PHYSICAL REVIEW D **96,** 014018 (2017)

Hidden-charm pentaquarks as a meson-baryon molecule with coupled channels for $\bar{D}^{(*)}\Lambda_c$ and $\bar{D}^{(*)}\Sigma_c^{(*)}$

Yasuhiro Yamaguchi* and Elena Santopinto†

Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy

(Received 11 November 2016; revised manuscript received 14 June 2017; published 21 July 2017)

The recent observation of two hidden-charm pentaquark states by LHCb collaborations prompted us to investigate the exotic states close to the $\bar{D}\Lambda_c$, $\bar{D}^*\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Sigma_c^*$ thresholds. We therefore studied the hadronic molecules that form the coupled-channel system of $\bar{D}^{(*)}\Lambda_c$ and $\bar{D}^{(*)}\Sigma_c^{(*)}$. As the heavy quark spin symmetry manifests the mass degenerations of $\bar{D}$ and $\bar{D}^*$ mesons, and of Σ_c and Σ_c^* baryons, the coupled channels of $\bar{D}^{(*)}\Sigma_c^{(*)}$ are important in these molecules. In addition, we consider the coupling to the $\bar{D}^{(*)}\Lambda_c$ channel whose thresholds are near the $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds, and the coupling to the state with nonzero orbital angular momentum mixed by the tensor force. This full coupled-channel analysis of $\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ with larger orbital angular momentum has never been performed before. By solving the coupled-channel Schrödinger equations with the one meson exchange potentials with respect to the heavy quark spin and chiral symmetries, we studied the hidden-charm hadronic molecules with $I(J^P) = 1/2(3/2^\pm)$ and $1/2(5/2^\pm)$. We conclude that the J^P assignment of the observed pentaquarks is $3/2^+$ for $P_c^+(4380)$ and $5/2^-$ for $P_c^+(4450)$, which is in agreement with the results of the LHCb analysis. In addition, we give predictions for other $J^P = 3/2^\pm$ states at 4136.0, 4307.9 and 4348.7 MeV in $J^P = 3/2^-$, and 4206.7 MeV in $J^P = 3/2^+$, which can be further investigated by means of experiment.

DOI: 10.1103/PhysRevD.96.014018

I. INTRODUCTION

In 2015, LHCb collaborations reported the two hidden-charm pentaquarks $P_c^+(4380)$ and $P_c^+(4450)$ in $\Lambda_b^0 \to J/\psi K^- p$ decay [1–3]. The reported masses and widths are $(M,\Gamma) = (4380 \pm 8 \pm 29, 205 \pm 18 \pm 86)$ MeV and $(4449.8 \pm 1.7 \pm 2.5, 39 \pm 5 \pm 19)$ MeV, respectively, which are close to $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c$ thresholds. Their significances are 9 and 12 standard deviations, respectively. The total angular momentum is $3/2$ for one state and $5/2$ for the other. These states have opposite parity. The minimal quark content of the pentaquarks is $c\bar{c}uud$ because the states decay into $J/\psi p$. In the literature there have been lively discussions about the structure of the hidden-charm pentaquarks. The compact pentaquark states have been discussed by the (di)quark model [4–8] and Gürsey-Radicati inspired formula [9]. The hadronic molecules have been studied by the meson-baryon coupled-channel approach [10–20] and the QCD sum rules [21,22]. On the other hand, the threshold enhancement by the anomalous triangle singularity is discussed in Refs. [23–25].

Near the thresholds, resonances are expected to have the exotic structure, like the hadronic molecule. In the strangeness sector, $\Lambda(1405)$ is considered to be generated by the $\bar{K}N$ and $\pi\Sigma$ [26–28]. In the heavy quark sectors, $X(3872)$ has the dominant component of the $D\bar{D}^*$ molecules [29–33]. The charged quarkonium states $Z_c(3900)$ [34] and $Z_b^{(\prime)}$ [35] are considered to be $D\bar{D}^*$ [36] and $B^{(*)}\bar{B}^*$ [37–39], respectively. The observed pentaquarks are found just below the $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c$ thresholds. Therefore the $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c$ molecular components are expected to be dominant.

Since the hadronic molecules are dynamically generated by the hadron-hadron interaction, the properties of the interaction are important in producing those structures. In the literature, the SU(4) flavor symmetric interaction has been applied to the charm sector. This is an extension of the interaction based on the SU(3) flavor symmetry applied to the strangeness sector. In the hidden-charm pentaquarks, the interactions based on the SU(4) flavor symmetry have been used [10–12]. However, the SU(4) symmetry is expected to be broken because the mass of the charm quark is much larger than those of light quarks.

In the heavy flavor sector, new symmetry of heavy quarks emerges which is called heavy quark symmetry [40–43]. This results from the suppression of the spin-dependent interaction among heavy quarks. It manifests the mass degeneracy of the states with different total spin, e.g. degeneracies of D and D^* mesons ($\Delta m_{DD^*} \sim 140$ MeV) and Σ_c and Σ_c^* baryons ($\Delta m_{\Sigma_c\Sigma_c^*} \sim 65$ MeV). Therefore, hadronic states should be a coupled-channel system. In that case, thresholds of $\bar{D}^{(*)}\Sigma_c^{(*)}$ ($\bar{D}^{(*)} = \bar{D}$ or $\bar{D}^*$, and $\Sigma_c^{(*)} = \Sigma_c$ or Σ_c^*) are close to the states we are going to study (see also Table I).

*yasuhiro.yamaguchi@ge.infn.it
†elena.santopinto@ge.infn.it

TABLE I. Various channels of the $\bar{D}^{(*)}\Lambda_c$ and $\bar{D}^{(*)}\Sigma_c^{(*)}$ states for the given total angular momentum and parity J^P and the corresponding thresholds. In each channel, the total spin S and orbital angular momentum L is represented as ${}^{2S+1}L$. The thresholds as a sum of the mass of the meson and baryon in the last row are given in the unit of MeV.

J^P	Channels
$3/2^-$	$\bar{D}\Lambda_c(^2D), \bar{D}^*\Lambda_c(^4S, ^2D, ^4D), \bar{D}\Sigma_c(^2D), \bar{D}\Sigma_c^*(^4S, ^4D), \bar{D}^*\Sigma_c(^4S, ^2D, ^4D), \bar{D}^*\Sigma_c^*(^4S, ^2D, ^4D, ^6D, ^6G)$
$3/2^+$	$\bar{D}\Lambda_c(^2P), \bar{D}^*\Lambda_c(^2P, ^4P, ^4F), \bar{D}\Sigma_c(^2P), \bar{D}\Sigma_c^*(^4P, ^4F), \bar{D}^*\Sigma_c(^2P, ^4P, ^4F), \bar{D}^*\Sigma_c^*(^2P, ^4P, ^6P, ^4F, ^6F)$
$5/2^-$	$\bar{D}\Lambda_c(^2D), \bar{D}^*\Lambda_c(^2D, ^4D, ^4G), \bar{D}\Sigma_c(^2D), \bar{D}\Sigma_c^*(^4D, ^4G), \bar{D}^*\Sigma_c(^2D, ^4D, ^4G), \bar{D}^*\Sigma_c^*(^6S, ^2D, ^4D, ^6D, ^4G, ^6G)$
$5/2^+$	$\bar{D}\Lambda_c(^2F), \bar{D}^*\Lambda_c(^4P, ^2F, ^4F), \bar{D}\Sigma_c(^2F), \bar{D}\Sigma_c^*(^4P, ^4F), \bar{D}^*\Sigma_c(^4P, ^2F, ^4F), \bar{D}^*\Sigma_c^*(^4P, ^6P, ^2F, ^4F, ^6F, ^6H)$
	Thresholds (MeV)
	$\bar{D}\Lambda_c(4153.5), \bar{D}^*\Lambda_c(4295.5), \bar{D}\Sigma_c(4320.5), \bar{D}\Sigma_c^*(4385.1), \bar{D}^*\Sigma_c(4462.5), \bar{D}^*\Sigma_c^*(4527.1)$

Moreover, we cannot ignore the $\bar{D}^{(*)}\Lambda_c$ channel. In the strangeness sector, the $\Lambda - \Sigma$ mixing is important in the hyperon-nucleon interaction [44]. In the early works [11,12,19,20], however, the coupling to $\bar{D}^{(*)}\Lambda_c$ is not considered in the hidden-charm pentaquarks. However, the $\bar{D}^*\Lambda_c$ threshold is 25 MeV below the $\bar{D}\Sigma_c$ threshold. Therefore, the $\bar{D}^*\Lambda_c$ channel is not irrelevant in the hidden-charm meson-baryon molecules.

The approximate mass degeneracy of heavy hadrons changes the aspect of interactions in the heavy quark sector. Indeed, the $\bar{D}N - \bar{D}^*N$ mixing enhances the effect of the one pion exchange potential (OPEP) in the $\bar{D}$ meson-nucleon ($\bar{D}N$) system, while the $KN - K^*N$ mixing is suppressed due to the large mass difference between K and K^* mesons ($\Delta m_{KK^*} \sim 400$ MeV) in the strangeness sector. In nuclear physics, the OPEP is the basic ingredient of the nuclear force that binds the atomic nuclei. Specifically, the tensor force mixing S-wave and D-wave components yields the strong attraction. This mechanism has been suggested to have an important role in the $\bar{D}^{(*)}N$ system in Refs. [45–52]. The coupled-channel analysis with the mixing of S-wave and D-wave was not performed in Refs. [11,12]. However, this mixing is helpful to produce the attraction in the hidden-charm molecules.

On the basis of the above discussions, we consider the coupled-channel systems of $\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ including states with larger orbital angular momentum, namely D-wave and G-wave for the negative parity state and F-wave and H-wave for the positive parity state, as summarized in Table I. This full-channel coupling has never been considered so far. The interaction is obtained by the one meson exchange potential that respects the heavy quark spin symmetry. The bound and resonant states in $I(J^P) = 1/2(3/2^\pm)$ and $1/2(5/2^\pm)$ are studied by solving the coupled-channel Schrödinger equations. In this study, the contribution of the decay processes $\Lambda_b \to K^- P_c$ and $P_c \to J/\psi N$ is not considered. In the literature [53], the selection rules due to the heavy quark symmetry have been discussed in the decay process $\Lambda_b \to K^- P_c$. Those rules would prefer the molecular components, e.g. $\bar{D}^{(*)}\Lambda_c$ and $J/\psi N$. For $\bar{D}^{(*)}\Lambda_c$, however, the $\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ transition can emerge, and our coupled-channel analysis includes this transition which is constrained by the heavy quark symmetry as long as we use the heavy hadron effective Lagrangians. On the other hand, the $J/\psi N$ component is not included in our calculation, because the model calculations in Refs. [11,12,15] obtained that the contribution of the couplings to $J/\psi N$ is too small to change the mass spectrum. The coupled-channel analysis including the $J/\psi N$ channel is left as the future work.

This paper is organized as follows. The meson exchange potentials between the charmed meson and baryon are shown in Sec. II. The numerical results are summarized in Sec. III. Section IV summarizes the work as a whole.

II. INTERACTIONS

The Lagrangians satisfying the heavy quark and chiral symmetries are employed. The Lagrangians for a heavy meson and a light meson are given [43,54–57] as

$$\mathcal{L}_{\pi HH} = g_\pi \mathrm{Tr}[H_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{H}_a], \tag{1}$$

$$\mathcal{L}_{vHH} = -i\beta \mathrm{Tr}[H_b v^\mu (\rho_\mu)_{ba} \bar{H}_a] + i\lambda \mathrm{Tr}[H_b \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \bar{H}_a], \tag{2}$$

$$\mathcal{L}_{\sigma HH} = g_s \mathrm{Tr}[H_a \sigma \bar{H}_a], \tag{3}$$

where the subscripts π, v and σ are for the pion, vector meson (ρ and ω) and sigma meson, respectively. v^μ is a four-velocity of a heavy quark. The heavy meson field constructed by the pseudoscalar meson P and vector meson P^* are represented [43,54–57] by

$$H_a = \frac{1+\not{v}}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5], \qquad \bar{H}_a = \gamma_0 H_a^\dagger \gamma_0, \tag{4}$$

where the subscripts a, b are for the light flavor. The axial vector current A_μ is given by

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \tag{5}$$

where $\xi = \exp(i\hat{\pi}/2f_\pi)$ with the pion decay constant $f_\pi = 92.3$ MeV. The pseudoscalar and vector meson fields are given by

$$\hat{\pi} = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \tag{6}$$

$$\rho_\mu = i\frac{g_V}{2}\hat{\rho}_\mu, \tag{7}$$

$$\hat{\rho}_\mu = \sqrt{2}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu, \tag{8}$$

$$F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu. \tag{9}$$

The gauge coupling constant g_V is obtained as $g_V = m_\rho/\sqrt{2}f_\pi$ [58].

The πPP^* coupling constant is determined by the strong decay of $D^* \to D\pi$ [59]. The coupling constants β and λ are fixed by the vector meson decays [60]. The coupling constant for the sigma meson is given by $g_s = -g'_\pi/2\sqrt{6}$ with the $0^+ \to 0^-\pi$ coupling constant $g'_\pi = 3.73$ [61]. These coupling constants are summarized in Table II.

The Lagrangians for a heavy baryon and a light meson [56,62] are given by

$$\mathcal{L}_{\pi BB} = \frac{3}{2}g_1 i v_\kappa \epsilon^{\mu\nu\lambda\kappa}\mathrm{tr}[\bar{S}_\mu A_\nu S_\lambda] + g_4\mathrm{tr}[\bar{S}^\mu A_\mu B_{\bar{3}}] + \mathrm{H.c.}, \tag{10}$$

$$\mathcal{L}_{vBB} = -i\beta_B\mathrm{tr}[\bar{B}_{\bar{3}}v^\mu\rho_\mu B_{\bar{3}}] - i\beta_S\mathrm{tr}[\bar{S}_\mu v^\alpha\rho_\alpha S^\mu] + \lambda_S\mathrm{tr}[\bar{S}_\mu F^{\mu\nu}S_\nu] + i\lambda_I\epsilon^{\mu\nu\lambda\kappa}v_\mu\mathrm{tr}[\bar{S}_\nu F_{\lambda\kappa}B_{\bar{3}}] + \mathrm{H.c.}, \tag{11}$$

$$\mathcal{L}_{\sigma BB} = \ell_B\mathrm{tr}[\bar{B}_{\bar{3}}\sigma B_{\bar{3}}] + \ell_S\mathrm{tr}[\bar{S}_\mu\sigma S^\mu]. \tag{12}$$

The superfield S_μ for $\Sigma_Q^{(*)}$ is represented by

$$S_\mu = B^*_{6\mu} + \frac{\delta}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 B_6, \qquad \bar{S}_\mu = \gamma_0 S^\dagger_\mu\gamma_0. \tag{13}$$

The phase factor is chosen by $\delta = -1$ as discussed in Ref. [62]. Heavy baryon fields are expressed by the 3×3 matrix form [56,62]:

$$B_6 = \begin{pmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^0 & \frac{1}{\sqrt{2}}\Xi_Q^{\prime +1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^0 & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime -1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q^{\prime +1/2} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime -1/2} & \Omega_Q \end{pmatrix}, \tag{14}$$

$$B_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_Q & \Xi_Q^{+1/2} \\ \Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{+1/2} & -\Xi_Q^{\prime -1/2} & 0 \end{pmatrix}. \tag{15}$$

The matrix for B_6^* is similar to B_6. The field of the B_6^* baryon is given by the Rarita-Schwinger field [62,63]. We use the coupling constants given by the quark model estimation discussed in Ref. [62].

In this study, we employ the leading order of the heavy hadron effective Lagrangians, and neglect the $1/M$ corrections with the heavy hadron mass M. Hence, the light degrees of freedom are separated from the heavy quark. In other words, the heavy quark is regarded as a spectator.

From the effective Lagrangians introduced above, we obtain the meson exchange potentials as

$$V_\pi^{ij}(r) = G_\pi^{ij}[\vec{\mathcal{O}}_1^i\cdot\vec{\mathcal{O}}_2^j C(r;m_\pi) + S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{r})T(r;m_\pi)], \tag{16}$$

$$V_v^{ij}(r) = G_v^{ij}C(r;m_v) + F_v^{ij}[-2\vec{\mathcal{O}}_1^i\cdot\vec{\mathcal{O}}_2^j C(r;m_v) + S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{r})T(r;m_v)], \tag{17}$$

$$V_\sigma^{ij}(r) = G_\sigma^{ij}C(r;m_\sigma). \tag{18}$$

m_α ($\alpha = \pi, \rho, \omega, \sigma$) is the mass of the exchanged light meson. In this study, we suppress the potential term, which

TABLE II. Masses of the exchanged mesons and coupling constants of the interaction Lagrangians for the heavy mesons and heavy baryons [46,60–62].

	m_α [MeV]	Meson	Baryon
π	137.27	$g_\pi = 0.59$	$g_1 = (3/\sqrt{8})g_4 = 1.0$
ρ	769.9	$\beta = 0.9$	$\beta_S = -2\beta_B = 12.0/g_V$
		$\lambda = 0.59[\mathrm{GeV}^{-1}]$	$\lambda_S = -2\sqrt{2}\lambda_I = 19.2/g_V[\mathrm{GeV}^{-1}]$
ω	781.94	$\beta = 0.9$	$\beta_S = -2\beta_B = 12.0/g_V$
		$\lambda = 0.59[\mathrm{GeV}^{-1}]$	$\lambda_S = -2\sqrt{2}\lambda_I = 19.2/g_V[\mathrm{GeV}^{-1}]$
σ	550.0	$g_s = -0.76$	$\ell_S = -2\ell_B = 7.30$

is proportional to the inverse of the heavy baryon mass. i and j stand for the indices of the channels. G_α^{ij} is the constant of the (i, j) component given by the coupling constants of the Lagrangians. $\mathcal{O}_1^i$ and $\mathcal{O}_2^j$ are the (transition) spin operator of the heavy meson and heavy baryon vertices, respectively [45–47,62]. $S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{r})$ is the tensor operator $S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{r}) = 3\vec{\mathcal{O}}_1^i\cdot\hat{r}\vec{\mathcal{O}}_2^j\cdot\hat{r} - \vec{\mathcal{O}}_1^i\cdot\vec{\mathcal{O}}_2^j$. The potential for the isovector mesons, π and ρ, is multiplied by the isospin factor, $\sqrt{6}$ for $\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ and -2 for $\bar{D}^{(*)}\Sigma_c^{(*)} - \bar{D}^{(*)}\Sigma_c^{(*)}$ with $I = 1/2$. The explicit form of the potentials is summarized in the Appendix.

The functions $C(r; m_\alpha)$ and $T(r; m_\alpha)$ are given by

$$C(r; m_\alpha) = \int \frac{d^3q}{(2\pi)^3}\frac{m_\alpha^2}{\vec{q}^2 + m_\alpha^2} \times e^{i\vec{q}\cdot\vec{r}}F_\alpha(\Lambda, \vec{q})S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{r})T(r; m_\alpha) \tag{19}$$

$$= \int \frac{d^3q}{(2\pi)^3}\frac{-\vec{q}^2}{\vec{q}^2 + m_\alpha^2}S_{\mathcal{O}_1^i\mathcal{O}_2^j}(\hat{q})e^{i\vec{q}\cdot\vec{r}}F_\alpha(\Lambda, \vec{q}), \tag{20}$$

where $\vec{q}$ is the momentum of the exchanged light meson. We introduce the standard dipole-type form factor $F_\alpha(\Lambda, \vec{q})$ for spatially extended hadrons [20,45–47,62]

$$F_\alpha(\Lambda, \vec{q}) = \frac{\Lambda_P^2 - m_\alpha^2}{\Lambda_P^2 + \vec{q}^2}\frac{\Lambda_B^2 - m_\alpha^2}{\Lambda_B^2 + \vec{q}^2}, \tag{21}$$

with the cutoff parameters Λ_P and Λ_B for the heavy meson and the heavy baryon, respectively. In this study, we employ the common cutoff parameter $\Lambda = \Lambda_P = \Lambda_B$ for simplicity, as discussed in Refs. [20,62]. In this study, only the cutoff Λ is a free parameter. We determine Λ in order to reproduce the mass spectra of the observed pentaquarks.

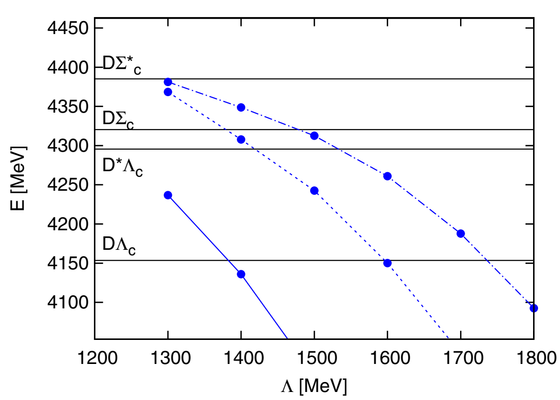

FIG. 1. Cutoff Λ dependence of the obtained energies E in $I(J^P) = 1/2(3/2^-)$. The solid, dashed and dot-dashed lines correspond to the obtained energy spectra of first, second and third states, respectively. The explicit values are shown in Table III. The black horizontal lines show the meson-baryon thresholds.

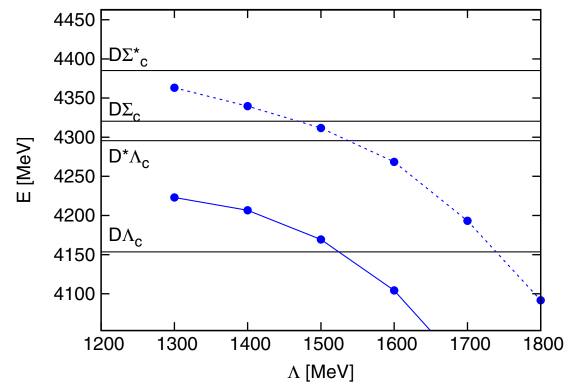

FIG. 2. The same as Fig. 1 for $I(J^P) = 1/2(3/2^+)$.

III. NUMERICAL RESULTS

The total Hamiltonian is given by the sum of the kinetic term and the meson exchange potential between the heavy meson and the heavy baryon for the coupled-channels in Eqs. (16)–(18). The interaction is the heavy quark spin symmetric. However, the breaking effect of the symmetry is given by the mass splittings of $\bar{D}$ and $\bar{D}^*$, and Σ_c and Σ_c^* in this calculation. By diagonalizing the Hamiltonian, we obtain the energy of the bound and resonant states.

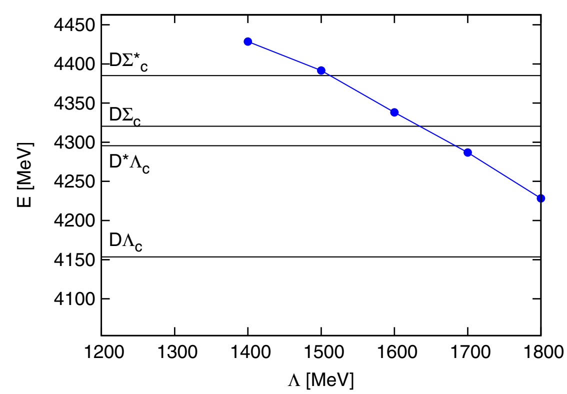

FIG. 3. The same as Fig. 1 for $I(J^P) = 1/2(5/2^-)$.

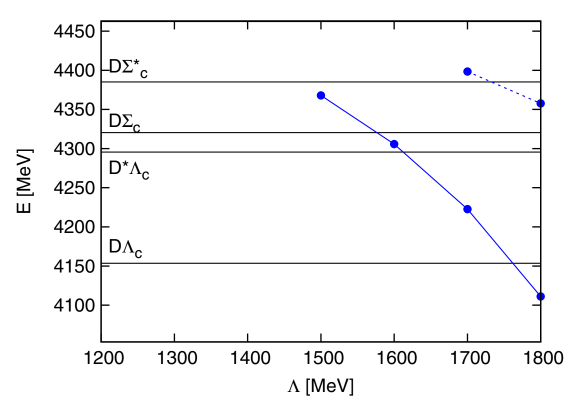

FIG. 4. The same as Fig. 1 for $I(J^P) = 1/2(5/2^+)$.

TABLE III. Obtained energies in $J^P = 3/2^\pm$ and $5/2^\pm$ with the various cutoffs Λ. The real energy gives the binding energy when the value of $\bar{D}\Lambda_c$ threshold (= 4153.5 MeV) is subtracted. The complex energy is given by $E = E_{\rm re} - i\Gamma/2$ with the resonance energy $E_{\rm re}$ and the decay width Γ. Note that the decay to $J/\psi p$ is not considered in this study.

Λ [MeV]	1300	1400	1500	1600	1700	1800
$J^P = 3/2^-$	$4236.9 - i0.8$	4136.0	4006.3	3848.2	3660.0	3438.26
	$4368.5 - i64.9$	$4307.9 - i18.8$	$4242.6 - i1.4$	4150.1	4035.2	3897.3
	$4381.3 - i11.4$	$4348.7 - i21.1$	$4312.7 - i16.0$	$4261.0 - i7.0$	$4187.7 - i0.9$	4092.5
$J^P = 3/2^+$	$4223.0 - i97.9$	$4206.7 - i41.2$	$4169.3 - i5.3$	4104.2	3996.7	3855.8
	$4363.3 - i57.0$	$4339.7 - i26.8$	$4311.8 - i6.6$	$4268.5 - i1.3$	$4193.2 - i0.1$	4091.6
$J^P = 5/2^-$	…	$4428.6 - i89.1$	$4391.7 - i88.8$	$4338.2 - i56.2$	$4286.8 - i27.3$	$4228.3 - i7.4$
$J^P = 5/2^+$	…	…	$4368.0 - i9.2$	$4305.8 - i1.9$	$4222.7 - i1.4$	4111.1
	…	…	…	…	$4398.5 - i15.0$	$4357.8 - i8.2$

The wave function is expressed by the Gaussian expansion method [64]. In order to obtain a complex energy of a resonance, the complex scaling method is used in this study [65–68].

We study the molecular states of $\bar{D}^{(*)}\Lambda_c^{(*)} - \bar{D}^{(*)}\Sigma^{(*)}$ with $J^P = 3/2^\pm, 5/2^\pm$ and isospin $I = 1/2$. The obtained energies with various cutoffs Λ are summarized in Figs. 1–4 and Table III. The energy above the $\bar{D}\Lambda_c$ threshold (= 4153.5 MeV) is given by the complex value $E = E_{\rm re} - i\Gamma/2$ with the resonance energy $E_{\rm re}$, and the decay width Γ for the meson-baryon scattering states considered in this analysis. The real energy below the $\bar{D}\Lambda_c$ threshold gives the binding energy by subtracting the value of the $\bar{D}\Lambda_c$ threshold. Figures 1–4 do not show the bound states with large binding energy because the hadronic molecular picture is not applicable to the deeply bound state [38,48]. Figures 1–4 show that the energy of states decreases when the cutoff Λ increases. In large Λ regions, the deeply bound state appears.

The cutoff parameter Λ is fixed to reproduce the observed pentaquarks, $P_c^+(4380)$ and $P_c^+(4450)$. We then focus on the narrow resonance $P_c^+(4450)$ whose significance is 12 standard deviations. In our calculations, the state close to the mass of $P_c^+(4450)$, $4449.8 \pm 1.7 \pm 2.5$ MeV is the $J^P = 5/2^-$ state with the resonance energy 4428.6 MeV in $\Lambda = 1400$ as shown in Fig. 3. Hence, the cutoff Λ is determined as $\Lambda = 1400$, and the J^P assignment of $P_c^+(4450)$ is $J^P = 5/2^-$. This result shows that the state corresponding to $P_c^+(4380)$ has $J = 3/2$ and the opposite parity of $P_c^+(4450)$, namely $J^P = 3/2^+$. At $\Lambda = 1400$ MeV in Fig. 2, the mass of the second state of $J^P = 3/2^+$ is 4339.7 MeV, which is near the mass of $P_c^+(4380)$, $4380 \pm 8 \pm 29$ MeV. Therefore, the state obtained with $J^P = 3/2^+$ corresponds to the observed pentaquark $P_c^+(4380)$. On the other hand, we find resonances other than the observed pentaquarks in $\Lambda = 1400$ MeV. These states are new predictions in this study. As shown in Table III and in Figs. 1 and 2, the $J^P = 3/2^-$ state has three states, whose masses are 4136.0 MeV, 4307.9 MeV and 4348.7 MeV, respectively, and the $J^P = 3/2^+$ state has one state with the mass, 4206.7 MeV. By contrast, the state with $J^P = 5/2^+$ in Fig. 4 is absent in $\Lambda = 1400$ MeV. The obtained cutoff $\Lambda = 1400$ MeV is close to the one employed in the Bonn NN potential [69], and not far from those in the model calculations in Refs. [13,18–20].

Our results are compared with those of the earlier studies on hidden-charm molecular states with $J^P = 3/2^-$ [11,12,15]. As summarized in Table IV, the energies obtained in this study are slightly greater than the results of the earlier works, where the full-channel coupling of

TABLE IV. Comparison between the lowest mass of hidden-charm meson-baryon molecules with $I(J^P) = 1/2(3/2^-)$ yielded by this work and those yielded by the early works [11,12,15]. The masses are shown in the second column in the unit of MeV. In this work, the value of Λ is 1400 MeV. The third column gives the channels which are considered in those works.

Reference	Mass [MeV]	Channels
This work	4136.0	$\bar{D}\Lambda_c, \bar{D}^*\Lambda_c, \bar{D}\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$
[11]	4415	$\bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$ with only S-wave
[12]	4454	$\bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$ with only S-wave
[15]	4334.5	$J/\psi N, \bar{D}^*\Lambda_c, \bar{D}^*\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c^*$ with only S-wave

TABLE V. Obtained masses with full channel coupling (Full), without $\bar{D}^{(*)}\Lambda_c$ (w/o $\bar{D}^{(*)}\Lambda_c$) and without large orbital angular momentum ℓ (w/o $\ell > 0$ or w/o $\ell > 1$) in $\Lambda = 1400$ MeV. The masses with full channel coupling are given in Table III.

J^P	Channels	Mass [MeV]
$3/2^-$	Full	4136.0, 4307.9, 4348.7
	w/o $\bar{D}^{(*)}\Lambda_c$	4278.4, 4400.4
	w/o $\ell > 0$	4220.4, 4376.6
$3/2^+$	Full	4206.7, 4339.7
	w/o $\bar{D}^{(*)}\Lambda_c$	…
	w/o $\ell > 1$	4275.3
$5/2^-$	Full	4428.6
	w/o $\bar{D}^{(*)}\Lambda_c$	…
	w/o $\ell > 0$	…

$\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ was not considered. In our calculation, we find that the masses increase by tens of MeV and some of the states disappear when the $\bar{D}^{(*)}\Lambda_c$ channel or the states with large orbital angular momentum are ignored, as summarized in Table V. Specifically the $J^P = 5/2^-$ state corresponding to $P_c^+(4450)$ disappears when the analysis with the full channel coupling is not performed.

IV. SUMMARY

We studied the hidden-charm pentaquarks as meson-baryon molecules. We took into account the coupled channels of $\bar{D}^{(*)}\Sigma_c^{(*)}$ whose thresholds are close to each other owing to the heavy quark spin symmetry. In addition, the couplings to $\bar{D}^{(*)}\Lambda_c$ near the $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds, and to the states with larger orbital angular momentum mixed by the tensor force were considered. Therefore, the analysis of the hidden-charm molecular systems involved by the full coupled channel for $\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$, which had not been performed in the early works. As for the meson-baryon interaction, the meson exchange potential was obtained by the effective Lagrangians that respects the heavy quark and chiral symmetries. By solving the coupled-channel Schrödinger equations, we studied the bound and resonant states in $I(J^P) = 1/2(3/2^\pm)$ and $1/2(5/2^\pm)$. The results show that the J^P assignments of $P_c^+(4380)$ and $P_c^+(4450)$ are $3/2^+$ and $5/2^-$, respectively. We also found new states in $J^P = 3/2^\pm$. In the molecular states obtained, we found that the coupling to the $\bar{D}^{(*)}\Lambda_c$ channel and to the state with large orbital angular momentum produced the attraction. The predicted states can be sought in future experiments by the relativistic heavy ion collision in LHC, the production via the hadron beam in J-PARC [70–72], the photoproduction in Jefferson Lab [73–75] and so on.

ACKNOWLEDGMENTS

This work was supported in part by the INFN Fellowship Programme.

APPENDIX: MESON EXCHANGE POTENTIAL

In this Appendix, the explicit form of the meson exchange potentials (16)–(18) is summarized.

1. π exchange potential

From the effective Lagrangians (1) and (10), the π exchange potentials for $I = 1/2$ are given by

$$V^{\pi}_{\bar{D}^*\Sigma_c-\bar{D}\Lambda_c}(r) = -\frac{gg_4}{3\sqrt{2}f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\sigma}C_{m_\pi} + S_{\varepsilon\sigma}(\hat{r})T_{m_\pi}], \tag{A1}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}\Lambda_c}(r) = \frac{gg_4}{\sqrt{6}f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}C_{m_\pi} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A2}$$

$$V^{\pi}_{\bar{D}\Sigma_c-\bar{D}^*\Lambda_c}(r) = -\frac{gg_4}{3\sqrt{2}f_\pi^2}[\vec{\varepsilon}\cdot\vec{\sigma}C_{m_\pi} + S_{\varepsilon\sigma}(\hat{r})T_{m_\pi}], \tag{A3}$$

$$V^{\pi}_{\bar{D}\Sigma_c^*-\bar{D}^*\Lambda_c}(r) = \frac{gg_4}{\sqrt{6}f_\pi^2}[\vec{\varepsilon}\cdot\vec{\bar{\Sigma}}C_{m_\pi} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A4}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c-\bar{D}^*\Lambda_c}(r) = -\frac{gg_4}{3\sqrt{2}f_\pi^2}[\vec{S}\cdot\vec{\sigma}C_{m_\pi} + S_{S\sigma}(\hat{r})T_{m_\pi}], \tag{A5}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Lambda_c}(r) = \frac{gg_4}{\sqrt{6}f_\pi^2}[\vec{S}\cdot\vec{\bar{\Sigma}}^\dagger C_{m_\pi} + S_{S\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A6}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c-\bar{D}\Sigma_c}(r) = \frac{gg_1}{3f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\sigma}C_{m_\pi} + S_{\varepsilon\sigma}(\hat{r})T_{m_\pi}], \tag{A7}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}\Sigma_c}(r) = \frac{gg_1}{2\sqrt{3}f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}^\dagger C_{m_\pi} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A8}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c-\bar{D}\Sigma_c^*}(r) = \frac{gg_1}{2\sqrt{3}f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}C_{m_\pi} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A9}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}\Sigma_c^*}(r) = \frac{gg_1}{3f_\pi^2}[\vec{\varepsilon}^\dagger\cdot\vec{\Sigma}C_{m_\pi} + S_{\varepsilon\Sigma}(\hat{r})T_{m_\pi}], \tag{A10}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c-\bar{D}^*\Sigma_c}(r) = -\frac{gg_1}{3f_\pi^2}[\vec{S}\cdot\vec{\sigma}C_{m_\pi} + S_{S\sigma}(\hat{r})T_{m_\pi}], \tag{A11}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Sigma_c}(r) = \frac{gg_1}{2\sqrt{3}f_\pi^2}[\vec{S}\cdot\vec{\bar{\Sigma}}^\dagger C_{m_\pi} + S_{S\bar{\Sigma}}(\hat{r})T_{m_\pi}], \tag{A12}$$

$$V^{\pi}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Sigma_c^*}(r) = \frac{gg_1}{3f_\pi^2}[\vec{S}\cdot\vec{\Sigma}C_{m_\pi} + S_{S\Sigma}(\hat{r})T_{m_\pi}], \tag{A13}$$

with $C_{m_\pi} \equiv C(r; m_\pi)$ and $T_{m_\pi} \equiv T(r; m_\pi)$. The tensor operator is defined by $S_{\mathcal{O}_{\bar{D}}\mathcal{O}_{Y_c}}(\hat{r}) = 3\vec{\mathcal{O}}_{\bar{D}}\cdot\hat{r}\vec{\mathcal{O}}_{Y_c}\cdot\hat{r} - \vec{\mathcal{O}}_{\bar{D}}\cdot\vec{\mathcal{O}}_{Y_c}$. The operator of the heavy meson $\mathcal{O}_{\bar{D}}$ is $\vec{\varepsilon}$ or S with the polarization vector $\vec{\varepsilon}^{(\pm)} = (\mp 1/\sqrt{2}, \pm i/\sqrt{2}, 0)$ and $\vec{\varepsilon}^{(0)} = (0,0,1)$, and the spin-one operator $\vec{S} = i\vec{\varepsilon}\times\vec{\varepsilon}^\dagger$. The operator of the heavy hadrons is $\mathcal{O}_{Y_c} = \sigma, \bar{\Sigma}, \Sigma$, where $\vec{\sigma}$ is the Pauli matrices, $\bar{\Sigma}^\mu$ is given by

$$\bar{\Sigma}^\mu = \begin{pmatrix} \vec{\varepsilon}^{(+)} & \sqrt{2/3}\vec{\varepsilon}^{(0)} & \sqrt{1/3}\vec{\varepsilon}^{(-)} & 0 \\ 0 & \sqrt{1/3}\vec{\varepsilon}^{(+)} & \sqrt{2/3}\vec{\varepsilon}^{(0)} & \vec{\varepsilon}^{(-)} \end{pmatrix}^\mu, \tag{A14}$$

and $\vec{\Sigma} = \frac{3}{2}i\vec{\bar{\Sigma}}\times\vec{\bar{\Sigma}}^\dagger$.

2. Vector meson exchange potential

From the effective Lagrangians (2) and (11), the vector meson exchange potentials for $I = 1/2$ are given by

$$V^{v}_{\bar{D}\Lambda_c-\bar{D}\Lambda_c}(r) = -\frac{\beta\beta_B g_V^2}{m_v^2}C_{m_v}, \tag{A15}$$

$$V^{v}_{\bar{D}^*\Sigma_c-\bar{D}\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2\delta}{3\sqrt{2}}[-2\vec{\varepsilon}^\dagger\cdot\vec{\sigma}C_{m_v} + S_{\varepsilon\sigma}(\hat{r})T_{m_v}], \tag{A16}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2}{\sqrt{6}}[-2\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}^\dagger C_{m_v} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A17}$$

$$V^{v}_{\bar{D}^*\Lambda_c-\bar{D}^*\Lambda_c}(r) = -\frac{\beta\beta_B g_V^2}{m_v^2}C_{m_v}, \tag{A18}$$

$$V^{v}_{\bar{D}\Sigma_c-\bar{D}^*\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2}{3\sqrt{2}}[-2\vec{\varepsilon}\cdot\vec{\sigma}C_{m_v} + S_{\varepsilon\sigma}(\hat{r})T_{m_v}], \tag{A19}$$

$$V^{v}_{\bar{D}\Sigma_c^*-\bar{D}^*\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2}{\sqrt{6}}[-2\vec{\varepsilon}\cdot\vec{\bar{\Sigma}}^\dagger C_{m_v} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A20}$$

$$V^{v}_{\bar{D}^*\Sigma_c-\bar{D}^*\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2\delta}{3\sqrt{2}}[-2\vec{S}\cdot\vec{\sigma}C_{m_v} + S_{S\sigma}(\hat{r})T_{m_v}], \tag{A21}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Lambda_c}(r) = -\frac{4\lambda\lambda_I g_V^2}{\sqrt{6}}[-2\vec{S}\cdot\vec{\bar{\Sigma}}^\dagger C_{m_v} + S_{S\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A22}$$

$$V^{v}_{\bar{D}\Sigma_c-\bar{D}\Sigma_c}(r) = \frac{\beta\beta_S g_V^2}{2m_v^2}C_{m_v}, \tag{A23}$$

$$V^{v}_{\bar{D}^*\Sigma_c-\bar{D}\Sigma_c}(r) = \frac{2\lambda\lambda_S g_V^2}{9}[-2\vec{\varepsilon}^\dagger\cdot\vec{\sigma}C_{m_v} + S_{\varepsilon\sigma}(\hat{r})T_{m_v}], \tag{A24}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}\Sigma_c}(r) = -\frac{\lambda\lambda_S g_V^2\delta}{3\sqrt{3}}[-2\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}^\dagger C_{m_v} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A25}$$

$$V^{v}_{\bar{D}\Sigma_c^*-\bar{D}\Sigma_c^*}(r) = \frac{\beta\beta_S g_V^2}{2m_v^2}C_{m_v}, \tag{A26}$$

$$V^{v}_{\bar{D}\Sigma_c^*-\bar{D}^*\Sigma_c}(r) = -\frac{\lambda\lambda_S g_V^2\delta}{3\sqrt{3}}[-2\vec{\varepsilon}^\dagger\cdot\vec{\bar{\Sigma}}C_{m_v} + S_{\varepsilon\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A27}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}\Sigma_c^*}(r) = \frac{2\lambda\lambda_S g_V^2}{9}[-2\vec{\varepsilon}^\dagger\cdot\vec{\Sigma}C_{m_v} + S_{\varepsilon\Sigma}(\hat{r})T_{m_v}], \tag{A28}$$

$$V^{v}_{\bar{D}^*\Sigma_c-\bar{D}^*\Sigma_c}(r) = \frac{\beta\beta_S g_V^2}{2m_v^2}C_{m_v} + \frac{2\lambda\lambda_S g_V^2}{9} \times[-2\vec{S}\cdot\vec{\sigma}C_{m_v} + S_{S\sigma}(\hat{r})T_{m_v}], \tag{A29}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Sigma_c}(r) = +\frac{\lambda\lambda_S g_V^2\delta}{3\sqrt{3}}[-2\vec{S}\cdot\vec{\bar{\Sigma}}^\dagger C_{m_v} + S_{S\bar{\Sigma}}(\hat{r})T_{m_v}], \tag{A30}$$

$$V^{v}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Sigma_c^*}(r) = \frac{\beta\beta_S g_V^2}{2m_v^2}C_{m_v} + \frac{2\lambda\lambda_S g_V^2}{9} \times[-2\vec{S}\cdot\vec{\Sigma}C_{m_v} + S_{S\Sigma}(\hat{r})T_{m_v}], \tag{A31}$$

with $C_{m_v} \equiv C(r;m_v)$ and $T_{m_v} \equiv T(r;m_v)$. For Eqs. (A23)–(A31), the ρ exchange potential is multiplied by (-2).

3. σ exchange potential

From the effective Lagrangians (3) and (12), the σ exchange potential is given by

$$V^{\sigma}_{\bar{D}\Lambda_c-\bar{D}\Lambda_c} = \frac{4g_s\ell_B}{m_\sigma^2}C_{m_\sigma}, \tag{A32}$$

$$V^{\sigma}_{\bar{D}^*\Lambda_c-\bar{D}^*\Lambda_c} = \frac{4g_s\ell_B}{m_\sigma^2}C_{m_\sigma}, \tag{A33}$$

$$V^{\sigma}_{\bar{D}\Sigma_c-\bar{D}\Sigma_c} = -\frac{2g_s\ell_S}{m_\sigma^2}C_{m_\sigma}, \tag{A34}$$

$$V^{\sigma}_{\bar{D}\Sigma_c^*-\bar{D}\Sigma_c^*} = -\frac{2g_s\ell_S}{m_\sigma^2}C_{m_\sigma}, \tag{A35}$$

$$V^{\sigma}_{\bar{D}^*\Sigma_c-\bar{D}^*\Sigma_c} = -\frac{2g_s\ell_S}{m_\sigma^2}C_{m_\sigma}, \tag{A36}$$

$$V^{\sigma}_{\bar{D}^*\Sigma_c^*-\bar{D}^*\Sigma_c^*} = -\frac{2g_s\ell_S}{m_\sigma^2}C_{m_\sigma}, \tag{A37}$$

with $C_{m_\sigma} \equiv C(r;m_\sigma)$.

[1] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **115**, 072001 (2015).
[2] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **117**, 082002 (2016).
[3] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **117**, 082003 (2016).
[4] W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou, Phys. Rev. C **84**, 015203 (2011).
[5] S. G. Yuan, K. W. Wei, J. He, H. S. Xu, and B. S. Zou, Eur. Phys. J. A **48**, 61 (2012).
[6] L. Maiani, A. D. Polosa, and V. Riquer, Phys. Lett. B **749**, 289 (2015).
[7] G.-N. Li, X.-G. He, and M. He, J. High Energy Phys. 12 (2015) 128.
[8] G. Yang and J. Ping, Phys. Rev. D **95**, 014010 (2017).
[9] E. Santopinto and A. Giachino, arXiv:1604.03769.
[10] J. Hofmann and M. Lutz, Nucl. Phys. **A763**, 90 (2005).
[11] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. Lett. **105**, 232001 (2010).
[12] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. C **84**, 015202 (2011).
[13] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, and S.-L. Zhu, Chin. Phys. C **36**, 6 (2012).
[14] C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo, and L. Tolos, Phys. Rev. D **87**, 074034 (2013).
[15] C. W. Xiao, J. Nieves, and E. Oset, Phys. Rev. D **88**, 056012 (2013).
[16] T. Uchino, W.-H. Liang, and E. Oset, Eur. Phys. J. A **52**, 43 (2016).
[17] M. Karliner and J. L. Rosner, Phys. Rev. Lett. **115**, 122001 (2015).
[18] J. He, Phys. Lett. B **753**, 547 (2016).
[19] R. Chen, X. Liu, and S.-L. Zhu, Nucl. Phys. **A954**, 406 (2016).
[20] Y. Shimizu, D. Suenaga, and M. Harada, Phys. Rev. D **93**, 114003 (2016).
[21] H.-X. Chen, W. Chen, X. Liu, T. G. Steele, and S.-L. Zhu, Phys. Rev. Lett. **115**, 172001 (2015).
[22] Z.-G. Wang, Eur. Phys. J. C **76**, 70 (2016).
[23] F.-K. Guo, U.-G. Meißner, W. Wang, and Z. Yang, Phys. Rev. D **92**, 071502 (2015).
[24] X.-H. Liu, Q. Wang, and Q. Zhao, Phys. Lett. B **757**, 231 (2016).
[25] F.-K. Guo, U.-G. Meißner, J. Nieves, and Z. Yang, Eur. Phys. J. A **52**, 318 (2016).
[26] E. Oset and A. Ramos, Nucl. Phys. **A635**, 99 (1998).
[27] B. Krippa, Phys. Rev. C **58**, 1333 (1998).
[28] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. **67**, 55 (2012).
[29] S.-K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[30] E. S. Swanson, Phys. Rep. **429**, 243 (2006).
[31] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011).
[32] D. Gamermann and E. Oset, Eur. Phys. J. A **33**, 119 (2007).
[33] J. Ferretti, G. Galata, and E. Santopinto, Phys. Rev. C **88**, 015207 (2013).
[34] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013).
[35] A. Bondar *et al.* (Belle Collaboration), Phys. Rev. Lett. **108**, 122001 (2012).
[36] Q. Wang, C. Hanhart, and Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013).
[37] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Phys. Rev. D **84**, 054010 (2011).
[38] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D **86**, 014004 (2012).
[39] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Prog. Theor. Exp. Phys. **2016**, 062C01 (2016).
[40] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[41] N. Isgur and M. B. Wise, Phys. Lett. B **237**, 527 (1990).
[42] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).
[43] A. V. Manohar and M. B. Wise, *Heavy Quark Physics*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, 1st ed. (Cambridge University Press, Cambridge, England, 2007).
[44] I. R. Afnan and B. F. Gibson, Phys. Rev. C **40**, R7 (1989).
[45] S. Yasui and K. Sudoh, Phys. Rev. D **80**, 034008 (2009).
[46] Y. Yamaguchi, S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D **84**, 014032 (2011).
[47] Y. Yamaguchi, S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D **85**, 054003 (2012).
[48] Y. Yamaguchi, S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D **87**, 074019 (2013).
[49] T. D. Cohen, P. M. Hohler, and R. F. Lebed, Phys. Rev. D **72**, 074010 (2005).
[50] D. Gamermann, C. García-Recio, J. Nieves, L. L. Salcedo, and L. Tolos, Phys. Rev. D **81**, 094016 (2010).
[51] W. H. Liang, T. Uchino, C. W. Xiao, and E. Oset, Eur. Phys. J. A **51**, 16 (2015).
[52] A. Hosaka, T. Hyodo, K. Sudoh, Y. Yamaguchi, and S. Yasui, arXiv:1606.08685.
[53] A. Ali, I. Ahmed, M. J. Aslam, and A. Rehman, Phys. Rev. D **94**, 054001 (2016).
[54] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[55] A. F. Falk and M. E. Luke, Phys. Lett. B **292**, 119 (1992).
[56] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, and H.-L. Yu, Phys. Rev. D **46**, 1148 (1992).
[57] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[58] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
[59] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[60] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).
[61] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[62] Y.-R. Liu and M. Oka, Phys. Rev. D **85**, 014015 (2012).
[63] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[64] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).
[65] J. Aguilar and J. M. Combes, Commun. Math. Phys. **22**, 269 (1971).
[66] E. Balslev and J. M. Combes, Commun. Math. Phys. **22**, 280 (1971).
[67] B. Simon, Commun. Math. Phys. **27**, 1 (1972).
[68] S. Aoyama, T. Myo, K. Kato, and K. Ikeda, Prog. Theor. Phys. **116**, 1 (2006).

[69] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[70] E. J. Garzon and J.-J. Xie, Phys. Rev. C **92**, 035201 (2015).
[71] X.-H. Liu and M. Oka, Nucl. Phys. **A954**, 352 (2016).
[72] S.-H. Kim, H.-C. Kim, and A. Hosaka, Phys. Lett. B **763**, 358 (2016).
[73] Y. Huang, J. He, H.-F. Zhang, and X.-R. Chen, J. Phys. G **41**, 115004 (2014).
[74] Q. Wang, X.-H. Liu, and Q. Zhao, Phys. Rev. D **92**, 034022 (2015).
[75] Y. Huang, J.-J. Xie, J. He, X. Chen, and H.-F. Zhang, Chin. Phys. C **40**, 124104 (2016).