Loop suppressed light fermion masses with $U(1)_R$ gauge symmetry

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(Received 18 April 2017; published 14 July 2017)

We propose a model with a two-Higgs doublet, where quark and charged-lepton masses in the first and second families are induced at one-loop level, and neutrino masses are induced at the two-loop level. In our model, we introduce an extra $U(1)_R$ gauge symmetry that plays a crucial role in achieving desired terms in no conflict with anomaly cancellation. We show the mechanism to generate fermion masses, the resultant mass matrices, and Yukawa interactions in mass eigenstates, and we discuss several interesting phenomenologies such as the muon anomalous magnetic dipole moment and the dark matter candidate that arise from this model.

DOI: 10.1103/PhysRevD.96.015016

I. INTRODUCTION

Radiatively induced mass scenarios have been applied widely and successfully to various models as theories at the low-energy scale (~TeV) that induce masses of light fermions such as neutrinos, include the dark matter (DM) candidate, and explain the muon anomalous magnetic dipole moment (muon $g-2$) without conflicts with various constraints such as flavor changing neutral currents (FCNCs), lepton flavor violations (LFVs), and quark and lepton masses and their mixings. Thus, many authors have been working with these ideas. Here, we classify such radiative models as the number of loops; i.e., Refs. [1–90] mainly focus on the scenarios at the one-loop level, Refs. [91–125] concentrate at the two-loop level, and Refs. [126–128] discuss the systematic analysis of (Dirac) neutrino oscillation, charged lepton flavor violation, and collider physics in the framework of the neutrinophilic and inert two-Higgs-doublet model (THDM), respectively.

One of the mysteries in the standard model (SM) is the hierarchical structure of fermion masses in both the quark and lepton sectors, which indicates the large hierarchy of the Yukawa coupling constants. In particular, masses of the SM neutrinos are very small compared to the other fermion masses. It is, thus, challenging to understand the hierarchical structure of fermion masses applying a scenario of radiatively induced mass; some attempts to resolve flavor hierarchies in THDM are found, for example, in Refs. [129–132].

In this paper, we propose a new type of THDM scenario that can explain the small fermion masses in the SM, *i.e.*, the first and second families in the quark and charged lepton sectors, and the tiny masses of active neutrinos, by applying a radiatively induced mass mechanism. Here, the second isospin doublet Higgs has small vacuum expectation value (VEV), which provides such lighter fermion masses in the first and second families, while the SM-like Higgs provides the mass of third family fermions in the SM—the top quark, bottom quark, and tauon. To realize such a small VEV and family dependence, we impose a $U(1)_R$ gauge symmetry in a family-dependent way and introduce extra scalar fields with $U(1)_R$ charges. Then the VEV of the second Higgs doublet is induced at the one-loop level, which could be an appropriate reason for the smallness due to the loop suppression. In addition, active neutrino masses are induced at the two-loop level with the canonical seesaw mechanism. As a bonus to introducing the extra scalars, we can also explain the muon $g-2$ and obtain a dark matter candidate, as is often the case with radiatively induced mass models.

This paper is organized as follows. In Sec. II, we show our model, establish the quark and lepton sector, and derive the analytical forms of FCNCs, LFVs, and muon anomalous magnetic dipole moment. We conclude and discuss in Sec. III.

II. MODEL SETUP

In this section, we introduce our model, analyze mass matrices in the quark and lepton sector, and discuss some phenomenologies. First of all, we impose an additional $U(1)_R$ gauge symmetry, where only the first and second families of right-handed SM fermions and N_R have nonzero charge x and N_R constitutes the Majorana field after the spontaneous $U(1)_R$ gauge symmetry breaking. All of the fermion contents and their assignments are summarized in Table I, in which $i = 1, 2$ and $\alpha = 1-3$ represent the number of the family. Notice here that the number of the family for N_R is two, since the anomaly arising from the $U(1)_R$ gauge symmetry cancels out in each of one generation [73,121].

For the scalar sector with nonzero VEVs, we introduce two $SU(2)_L$ doublet scalars Φ_1 and Φ_2, and two $SU(2)_L$ singlet scalars φ_1 and φ_2 which are charged under $U(1)_R$. Here, Φ_1 is supposed to be the SM-like Higgs doublet, while Φ_2 is the additional Higgs doublet with tiny VEV and has

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nonzero $U(1)_R$ charge. For SM singlet scalars, φ_1 plays a role in inducing the tiny VEV of Φ_2 at the one-loop level, and φ_2 provides the Majorana fermions N_R after the spontaneous $U(1)_R$ breaking. On the other hand, $SU(2)_L$ singlet scalars S, χ, and doublet scalar η are inert scalars because of odd parity under the Z_2, and they play a role in generating the tiny VEV of Φ_2 by running inside a loop diagram. In addition, the lightest state of these neutral scalars can be a dark matter candidate [28]. All of the scalar contents and their assignments are summarized in Table II, where we assume S to be a real field for simplicity. We also note that massive Z' boson appears after $U(1)_R$ symmetry breaking. In this paper, we omit detailed analysis for phenomenology of Z' and just assume mass of Z' is sufficiently heavy to avoid constraints from collider experiments.

TABLE I. Field contents of fermions and their charge assignments under $SU(2)_L \times U(1)_Y \times U(1)_R \times Z_2$, where each of the flavor index is defined as $\alpha \equiv 1-3$ and $i = 1, 2$.

	Quarks					Leptons			
Fermions	Q_L^α	u_R^i	d_R^i	t_R	b_R	L_L^α	e_R^i	N_R^i	τ_R
$SU(3)_C$	**3**	**3**	**3**	**3**	**3**	**1**	**1**	**1**	**1**
$SU(2)_L$	**2**	**1**	**1**	**1**	**1**	**2**	**1**	**1**	**1**
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	-1
$U(1)_R$	0	x	$-x$	0	0	0	$-x$	x	0
Z_2	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

TABLE II. Boson sector, where all the bosons are $SU(3)_C$ singlet.

	VEV $\neq 0$				Inert		
Bosons	Φ_1	Φ_2	φ_1	φ_2	η	S	χ
$SU(2)_L$	**2**	**2**	**1**	**1**	**2**	**1**	**1**
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
$U(1)_R$	0	x	$\frac{x}{3}$	$2x$	$\frac{x}{3}$	0	$\frac{x}{3}$
Z_2	$+$	$+$	$+$	$+$	$-$	$-$	$-$

A. Yukawa interactions and scalar sector

Yukawa Lagrangian: Under our fields and symmetries, the renormalizable Lagrangians for the quark and lepton sector are given by

$$-\mathcal{L}_Q = (y_u)_{\alpha j}\bar{Q}_{L_\alpha} u_{R_j}\tilde{\Phi}_2 + (y_d)_{\alpha j}\bar{Q}_{L_\alpha}\Phi_2 d_{R_j} + (y_t)_{\alpha 3}\bar{Q}_{L_\alpha} t_{R_3}\tilde{\Phi}_1 + (y_b)_{\alpha 3}\bar{Q}_{L_\alpha}\Phi_1 b_{R_3} + \text{c.c.}, \tag{2.1}$$

$$-\mathcal{L}_L = (y_\nu)_{\alpha j}\bar{L}_{L_\alpha} N_{R_j}\tilde{\Phi}_2 + (y_\ell)_{\alpha j}\bar{L}_{L_\alpha}\Phi_2 e_{R_j} + (y_\tau)_{\alpha 3}\bar{L}_{L_\alpha} e_{R_3}\tilde{\Phi}_1 + (y_N)_{ii}\bar{N}_{R_i} N_{R_i}^C \varphi_2 + \text{c.c.}, \tag{2.2}$$

where $\tilde{\Phi}_{1,2} \equiv (i\sigma_2)\Phi_{1,2}^*$ with σ_2 being the second Pauli matrix. Here, we note that the SM-like Higgs doublet Φ_1 only couples to third-family right-handed fermions while Φ_2 couples first and second families right-handed fermions because of the gauge invariance under $U(1)_R$.

Scalar potential: In our model, scalar potential is given by

$$\begin{aligned}
V = &\sum_{a=1-2}(\mu_{\varphi_a}^2|\varphi_a|^2) + \mu_S^2 S^2 + \mu_\chi^2|\chi|^2 + \mu_\eta^2|\eta|^2 + \lambda_0[(\Phi_2^\dagger\eta)\chi\varphi_1 + \text{c.c.}] + \lambda_0'[(\Phi_1^\dagger\eta)S\varphi_1^* + \text{c.c.}] \\
&+ \mu(\chi S\varphi_1^* + \text{c.c.}) + \sum_{\mathrm{a}=1-2}(\lambda_{\varphi_\mathrm{a}}|\varphi_\mathrm{a}|^4 + \lambda_{\varphi_\mathrm{a}S}|\varphi_\mathrm{a}|^2 S^2 + \lambda_{\varphi_\mathrm{a}\chi}|\varphi_\mathrm{a}|^2|\chi|^2 + \lambda_{\varphi_\mathrm{a}\eta}|\varphi_\mathrm{a}|^2|\eta|^2) \\
&+ \lambda_S S^4 + \lambda_\chi|\chi|^4 + \lambda_\eta|\eta|^4 + \lambda_{S\chi}S^2|\chi|^2 + \lambda_{S\eta}S^2|\eta|^2 + \lambda_{\chi\eta}|\chi|^2|\eta|^2 \\
&+ \sum_{i=1,2}\left[\sum_{a=1,2}(\lambda_{\varphi_a\Phi_i}|\varphi_a|^2|\Phi_i|^2) + \lambda_{S\Phi_i}S^2|\Phi_i|^2 + \lambda_{\chi\Phi_i}|\chi|^2|\Phi_i|^2 + \lambda_{\Phi_i\eta}|\Phi_i|^2|\eta|^2 + \lambda_{\Phi_i\eta}'|\Phi_i^\dagger\eta|^2\right] \\
&+ \mu_{11}^2|\Phi_1|^2 + \mu_{22}^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2,
\end{aligned} \tag{2.3}$$

where we choose some parameters in the potential so that $\langle\Phi_2\rangle \equiv v_2/\sqrt{2} = 0$ at the tree level. After the spontaneous $U(1)_R$ symmetry breaking, the effective mass term $\mu_{12}^2\Phi_2^\dagger\Phi_1$ is given via Eq. (2.3), and μ_{12} is given by

$$\mu_{12}^2 = -\frac{\lambda_0\lambda_0'\mu v_{\varphi_1}^3}{\sqrt{2}(4\pi)^2}\frac{m_\chi^2 m_S^2\ln[\frac{m_\chi}{m_S}] + m_\eta^2 m_S^2\ln[\frac{m_S}{m_\eta}] + m_\chi^2 m_\eta^2\ln[\frac{m_\eta}{m_\chi}]}{(m_\chi^2 - m_S^2)(m_\chi^2 - m_\eta^2)(m_S^2 - m_\eta^2)}, \tag{2.4}$$

$$m_\chi^2 = \mu_\chi^2 + \frac{\lambda_{\chi\Phi_1}}{2}v_1^2 + \frac{\lambda_{\chi\Phi_2}}{2}v_2^2 + \frac{\lambda_{\varphi_1\chi}}{2}v_{\varphi_1}^2 + \frac{\lambda_{\varphi_2\chi}}{2}v_{\varphi_2}^2, \tag{2.5}$$

$$m_S^2 = \mu_S^2 + \frac{\lambda_{S\Phi_1}}{2}v_1^2 + \frac{\lambda_{S\Phi_2}}{2}v_2^2 + \frac{\lambda_{\varphi_1 S}}{2}v_{\varphi_1}^2 + \frac{\lambda_{\varphi_2 S}}{2}v_{\varphi_2}^2, \tag{2.6}$$

$$m_\eta^2 = \mu_\eta^2 + \frac{\lambda_{\Phi_1\eta}}{2}v_1^2 + \frac{\lambda_{\Phi_2\eta}}{2}v_2^2 + \frac{\lambda_{\Phi_1\eta}'}{2}v_1^2 + \frac{\lambda_{\Phi_2\eta}'}{2}v_2^2 + \frac{\lambda_{\varphi_1\eta}}{2}v_{\varphi_1}^2 + \frac{\lambda_{\varphi_2\eta}}{2}v_{\varphi_2}^2, \tag{2.7}$$

where $\langle \varphi_i \rangle \equiv v_{\varphi_i}/\sqrt{2}\,(i = 1-2)$ and $v_2 \neq 0$. The resultant scalar potential in the THDM sector is given by

$$V_{\rm THDM} = \mu_{12}^2(\Phi_1^\dagger \Phi_2 + {\rm c.c.}) + \mu_{11}^2|\Phi_1|^2 + \mu_{22}^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2, \tag{2.8}$$

where $\langle \Phi_i \rangle \equiv v_i/\sqrt{2}$ $(i = 1-2)$ and we chose that μ_{12}^2 is negative and μ_{11}^2 is positive. Taking $v_2/v_1 \ll 1$ and $v_2/v_S \ll 1$,[1] we finally obtain the formula of Φ_2 VEV as

$$v_2 \approx \frac{2v_1\mu_{12}^2}{2\mu_{22}^2 + v_1^2(\lambda_3+\lambda_4) + v_{\varphi_1}^2\lambda_{\varphi_1\Phi_2} + v_{\varphi_2}^2\lambda_{\varphi_2\Phi_2}}. \tag{2.9}$$

Notice that our THD potential Eq. (2.8) is that of THDM which has a softly broken Z_2 symmetry and no $\lambda_5[(\Phi_1^\dagger\Phi_2) + {\rm H.c.}]$ term [133].

Including their VEVs, the scalar fields are parametrized as

$$\Phi_i = \begin{bmatrix} h_1{}^+ \\ \frac{v_1+h_1+ia_1}{\sqrt{2}} \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} h_2^+ \\ \frac{v_2+h_2+ia_2}{\sqrt{2}} \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta^+ \\ \frac{\eta_R+i\eta_I}{\sqrt{2}} \end{bmatrix}, \tag{2.10}$$

$$\varphi_a = \frac{v_{\varphi_a} + \varphi_{R_a} + i\varphi_{I_a}}{\sqrt{2}}, \quad (a = 1, 2),$$
$$\chi = \frac{\chi_R + i\chi_I}{\sqrt{2}}, \qquad S = \frac{s_R}{\sqrt{2}}, \tag{2.11}$$

where φ_{I_a} does not have nonzero mass eigenvalues, and either of them is absorbed by the longitudinal degrees of freedom of Z' gauge boson.[2] After the spontaneous symmetry breaking, neutral bosons mix each other and their mass eigenstates and eigenvalues are defined by:

$$\begin{aligned} {\rm Diag}(m_{H_1^0}^2, m_{H_2^0}^2, m_{H_3^0}^2, m_{H_4^0}^2) &= O_H m^2(\varphi_{R_1}, \varphi_{R_2}, h_1, h_2) O_H^T, \\ {\rm Diag}(m_{G^0}^2, m_{A^0}^2) &= O_C m^2(a_1, a_2) O_C^T, \\ {\rm Diag}(m_{\omega^\pm}^2, m_{H^\pm}^2) &= O_C m^2(h_1^\pm, h_2^\pm) O_C^T, \\ {\rm Diag}(m_{\eta_{R_1}}^2, m_{\eta_{R_2}}^2, m_{\eta_{R_3}}^2) &= O_R m^2(\eta_R, s_R, \chi_R) O_R^T, \\ {\rm Diag}(m_{\eta_{I_1}}^2, m_{\eta_{I_2}}^2) &= O_I m^2(\eta_I, \chi_I) O_I^T, \end{aligned} \tag{2.12}$$

where $O_{H,C,R,I}$ denotes the mixing matrices which diagonalize the mass matrices accordingly. Here, G^0 and $\omega^\pm$ do not have nonzero mass eigenvalue, and they are absorbed by the longitudinal degrees of freedom of neutral SM gauge boson Z and charged gauge boson $W^\pm$ respectively as Numbu-Goldstone (NG) bosons. The mass matrices in the right-hand side of Eq. (2.12) are given by the parameters in scalar potential. For neutral CP-even components we obtain

$$m^2(\varphi_{R_1}, \varphi_{R_2}, h_1, h_2) = \begin{bmatrix} 2v_{\varphi_1}^2\lambda_{\varphi_1} & & & \\ 0 & 2v_{\varphi_2}^2\lambda_{\varphi_2} & & \\ v_1 v_{\varphi_1}\lambda_{\varphi_1\Phi_1} & v_1 v_{\varphi_2}\lambda_{\varphi_2\Phi_1} & v_1^2\lambda_1 - \frac{v_2\mu_{12}^2}{v_1} & \\ v_2 v_{\varphi_1}\lambda_{\varphi_1\Phi_2} & v_2 v_{\varphi_2}\lambda_{\varphi_2\Phi_2} & v_1v_2(\lambda_3+\lambda_4) + \mu_{12}^2 & v_2^2\lambda_2 - \frac{v_1\mu_{12}^2}{v_2} \end{bmatrix}, \tag{2.13}$$

where the matrix has symmetric structure. We also obtain the mass matrices for CP-odd and charged components as

$$m^2(a_1, a_2) = \begin{bmatrix} -\frac{v_2\mu_{12}^2}{v_1} & \mu_{12}^2 \\ \mu_{12}^2 & -\frac{v_1\mu_{12}^2}{v_2} \end{bmatrix}, \qquad m_{A^0}^2 = -\frac{(v_1^2+v_2^2)\mu_{12}^2}{v_1v_2}, \tag{2.14}$$

[1] To achieve it, one has to assume $0 < 2\mu_{22}^2 + \lambda_{22}v_2^2 - (\lambda_3+\lambda_4)v_1^2$, arising from the tadpole condition: $\frac{\partial V_{\rm THDM}}{\partial \Phi_2}\big|_{v_1,v_2} = 0$.

[2] A physical massless boson at the tree level seems to underlie our model. And it will be severely constrained by non-Newtonian forces, if its mass is extremely tiny compared to 1 eV [134]. However since its vanishing mass originates from an accidental global symmetry after all the gauge symmetry breaking, it can always be massive at higher-dimensional operators [135]. In our case, for example, five-dimensional operators, $\frac{1}{M_{pl}}(\Phi_2^\dagger\Phi_1)\varphi_1^3$ and $\frac{1}{M_{pl}}(\Phi_2^\dagger\Phi_1)^2\varphi_2$ that retain all the gauge symmetries, violate the accidental symmetry. Then one finds it nonzero mass with Planck scale suppression. Nevertheless, the mass scale can be generated up to 1 MeV in case $v_1 \ll v_{\varphi_{1(2)}}$. Thus, we can evade this constraint via this effect.

$$m^2(h_1^\pm, h_2^\pm) = \begin{bmatrix} -\frac{v_2(v_1 v_2 \lambda_4 + 2\mu_{12}^2)}{2v_1} & \frac{v_2 v_2 \lambda_4}{2} + \mu_{12}^2 \\ \frac{v_2 v_2 \lambda_4}{2} + \mu_{12}^2 & -\frac{v_1(v_1 v_2 \lambda_4 + 2\mu_{12}^2)}{2v_2} \end{bmatrix},$$

$$m_{H^\pm}^2 = -\frac{(v_1^2 + v_2^2)(v_1 v_2 \lambda_4 + 2\mu_{12}^2)}{2 v_1 v_2}. \tag{2.15}$$

The mass matrices for inert scalar sector are given by

$$m^2(m_{\eta_{R_1}}^2, m_{\eta_{R_2}}^2, m_{\eta_{R_3}}^2) = \begin{bmatrix} (m_{\eta_R}^2)_{11} & & \\ \frac{v_1 v_{\varphi_1} \lambda_0'}{2} & (m_{\eta_R}^2)_{22} & \\ \frac{v_2 v_{\varphi_1} \lambda_0}{2} & 0 & (m_{\eta_R}^2)_{33} \end{bmatrix},$$

$$(m_{\eta_R}^2)_{11} = m_\eta^2, \qquad (m_{\eta_R}^2)_{22} = m_S^2,$$
$$(m_{\eta_R}^2)_{33} = m_\chi^2, \tag{2.16}$$

$$m^2(\eta_I, \chi_I) = \begin{bmatrix} (m_{\eta_R}^2)_{11} & -\frac{v_2 v_{\varphi_1} \lambda_0}{2} \\ -\frac{v_2 v_{\varphi_1} \lambda_0}{2} & (m_{\eta_R}^2)_{33} \end{bmatrix},$$

$$m_{\eta_{I_1}}^2 = \frac{(m_{\eta_R}^2)_{11} + (m_{\eta_R}^2)_{33} - \sqrt{[(m_{\eta_R}^2)_{11} - (m_{\eta_R}^2)_{33}]^2 + v_2^2 v_{\varphi_1}^2 \lambda_0^2}}{2},$$

$$m_{\eta_{I_2}}^2 = \frac{(m_{\eta_R}^2)_{11} + (m_{\eta_R}^2)_{33} + \sqrt{[(m_{\eta_R}^2)_{11} - (m_{\eta_R}^2)_{33}]^2 + v_2^2 v_{\varphi_1}^2 \lambda_0^2}}{2}. \tag{2.17}$$

Here, we explicitly show the 2 by 2 matrices, O_C and O_I, as

$$O_C \equiv \begin{bmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{bmatrix}, \qquad s_\beta = \frac{v_2}{\sqrt{v_1^2 + v_2^2}}, \tag{2.18}$$

$$O_I \equiv \begin{bmatrix} c_a & s_a \\ -s_a & c_a \end{bmatrix}, \qquad s_{2a} = -\frac{v_2 v_{\varphi_2} \lambda_0}{m_{\eta_{I_2}}^2 - m_{\eta_{I_1}}^2}, \tag{2.19}$$

where $c_a \equiv \cos a$ and $s_a \equiv \sin a$, and we define $v \equiv \sqrt{v_1^2 + v_2^2}$ and $\tan\beta \equiv \frac{v_2}{v_1}$ which lead $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$ as in the other THDMs. The mass eigenvalues $m_{H_a^0}(a = 1-4)$ are found to be numerical form only. In our notation, $H_3^0(\equiv h_{\rm SM})$ is the SM-like Higgs and the other three neutral bosons are the additional(heavier) Higgs bosons. Here, $\eta_{R_i}(i = 1-3)$ is the mass eigenstate of the real part of inert neutral boson, and $\eta_{I_i}(i = 1-2)$ is the mass eigenstate of the imaginary part of inert neutral boson. All of the mass eigenvalues and mixings are written in terms of VEVs, and quartic couplings in the Higgs potential after inserting the tadpole conditions: $\partial V/\partial\phi|_{v_1, v_2, v_{\varphi_1}, v_{\varphi_2}} = 0$ and $\partial V/\partial\varphi_R|_{v_1, v_2, v_{\varphi_1}, v_{\varphi_2}} = 0$. Also the mass of $\eta^\pm$ is given by

$$m_{\eta^\pm} = \frac{v_{\varphi_1}^2 \lambda_{\varphi_1\eta} + v_1^2 \lambda_{\Phi_1\eta} + v_{\varphi_2}^2 \lambda_{\varphi_2\eta} + v_2^2 \lambda_{\Phi_2\eta} + 2\mu_\eta^2}{2}. \tag{2.20}$$

We note that in THD sector SM-like couplings are preferred for gauge interactions of $h_{\rm SM}$ by the current Higgs data [136]. Note also that a mixing between Higgs and extra scalar singlet modifies the SM Higgs couplings which is tested by the Higgs measurements at the LHC. The mixing angle is constrained as $\sin\theta \lesssim 0.4$ by global analysis in terms of LHC data for SM Higgs production cross section and decay branching ratio [137–140]. In this paper, we simply assume the mixing is small to satisfy the constraints. In general, we can fit the data by choosing the parameters in the potential accordingly. However the detailed analysis of the constraints is beyond the scope of this paper.

B. Quark sector

In this subsection, we will analyze the quark sector. First of all, let us focus on the Yukawa sector, in which the measured SM quark masses and their mixings are induced. Up- and down-quark mass matrices are diagonalized by $D_u \equiv (M_u^{\rm diag}) = V_{u_L} M_u V_{u_R}^\dagger$, and $D_d(\equiv M_d^{\rm diag}) = V_{d_L} M_d V_{d_R}^\dagger$, where $V's$ are unitary matrix to give their diagonalization matrices. Then CKM matrix is defined by $V_{\rm CKM} \equiv V_{d_L}^\dagger V_{u_L}$, where it can be parametrized by three mixings with one phase as follows:

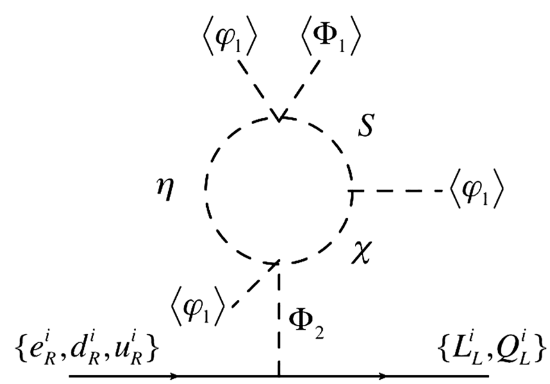

FIG. 1. The one-loop diagram which induces masses first and second families of quarks and charged leptons.

$$V_{u(d)_L} \equiv \begin{bmatrix} c_{u(d)_{13}}c_{u(d)_{12}} & c_{u(d)_{13}}s_{u(d)_{12}} & s_{u(d)_{13}} \\ -c_{u(d)_{23}}s_{u(d)_{12}} - s_{23}s_{u(d)_{13}}c_{u(d)_{12}} & c_{u(d)_{23}}c_{u(d)_{12}} - s_{u(d)_{23}}s_{u(d)_{13}}s_{u(d)_{12}} & s_{u(d)_{23}}c_{u(d)_{13}} \\ s_{u(d)_{23}}s_{u(d)_{12}} - c_{u(d)_{23}}s_{u(d)_{13}}c_{u(d)_{12}} & -s_{u(d)_{23}}c_{u(d)_{12}} - c_{u(d)_{23}}s_{u(d)_{13}}s_{u(d)_{12}} & c_{u(d)_{23}}c_{u(d)_{13}} \end{bmatrix}, \tag{2.21}$$

The mass matrix in our form is written in terms of the dominant contribution $(M^{(1)}_{t(b)})$ that is proportional to v_1 and the subdominant one $(M^{(2)}_{u(d)})$ that is proportional to v_2. Also we can write the left-handed mixing matrix in terms of linear combination as $V_{u(d)_L} \equiv V^{(1)}_{t(b)_L} + V^{(2)}_{u(d)_L}$, where $V^{(1)}_{t(b)_L}(V^{(2)}_{u(d)_L})$ corresponds to $M^{(1)}_{t(b)}(M^{(2)}_{u(d)})$. Then we consider the product of the mass matrix given by

$$\begin{aligned}(M_{u(d)}M^{\dagger}_{u(d)})_{\alpha\beta} &= ((M^{(1)}_{t(b)})(M^{(1)}_{t(b)})^{\dagger})_{\alpha\beta} + ((M^{(2)}_{u(d)})(M^{(2)}_{u(d)})^{\dagger})_{\alpha\beta} \\ &= \frac{v_1^2}{2}\begin{bmatrix} ((y_{t(b)})_{13})^2 & (y_{t(b)})_{13}(y_{t(b)})_{23} & (y_{t(b)})_{13}(y_{t(b)})_{33} \\ (y_{t(b)})_{13}(y_{t(b)})_{23} & ((y_{t(b)})_{23})^2 & (y_{t(b)})_{23}(y_{t(b)})_{33} \\ (y_{t(b)})_{13}(y_{t(b)})_{33} & (y_{t(b)})_{23}(y_{t(b)})_{33} & (y_{t(b)})_{33})^2 \end{bmatrix} \\ &+ \frac{v_2^2}{2}\begin{bmatrix} (y_{u(d)})^2_{11} + (y_{u(d)})^2_{12} & & \\ (y_{u(d)})_{11}(y_{u(d)})_{21} + (y_{u(d)})_{12}(y_{u(d)})_{22} & (y_{u(d)})^2_{21} + (y_{u(d)})^2_{22} & \\ (y_{u(d)})_{11}(y_{u(d)})_{31} + (y_{u(d)})_{12}(y_{u(d)})_{32} & (y_{u(d)})_{21}(y_{u(d)})_{31} + (y_{u(d)})_{22}(y_{u(d)})_{32} & (y_{u(d)})^2_{31} + (y_{u(d)})^2_{32} \end{bmatrix},\end{aligned} \tag{2.22}$$

which is diagonalized by $V_{u(d)L}$. When we redefine $a_{t(b)} \equiv \frac{(y_{t(b)})_{13}}{(y_{t(b)})_{33}}$ and $b_{t(b)} \equiv \frac{(y_{t(b)})_{23}}{(y_{t(b)})_{33}}$ in $(M^{(1)}_{t(b)})_{\alpha\beta}$, we can rewrite the leading term as

$$((M^{(1)}_{t(b)})(M^{(1)}_{t(b)})^{\dagger})_{\alpha\beta} = \frac{(v_1(y_{t(b)})_{33})^2}{2}\begin{bmatrix} a^2_{t(b)} & a_{t(b)}b_{t(b)} & a_{t(b)} \\ a_{t(b)}b_{t(b)} & b^2_{t(b)} & b_{t(b)} \\ a_{t(b)} & b_{t(b)} & 1 \end{bmatrix}. \tag{2.23}$$

Its resulting mass eigenvalues and mixing matrix are given by

$$|D^{(1)}_{t(b)}|^2 = \mathrm{Diag}\left(0, 0, \frac{(v_1(y_{t(b)})_{33})^2}{2}(1 + a^2_{t(b)} + b^2_{t(b)})\right) \equiv (0, 0, |m_{t(b)}|^2), \tag{2.24}$$

$$V^{(1)}_{t(b)_L} = \begin{bmatrix} -\frac{1}{\sqrt{1+a^2_{t(b)}}} & 0 & \frac{a_{t(b)}}{\sqrt{1+a^2_{t(b)}}} \\ -\frac{a_{t(b)}b_{t(b)}}{\sqrt{1+a^2_{t(b)}}\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & \frac{\sqrt{1+a^2_{t(b)}}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & -\frac{b_{t(b)}}{\sqrt{1+a^2_{t(b)}}\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} \\ \frac{a_{t(b)}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & \frac{b_{t(b)}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & \frac{1}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} \end{bmatrix}. \tag{2.25}$$

It suggests that the leading term provides the top and bottom masses only. Thus, the first and second masses are generated via subleading matrix $(M^{(2)}_{u(d)})$, where it arises at the one-loop level as can be seen in Fig. 1.

The first and second quark mass eigenvalues are calculated by solving the secular equation

$$\begin{bmatrix} \delta m^2_{q11} & \delta m^2_{q12} \\ \delta m^2_{q21} & \delta m^2_{q22} \end{bmatrix} \equiv \begin{bmatrix} (V^{(1)}_{q_L})_{1i}((M^{(2)}_{u(d)})(M^{(2)}_{u(d)})^{\dagger})_{ij}(V^{(1)\dagger}_{q_L})_{j1} & (V^{(1)}_{q_L})_{1i}((M^{(2)}_{u(d)})(M^{(2)}_{u(d)})^{\dagger})_{ij}(V^{(1)\dagger}_{q_L})_{j2} \\ (V^{(1)}_{q_L})_{2i}((M^{(2)}_{u(d)})(M^{(2)}_{u(d)})^{\dagger})_{ij}(V^{(1)\dagger}_{q_L})_{j1} & (V^{(1)}_{q_L})_{2i}((M^{(2)}_{u(d)})(M^{(2)}_{u(d)})^{\dagger})_{ij}(V^{(1)\dagger}_{q_L})_{j2} \end{bmatrix}, \tag{2.26}$$

where $\delta m_{qij}(i,j=1,2)$ is written in terms of bi-linear combinations of $a(b)_{t(b)}$ and $(y_{u(d)})_{k,\ell}(k=1-3),(\ell=1,2)$. The resultant mass eigenvalues and mixing matrix are then given by

$$|D^{(2)}_{u(d)}|^2 \equiv \mathrm{Diag}(|m_{u(d)}|^2, |m_{c(s)}|^2, 0) = \mathrm{Diag}(\delta m^2_{q22} - \delta m^2_{q0}, \delta m^2_{q11} + \delta m^2_{q0}, 0), \tag{2.27}$$

$$V^{(2)}_{u(d)_L} = \begin{bmatrix} -\frac{\delta m^2_{q0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{21}}} & \frac{\delta m^2_{q21}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & 0 \\ \frac{\delta m^2_{21}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & \frac{\delta m^2_{q0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & 0 \\ \mathcal{O}\left(\frac{v_2}{v_1}\right)^2 & \mathcal{O}\left(\frac{v_2}{v_1}\right)^2 & \mathcal{O}\left(\frac{v_2}{v_1}\right)^2 \end{bmatrix} \approx \begin{bmatrix} -\frac{\delta m^2_{0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & \frac{\delta m^2_{q21}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & 0 \\ \frac{\delta m^2_{21}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & \frac{\delta m^2_{q0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2.28}$$

where $\delta m^2_{q0} \equiv (\sqrt{(\delta m^2_{q11} - \delta m^2_{q22})^2 + 4\delta m^2_{q21}\delta m^2_{q12}} - \delta m^2_{q11} + \delta m^2_{q22})/2$, and δm_{ij} implies $\delta m_{u_{ij}}$ or $\delta m_{d_{ij}}$. Totally one finds

$$|D_{u(d)}|^2 = \mathrm{Diag}\left(\delta m^2_{q22} - \delta m^2_{q0}, \delta m^2_{q11} + \delta m^2_{q0}, \frac{(v_1 (y_{t(b)})_{33})^2}{2}(1 + a^2_{t(b)} + b^2_{t(b)})\right), \tag{2.29}$$

$$V_{u(d)_L} \approx \begin{bmatrix} \frac{-1}{\sqrt{1+a^2_{t(b)}}} - \frac{\delta m^2_{q0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & \frac{\delta m^2_{21}}{\sqrt{\delta m^4_{0}+\delta m^4_{21}}} & \frac{a_{t(b)}}{\sqrt{1+a^2_{t(b)}}} \\ \frac{-a_{t(b)} b_{t(b)}}{\sqrt{1+a^2_{t(b)}}\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} + \frac{\delta m^2_{21}}{\sqrt{\delta m^4_{0}+\delta m^4_{21}}} & \frac{\sqrt{1+a^2_{t(b)}}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} + \frac{\delta m^2_{q0}}{\sqrt{\delta m^4_{q0}+\delta m^4_{q21}}} & \frac{-b_{t(b)}}{\sqrt{1+a^2_{t(b)}}\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} \\ \frac{a_{t(b)}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & \frac{b_{t(b)}}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} & \frac{1}{\sqrt{1+a^2_{t(b)}+b^2_{t(b)}}} \end{bmatrix}. \tag{2.30}$$

Comparing Eq. (2.21) and Eq. (2.30), one finds the following relations:

$$s_{u(d)_{12}} \approx \sqrt{1 + a^2_{u(d)}}(V_{u(d)_L})_{12}, \qquad s_{u(d)_{23}} \approx -\frac{b_{u(d)}}{\sqrt{1 + a^2_{u(d)} + b^2_{u(d)}}}, \qquad s_{u(d)_{23}} \approx -\frac{a_{u(d)}}{\sqrt{1 + a^2_{u(d)}}}. \tag{2.31}$$

Since V_{CKM} is close to the unit matrix, one approximately finds to be $V_{\mathrm{CKM}} \approx V_{u_L} \approx V_{d_L}$. Here, we take $v_2 \approx 10$ GeV to explain the charm mass ~ 1.3 GeV, which is the maximal mass among the SM fermions except the third SM fermions.

FCNCs: Now that all the mass eigenstates have been derived in the quark sector, we rewrite the interacting Lagrangian in terms of the mass eigenstate as follows:

$$\begin{aligned} -\mathcal{L}^{Q}_{\mathrm{int}} &= -(V_{d_L})_{\beta\alpha}[(y_u)_{\alpha j}c_\beta - (y_t)_{\alpha 3}s_\beta]\bar{d}_{L_\beta}u_{R_\gamma}H^- + (V_{u_L})_{\beta\alpha}[(y_d)_{\alpha j}c_\beta - (y_b)_{\alpha 3}s_\beta]\bar{u}_{L_\beta}d_{R_\gamma}H^+ \\ &\quad + \frac{(V_{u_L})_{\beta\alpha}}{\sqrt{2}}[(y_u)_{\alpha j}(O^T_H)_{4a} - (y_t)_{\alpha 3}(O^T_H)_{3a}]\bar{u}_{L_\beta}u_{R_\gamma}H^0_a - i\frac{(V_{u_L})_{\beta\alpha}}{\sqrt{2}}[(y_u)_{\alpha j}c_\beta - (y_t)_{\alpha 3}s_\beta]\bar{u}_{L_\beta}u_{R_\gamma}A^0 \\ &\quad + \frac{(V_{d_L})_{\beta\alpha}}{\sqrt{2}}[(y_d)_{\alpha j}(O^T_H)_{4a} + (y_b)_{\alpha 3}(O^T_H)_{3a}]\bar{d}_{L_\beta}d_{R_\gamma}H^0_a - i\frac{(V_{d_L})_{\beta\alpha}}{\sqrt{2}}[(y_d)_{\alpha j}c_\beta - (y_b)_{\alpha 3}s_\beta]\bar{d}_{L_\beta}d_{R_\gamma}A^0 + \mathrm{c.c.} \\ &\equiv -(Y_u)_{\beta\gamma}\bar{d}_{L_\beta}u_{R_\gamma}H^- + (Y_d)_{\beta\gamma}\bar{u}_{L_\beta}d_{R_\gamma}H^+ + (Y'_u)^a_{\beta\gamma}\bar{u}_{L_\beta}u_{R_\gamma}H^0_a - i(Y''_u)_{\beta\gamma}\bar{u}_{L_\beta}u_{R_\gamma}A^0 \\ &\quad + (Y'_d)^a_{\beta\gamma}\bar{d}_{L_\beta}d_{R_\gamma}H^0_a - i(Y''_d)_{\beta\gamma}\bar{d}_{L_\beta}d_{R_\gamma}A^0 + \mathrm{c.c.}, \end{aligned} \tag{2.32}$$

where $a = 1-4$ should be summed up.

$M-\bar{M}$ mixing: It is given in terms of the above Lagrangian, where the leading contribution of Y is induced at the one-loop level, which are found in the Appendix. While the one of Y' and Y'' is done at the tree level. Then its resulting form is found to be

$$\Delta m_M(d_a\bar{d}_c \to \bar{d}_b d_d)(Y'_d, Y''_d)$$
$$\approx \frac{5}{24}\left(\frac{m_M}{m_{d_a}+m_{d_c}}\right)^2 m_M f_M^2 \times \mathrm{Re}\left[\sum_i^4 \frac{[(Y'^i_d)_{ca}(Y'^i_d)_{bd} + (Y'^{i\dagger}_d)_{ca}(Y'^{i\dagger}_d)_{bd}]}{m^2_{H^0_i}} - \frac{[(Y''_d)_{ca}Y''_d)_{bd} + (Y''^{\dagger}_d)_{ca}(Y''^{\dagger}_d)_{bd}]}{m^2_{A_0}}\right]$$
$$-\left(\frac{1}{24}+\frac{1}{4}\left(\frac{m_M}{m_{d_a}+m_{d_c}}\right)^2\right) m_M f_M^2 \times \mathrm{Re}\left[\sum_i^4 \frac{[(Y'^i_d)_{ca}(Y'^{i\dagger}_d)_{bd} + (Y'^{i\dagger}_d)_{ca}(Y'^i_d)_{bd}]}{m^2_{H^0_i}} + \frac{[(Y''_d)_{ca}(Y''^{\dagger}_d)_{bd} + (Y''^{\dagger}_d)_{ca}(Y''_d)_{bd}]}{m^2_{A_0}}\right], \tag{2.33}$$

where $\Delta m_M(u_a\bar{u}_c \to \bar{u}_b u_d) = \Delta m_M(d_a\bar{d}_c \to \bar{d}_b d_d)(Y'_u, Y''_d)$ $(u \leftrightarrow d)$ and $x_{ab} \equiv \frac{m_a^2}{m_b^2}$. The experimental values for the mixing are given in Table III and we apply phenomenological constraint $\Delta m_M \le \Delta m_M^{\mathrm{exp}}$. From the above current bounds, severe constraints are found. Here, we conservatively discuss the order of the Yukawa couplings and masses of scalar bosons allowed by the constraints. The flavor violating components of $(Y'_{u(d)})^a$ is strongly constrained to be less than $\mathcal{O}(10^{-5})$ when corresponding H^0_a is the SM Higgs boson.[3] On the other hand one can take $Y''_{u(d)} = \mathcal{O}(10^{-3})$ if $m_{A_0} = \mathcal{O}(100)$ GeV. $Y_{u(d)}$ contribute $M-\bar{M}$ mixing at the one-loop level as shown in the Appendix, and $Y'_{u(d)} = \mathcal{O}(0.1)$ if $m_{H^\pm} = \mathcal{O}(1)$ TeV. Notice here that above estimations are that for Yukawa couplings which violate flavors and flavor conserving couplings are less constrained. The masses of extra bosons are, thus, preferred to be heavier than SM Higgs to avoid the constraints.

Before closing this subsection, it is worthwhile to discuss the rare decay processes of the quark sector such as $b \to s\mu^-\mu^+$ and $b \to c\ell^-_i\bar{\nu}_j$. The lepton universality violating decay $b \to s\mu^-\mu^+$ is measured as the ratio $R_K \equiv \frac{B(B\to K\mu\mu)}{B(B\to Kee)} = 0.745^{+0.090}_{-0.074} \pm 0.036$ by LHCb [141], which has deviation from the SM prediction. This process is found by the following effective Hamiltonian in our model:

$$H_{\mathrm{eff}} = -\frac{1}{\sqrt{2}}\left[\sum_i^4\left(\frac{(Y'^i_d)_{\beta\alpha}(Y^i_L)_{cd}}{m^2_{H^0_i}} + \frac{(Y''_d)_{\beta\alpha}(Y'_\ell)_{cd}}{m^2_{A_0}}\right)(\bar{d}_\beta P_R d_\alpha)(\bar{\ell}_c P_R \ell_d)\right.$$
$$+\left(\sum_i^4\frac{(Y'^i_d)_{\beta\alpha}(Y^{i\dagger}_L)_{cd}}{m^2_{H^0_i}} + \frac{(Y''_d)_{\beta\alpha}(Y'^{\dagger}_\ell)_{cd}}{m^2_{A_0}}\right)(\bar{d}_\beta P_R d_\alpha)(\bar{\ell}_c P_L \ell_d)$$
$$+\left(\sum_i^4\frac{(Y'^{i\dagger}_d)_{\beta\alpha}(Y^i_L)_{cd}}{m^2_{H^0_i}} + \frac{(Y''^{\dagger}_d)_{\beta\alpha}(Y'_\ell)_{cd}}{m^2_{A_0}}\right)(\bar{d}_\beta P_L d_\alpha)(\bar{\ell}_c P_R \ell_d)$$
$$\left.+\left(\sum_i^4\frac{(Y'^{i\dagger}_d)_{\beta\alpha}(Y^{i\dagger}_L)_{cd}}{m^2_{H^0_i}} + \frac{(Y''^{\dagger}_d)_{\beta\alpha}(Y'^{\dagger}_\ell)_{cd}}{m^2_{A_0}}\right)(\bar{d}_\beta P_L d_\alpha)(\bar{\ell}_c P_L \ell_d)\right].$$

The semi-leptonic decay $b \to c\ell^-_i\bar{\nu}_j$ is measured as the ratio $R_D \equiv \frac{B(\bar{B}\to D\tau\nu)}{B(\bar{B}\to D\ell\nu)} = 0.403 \pm 0.040 \pm 0.024$ by flavor averaging group (HFAG) [142], which also has deviation from the SM prediction. This process is also found by the following effective Hamiltonian in our model:

$$H_{\mathrm{eff}} = \frac{(Y^{\dagger}_{\nu\ell})_{ij}}{m^2_{H^\pm}}[(Y^{\dagger}_u)_{ba}(\bar{u}_b P_L d_a)(\bar{\ell}_i P_L \nu_j) - (Y_d)_{ba}(\bar{u}_b P_R d_a)(\bar{\ell}_i P_L \nu_j)].$$

However since all the effective Hamiltonians discussed above depend on the $Y^{(\prime,\prime\prime)}_{u(d)}$ and $1/m^2_{A_0}$, $1/m^2_{H^0_i}$, $1/m^2_{H^\pm}$, which are severely restricted by the bounds of $M-\bar{M}$ mixings. Hence it could be difficult to explain such anomalies in our order estimations.

TABLE III. The experimental values for $M-\bar{M}$ mixing.

Meson	(a,b,c,d)	m_M [GeV]	f_M [GeV]	$\Delta m_M^{\mathrm{exp}}$ [GeV]
D^0	$(c,u,\bar{u},\bar{c})$	1.865	0.212	6.25×10^{-15}
B^0	$(d,b,\bar{b},\bar{d})$	5.280	0.191	3.36×10^{-13}
B^0_s	$(s,b,\bar{b},\bar{s})$	5.367	0.200	1.17×10^{-11}
K^0	$(d,s,\bar{s},\bar{d})$	0.488	0.160	3.48×10^{-15}

[3]In case where H^0_a is not the SM Higgs, one can take the same order as $Y''_{u(d)}$ and m_{A_0}.

C. Lepton sector

In this subsection, we will discuss the lepton sector, where neutrinos are canonical seesaw type. Thus, the process to induce the mass matrix in the charged-lepton sector is the same as the down-quark sector, by changing $b \to \tau$ and $d \to \ell$ in the quark sector. The mass matrix is diagonalized by $D_\ell (\equiv M_\ell^{\rm diag}) = V_{\ell_L} M_\ell V_{\ell_R}^\dagger$, while the neutrino mass matrix is diagonalized by $D_\nu (\equiv M_\nu^{\rm diag}) = U_\nu M_\nu U_\nu^T$, where V_{ℓ_L} and U_ν are unitary matrix to give their diagonalization matrices. Then MNS matrix is defined by $V_{\rm MNS} \equiv V_{\ell_L}^\dagger U_\nu$.

Then the charged-lepton mass matrix arises at the one-loop level as can be seen in Fig. 1, and the resulting form is straightforwardly written as

$$\begin{aligned}((M_\ell)(M_\ell)^\dagger)_{\alpha\beta} &= ((M_\ell^{(1)})(M_\ell^{(1)})^\dagger)_{\alpha\beta} + ((M_\ell^{(2)})(M_\ell^{(2)})^\dagger)_{\alpha\beta} \\ &= \frac{v_1^2}{2}\begin{bmatrix} (y_\tau^2)_{13} & (y_\tau)_{13}(y_\tau)_{23} & (y_\tau)_{13}(y_\tau)_{33} \\ (y_\tau)_{13}(y_\tau)_{23} & (y_\tau^2)_{23} & (y_\tau)_{23}(y_\tau)_{33} \\ (y_\tau)_{13}(y_\tau)_{33} & (y_\tau)_{23}(y_\tau)_{33} & (y_\tau^2)_{33} \end{bmatrix} \\ &\quad + \frac{v_2^2}{2}\begin{bmatrix} (y_\ell)_{11}^2 + (y_\ell)_{12}^2 & & \\ (y_\ell)_{11}(y_\ell)_{21} + (y_\ell)_{12}(y_\ell)_{22} & (y_\ell)_{21}^2 + (y_\ell)_{22}^2 & \\ (y_\ell)_{11}(y_\ell)_{31} + (y_\ell)_{12}(y_\ell)_{32} & (y_\ell)_{21}(y_\ell)_{31} + (y_\ell)_{22}(y_\ell)_{32} & (y_\ell^2)_{31} + (y_\ell^2)_{32} \end{bmatrix}. \end{aligned} \tag{2.34}$$

Following the quark sector, the mass eigenvalues $D_\ell \equiv {\rm Diag}(m_e, m_\mu, m_\tau)$ and eigenstate are respectively given by

$$|D_\ell|^2 = {\rm Diag}\left(\delta m_{\ell 22}^2 - \delta m_{\ell 0}^2, \delta m_{\ell 11}^2 + \delta m_{\ell 0}^2, \frac{(v_1 (y_{t(b)})_{33})^2}{2}(1 + a_\tau^2 + b_\tau^2)\right), \tag{2.35}$$

$$V_{\ell_L} \approx \begin{bmatrix} -\frac{1}{\sqrt{1+a_\tau^2}} - \frac{\delta m_{\ell 0}^2}{\sqrt{\delta m_0^4 + \delta m_{\ell 21}^4}} & \frac{\delta m_{21}^2}{\sqrt{\delta m_{\ell 0}^4 + \delta m_{\ell 21}^4}} & \frac{a_\tau}{\sqrt{1+a_\tau^2}} \\ -\frac{a_\tau b_\tau}{\sqrt{1+a_\tau^2}\sqrt{1+a_\tau^2+b_\tau^2}} + \frac{\delta m_{\ell 21}^2}{\sqrt{\delta m_{\ell 0}^4 + \delta m_{\ell 21}^4}} & \frac{\sqrt{1+a_{t(b)}^2}}{\sqrt{1+a_\tau^2+b_\tau^2}} + \frac{\delta m_{\ell 0}^2}{\sqrt{\delta m_{\ell 0}^4 + \delta m_{\ell 21}^4}} & -\frac{b_\tau}{\sqrt{1+a_\tau^2}\sqrt{1+a_\tau^2+b_\tau^2}} \\ \frac{a_\tau}{\sqrt{1+a_\tau^2+b_\tau^2}} & \frac{b_\tau}{\sqrt{1+a_\tau^2+b_\tau^2}} & \frac{1}{\sqrt{1+a_\tau^2+b_\tau^2}} \end{bmatrix}, \tag{2.36}$$

where $a_\tau \equiv \frac{(y_\tau)_{13}}{(y_\tau)_{33}}$, $b_\tau \equiv \frac{(y_\tau)_{23}}{(y_\tau)_{33}}$, $\delta m_{\ell 0}$, and $\delta m_{\ell ij}, (i,j) = 1, 2$ is the same as the one of the quark sector. Comparing Eq. (2.21) and Eq. (2.36), one finds the following relations:

$$\begin{aligned} &s_{\ell_{12}} \approx \sqrt{1 + a_\tau^2}(V_{\ell_L})_{12}, \\ &s_{\ell_{23}} \approx -\frac{b_\tau}{\sqrt{1 + a_\tau^2 + b_\tau^2}}, \qquad s_{\ell_{23}} \approx -\frac{a_\tau}{\sqrt{1 + a_\tau^2}}. \end{aligned} \tag{2.37}$$

The neutrino mass matrix arises at the two-loop level as can be seen in Fig. 2, and the resulting form is given by

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{v_2^2}{4}\sum_{i=1-2}(y_\nu)_{\alpha i}(M_N^{-1})_{ii}(y_\nu)_{i\beta}^T, \tag{2.38}$$

where $M_N \equiv y_N v_{\varphi_2}/\sqrt{2}$. We apply Casas-Ibarra parametrization [143] to reproduce neutrino oscillation data, then one finds the following relation:

$$y_\nu = \frac{2}{v_2} V_{\rm MNS}^\dagger V_{\ell_L}^\dagger \sqrt{D_\nu}\mathcal{O}\sqrt{M_N}, \tag{2.39}$$

where $\mathcal{O}(= \mathcal{O}\mathcal{O}^T = 1)$ is an arbitrary orthogonal matrix with complex values.

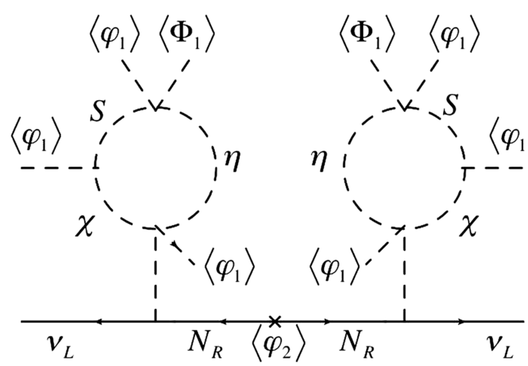

FIG. 2. The two-loop diagram which induces masses of active neutrinos.

LFVs: Now that all the mass eigenstates have been derived in the lepton sector, we rewrite the interacting Lagrangian in terms of the mass eigenstate as follows:

$$\begin{aligned}-\mathcal{L}^L_{\rm int} &= -c_\beta (V_{\ell_L})_{\beta\alpha}(y_\nu)_{\alpha j}\bar{\ell}_{L_\beta} N_{R_j} H^- + (U_\nu)_{\beta\alpha}[(y_\ell)_{\alpha j} c_\beta - (y_\tau)_{\alpha 3} s_\beta]\bar{\nu}_{L_\beta} e_{R_\gamma} H^+ \\ &\quad + \frac{1}{\sqrt{2}}[(V_{\ell_L})_{\beta\alpha}(y_\ell)_{\alpha j}(O_H^T)_{4a} + (V_{\ell_L})_{\beta\alpha}(y_\tau)_{\alpha 3}(O_H^T)_{3a}]\bar{\ell}_{L_\beta} e_{R_\gamma} H^0_a \\ &\quad + \frac{i}{\sqrt{2}}[(V_{\ell_L})_{\beta\alpha}(y_\ell)_{\alpha j} c_\beta - (V_{\ell_L})_{\beta\alpha}(y_\tau)_{\alpha 3} s_\beta]\bar{\ell}_{L_\beta} e_{R_\gamma} A^0 + \text{c.c.}\end{aligned} \tag{2.40}$$

$$\equiv -(Y_\nu)_{\beta j}\bar{\ell}_{L_\beta} N_{R_j} H^- + (Y_{\nu\ell})_{\beta\gamma}\bar{\nu}_{L_\beta} e_{R_\gamma} H^+ + (Y_L)^a_{\beta\gamma}\bar{\ell}_{L_\beta} e_{R_\gamma} H^0_a + i(Y_{\ell}')_{\beta\gamma}\bar{\ell}_{L_\beta} e_{R_\gamma} A^0 + \text{c.c.}, \tag{2.41}$$

where $a = 1-4$ should be summed up, and Y_ν, $Y_{\nu\ell}$, Y_ν, and Y'_ℓ can respectively be arbitral scale by controlling the parameters $\mathcal{O}$, U_ν, O_H and V_{ℓ_L}.

$\ell_\alpha \to \ell_\beta \gamma$: The lepton flavor (LFVs) violation processes give the constraints on our parameters. The experimental bounds are found in Table. IV. The most known processes are $\ell_\alpha \to \ell_\beta\gamma$, and its branching ratio is given by

$$\text{BR}(\ell_\alpha \to \ell_\beta\gamma) \approx \frac{48\pi^3 \alpha_{em} C_{\alpha\beta}}{G_F^2 m^2_{\ell_\alpha}}(|a_{R_1} + a_{R_2} + a_{R_3}|^2 + |a_{L_1} + a_{L_2} + a_{L_3}|^2)_{\alpha\beta} \tag{2.42}$$

where $\alpha_{em} \approx 1/128$ is the fine-structure constant, $C_{\alpha\beta} = (1, 0.178, 0.174)$ for $((\alpha,\beta) = [(2,1),(3,2),(3,1)]$, $G_F \approx 1.17 \times 10^{-5}$ GeV^{-2} is the Fermi constant, and $a_{R_{\alpha\beta}}$ and $a_{L_{\alpha\beta}}$ are computed as

$$\begin{aligned}(a_{R_1})_{\alpha\beta} &= \frac{(Y_\nu)_{\beta j}(Y^\dagger_\nu)_{j\alpha} m_{\ell_\alpha}}{12(4\pi)^2}\frac{2M^6_{N_j} + 3M^4_{N_j} m^2_{H^\pm} - 6M^2_{N_j} m^4_{H^\pm} + m^6_{H^\pm} + 12 M^4_{N_j} m^2_{H^\pm}\ln[\frac{m_{H^\pm}}{M_{N_j}}]}{(M^2_{N_j} - m^2_{H^\pm})^4}, \\ (a_{L_1})_{\alpha\beta} &= \frac{(Y_\nu)_{\beta j}(Y^\dagger_\nu)_{j\alpha} m_{\ell_\beta}}{12(4\pi)^2}\frac{2M^6_{N_j} + 3M^4_{N_j} m^2_{H^\pm} - 6M^2_{N_j} m^4_{H^\pm} + m^6_{H^\pm} + 12 M^4_{N_j} m^2_{H^\pm}\ln[\frac{m_{H^\pm}}{M_{N_j}}]}{(M^2_{N_j} - m^2_{H^\pm})^4}, \\ (a_{R_2})_{\alpha\beta} &= -\frac{(Y_L)^a_{\beta\gamma}(Y^\dagger_L)^a_{\gamma\alpha} m_{\ell_\alpha}}{6(4\pi)^2 m^2_{H^0_a}}, \qquad (a_{L_2})_{\alpha\beta} = -\frac{(Y_L)^a_{\beta\gamma}(Y^\dagger_L)^a_{\gamma\alpha} m_{\ell_\beta}}{6(4\pi)^2 m^2_{H^0_a}}, \\ (a_{R_3})_{\alpha\beta} &= -\frac{(Y'_\ell)_{\beta\gamma}(Y'^\dagger_\ell)_{\gamma\alpha} m_{\ell_\alpha}}{6(4\pi)^2 m^2_{A^0}}, \qquad (a_{L_3})_{\alpha\beta} = -\frac{(Y'_\ell)_{\beta\gamma}(Y'^\dagger_\ell)_{\gamma\alpha} m_{\ell_\beta}}{6(4\pi)^2 m^2_{A^0}}.\end{aligned} \tag{2.43}$$

Muon anomalous magnetic dipole moment $(g-2)_\mu$: Through the same process from the above LFVs, there exists the contribution to $(g-2)_\mu$, and its form Δa_μ is simply given by

$$\Delta a_\mu \approx -m_\mu (a_{R_1} + a_{R_2} + a_{R_3} + a_{L_1} + a_{L_2} + a_{L_3})_{\mu\mu}. \tag{2.44}$$

TABLE IV. Summary of $\ell_\alpha \to \ell_\beta\gamma$ process and the lower bound of experimental data.

Process	(α,β)	Experimental bounds (90% C.L.)	References
$\mu^- \to e^-\gamma$	(2,1)	$\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$	[144]
$\tau^- \to e^-\gamma$	(3,1)	$\text{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$	[145]
$\tau^- \to \mu^-\gamma$	(3,2)	$\text{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$	[145]

This value can be tested by current experiments $\Delta a_\mu = (28.8 \pm 8.0) \times 10^{-10}$ [146]. As can be seen in Eq. (2.43), one finds that the first two forms $a_{R(L)_1}$ give negative contribution, while the others provide positive contribution. Note that from the flavor violation in the quark sector, extra scalar bosons are preferred to be heavier than SM Higgs. Thus, we here assume the dominant contribution to the muon $g-2$ and $\mu \to e\gamma$, the stringent constraint $\text{BR}(\mu \to e\gamma)$, are approximately given by SM Higgs as

$$\Delta a_\mu \sim -m_\mu (a_{R_2} + a_{L_2})_{\mu\mu} = \sum_{\gamma=1}^{3}\frac{(Y_L)^3_{2\gamma}(Y^\dagger_L)^3_{\gamma 2}}{3(4\pi)^2}\frac{m^2_\mu}{m^2_{H^0_3}}, \tag{2.45}$$

$$\mathrm{BR}(\mu \to e\gamma) \sim \frac{48\pi^3\alpha_{em}}{G_F^2 m_\mu^2}|(a_{R_2})_{\mu e}|^2 = \frac{|\sum_{\gamma=1}^{3}(Y_L)^3_{1\gamma}(Y_L^\dagger)^3_{\gamma 2}|^2}{192\pi G_F^2 m_{H_3^0}^4}, \tag{2.46}$$

where $m_{H_3^0}(\approx 125\ \mathrm{GeV})$ is the mass of the SM Higgs. As can be seen in Eqs. (2.45) and (2.46), one can satisfy the constraint of LFV due to the independent parameters. Thus, we show the allowed range of the current measurement of muon $g-2$ in terms of Yukawa couplings $(Y_L)^3_{2\gamma}(Y_L^\dagger)^3_{\gamma 2}$:

$$2.76 \lesssim \sum_{\gamma=1}^{3}(Y_L)^3_{2\gamma}(Y_L^\dagger)^3_{\gamma 2} \lesssim 4.88. \tag{2.47}$$

D. Dark matter

In our scenario, real scalar S is considered as a DM candidate, where we assume to be no mixing between S and η_R that is natural assumption because of $v_2 \ll v_1$.

Our DM candidate S can interact via a Higgs portal coupling S-S-h_{SM}. However the Higgs portal coupling is strongly constrained by the direct detection search at the LUX experiment [147]. We then assume the SM Higgs portal coupling is negligibly small by choosing some parameters in the scalar potential to avoid the constraint from the direct detection. We then consider that S dominantly interacts with one of the extra scalar singlets $H_2^0 \simeq \varphi_2$, assuming small mixing among CP-even scalars. Then the dominant annihilation process is $2S \to 2H_2^0$ via four point coupling of S-S-H_2^0-H_2^0 taking mass relation $m_{H_2^0} < m_S$.[4] Note also that constraint on mass of H_2^0 is not strict for small mixing case since H_2^0 production cross section is small at the colliders. To estimate the relic density, we parametrize the interaction as

$$\mathcal{L} \supset \lambda_{SSHH} S S H_2^0 H_2^0, \tag{2.48}$$

where the coupling λ_{SSHH} is given by combination of couplings in the potential Eq. (2.3). In case of small mixing limit, it is $\lambda_{SSHH} \sim \lambda_{\varphi_2 S}$. The relic density of DM is then given by [148]

$$\Omega h^2 \approx \frac{1.07 \times 10^9}{\sqrt{g_*(x_f)} M_{\mathrm{Pl}} J(x_f)[\mathrm{GeV}]}, \tag{2.49}$$

where $g^*(x_f \approx 25) \approx 100$, $M_{\mathrm{Pl}} \approx 1.22 \times 10^{19}$, and $J(x_f)(\equiv \int_{x_f}^{\infty} dx \frac{\langle \sigma v_{\mathrm{rel}} \rangle}{x^2})$ is given by

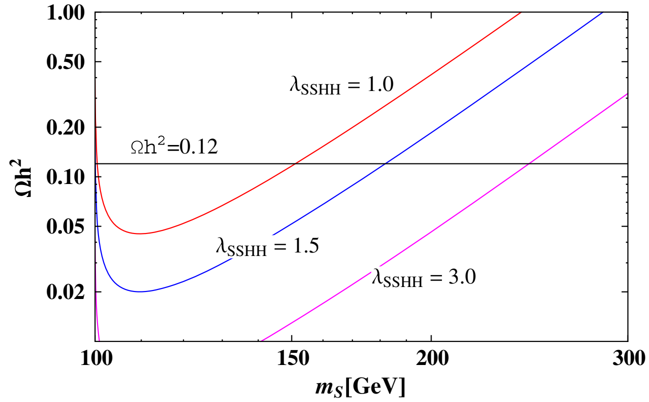

FIG. 3. Relic density of DM in terms of the DM mass, where $\lambda_{SSHH} = (1.0, 1.5, 3.0)$ represent the lines of red, blue, and magenta, respectively. Here, we fixed $m_{H_2^0} = 100$ GeV for simplicity.

$$J(x_f) = \int_{x_f}^{\infty} dx \left[\frac{\int_{4m_S^2}^{\infty} ds \sqrt{s - 4m_S^2}\, s(\sigma v_{\mathrm{rel}}) K_1(\frac{\sqrt{s}}{m_S} x)}{16 m_S^5 x [K_2(x)]^2} \right],$$

$$(\sigma v_{\mathrm{rel}}) = \frac{|\lambda_{SSHH}|^2}{8\pi^2 s}\sqrt{1 - \frac{4m_{H_2^0}^2}{s}}. \tag{2.50}$$

Here, s is a Mandelstam variable, and $K_{1,2}$ are the modified Bessel functions of the second kind of order one and two, respectively. The observed relic density is $\Omega h^2 \approx 0.12$ [149]. We show the relic density in terms of the DM mass in Fig. 3 for several values of the coupling constant fixing $m_{H_2^0} = 100$ GeV, which suggests that the order-one quartic coupling is needed.

III. CONCLUSIONS AND DISCUSSIONS

We have proposed a model with the two-Higgs doublet $\Phi_{1,2}$ in which the quark and charged-lepton masses in the first and second families are induced at the one-loop level and neutrino masses are induced at the two-loop level. In the model, we have introduced an extra $U(1)_R$ gauge symmetry in a family-dependent way that plays a crucial role in achieving desired interaction terms in no conflict with anomaly cancellation. The second Higgs doublet Φ_2 is also charged under $U(1)_R$ and couples to only the first and second families of right-handed fermions. We have then considered the scenario in which vacuum expectation value of Φ_2 is absent at tree level and induced at the one-loop level via spontaneous symmetry breaking of gauge symmetries.

After the gauge symmetry breaking, we have obtained the scalar potential of THDM with softly broken Z_2 symmetry where $\Phi_1^\dagger \Phi_2$ term is suppressed by loop effect and $\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + \mathrm{H.c.}]$ term is absent at tree level. We have shown the fermion masses where first and second

[4]Here, we assume the DM pair annihilate into a H_2^0 pair but an annihilation mode into a H_1^0 pair is also possible if we set H_1^0 lighter than DM.

families are loop suppressed and discussed structure of the mass matrices. Here, we emphasize that our original Yukawa couplings could be less hierarchical compared to the SM or general THDM because of the loop suppression effect for the first and second families. The Yukawa couplings with mass eigenstates are also derived and we discussed several phenomenologies such as flavor changing neutral current in the quark sector, lepton flavor violations, muon $g-2$. In addition, we have analyzed relic density for the dark matter candidate in this model which can be accommodated with observed data.

In the model, rich phenomenologies can be considered such as flavor violating SM Higgs decay and collider physics although we have not discussed. It will be also interesting to investigate difference from other THDMs in detail since we have specific structure of Yukawa couplings where one Higgs doublet couples to third family right-handed fermions and the second doublet couples to other families of right-handed fermion. In addition, we can discuss physics of extra Z' gauge boson which comes from our $U(1)_R$. More detailed analysis of the model will be done elsewhere.

ACKNOWLEDGMENTS

H. O. is sincerely grateful for all the Korea Institute for Advanced Study (KIAS) members and their hospitality.

APPENDIX: $M-\bar{M}$ mixing

The one-loop contribution that is proportional to Y' and Y'' is found to be

$$\Delta m_d^{(2)}(Y_u, Y_d) = \frac{m_{u_\alpha} m_{u_\beta} m_M f_M^2}{24(4\pi)^2 m_{H^\pm}^4}\left(\frac{m_M}{m_{d_a}+m_{d_c}}\right)^2 \tag{A1}$$

$$\times \mathrm{Re}[(Y_u Y_d)_{ba}(Y_u Y_d)_{cd} + (Y_d^\dagger Y_u^\dagger)_{ba}(Y_d^\dagger Y_u^\dagger)_{cd}]$$
$$\times F_{II}(x_{u_\alpha H^\pm}, x_{u_\beta H^\pm}),$$

$$F_{II}(x_1, x_2) = \int dadbdc \frac{\delta(a+b+c-1)a}{(a+bx_1+cx_2)^2}, \tag{A2}$$

where $\Delta m_u^{(2)}(Y_d, Y_u) = \Delta m_d^{(2)}(Y_u, Y_d)(u \leftrightarrow d)$, $x_{ab} \equiv \frac{m_a^2}{m_b^2}$, and

$$(a,b,c,d) = (c,u,\bar{u},\bar{c}), \quad \text{for } D^0, \tag{A3}$$

$$(a,b,c,d) = (d,b,\bar{b},\bar{d}), \quad \text{for } B^0, \tag{A4}$$

$$(a,b,c,d) = (s,b,\bar{b},\bar{s},), \quad \text{for } B_S^0, \tag{A5}$$

$$(a,b,c,d) = (d,s,\bar{s},\bar{d}), \quad \text{for } K^0. \tag{A6}$$

In the order estimation of $M-\bar{M}$ mixing, it is satisfactory if we take $Y_{u(d)} \lesssim \mathcal{O}(1)$ and $m_{H^\pm} = \mathcal{O}(1)$ TeV.

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