penguins. In the latter scenarios, the *Z*-penguin contributions to ϵ'_K is proportional to $(\mathrm{Im}\Delta_L + 3.3\mathrm{Im}\Delta_R)$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) - \mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu})$ is proportional to $-(\mathrm{Im}\Delta_L + \mathrm{Im}\Delta_R)$. Therefore, a suppression of the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$ (numerically $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})/\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 0.7$ [15]) is in general predicted if there is no cancellation between $\mathrm{Im}\Delta_L$ and $\mathrm{Im}\Delta_R$ [10]. Here, $\Delta_{L(R)}$ denotes the effective coupling of $\bar{s}\gamma_\mu P_{L(R)} dZ^\mu$ originating from NP interactions. This means that an accurate measurement of $K_L \to \pi^0\nu\bar{\nu}$ would be able to distinguish these scenarios.

For our analysis, we assume GUT relations among the gauginos. Relaxing this assumption allows for larger, but less correlated, effects in $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$. Such an analysis together with a presentation of the complete analytic expressions for ϵ'_K/ϵ_K, $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ will be presented in a forthcoming article.

Finally, we discuss the Higgs boson mass within the MSSM. In our phenomenological analysis ($\tan\beta = 10$ and $A_{ij} = 0$), the Higgs boson mass of 125 GeV can be achieved for stop masses around 5 TeV [46]. To accommodate for the measured Higgs mass with lighter stops we have checked that one can choose large diagonal trilinear A_{ii} terms (defined with the Yukawa couplings factored out) without relevant effect on the studied observables. In particular, diagonal A_{ii} terms neither generate sizable *Z*-penguins nor effects in nucleon EDM. Furthermore, promoting the MSSM to the NMSSM or adding additional *D*-term contributions to the Higgs boson mass would leave our analysis unchanged. (The patterns of flavor observables in the MSSM and NMSSM are essentially identical.) Therefore, one can account for the measured value of 125 GeV for the light Higgs boson mass within our setup.

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