# Interpretation of the newly observed  $\Omega_c^0$  resonances

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We study the charmed and bottomed doubly strange baryons within the heavy-quark-light-diquark framework. The two strange quarks are assumed to lie in S wave and thus their total spin is 1. We calculate the mass spectra of the S- and P-wave orbitally excited states and find the  $\Omega_c^0(2695)$  and  $\Omega_c^0(2770)$  fit well as the S-wave states of charmed doubly strange baryons. The five newly  $\Omega_c^0(X)$  resonances observed by the LHCb Collaboration i.e.  $\Omega_c^0(2000)$ ,  $\Omega_c^0(2050)$ ,  $\Omega_c^0(2000)$ , and  $\Omega_c^0(2110)$ , and be interpreted as LHCb Collaboration, i.e.,  $\Omega_c^0(3000)$ ,  $\Omega_c^0(3050)$ ,  $\Omega_c^0(3066)$ ,  $\Omega_c^0(3090)$ , and  $\Omega_c^0(3119)$ , can be interpreted as the P-wave orbitally excited states. In heavy quark effective theory, we analyze their decays into the  $\Xi_c^+ K^$ and  $\Xi_c^{\prime+} K^-$ , and point out that decays of the five P-wave  $\Omega_c^0$  states into the  $\Xi_c^+ K^-$  and  $\Xi_c^{\prime+} K^-$  are suppressed by either heavy quark symmetry or phase space. The narrowness of the five newly observed  $\Omega_c^0(X)$  states can then be naturally interpreted with heavy quark symmetry can then be naturally interpreted with heavy quark symmetry.

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### I. INTRODUCTION

The hadron spectroscopy plays an important role in understanding the fundamental theory of strong interactions, i.e., the quantum chromodynamics (QCD). In the naive quark model, the mesons are bound states of a quark-antiquark pair while the baryons are composed of three quarks. However, the structure of hadrons is more complicated than the description in the naive quark model. There might be hybrids, glueballs, and multiquark states, which are also allowed under the principle of color confinement. Take the exotic baryon states as an example; the LHCb Collaboration has observed two pentaquark candidates  $P_c(4380)$  and  $P_c(4450)$  in  $\Lambda_b^0 \to J/\psi K^- p$ <br>decays [1] which also have been analyzed in  $\Lambda_c^0 \to$ decays [\[1\],](#page-5-0) which also have been analyzed in  $\Lambda_b^0 \rightarrow$  $J/\psi \pi$ <sup>-</sup>p decays [\[2\].](#page-5-1) The general studies of hadron inner structures enhance our knowledge on the properties of QCD color confinement.

The charmed doubly strange baryon  $\Omega_c^0(2695)$  with<br>senin and spin parity  $I(I^P) = O(\frac{1}{2})$  was first observed isospin and spin parity  $I(J^P) = 0(\frac{1}{2}^+)$  was first observed<br>in the hyperon beam experiment WA62 [3]. Later it was in the hyperon beam experiment WA62 [\[3\].](#page-5-2) Later it was confirmed in the electron-positron collider experiment [\[4\]](#page-5-3) and the photon beam experiment [\[5\].](#page-5-4) The excited state  $\Omega_c^0(2770)$  with  $I(J^P) = 0(\frac{3}{2}^+)$  was first observed in the redictive doesn  $\Omega_c^0(2770)$ ,  $\Omega_c^0(2605)$ , why the *PAPAP* radiative decay  $\Omega_c^0(2770) \to \Omega_c^0(2695) + \gamma$  by the *BABAR*<br>Collaboration [6] and then confirmed by the Belle Collaboration [\[6\],](#page-5-5) and then confirmed by the Belle Collaboration [\[7\].](#page-5-6)

in the  $\Xi_c^+ K^-$  invariant mass spectrum [\[8\].](#page-5-7) The authors have determined the masses and decay widths of the five new  $\Omega_c^0(X)$  states [\[8\]](#page-5-7) and the results are collected in Table [I.](#page-0-1)<br>After these discoveries it is natural to ask ourselves thre

After these discoveries, it is natural to ask ourselves three questions: (1) Why are there so small mass differences among these five new states? (2) What are the spin parities for these five new states? (3) Why are the decay widths so narrow for these five new states?

Investigating their mass spectra and decay properties answers these questions accordingly. In theoretical aspects, there have already been some attempts to interpret of the newly observed  $\Omega_{c}^{0}(X)$  resonances. Agaev *et al.* proposed<br>to assign  $O^{0}(3066)$  and  $O^{0}(3110)$  states as the first radially to assign  $\Omega_c^0(3066)$  and  $\Omega_c^0(3119)$  states as the first radially<br>excited  $(25.1+)$  and  $(25.3+)$  charmed baryons in OCD sum excited  $(2S, \frac{1}{2}^+)$  and  $(2S, \frac{3}{2}^+)$  charmed baryons in QCD sum<br>while  $[0]$ . Chan at al. analyzed the newly  $O(N)$  atore with rules [\[9\].](#page-5-8) Chen *et al.* analyzed the newly  $\Omega_c^0(X)$  states with different spins and obtained the related decay widths into different spins and obtained the related decay widths into  $\Xi_c^+ K^-$ ,  $\Xi_c^{\prime +} K^-$  and  $\Xi_c^{*+} K^-$  in QCD sum rules [\[10\].](#page-5-9) Karliner *et al.* proposed to assign the newly  $\Omega_c^0(X)$  states as bound<br>states of a charm quark and a *P*-wave ss diquark [11] states of a charm quark and a P-wave ss diquark [\[11\]](#page-5-10). Wang *et al.* studied the strong and radiative decays of the

<span id="page-0-1"></span>TABLE I. Masses and widths (MeV) of the  $\Omega_c^0(X)$  baryons observed by the LHCb Collaboration. The first uncertainty is observed by the LHCb Collaboration. The first uncertainty is statistical and the second one is systematic; and the third uncertainty in masses of  $\Omega_c^0(X)$  baryons is from the  $\Xi_c^+$  mass.

Ilaboration [7]. Using a sample of $pp$ collision data corresponding to an	<b>State</b>	<b>Mass</b>	Width				
egrated luminosity of 3.3 $fb^{-1}$ , the LHCb Collaboration	$\Omega_c(3000)$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$				
s recently observed five new narrow excited $\Omega_c^0(X)$ states	$\Omega_c^0(3050)$	$3050.2 \pm 0.1 \pm 0.1_{-0.5}^{+0.3}$	$0.8 \pm 0.2 \pm 0.1$				
	$\Omega_c^0(3066)$	$3065.6 \pm 0.1 \pm 0.3_{-0.5}^{+0.3}$	$3.5 \pm 0.4 \pm 0.2$				
	$\Omega_c^0(3090)$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$				
*Corresponding author. hu@njnu.edu.cn	$\Omega_c^0(3119)$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$				

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 $\Omega_c^0(X)$  states in a constituent quark model [\[12\]](#page-5-11). Besides,<br>Yang et al. proposed to assign some of the newly  $O^0(Y)$ Yang *et al.* proposed to assign some of the newly  $\Omega_c^0(X)$  states as the possible pentaquark states [13] states as the possible pentaquark states [\[13\].](#page-5-12)

In this paper, we interpret the five new observed  $\Omega_c^0(X)$ <br>tes as the *P*-wave orbitally excited states of charmed states as the P-wave orbitally excited states of charmed doubly strange baryons in the heavy-quark-light-diquark picture. The spectra of the bottom partners of  $\Omega_{\rm c}^0(X)$  states<br>are also predicted. In the end, the decay properties of are also predicted. In the end, the decay properties of charmed doubly strange baryons are discussed in the heavy quark effective theory.

### II. INTERPRETATION OF THE NEWLY OBSERVED  $\Omega_c^0$  RESONANCES

The notion of diquark is as old as the quark model where Gell-Mann mentioned the possibility of diquarks in the original paper on quarks [\[14\].](#page-5-13) According to the color SU(3) group, the color configuration of a diquark can be represented either by an antitriplet or sextet in the decomposition of 3  $\otimes$  3 =  $\overline{3}$   $\oplus$  6. The binding of the  $q_1\overline{q}_2$  or  $q_1q_2$  system<br>depends solely on the quadratic Casimir  $C_2(R)$  of the depends solely on the quadratic Casimir  $C_2(R)$  of the product color representation R to which the quarks couple according to the discriminator  $I = \frac{1}{2}(C_2(R) - C_2(R_1) - C_1(R_2))$ , where *P*, denotes the color representations of two  $C_2(R_2)$ , where  $R_i$  denotes the color representations of two quarks [\[15\]](#page-5-14). The discriminators are then determined as  $I = \frac{1}{6}(-8, -4, +2, +1)$  for  $R = (1, \bar{3}, 6, 8)$ , respectively. The interaction force becomes attractive when the discriminator is negative, which is somewhat analogous to the Coulomb force in QED. Thus, the only color attractive configuration of  $q_1\bar{q}_2$  is in the color-singlet 1, whereas the color attractive configuration of  $q_1q_2$  is in the color antitriplet  $\bar{3}$ . The attractive force strength in the color antitriplet diquark is half of that in the color-singlet quark-antiquark pair in the one-gluon-exchange model. Thus two quarks in the color antitriplet  $\bar{3}$  have a large possibility to bind into a diquark [\[15](#page-5-14)–17], and thus a baryon can be treated as a quarkdiquark system.

In the css system, two strange quarks can form a light diquark system, while the charm and strange quarks may also form a cs diquark. The strength of the attractive force between two quarks is reflected by a coupling constant as given below. A fit of the experimental data has indicated that the coupling constant for the two strange quarks is much larger than that for the cs system, for instance,  $\kappa_{ss} = 72$  MeV and  $\kappa_{cs} \approx (24-25)$  MeV [\[18](#page-5-15)–20]. Following this scheme, we treat the charmed doubly strange baryons as heavy-quark-light-diquark bound states in order to explain the newly observed five narrow  $\Omega_c^0(X)$ states.

The wave function of the charmed doubly strange baryon is composed of four parts, coordinate-space, color, flavor, and spin subspaces [\[21\]](#page-5-16)

$$
\Psi(c, s, s) = \psi(x_1, x_2, x_3) \otimes \chi_{123} \otimes f_{123} \otimes s_1 s_2 s_3, \qquad (1)
$$

where we give numbers 1, 2, 3 to denote charm and two strange quarks, respectively;  $\psi(x_i)$ ,  $\chi$ , f, and  $s_i$  denote the coordinate-space, color, flavor, and spin wave functions, respectively. The total wave function should satisfy the Pauli exclusion principle when we interchange the two strange quarks. We restrict ourselves to the ground state of the diquark, namely, the coordinate-space wave function is in the S wave with  $L = 0$ , and thus symmetric. The color wave function is antisymmetrical because the baryon system is in the color singlet. The flavor wave function is also symmetrical to the interchange of the two strange quarks. Thus the spin wave function should be also symmetrical, i.e., the spin of two strange quarks should be 1 in the charmed doubly strange baryon.

The charmed doubly strange baryons are composed of a charm quark and two strange quarks. We assume the two strange quarks form a diquark  $\delta = ss$ , which along with the charm quark make it true for the stable spectra of the  $\Omega_c^0(X)$ <br>system. The baryon mass splitting  $\Lambda M$  can be estimated as system. The baryon mass splitting  $\Delta M$  can be estimated as [\[16,18\]](#page-5-17)

<span id="page-1-0"></span>
$$
\Delta M = 2(\kappa_{cs})_{\bar{3}} (\mathbf{S}_c \cdot \mathbf{S}_\delta) + 2(\kappa_{ss})_{\bar{3}} (\mathbf{S}_s \cdot \mathbf{S}_s)
$$
  
+ 2A<sub>c</sub> ( $\mathbf{S}_c \cdot \mathbf{L}$ ) + 2A<sub>δ</sub> ( $\mathbf{S}_\delta \cdot \mathbf{L}$ )  
+ B  $\frac{L(L+1)}{2}$ , (2)

where the first two terms are a spin-spin interaction between the diquark and charm quarks and inside the diquark. The third and fourth terms are the spin-orbital interactions. The fifth term is the pure orbital interactions. The  $S_\delta$  corresponds to the spin operator of diquark. The spin operators of the strange quark and charm quark are given by  $S_s$  and  $S_c$ , respectively. The coefficients  $(\kappa_{q_1q_2})_3$ <br>are the spin-spin couplings for two quarks in color are the spin-spin couplings for two quarks in color antitriplet, respectively.

Unlike the case in the  $\Omega^-$  where the total angular momentum J is  $3/2$  with  $L = 0$ , the S-wave states of the  $\Omega_c^0$  system have two states where the total angular momentum J can be either  $1/2$  or  $3/2$ .

$$
\left| L = 0, \frac{1}{2J} \right\rangle = \left| \frac{1}{2c}, 1_{\delta}; \frac{1}{2c\delta}; L = 0; \frac{1}{2J} \right\rangle
$$

$$
= \frac{\sqrt{2}}{\sqrt{3}} (\downarrow)_{c} (\uparrow)_{s} (\uparrow)_{s}, \tag{3}
$$

$$
\left| L = 0, \frac{3}{2J} \right\rangle = \left| \frac{1}{2c}, 1_{\delta}; \frac{3}{2c\delta}; L = 0; \frac{3}{2J} \right\rangle
$$
  
=  $(\uparrow)_c (\uparrow)_s (\uparrow)_s,$  (4)

where  $|S_c, S_\delta; S_{c\delta}; L = 0; N_J\rangle$  stands for the baryon; the  $S_\delta$ and  $S_c$  denote the spin of the diquark [ss] and the charm quark, respectively, and the  $N<sub>J</sub>$  denotes the total angular momentum of the baryon.

There are five P-wave states of the  $\Omega_c^0$  system with  $L = 1$  and negative parity,

$$
\left| L = 1, \frac{1}{2J} \right\rangle_{1} = \left| \frac{1}{2c}, 1_{\delta}; \frac{1}{2c\delta}; L = 1; \frac{1}{2J} \right\rangle, \tag{5}
$$

$$
\left| L = 1, \frac{1}{2J} \right\rangle_{2} = \left| \frac{1}{2c}, 1_{\delta}; \frac{3}{2c\delta}; L = 1; \frac{1}{2J} \right\rangle, \tag{6}
$$

$$
\left| L = 1, \frac{3}{2J} \right\rangle_{1} = \left| \frac{1}{2c}, 1_{\delta}; \frac{1}{2c\delta}; L = 1; \frac{3}{2J} \right\rangle, \tag{7}
$$

$$
\left| L = 1, \frac{3}{2J} \right\rangle_{2} = \left| \frac{1}{2c}, 1_{\delta}; \frac{3}{2c\delta}; L = 1; \frac{3}{2J} \right\rangle, \tag{8}
$$

<span id="page-2-2"></span>
$$
\left| L = 1, \frac{5}{2J} \right\rangle = \left| \frac{1}{2c}, 1_{\delta}; \frac{3}{2c\delta}; L = 1; \frac{5}{2J} \right\rangle. \tag{9}
$$

<span id="page-2-0"></span>There are some simple relations among the S- and P-wave states of the  $\Omega_c^0$  system when using the mass splitting formulas. Their relations are

$$
M_{|L=0,\frac{3}{2}y\rangle} = M_{|L=0,\frac{1}{2}y\rangle} + 3(\kappa_{cs})_{\bar{3}},\tag{10}
$$

$$
M_{|L=1,\frac{1}{2}+\}} = M_{|L=0,\frac{1}{2}\}} - 2A_c + B,\tag{11}
$$

$$
M_{|L=1,\frac{1}{2}j\rangle_2} = M_{|L=0,\frac{1}{2}j\rangle} + 3(\kappa_{cs})_{\bar{3}} - 5A_c + B, \quad (12)
$$

$$
M_{|L=1,\frac{3}{2}+\}} = M_{|L=0,\frac{1}{2}+\}} + A_c + B,\tag{13}
$$

$$
M_{|L=1,\frac{3}{2}j\rangle_2} = M_{|L=0,\frac{1}{2}j\rangle} + 3(\kappa_{cs})_{\bar{3}} - 2A_c + B, \quad (14)
$$

<span id="page-2-1"></span>
$$
M_{|L=1,\frac{5}{2}\rangle} = M_{|L=0,\frac{1}{2}\rangle} + 3(\kappa_{cs})_{\bar{3}} + 3A_c + B, \quad (15)
$$

where we simply assume  $A_{\delta} = A_c$ .

For convenience, we write the possible states into the corresponding form  $|n^{2S+1}L_J\rangle$ , i.e.,  $|1^2S_1\rangle = |L = 0, \frac{1}{2J}\rangle$ ,  $|1^4S_1\rangle = |I = 0, \frac{3}{2J}\rangle$ ,  $|1^2B_1\rangle = |I = 1, \frac{1}{2J}\rangle$ ,  $|1^4B_1\rangle = |I = 1, \frac{1}{2J}\rangle$  $|1^4S_2\rangle = |L = 0, \frac{3}{2J}\rangle, |1^2P_1\rangle = |L = 1, \frac{1}{2J}\rangle_1^2, |1^4P_1\rangle = |L =$  $1, \frac{1}{2}$ ,  $2\left(1^2P_2\right) = |L = 1, \frac{3}{2}$ ,  $|1^4P_2\right) = |L = 1, \frac{3}{2}$ , and  $|1^4P_{\frac{5}{2}}\rangle = |L = 1, \frac{5}{2}J\rangle$ . Assuming the  $\Omega_c^0(2695)$  is the ground state with  $1^2S_1$  and then the  $\Omega_c^0(2695)$  is the lightest state, the mass spectra of the S- and P-wave states of  $\Omega_c^0(X)$  baryons can be obtained from the relations in  $E_{\text{GS}}(10)$  (15) Eqs.  $(10)$ – $(15)$ .

The coupling constants in Eq. [\(2\)](#page-1-0) are described in detail in Refs. [\[17,18,22](#page-5-18)–25]. In order to give more information of the coupling constants, we extract the coupling constants from the baryon mass relations [\[17\]](#page-5-18)

$$
(\kappa_{cs})_{\bar{3}} = 2K(c, \{u, s\}) - K(c, \{u, d\}), \qquad (16)
$$

$$
K(c, \{u, d\}) = \frac{1}{3}(m_{\Sigma_c^{*+}} - m_{\Sigma_c^+}),
$$
 (17)

$$
K(c, \{u, s\}) = \frac{1}{6} (2m_{\Xi_c^{*0}} - m_{\Omega_c^0} - m_{\Sigma_c^+}).
$$
 (18)

Inputting the related charmed baryon masses [\[26\],](#page-5-19) i.e.,  $m_{\Xi_c^{*0}} = (2645.9 \pm 0.5) \text{ MeV}, m_{\Omega_c^{0}} = (2695.2 \pm 1.7) \text{ MeV},$ <br> $m_{\Sigma_c^{*0}} = (2452.9 \pm 0.4) \text{ MeV}, \text{ and } m_{\Sigma_c^{*0}} = (2517.5 \pm 2.3) \text{ MeV}.$  $m_{\Sigma_c^+} = (2452.9 \pm 0.4) \text{MeV}$ , and  $m_{\Sigma_c^{++}} = (2517.5 \pm 2.3) \text{MeV}$ ,<br>the value of the coupling constant  $(\kappa_c)$ , can be extracted the value of the coupling constant  $(\kappa_{cs})_{\bar{3}}$  can be extracted as  $(\kappa_{cs})_{\bar{3}} = (26 \pm 1.5)$  MeV.

The parameters  $A_c$  and B that describe the orbital couplings of the excited states can be estimated by the comparison with the observed spin-orbital splitting in the  $\Xi_c^0(X)$  states. We have the estimation

$$
-2A_c + B \simeq m_{\Xi_c^0(\frac{1}{2})} - m_{\Xi_c^0(\frac{1}{2})}, \tag{19}
$$

$$
A_c + B \simeq m_{\Xi_c^0(\frac{3}{2}^-)} - m_{\Xi_c^0(\frac{1}{2}^+)}.
$$
 (20)

Inputting the related charmed baryon masses [\[26\]](#page-5-19), i.e.,  $m_{\Xi_c^0(\frac{1}{2})} = (2470.85_{-0.49}^{+0.28}) \text{ MeV}, m_{\Xi_c^0(\frac{1}{2})} = (2791.9 \pm 0.23) \text{ MeV}$ 3.3) MeV, and  $m_{\Xi_c^0(\frac{3}{2})} = (2819.6 \pm 1.2)$  MeV, the value of the coupling constants can be extracted as  $A_c(\Omega_c)$  =  $(9 \pm 1.5)$  MeV and  $B(\Omega_c) = (340 \pm 2)$  MeV.

Considering the uncertainties of the inputting parameters, the mass spectra of the S- and P-wave states of  $\Omega_c^0(X)$  baryons are given in Table [II](#page-3-0). In this table, the assignment of  $\Omega_c^0$  baryons to  $\ln^{2S+1}I$ , is by no means assignment of  $\Omega_c^0$  baryons to  $|n^{2S+1}L_J\rangle$  is by no means<br>conclusive. For instance, the O (2695) has been assigned conclusive. For instance, the  $\Omega_c(2695)$  has been assigned as the ground state only due to the fact there is no other lower state that has been established on the experimental side. In Table [II](#page-3-0) we also list the experimental data and other theoretical predictions. Most of them are based on the potential model, QCD sum rules, and lattice QCD simulation. Besides, some excited states of  $\Omega_c^0(X)$  baryons are<br>also predicted from meson-baryon unitarization starting also predicted from meson-baryon unitarization starting from a lowest order potential in Refs. [\[27,28\],](#page-5-20) where the existence of a bound state at 2959 MeV, near the lowest threshold, and two resonances placed at 2966 and 3117 MeV are predicted in this scheme. The widths of the two resonances are calculated as  $\Gamma(2966) = 1.1$  MeV and  $\Gamma(3117) = 16$  MeV.

The bottom partners of the  $\Omega_c^0(X)$  baryons can also<br>predicted. Assuming the  $Q_c^-(6046)$  with the mass be predicted. Assuming the  $\Omega_b^-(6046)$  with the mass (6046 + 1.0) MeV is the lightest state with 1<sup>2</sup>S, the  $(6046 \pm 1.9)$  MeV is the lightest state with  $1^{2}S_{1}$ , the spectra of  $\Omega_b^-(X)$  baryons are very similar to that of  $\Omega_b^0(Y)$  baryons. Their masses and spin parities are esti- $\Omega_c^0(X)$  baryons. Their masses and spin parities are esti-<br>mated as mated as

$$
M_{\Omega_b}(1^4 S_{\frac{3}{2}}) = (6121 \pm 8) \text{ MeV},
$$
  
\n
$$
M_{\Omega_b}(1^2 P_{\frac{1}{2}}) = (6444 \pm 10) \text{ MeV},
$$
  
\n
$$
M_{\Omega_b}(1^2 P_{\frac{3}{2}}) = (6459 \pm 8) \text{ MeV},
$$
  
\n
$$
M_{\Omega_b}(1^4 P_{\frac{1}{2}}) = (6504 \pm 22) \text{ MeV},
$$
  
\n
$$
M_{\Omega_b}(1^4 P_{\frac{3}{2}}) = (6519 \pm 16) \text{ MeV},
$$
  
\n
$$
M_{\Omega_b}(1^4 P_{\frac{5}{2}}) = (6544 \pm 18) \text{ MeV},
$$
  
\n(21)

<span id="page-3-0"></span>TABLE II. The mass spectra (MeV) of  $\Omega_c^0(X)$  baryons. The uncertainties of the experimental measurements are squared averages of those from the statistical and systematic, and the  $F^+$  mass those from the statistical and systematic, and the  $\Xi_c^+$  mass.

$n^{2S+1}L_1$	This work	Experiment $[8,26]$ <sup>a</sup>	[29]	[30]	[31]	$\lceil 32 \rceil$	[33]	[34]	[35]	[36]
$1^2S_{\frac{1}{2}}$	$2695.2 \pm 1.7$	$2695.2 \pm 1.7$	2695	2698	2718	2731	$\cdots$	2699	2648	2718
$1^4S_{\frac{3}{2}}$	$2773 \pm 6$	$2765.9 \pm 2.0$	2767	2768	2776	2779	$\cdots$	2767	$\cdots$	$\cdots$
$1^2P_{\frac{1}{2}}$	$3068 \pm 16$	$3050.2 \pm 0.5$	3011	3055	2977	3030	3250	2980	2995	3046
$1^2P_{\frac{3}{2}}$	$3095 \pm 11$	$3090.2 \pm 0.8$	2976	3054	2986	3033	3260	2980	3016	2986
$1^4P_1$	$3017 \pm 7$	$3000.4 \pm 0.5$	3028	2966	2977	$\cdots$	$\cdots$	3035	$\cdots$	$\cdots$
$1^4P_{\frac{3}{2}}$	$3044 \pm 5$	$3065.6 \pm 0.6$	2993	3029	2959	$\cdot$ $\cdot$ $\cdot$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$1^4P_5$	$3140 \pm 13$	$3119.1 \pm 1.1$	2947	3051	3014	3057	3320	$\cdots$	$\cdots$	3014

<sup>a</sup>The following assignment of  $\Omega_c^0$  baryons to  $|n^{2S+1}L_J\rangle$  is by no means conclusive. For instance, the  $\Omega_c(2695)$  has been assigned as experimental side. the ground state only due to the fact that there is no other lower state that has been established on the experimental side.

where the parameters are adopted as  $(\kappa_{bs})_{\bar{3}} = 25 \pm 2$  MeV,  $A_b(\Omega_b) = 5 \pm 2$  MeV, and  $B(\Omega_b) = 408 \pm 4$  MeV [\[19,20,22\]](#page-5-21). Since the observed spin-orbital splitting in the  $\Xi_b^-(X)$  states<br>is limited we only give the approximate error and discuss is limited, we only give the approximate error and discuss the uncertainties of the coupling constants in future works. The mass splitting for the P-wave orbitally excited states is very small. Currently, only the  $\Omega_b(1^2S_1)$  has been observed [\[26\]](#page-5-19). The S-wave orbitally excited state  $\Omega_b(1^4S_3)$  and the five B wave orbitally excited states are also be reason five P-wave orbitally excited states can also be reconstructed by the electroweak decay channel  $\Omega_b^{-}(X) \to$ <br> $J/\mu \to \Omega^-$  with the subdecays  $J/\mu \to \mu^+ \mu^- (e^+e^-)$  and  $J/\psi + \Omega^-$  with the subdecays  $J/\psi \rightarrow \mu^+\mu^-(e^+e^-)$  and  $\Omega^{-} \to \Lambda K^{-}(\Xi^{0}\pi^{-}) \to p\pi^{-}K^{-}(p\pi^{-}\pi^{0}\pi^{-})$ . This can be examined in the future.

## III. DECAYS INTO  $\Xi_c K$  AND  $\Xi_c' K$

In the heavy quark limit, the static heavy quark can only interact with gluons via its chromoelectric charge, which leads to the heavy quark spin symmetry. In this heavy quark limit, the spin of the heavy quark and the light degrees of freedom  $S_{\ell} = J - S_{Q}$  with  $Q = c$ , b are conserved, respectively. Thus some relations for the strong decays can be obtained.

In the heavy quark limit, the five P-wave baryonic states are given as

$$
\left|\frac{1}{2J}\right\rangle_{1'} = \left|\frac{1}{2c}; S_{\ell} = 0\right\rangle, \tag{22}
$$

$$
\left|\frac{1}{2J}\right\rangle_{2'} \equiv \left|\frac{1}{2c}; S_{\ell} = 1\right\rangle_{1'},
$$
\n(23)

$$
\left|\frac{3}{2J}\right\rangle_{1'} = \left|\frac{1}{2c}; S_{\ell} = 1\right\rangle_{2'},
$$
\n(24)

$$
\left|\frac{3}{2J}\right\rangle_{2'} = \left|\frac{1}{2c}; S_{\ell} = 2\right\rangle_{1},\tag{25}
$$

$$
\left|\frac{5}{2J}\right\rangle \equiv \left|\frac{1}{2c}; S_{\ell} = 2\right\rangle_{2}.
$$
 (26)

Apparently, the spin- $5/2$  baryonic state is the same as the one in Eq. [\(9\)](#page-2-2), while the two spin- $1/2$  and  $3/2$  states mix with each other respectively. The mixing matrix is given as

$$
\left|\frac{1}{2J}\right\rangle_{1'} = -\sqrt{\frac{1}{3}}\left|L=1,\frac{1}{2J}\right\rangle_{1} + \sqrt{\frac{2}{3}}\left|L=1,\frac{1}{2J}\right\rangle_{2},\tag{27}
$$

$$
\left|\frac{1}{2J}\right\rangle_{2'} = -\sqrt{\frac{2}{3}}\left|L=1,\frac{1}{2J}\right\rangle_{1} - \sqrt{\frac{1}{3}}\left|L=1,\frac{1}{2J}\right\rangle_{2}, \quad (28)
$$

for the two spin- $1/2$  states and

$$
\left|\frac{3}{2J}\right\rangle_{1'} = \sqrt{\frac{1}{6}}\left|L=1,\frac{3}{2J}\right\rangle_{1} + \sqrt{\frac{5}{6}}\left|L=1,\frac{3}{2J}\right\rangle_{2},\tag{29}
$$

$$
\left|\frac{3}{2J}\right\rangle_{2'} = \sqrt{\frac{5}{6}} \left| L = 1, \frac{3}{2J} \right\rangle_{1} - \sqrt{\frac{1}{6}} \left| L = 1, \frac{3}{2J} \right\rangle_{2}, \quad (30)
$$

for the spin- $3/2$  baryons.

In the heavy quark limit, the amplitudes of  $\Omega_c^0(X) \to (E^{\prime+})K^-$  can be expressed as  $\Xi_c^+(\Xi_c^{\prime+})K^-$  can be expressed as

$$
\mathcal{A}(\Omega_c(J, J_z) \to \Xi_c^{(\prime)}(J', J'_z)K(L, L_z))
$$
\n
$$
= \sum \left\langle \frac{1}{2}, S_{cz}; S_{\ell}, S_{\ell z} | J, J_z \right\rangle
$$
\n
$$
\times \left\langle \frac{1}{2}, S_{cz}; S'_{\ell}, S'_{\ell z} | J', J'_z \right\rangle
$$
\n
$$
\times \langle L, S'_{\ell}; ||\mathcal{H}_{eff}||S_{\ell}\rangle \langle L, L_z; S'_{\ell}, S'_{\ell z}|S_{\ell}, S_{\ell z}\rangle,
$$
\n(31)

where the quantum numbers  $S_{\ell}$  and  $S_{\ell}'$  are the spin of the light degrees of freedom in  $\Omega_c^0(X)$  and  $\Xi_c^+(\Xi_c^{+\})$ , respectively and the quantum numbers *L* and *L'* are the total tively, and the quantum numbers  $J$  and  $J'$  are the total angular momentum of  $\Omega_c^0(X)$  and  $\Xi_c^+(\Xi_c^{+})$ , respectively.

#### INTERPRETATION OF THE NEWLY OBSERVED … PHYSICAL REVIEW D 96, 014024 (2017)

The decay widths of  $\Omega_c^0(X) \to \Xi_c^+(\Xi_c^{\prime+})K^-$  are propor-<br>nal to Clebsch-Gordan coefficients tional to Clebsch-Gordan coefficients

$$
\Gamma \propto (2S_{\ell} + 1)(2J' + 1) \left| \begin{Bmatrix} L & S'_{\ell} & S_{\ell} \\ \frac{1}{2} & J & J' \end{Bmatrix} \right|^2, \quad (32)
$$

where the product of Clebsch-Gordan coefficients is in terms of 6j symbols.

For  $\Omega_c^0(X) \to \Xi_c^+ K^-$ , the quantum numbers are

$$
S'_{\ell} = 0,
$$
  $S_{\ell} = (0, 1, 2),$   $J' = \frac{1}{2},$   $J = \left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right).$  (33)

We find the following results:

- (i) Due to the parity conservation, the decays can proceed through the S wave or D wave.
- (ii) Only the lowest-lying state,  $\frac{1}{2}S_{\xi=0}$ , can decays into the  $\Xi$  K in the S wave. The  $\frac{1}{2}$  may mix with the  $\Xi_c K$  in the S wave. The  $\left(\frac{1}{2}\right)_{S_\ell=0}$  may mix with  $\left(\frac{1}{2}\right)_{S_{\ell}=1}$  in QCD. However we expect that their low<br>masses do not allow a large phase space. So the 1/2 masses do not allow a large phase space. So the  $1/2$ states do not have large decay widths.
- (iii) The  $\left|\frac{3}{2}\right\rangle_{S_{\ell}=2}$  and  $\left|\frac{5}{2}\right\rangle_{S_{\ell}=2}$  can decay into the  $\Xi_c K$ <br>through the D wave For the  $\left|\frac{5}{2}\right\rangle$  this is guarthrough the D wave. For the  $\frac{5}{2}S_{\epsilon=2}$ , this is guaranteed by the angular momentum conservation, while the heavy quark symmetry relates the decays of  $\left|\frac{3}{2}\right\rangle_{S_e=2}$ . Such amplitudes are also suppressed due to the phase space. Thus the total widths are expected to be small again.
- (iv) The breaking of heavy quark symmetry may induce small contributions to decay widths.

For the channel  $\Omega_c^0(X) \to \Xi_c^{\prime +} K^-$ , the related quantum<br>mbers of the initial and final states are numbers of the initial and final states are

$$
S'_{\ell} = 1,
$$
  $S_{\ell} = (0, 1, 2),$   $J' = \frac{1}{2},$   $J = \left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right).$  (34)

The following remarks are given in order.

- (i) The threshold of  $\Xi_c^{\prime+} K^-$  is about 3069 MeV, which prohibits decays of the lower three baryons.
- (ii) Decays of  $\Omega$ (3090) and  $\Omega$ (3119) into  $\Xi_c^{+K^-}$  have some phase space some phase space.
- (iii) From the 6j symbol, we find that the S-wave decay is through  $|1/2\rangle_{S_{\ell}=1} \rightarrow \Xi_{c}^{\prime} K$ . But considering the threshold of  $\Xi^{\prime+} K^-$  is about 3069 MeV this is not threshold of  $\Xi_c^{\prime + K^-}$  is about 3069 MeV, this is not kinematically allowed.
- (iv) There are D-wave decay amplitudes for  $|1/2\rangle_{S_{\ell}=1} \rightarrow$  $\Xi_c' K$ ,  $|3/2\rangle_{S_c=2} \to \Xi_c' K$ , and  $|5/2\rangle_{S_c=2} \to \Xi_c' K$ .<br>However these contributions are not big since the However, these contributions are not big since the phase space is limited.

Since both decays into  $\Xi_c^+ K^-$  and  $\Xi_c^{\prime+} K^-$  are suppressed, the narrowness of the five newly observed  $\Omega_c$  states can be understood using heavy quark symmetry.

In the heavy-quark-light-diquark model, the decay of  $\Omega_c$  into  $\Xi_c^+ K^-$  requires tearing the ss diquark apart, and thus the calculation of the width decay into  $\Xi_c K$  is beyond the quark-diquark scheme mainly used in this work. A tool to estimate the decay width might be using the flavor SU(3) symmetry to relate to other charmed baryons, for instance  $\Gamma(\Lambda_c(2595)) = (2.6 \pm 0.6) \text{ MeV}$ ,<br>  $\Gamma(\Lambda_c(2625)) < 0.97 \text{ MeV}$  [26],  $\Xi_c^+(2645) = (2.1 \pm 0.6) \text{ MeV}$  $\Gamma(\Lambda_c(2625)) < 0.97 \text{ MeV}$  [\[26\]](#page-5-19),  $\Xi_c^+$ <br>
(2020) MeV  $\Xi^+(2790) - (8.9 + 1.0)$  [37] [26],  $\Xi_c^+(2645)=(2.1\pm$ 0.2) MeV,  $\Xi_c^+(2790) = (8.9 \pm 1.0)$  [\[37\].](#page-5-30) This can give us a bint that the corresponding O states might be narrow us a hint that the corresponding  $\Omega_c$  states might be narrow. However, a conclusive result requires the classification of the  $\Lambda_c$  and  $\Xi_c$  baryons and a more comprehensive analysis to be published in the future.

### IV. CONCLUSION

In this work, we have studied the charmed and bottomed baryons with two strange quarks in a quark-diquark model. The two strange quarks lie in the S wave and thus their total spin is 1. Within the heavy-quark-light-diquark framework, we calculate the mass spectra of the S- and P-wave orbitally excited states. We find that the  $\Omega_c^0(2695)$  and  $\Omega_c^0(2770)$  fit<br>well as the S-wave states of charmed doubly strange well as the S-wave states of charmed doubly strange baryons. There are five P-wave states. The five newly  $\Omega_c^0$  resonances observed by the LHCb Collaboration, i.e.,  $\Omega_c^0(3000)$ ,  $\Omega_c^0(3050)$ ,  $\Omega_c^0(3066)$ ,  $\Omega_c^0(3090)$ , and  $\Omega_c^0(3119)$ , can be interpreted as the *P*-wave orbitally excited states of can be interpreted as the P-wave orbitally excited states of charmed doubly strange baryons. We have analyzed their decays into the  $\Xi_c K$  and  $\Xi_c K$  in the heavy quark effective theory. We find that decays of the five new  $\Omega_c$  states into the  $\Xi_c K$  and  $\Xi_c K$  are suppressed by the heavy quark symmetry or the phase space. The narrowness of the five newly observed  $\Omega_c$  states can be understood using heavy quark symmetry.

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Note added.—Recently, there were studies of the masses and/or decay properties of the newly observed  $\Omega_c^0(X)$  states<br>using different approaches: the OCD sum rules [38–421] using different approaches: the QCD sum rules [\[38](#page-5-31)–42], heavy hadron chiral perturbation theory [\[43\]](#page-5-32), the chiral quark-soliton model [\[44\],](#page-5-33) lattice QCD [\[45\],](#page-5-34) the constituent quark models, and treatment as pentaquarks [\[46,47\].](#page-5-35)

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