

Nonleptonic charmless decays of $B_c \rightarrow TP, TV$ in the perturbative QCD approach

Xin Liu^{*}*School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou 221116, China*Run-Hui Li[†]*School of Physical Science and Technology, Inner Mongolia University, Hohhot 010021, China*Zhi-Tian Zou[‡]*Department of Physics, Yantai University, Yantai 264005, China*Zhen-Jun Xiao[§]*Department of Physics and Institute of Theoretical Physics,
Nanjing Normal University, Nanjing 210023, China*

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Two-body charmless hadronic B_c decays involving a light 1^3P_2 -tensor (T) meson are investigated for the first time within the framework of perturbative QCD (pQCD) at leading order, in which the other meson is the lightest pseudoscalar (P) or vector (V) state. The concerned processes can only occur through the pure weak annihilation topology in the standard model. We predict the CP -averaged branching ratios and polarization fractions of those considered decays in Cabibbo-Kobayashi-Maskawa (CKM) favored and suppressed modes. Phenomenologically, several modes—such as the $B_c \rightarrow K_2^*(1430)K$ and the CKM-favored $B_c \rightarrow TV$ —have large decay rates of 10^{-6} , which are expected to be detected at Large Hadron Collider experiments in the near future. Moreover, all of the $B_c \rightarrow TV$ modes are governed by the longitudinal amplitudes in the pQCD calculations and the corresponding fractions vary around 78–98%. A confirmation of these results could prove the reliability of the pQCD approach used here and further shed some light on the annihilation decay mechanism.

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Heavy flavor physics has played an important role in the precision tests of the standard model (SM), as well as in investigating the properties of involved light hadrons after the advent of two B factories, i.e., *BABAR* at SLAC and *Belle* at KEK. An increasing number of interesting mesons have been observed in the decay channels of the heavy mesons—specifically $D_{(s)}$ mesons with a c quark and $B_{(s)}$ mesons with a b quark [1]—which provide a fertile ground for probing the perturbative and nonperturbative QCD dynamics in the SM. With the advent of the Large Hadron Collider (LHC) at CERN, a new territory has been developed since a great number of B_c meson events can be observed. The properties of the B_c meson and the dynamics involved in B_c decays could be fully exploited through the precision measurements at the LHC with its high collision energy and high luminosity. Therefore, the B_c meson decays will open a window to richer physics, which could start a new golden era of heavy flavor physics with the LHC experiments [2,3].

Tensor mesons with quantum number $J^P = 2^+$ have recently become a hot topic. On the one hand, experimentally, *BABAR* and *Belle* have measured several charmless hadronic B decays involving a light tensor meson in the final states [1,4]. Furthermore, the measurements on the polarization fractions of $B \rightarrow \phi K_2^*(1430)$ decays showed that these two modes are dominated by the longitudinal polarization amplitudes, which is contrary to the same $b \rightarrow s\bar{s}s$ -transition-induced $B \rightarrow \phi K^*$ processes. This phenomenology makes the well-known “polarization puzzle” more confusing. On the other hand, theoretically, the tensor meson cannot be produced through either local vector or axial-vector operators, or via the tensor current, which implies that large nonfactorizable amplitudes or annihilation diagrams would contribute to the tensor meson emitted modes with experimentally sizable branching ratios and the relevant investigations should go beyond the naive factorization. Of course, the polarization studies on the tensor-vector, tensor-axial-vector, and even tensor-tensor modes in heavy flavor decays can further shed light on the underlying helicity structure of the decay mechanism [5]. According to the counting rule, the annihilation contributions are usually power suppressed, compared to other spectator diagrams. Nevertheless, the annihilation contributions are not negligible and the size is

^{*}liuxin@jsnu.edu.cn[†]lirh@imu.edu.cn[‡]zouzt@ytu.edu.cn[§]xiaozenjun@njnu.edu.cn

still an important issue in B meson physics (see, e.g., Refs. [2,3,6–18]). Indeed, the experiments have confirmed some large annihilation decay modes, for example, the well-known $B_d \rightarrow K^+ K^-$ and $B_s \rightarrow \pi^+ \pi^-$ decays [19]. Moreover, phenomenologically, the theoretical studies on the $B \rightarrow \phi K^*$ [10,11,18] and $B \rightarrow \phi K_2^*(1430)$ decays [5,20] have provided important improvements in the explanation of the “polarization puzzle” by including the annihilation effects, though the authors claimed that $f_L(B_d \rightarrow \phi K_2^*(1430)) \sim \mathcal{O}(1)$ with or without the annihilation effects [21].

Compared to the annihilation amplitudes in the charmless B decays, the magnitude in the B_c decays would be roughly enlarged by a factor $|V_{cb}/V_{ub}| \sim 11.5$, which would consequently result in a 100 times enhancement to the branching ratios. Therefore, the annihilation B_c modes could possibly provide a promising and more appropriate platform to study the contributions from the annihilation diagrams, and even further uncover the annihilation decay mechanism. It is great to find that the measurements on the pure annihilation B_c decay modes have been initiated by the LHCb Collaboration, for example, $B_c \rightarrow K^+ \bar{K}^0$ [22], $B_c \rightarrow K^+ K^- \pi^+$ [23], and $B_c \rightarrow p \bar{p} \pi^+$ [24], etc. Certainly, with the increasing number of B_c events being collected, more and more annihilation types of B_c decay channels will be opened. Sequentially, much more information on the annihilation decay mechanism must be obtained.

To date, an agreement on how to calculate the Feynman diagrams with annihilation topology reliably has not been achieved among the theorists. At least, the perturbative QCD (pQCD) approach [6,7,25] and soft-collinear effective theory (SCET) [26], as two popular tools for calculating hadronic matrix elements based on QCD dynamics,¹ have rather different viewpoints: the almost imaginary annihilation amplitudes with a large strong phase obtained through keeping the parton’s transverse momentum in the pQCD framework [14], and the almost real annihilation amplitudes with a tiny strong phase obtained by considering the zero-bin subtraction in the SCET framework [32]. However, objectively speaking, the confirmation of the predicted branching ratios for the pure annihilation $B_d \rightarrow K^+ K^-$ and $B_s \rightarrow \pi^+ \pi^-$ decays provided by the CDF [33] and LHCb [19,34] collaborations provided firm support to the current pQCD approach.

In this work, we will study the two-body nonleptonic charmless B_c decays involving a light tensor meson (T) and a light pseudoscalar (P) or vector meson (V) in the final states by employing the pQCD approach at leading order. These considered decays can only occur through weak

annihilation interactions in the SM. Here, the light pseudoscalar (vector) meson includes π , K , η , and η' (ρ , K^* , ω , and ϕ). In the quark model, the observed light tensor meson contains the isovector states $a_2(1320)$, the isodoublet states $K_2^*(1430)$, and the isoscalar singlet states $f_2(1270)$ and $f_2'(1525)$, which have been well established in various processes [1]. Hereafter, for the sake of simplicity, we will adopt a_2 , K_2^* , f_2 , and f_2' to denote the light tensor mesons correspondingly, unless otherwise specified. It is worth mentioning that, just like the $\eta - \eta'$ mixing in the pseudoscalar sector, the two isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ also have a mixing as

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos \phi_{f_2} & -\sin \phi_{f_2} \\ \sin \phi_{f_2} & \cos \phi_{f_2} \end{pmatrix} \begin{pmatrix} f_{2q} \\ f_{2s} \end{pmatrix}, \quad (1)$$

with $f_{2q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{2s} \equiv s\bar{s}$. The angle between $f_2(1270)$ and $f_2'(1525)$ mixing should be small due to a fact that the former (latter) predominantly decays into $\pi\pi$ ($K\bar{K}$) [1]. Specifically, the mixing angle ϕ_{f_2} lies in the range $6^\circ - 10^\circ$ [1,35,36]. Therefore, analogous to ω and ϕ mesons in the vector sector, we will first approximately assume $f_2(f_2')$ as the pure $f_{2q}(f_{2s})$ state. The mixture of $f_2 - f_2'$ with the mixing angle ϕ_{f_2} will be left for future studies associated with experimentally precise measurements.

As mentioned above, the pQCD approach is an appropriate tool to effectively calculate the hadronic matrix elements of annihilation topology in the nonleptonic weak B meson decays. The most important feature of the pQCD approach is that it picks up the intrinsic transverse momentum k_T of the valence quarks in light of the end-point divergences that exist in the collinear factorization. Then, based on the k_T factorization theorem, by utilizing the technique of resummation the double logarithmic divergences factored out from the hard part can be grouped into a Sudakov factor (e^{-S}) [37] and a threshold factor $[S_T(x)]$ [38], which consequently make the pQCD approach more self-consistent. Then, the single logarithmic divergences separated from the hard kernel can be reabsorbed into the meson wave functions using the eikonal approximation [39]. The interested reader can refer to the review paper [25] for more details about this approach. Presently, many quantitative annihilation-type-diagram calculations have been made with this pQCD approach.

The Feynman diagrams for the nonleptonic charmless $B_c \rightarrow TP, TV$ decays in the pQCD approach at leading order are illustrated in Fig. 1: Figs. 1(a) and 1(b) use the factorizable annihilation topology, while Figs. 1(c) and 1(d) use the nonfactorizable annihilation topology. For a spin-2 tensor meson, the polarization can be specified by a symmetric and traceless tensor $\epsilon_{(\lambda)}^{\mu\nu}$ with helicity λ that satisfies the relation $\epsilon_{(\lambda)}^{\mu\nu} P_\mu = \epsilon_{(\lambda)}^{\mu\nu} P_\nu = 0$, with P being its momentum. Furthermore, this polarization tensor can be constructed through the spin-1 polarization vector ϵ_V [40].

¹Another popular method is the QCD factorization approach [27,28], which cannot make effective calculations on the annihilation diagrams since there exist end-point singularities in the integrals. However, data fitting has been broadly adopted in this approach to make theoretical predictions in the $B_{(s)}$ decays; see, for example, Refs. [27–31].

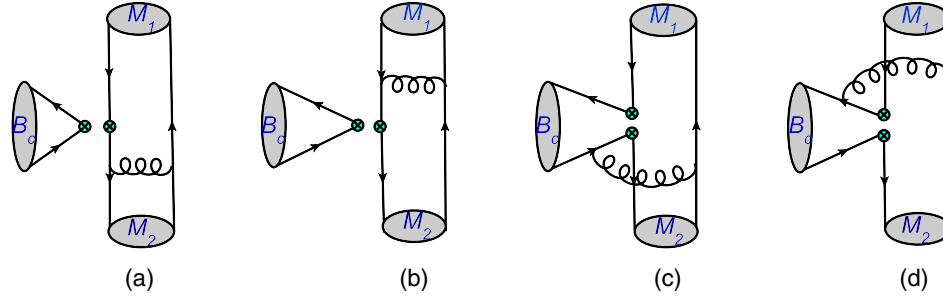


FIG. 1. Typical Feynman diagrams contributing to charmless decays of $B_c \rightarrow M_1 M_2$ in the pQCD approach at leading order, where the $M_1 M_2$ pair denotes TP, PT, TV , and VT in this work.

Although a tensor meson contains five spin degrees of freedom, only $\lambda = 0$ will give a nonzero contribution in the $B_c \rightarrow TP$ modes, since the mother B_c meson is spinless and the daughter T and P mesons should obey the conservation law of angular momentum. Likewise, the $B_c \rightarrow TV$ decays will be contributed from $\lambda = 0$ and $\lambda = \pm 1$ helicities. Then, one can intuitively postulate that the considered $B_c \rightarrow TP, TV$ decays appear more like $B_c \rightarrow VP, VV$ ones by elaborating a new polarization vector ϵ_T for the tensor meson [41,42]. Actually, ϵ_T has been explicitly presented in the literature (see, for example, Refs. [5,41,42]) with $\epsilon_T(L) = \sqrt{\frac{2}{3}}\epsilon_V(L)$ and $\epsilon_T(T) = \sqrt{\frac{1}{2}}\epsilon_V(T)$.² Here, the capital L and T in the parentheses describe the longitudinal and transverse polarizations, respectively (not to be confused with the abbreviation T for the light tensor meson). The decay amplitudes of $B \rightarrow TP$ and $B \rightarrow TV$ modes presented in Refs. [5,20,43] have confirmed the above postulation. Therefore, the decay amplitudes of the $B_c \rightarrow TP, TV$ decays considered in this work can be straightforwardly obtained by replacing the polarization vector ϵ_V of the vector meson with the corresponding ϵ_T of the tensor one in the $B_c \rightarrow VP, VV$ [3] modes. That is:

- (1) Equations (28)–(31) in Ref. [3] with a factor $\sqrt{\frac{2}{3}}$ will give the analytic Feynman amplitudes of the $B_c \rightarrow PT, TP$ decays with only longitudinal polarization, in which the vector meson mass and distribution amplitudes should be replaced with the tensor state.
- (2) Equations (49)–(50) [Eqs. (51)–(54)] in Ref. [3] with a factor $\sqrt{\frac{2}{3}}$ [$\sqrt{\frac{1}{2}}$] can contribute to the analytic Feynman amplitudes of the $B_c \rightarrow VT, TV$ decays in longitudinal [transverse] polarizations, where the corresponding quantities of the tensor state will be substituted for those of one of the two vector mesons. Because no $f_2 - f'_2$ mixing is considered, the $B_c \rightarrow \pi^+ f'_2$ and $B_c \rightarrow \rho^+ f'_2$ decays will naturally be absent. Also forbidden is the $B_c \rightarrow a_2^+ \phi$ mode as a result of not including

$\omega - \phi$ mixing effects. Therefore, here we will not present the factorization formulas and the expressions for total decay amplitudes explicitly for those considered decays. The readers can refer to Ref. [3] for details.

Now we can turn to the numerical calculations of these $B_c \rightarrow TP, TV$ decays in the pQCD approach. Before proceeding, some comments on the input quantities are in order:

- (1) For the lightest pseudoscalar and vector mesons, the (chiral) masses, the decay constants, the QCD scale, and the light-cone distribution amplitudes including Gegenbauer moments are the same as those used in Ref. [3]. Please refer to Appendix A of Ref. [3] for details.
- (2) For the B_c meson, the distribution amplitude and the decay constant are the same as those adopted in Ref. [3] but with the up-to-date mass $m_{B_c} = 6.275$ GeV and lifetime $\tau_{B_c} = 0.507$ ps, which have been updated in the latest version of the Review of Particle Physics [1].
- (3) For the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, we also adopt the Wolfenstein parametrization at leading order, but with the updated parameters $A = 0.811$ and $\lambda = 0.22506$ [1].
- (4) For the light tensor meson, the decay constants with longitudinal and transverse polarizations are collected in Table I. The related masses are $m_{a_2} = 1.318$ GeV, $m_{K_2^*} = 1.426$ GeV, $m_{f_2} = 1.275$ GeV, and $m_{f'_2} = 1.525$ GeV. Again, the $f_2 - f'_2$ mixing is not considered in this work. Therefore, the mass of the pure flavor f_{2q} (f_{2s}) state is taken as that of the physical f_2 (f'_2) for convenience.

The related light-cone distribution amplitudes have been recently investigated in the QCD sum rules [44]. Analogous to the light vector meson, the asymptotic forms of the tensor meson distribution amplitudes are adopted. Here, we present the expressions of the light-cone distribution amplitudes for the light tensor mesons following Ref. [42]:

$$\phi_T(x) = \frac{3f_T}{\sqrt{2N_c}}\phi_{\parallel}(x), \quad \phi_T^T(x) = \frac{3f_T^T}{\sqrt{2N_c}}\phi_{\perp}(x), \quad (2)$$

²Since only three helicities $\lambda = 0, \pm 1$ contribute to the $B_c \rightarrow TV$ decays, the involved light tensor meson can be treated as a vector-like meson with tensor meson mass.

TABLE I. Decay constants of the light tensor mesons (in GeV) [44].

f_{a_2}	$f_{a_2}^T$	$f_{K_2^*}$	$f_{K_2^*}^T$	f_{f_2}	$f_{f_2}^T$	$f_{f_2'}$	$f_{f_2'}^T$
0.107 ± 0.006	0.105 ± 0.021	0.118 ± 0.005	0.077 ± 0.014	0.102 ± 0.006	0.117 ± 0.025	0.126 ± 0.004	0.065 ± 0.012

$$\phi_T^l(x) = \frac{f_T^T}{2\sqrt{2N_c}} h_{\parallel}^l(x), \quad \phi_T^s(x) = \frac{f_T^T}{4\sqrt{2N_c}} \frac{d}{dx} h_{\parallel}^s(x), \quad (3)$$

$$\phi_T^v(x) = \frac{f_T}{2\sqrt{2N_c}} g_{\parallel}^v(x), \quad \phi_T^a(x) = \frac{f_T}{8\sqrt{2N_c}} \frac{d}{dx} g_{\perp}^a(x), \quad (4)$$

with

$$\phi_{\parallel}(x) = \phi_{\perp}(x) = x(1-x)[a_1 C_1^{3/2}(t)], \quad (5)$$

$$h_{\parallel}^l(x) = \frac{15}{2}(1-6x+6x^2)t, \quad h_{\parallel}^s(x) = 15x(1-x)t, \quad (6)$$

$$g_{\perp}^v(x) = 5t^3, \quad g_{\perp}^a(x) = 20x(1-x)t, \quad (7)$$

where the Gegenbauer moment $a_1 = \frac{5}{3}$ for the first rough estimates and the Gegenbauer polynomial $C_1^{3/2}(t) = 3t$ with $t = 2x - 1$. It is worth commenting that, in principle, the Gegenbauer moments for different meson distribution amplitudes should usually be different, due to the expected $SU(3)$ flavor symmetry-breaking effects. Therefore, the larger Gegenbauer moment a_1 adopted here will demand further improvements resultant from the near-future relevant measurements with good precision.

The pQCD predictions of the CP -averaged branching ratios in the $B_c \rightarrow TP, TV$ decays and of the polarization fractions in the $B_c \rightarrow TV$ modes are collected in Tables II, III, and IV, respectively.

- (1) For the $B_c \rightarrow TP$ decays, the main four errors arise from the uncertainties of the charm-quark mass $m_c = 1.5 \pm 0.15$ GeV in the B_c meson distribution amplitude, of the combined decay constants of f_T and f_T^T in the tensor meson distribution amplitudes, of the combined Gegenbauer moments a_1 and/or a_2 in the pseudoscalar meson distribution amplitudes, and of the chiral mass m_0^P of the pseudoscalar mesons.³ Of course, for the $B_c \rightarrow TP$ modes involving η and η' states, we also take the variations of the

mixing angle $\phi_P = 39.3^\circ \pm 1.0^\circ$ into account as the fifth error.

- (2) For the $B_c \rightarrow TV$ channels, the major four errors are induced by the uncertainties of the charm-quark mass $m_c = 1.5 \pm 0.15$ GeV in the B_c meson distribution amplitude, of the combined decay constants of f_T and f_T^T in the tensor meson distribution amplitudes, of the combined decay constants f_V and f_V^T in the vector meson distribution amplitudes, and of the combined Gegenbauer moments of $a_1^{\parallel(\perp)}$ and/or $a_2^{\parallel(\perp)}$ in the vector meson distribution amplitudes.

Here, we will specify the decay modes into two types: the CKM-favored channels with $\Delta S = 0$ (no strange or two strange mesons in the final states) and the CKM-suppressed modes with $\Delta S = 1$ (only one strange meson in the final states) for clarifications. Based on the pQCD predictions of the CP -averaged branching ratios for the considered decay channels $B_c \rightarrow TP$ presented in Table II, one can find the following results:

- (1) Relative to the suppressed CKM matrix element $V_{us} \sim 0.22506$ [1] in the $\Delta S = 1$ modes, the enhanced one $V_{ud} \sim 0.97434$ in the $\Delta S = 0$ modes makes their decay rates generally much larger around one order, which can be clearly seen in Table II.
- (2) Generally speaking, the nonleptonic charmless $B_c \rightarrow TP$ modes have decay rates from 10^{-7} (e.g., $B_c \rightarrow f_2 \pi^+$) to 10^{-8} (e.g., $B_c \rightarrow K_2^{*+} \pi^0$) in the pQCD framework, except for the two $B_c \rightarrow K_2^* K$ processes with large branching ratios,

$$\begin{aligned} \text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^0) &= 1.11_{-0.18}^{+0.31} \times 10^{-6}, \\ \text{Br}(B_c \rightarrow \bar{K}_2^{*0} K^+) &= 1.05_{-0.34}^{+0.50} \times 10^{-6}, \end{aligned} \quad (8)$$

which are expected to be tested in the near future since, as argued in Ref. [45], the B_c decays with the branching ratios of 10^{-6} can be measured at the LHC experiments. In light of the still large theoretical errors in these two modes, we usually provide a more precise ratio between these two CP -averaged branching ratios $\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^0)$ and $\text{Br}(B_c \rightarrow \bar{K}_2^{*0} K^+)$ as

$$R_{\bar{K}^0/K^+} \equiv \frac{\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^0)}{\text{Br}(B_c \rightarrow \bar{K}_2^{*0} K^+)} \approx 1.07_{-0.29-0.06-0.00-0.01}^{+0.40+0.05+0.13+0.02}, \quad (9)$$

³In order to estimate the theoretical uncertainties induced by the meson chiral mass, here we consider 10% variations of the central values for simplicity.

TABLE II. CP -averaged branching ratios of charmless decays $B_c \rightarrow TP$ in the pQCD approach.

Decay Modes ($\Delta S = 0$)	Branching ratios (10^{-7})	Decay modes ($\Delta S = 1$)	Branching ratios (10^{-8})
$B_c \rightarrow a_2^+ \pi^0$	$5.13^{+1.61+0.59+0.94+0.00}_{-1.40-0.56-0.68-0.01}$	$B_c \rightarrow K_2^{*0} \pi^+$	$3.73^{+1.66+0.31+0.37+0.04}_{-1.18-0.31-0.44-0.02}$
$B_c \rightarrow a_2^0 \pi^+$	$5.13^{+1.61+0.59+0.94+0.00}_{-1.40-0.56-0.68-0.01}$	$B_c \rightarrow K_2^{*+} \pi^0$	$1.87^{+0.82+0.15+0.18+0.01}_{-0.59-0.16-0.23-0.02}$
$B_c \rightarrow a_2^+ \eta$	$3.92^{+0.39+0.73+0.18+0.35+0.12}_{-0.31-0.70-0.28-0.34-0.11}$	$B_c \rightarrow K_2^{*+} \eta$	$5.37^{+1.77+0.54+0.46+0.04+0.15}_{-0.93-0.55-0.34-0.05-0.15}$
$B_c \rightarrow a_2^+ \eta'$	$2.56^{+0.25+0.48+0.11+0.22+0.11}_{-0.20-0.46-0.19-0.23-0.11}$	$B_c \rightarrow K_2^{*+} \eta'$	$6.51^{+0.90+1.21+0.42+0.39+0.14}_{-0.34-1.14-0.35-0.37-0.16}$
$B_c \rightarrow f_2 \pi^+$	$7.38^{+0.63+1.59+0.16+0.72}_{-0.39-1.53-0.31-0.70}$	$B_c \rightarrow a_2^+ K^0$	$9.04^{+1.52+1.62+1.11+0.52}_{-0.90-1.45-1.34-0.44}$
$B_c \rightarrow K_2^{*+} \bar{K}^0$	$11.14^{+0.80+1.64+2.42+0.49}_{-0.15-1.45-1.07-0.33}$	$B_c \rightarrow a_2^0 K^+$	$4.52^{+0.76+0.82+0.57+0.26}_{-0.45-0.72-0.67-0.21}$
$B_c \rightarrow \bar{K}_2^{*0} K^+$	$10.46^{+4.82+0.93+0.84+0.20}_{-2.96-0.87-1.50-0.22}$	$B_c \rightarrow f_2 K^+$	$4.56^{+0.72+0.93+0.55+0.30}_{-0.42-0.84-0.65-0.26}$
		$B_c \rightarrow f_2' K^+$	$6.33^{+2.79+0.40+0.46+0.11}_{-1.82-0.39-0.86-0.15}$

TABLE III. CP -averaged branching ratios and polarization fractions of CKM-favored $B_c \rightarrow TV$ modes in the pQCD approach.

Decay modes ($\Delta S = 0$)	Branching ratios (10^{-6})	Polarization fractions f_L (%)	Polarization fractions f_T (%)
$B_c \rightarrow a_2^+ \rho^0$	$1.29^{+0.33+0.18+0.04+0.08}_{-0.29-0.16-0.02-0.07}$	$78.2^{+4.4+1.0+0.3+1.3}_{-6.3-1.0-0.2-1.2}$	$21.8^{+6.3+1.0+0.2+1.2}_{-4.4-1.0-0.3-1.3}$
$B_c \rightarrow a_2^0 \rho^+$	$1.29^{+0.33+0.18+0.04+0.08}_{-0.29-0.16-0.02-0.07}$	$78.2^{+4.4+1.0+0.3+1.3}_{-6.3-1.0-0.2-1.2}$	$21.8^{+6.3+1.0+0.2+1.2}_{-4.4-1.0-0.3-1.3}$
$B_c \rightarrow a_2^+ \omega$	$0.87^{+0.11+0.12+0.03+0.02}_{-0.09-0.13-0.04-0.02}$	$97.4^{+0.3+0.3+0.1+0.1}_{-0.3-0.3-0.1-0.1}$	$2.6^{+0.3+0.3+0.1+0.1}_{-0.3-0.3-0.1-0.1}$
$B_c \rightarrow f_2 \rho^+$	$0.97^{+0.11+0.17+0.04+0.00}_{-0.11-0.16-0.04-0.02}$	$98.0^{+0.2+0.3+0.1+0.0}_{-0.2-0.4-0.1-0.1}$	$2.0^{+0.2+0.4+0.1+0.1}_{-0.2-0.3-0.1-0.0}$
$B_c \rightarrow K_2^{*+} \bar{K}^0$	$1.75^{+0.02+0.15+0.08+0.10}_{-0.08-0.15-0.07-0.07}$	$82.7^{+0.0+0.8+0.4+1.0}_{-0.7-0.7-0.2-0.6}$	$17.3^{+0.7+0.7+0.2+0.6}_{-0.0-0.8-0.4-1.0}$
$B_c \rightarrow \bar{K}_2^{*0} K^{*+}$	$1.54^{+0.57+0.14+0.08+0.12}_{-0.45-0.13-0.07-0.12}$	$81.2^{+5.2+0.1+0.1+1.3}_{-7.9-0.4-0.2-1.7}$	$18.8^{+7.9+0.4+0.2+1.7}_{-5.2-0.1-0.1-1.3}$

in which the uncertainties induced by the hadronic inputs could be greatly canceled. Of course, the largest error of the ratio $R_{\bar{K}^0/K^+}$ arising from the charm-quark mass in the B_c meson distribution amplitude ϕ_{B_c} indicates that much more effort should be devoted to better understanding the

nonperturbative QCD dynamics involved in the B_c meson, which will be helpful to further provide theoretical predictions with good precision for experiments. Compared to the $B_c \rightarrow K_2^* K$ modes, it is worth noticing the different phenomenologies exhibited in the $B_c \rightarrow K^* K$ decays [3]. The decay rate

TABLE IV. Same as Table III but for CKM-suppressed $B_c \rightarrow TV$ modes.

Decay modes ($\Delta S = 1$)	Branching ratios (10^{-8})	Polarization fractions f_L (%)	Polarization fractions f_T (%)
$B_c \rightarrow K_2^{*0} \rho^+$	$7.17^{+2.67+0.65+0.14+0.45}_{-2.11-0.62-0.13-0.47}$	$84.8^{+4.3+0.2+0.1+0.9}_{-6.5-0.2-0.0-1.1}$	$15.2^{+6.5+0.2+0.0+1.1}_{-4.3-0.2-0.1-0.9}$
$B_c \rightarrow K_2^{*+} \rho^0$	$3.59^{+1.33+0.31+0.07+0.22}_{-1.06-0.31-0.07-0.24}$	$84.8^{+4.3+0.2+0.1+0.9}_{-6.5-0.2-0.0-1.1}$	$15.2^{+6.5+0.2+0.0+1.1}_{-4.3-0.2-0.1-0.9}$
$B_c \rightarrow K_2^{*+} \omega$	$3.14^{+1.17+0.28+0.10+0.20}_{-0.92-0.27-0.09-0.20}$	$84.4^{+4.4+0.2+0.1+1.0}_{-6.5-0.2-0.0-1.0}$	$15.6^{+6.5+0.2+0.0+1.0}_{-4.4-0.2-0.1-1.0}$
$B_c \rightarrow K_2^{*+} \phi$	$11.80^{+0.15+1.11+0.41+0.26}_{-0.83-1.05-0.38-0.29}$	$80.2^{+0.0+1.1+0.3+0.4}_{-1.4-1.2-0.3-0.5}$	$19.8^{+1.4+1.2+0.3+0.5}_{-0.0-1.1-0.3-0.4}$
$B_c \rightarrow a_2^+ K^{*0}$	$9.20^{+0.00+1.11+0.38+0.26}_{-0.68-1.06-0.38-0.38}$	$87.4^{+0.0+0.9+0.2+0.4}_{-0.9-1.0-0.3-0.6}$	$12.6^{+0.9+1.0+0.3+0.6}_{-0.0-0.9-0.2-0.4}$
$B_c \rightarrow a_2^0 K^{*+}$	$4.60^{+0.00+0.55+0.19+0.12}_{-0.34-0.53-0.19-0.19}$	$87.4^{+0.0+0.9+0.2+0.4}_{-0.9-1.0-0.3-0.6}$	$12.6^{+0.9+1.0+0.3+0.6}_{-0.0-0.9-0.2-0.4}$
$B_c \rightarrow f_2 K^{*+}$	$4.38^{+0.00+0.58+0.17+0.09}_{-0.30-0.56-0.20-0.19}$	$88.6^{+0.0+1.0+0.2+0.2}_{-0.6-1.1-0.3-0.5}$	$11.4^{+0.6+1.1+0.3+0.5}_{-0.0-1.0-0.2-0.2}$
$B_c \rightarrow f_2' K^{*+}$	$9.60^{+3.42+0.66+0.46+0.77}_{-2.68-0.58-0.43-0.69}$	$79.4^{+5.6+0.3+0.1+1.5}_{-8.1-0.1-0.0-1.6}$	$20.6^{+8.1+0.1+0.0+1.6}_{-5.6-0.3-0.1-1.5}$

of $B_c \rightarrow \bar{K}^{*0}K^+$ is much larger than that of $B_c \rightarrow \bar{K}^0K^{*+}$ by a factor of about 5.5, in terms of the central values. The underlying reason is that, relative to the antisymmetric K_2^* light-cone distribution amplitudes [see Eq. (5)] in the $SU(3)$ limit [44], the significant $SU(3)$ flavor symmetry-breaking effects have been included in both K and K^* mesons, which can be seen evidently from the a_1 terms in their leading-twist distribution amplitudes [3].

- (3) The $B_c \rightarrow TP$ modes involving $\eta - \eta'$ mixing effects [i.e., $B_c \rightarrow a_2^+(\eta, \eta')$ and $B_c \rightarrow K_2^{*+}(\eta, \eta')$ decays] show different interferences between η_q and η_s flavor states. That is, there is constructive (destructive) interference in the $B_c \rightarrow a_2^+\eta$ ($B_c \rightarrow a_2^+\eta'$) mode, while the opposite occurs in the $B_c \rightarrow K_2^{*+}\eta$ ($B_c \rightarrow K_2^{*+}\eta'$) channel. Furthermore, one can deduce the dominance of η_q (η_s) contributions in the $B_c \rightarrow a_2^+\eta^{(l)}$ ($B_c \rightarrow K_2^{*+}\eta^{(l)}$) modes based on the numerical results of the branching ratios displayed in Table II. Similar interferences have also been observed in the $B_c \rightarrow \rho^+(\eta, \eta')$ and $B_c \rightarrow K^{*+}(\eta, \eta')$ decays [3]. We explicitly present four interesting ratios among the above-mentioned $B_c \rightarrow (\rho, K^*, a_2, K_2^*)(\eta, \eta')$ decays:

$$R_{\eta/\eta'}^{a_2} \equiv \frac{\text{Br}(B_c \rightarrow a_2^+\eta)}{\text{Br}(B_c \rightarrow a_2^+\eta')} = 1.53_{-0.02}^{+0.03},$$

$$R_{\eta/\eta'}^\rho \equiv \frac{\text{Br}(B_c \rightarrow \rho^+\eta)}{\text{Br}(B_c \rightarrow \rho^+\eta')} = 1.50_{-0.02}^{+0.00}, \quad (10)$$

$$R_{\eta'/\eta}^{K_2^*} \equiv \frac{\text{Br}(B_c \rightarrow K_2^{*+}\eta')}{\text{Br}(B_c \rightarrow K_2^{*+}\eta)} = 1.21_{-0.21}^{+0.22},$$

$$R_{\eta'/\eta}^{K^*} \equiv \frac{\text{Br}(B_c \rightarrow K^{*+}\eta')}{\text{Br}(B_c \rightarrow K^{*+}\eta)} = 4.22_{-1.59}^{+0.76}, \quad (11)$$

where various errors in the ratios have been added in quadrature. The good isospin symmetry makes the $R_{\eta/\eta'}^{a_2}$ approximately equal to the $R_{\eta/\eta'}^\rho$; however, the significant $SU(3)$ flavor symmetry-breaking effects in the K^* meson makes the $R_{\eta'/\eta}^{K^*}$ quite different from the $R_{\eta'/\eta}^{K_2^*}$. It is expected that future precise measurements of these ratios might be helpful to investigate the possible pseudoscalar glueball in the η' state [46,47].

- (4) As far as the $B_c \rightarrow (a_2^0, f_2)(\pi^+, K^+)$ channels are concerned, one can find that, according to the pQCD predictions for the branching ratios, the constructive (destructive) interferences between $u\bar{u}$ and $d\bar{d}$ components in the $f_2(a_2^0)$ meson with the same (opposite) sign result in a slightly larger (smaller) $\text{Br}(B_c \rightarrow f_2\pi^+) = 7.38_{-1.75}^{+1.86} \times 10^{-7}$

[$\text{Br}(B_c \rightarrow a_2^0\pi^+) = 5.13_{-1.65}^{+1.96} \times 10^{-7}$]. On the other hand, due to only the $u\bar{u}$ component in both a_2^0 and f_2 states giving contributions, the almost equivalent branching ratios $\text{Br}(B_c \rightarrow a_2^0K^+) \approx \text{Br}(B_c \rightarrow f_2K^+)$ can be obtained, which are more like that seen in the $B_c \rightarrow (\rho^0, \omega)K^+$ modes [3]. The negligibly tiny deviations between the $B_c \rightarrow a_2^0K^+$ and $B_c \rightarrow f_2K^+$ decays arise from the slightly different decay constants and hadron masses of the a_2^0 and f_2 states, as well as from the same QCD behavior at leading twist. Likewise, the similar phenomenologies of the branching ratios and polarization fractions can be seen clearly from the processes of $B_c \rightarrow K_2^{*+}(\rho^0, \omega)$ and $B_c \rightarrow (a_2^0, f_2)K^{*+}$ in Table IV.

- (5) Some simple relations and many other interesting ratios, which can shed light on the (non)validity of $SU(3)$ flavor symmetry in the considered decays, are given as follows:

$$\begin{aligned} \text{Br}(B_c \rightarrow \bar{K}^{*0}\pi^+) &= 2 \cdot \text{Br}(B_c \rightarrow K_2^{*+}\pi^0) \\ &= \text{Br}(B_c \rightarrow \bar{K}^{*0}K^+) \cdot \left(\left| \frac{V_{us}}{V_{ud}} \right| \cdot \frac{f_\pi}{f_K} \right)^2 \\ &\sim \text{Br}(B_c \rightarrow K_2^{*+}\bar{K}^0) \cdot \left(\left| \frac{V_{us}}{V_{ud}} \right| \cdot \frac{f_\pi}{f_K} \right)^2, \end{aligned} \quad (12)$$

$$\text{Br}(B_c \rightarrow a_2^+K^0) = 2 \cdot \text{Br}(B_c \rightarrow a_2^0K^+), \quad (13)$$

$$\begin{aligned} R_{K/\pi}^{a_2} &\equiv \frac{\text{Br}(B_c \rightarrow a_2^0K^+)}{\text{Br}(B_c \rightarrow a_2^0\pi^+)} \\ &= 0.088_{-0.010-0.005-0.004-0.004}^{+0.024+0.005+0.001+0.005}, \end{aligned} \quad (14)$$

$$\begin{aligned} R_{K/\pi}^{f_2} &\equiv \frac{\text{Br}(B_c \rightarrow f_2K^+)}{\text{Br}(B_c \rightarrow f_2\pi^+)} \\ &= 0.062_{-0.003-0.001-0.007-0.002}^{+0.004+0.001+0.006+0.002}. \end{aligned} \quad (15)$$

Moreover, the ratio between $\text{Br}(B_c \rightarrow f_2K^+)$ and $\text{Br}(B_c \rightarrow f_2'\pi^+)$ when confronted with future precision data can provide useful hints for the $f_2 - f_2'$ mixing, though the ideal mixing is assumed in this work,

$$R_{f_2/f_2'}^{K} \equiv \frac{\text{Br}(B_c \rightarrow f_2K^+)}{\text{Br}(B_c \rightarrow f_2'\pi^+)} = 0.72_{-0.14-0.09-0.01-0.02}^{+0.20+0.10+0.03+0.03}, \quad (16)$$

where the largest error of the ratio is also induced by the variations of the B_c meson distribution amplitude. Therefore, an in-depth understanding of the hadronization of the involved meson is the key to provide precise predictions in the pQCD approach for future experimental measurements.

Now we turn to the analyses of the branching ratios and polarization fractions of the $B_c \rightarrow TV$ decays in the pQCD approach. As stressed previously, due to the angular momentum conservation, the $B_c \rightarrow TV$ decays contain three helicities, which are more like the $B_c \rightarrow VV$ ones. Then the definitions of the related helicity amplitudes, polarization fractions, and relative phases are also the same as those of $B_c \rightarrow VV$ modes (see Ref. [3] for details). It should be noted that, as this is a first investigation of the nonleptonic charmless $B_c \rightarrow TV$ decays, only CP -averaged branching ratios and polarization fractions (whose values are collected in Tables III and IV) are presented in this work. Moreover, we specify the polarization fractions as longitudinal f_L and transverse $f_T (= 1 - f_L)$, not those f_L , f_{\parallel} , and f_{\perp} adopted previously [3]. Some remarks are in order:

- (1) Differently from the $B_c \rightarrow TP$ decays, all of the CKM-favored $B_c \rightarrow TV$ modes (which contain three polarization contributions with larger decay constants and hadron masses of vector mesons) have decay rates of 10^{-6} within theoretical errors in the pQCD approach at leading order. It is believed that the predictions of these large branching ratios can be confirmed soon by the LHC experiments at CERN [45]. The CP -averaged branching ratios of the CKM-suppressed $B_c \rightarrow TV$ modes are nearly 10^{-8} – 10^{-7} , which may have to await future tests with much larger data samples. Nevertheless, one can easily find that all of the $B_c \rightarrow TV$ modes are governed by the longitudinal decay amplitudes, which result in the large polarization fractions in the range of 78–98%, as presented in Tables III and IV.
- (2) As shown in Table III, the CP -averaged branching ratios and the polarization fractions of $B_c \rightarrow a_2^+ \omega$ and $B_c \rightarrow f_2 \rho^+$ channels are close to each other. The reason is that, on the one hand, the pure $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ component for the $f_2(1270)$ state is assumed which is same as the ω meson with ideal mixing, and on the other hand, the adopted decay constants and masses of the involved tensor and vector mesons are similar in magnitude with only small differences. More specifically,

$$\begin{aligned}
 m_{\rho} &= 0.770 \text{ GeV}, & m_{\omega} &= 0.782 \text{ GeV}, \\
 f_{\rho} &= 0.209 \pm 0.002 \text{ GeV}, \\
 f_{\omega} &= 0.195 \pm 0.003 \text{ GeV}; \\
 f_{\rho}^T &= 0.165 \pm 0.009 \text{ GeV}, \\
 f_{\omega}^T &= 0.145 \pm 0.010 \text{ GeV}
 \end{aligned} \tag{17}$$

for the light vector ρ and ω mesons, and

$$\begin{aligned}
 m_{a_2} &= 1.318 \text{ GeV}, & m_{f_2} &= 1.275 \text{ GeV}, \\
 f_{a_2} &= 0.107 \pm 0.006 \text{ GeV}, \\
 f_{f_2} &= 0.102 \pm 0.006 \text{ GeV}; \\
 f_{a_2}^T &= 0.105 \pm 0.021 \text{ GeV}, \\
 f_{f_2}^T &= 0.117 \pm 0.025 \text{ GeV}
 \end{aligned} \tag{18}$$

for the light tensor a_2 and f_2 states. Therefore, it is also understandable that these two decay rates are a bit smaller than that of the $B_c \rightarrow \rho^+ \omega$ channel [3].

- (3) It is interesting to note that the $B_c \rightarrow a_2^+ \rho^0$ and $B_c \rightarrow a_2^+ \omega$ decay rates indicate different interferences between the $u\bar{u}$ and $d\bar{d}$ components in the ρ^0 and ω mesons. As can be seen in Table III, it is evident that the constructive (destructive) interferences contribute to the former (latter) mode. Similar phenomenologies also appear in the $B_c \rightarrow a_2^0 \rho^+$ and $B_c \rightarrow f_2 \rho^+$ decays. Moreover, one can easily observe that the numerical results of the branching ratios are sensitive to the hadronic parameters such as the charm-quark mass, the decay constants of the light tensor meson, etc. We thus define some ratios among the branching ratios as follows:

$$R_{\omega/\rho^0}^{a_2} \equiv \frac{\text{Br}(B_c \rightarrow a_2^+ \omega)}{\text{Br}(B_c \rightarrow a_2^+ \rho^0)} = 0.67_{-0.07-0.02-0.02-0.02}^{+0.11+0.00+0.01+0.03}, \tag{19}$$

$$R_{f_2/a_2}^{\rho} \equiv \frac{\text{Br}(B_c \rightarrow f_2 \rho^+)}{\text{Br}(B_c \rightarrow a_2^0 \rho^+)} = 0.75_{-0.08-0.03-0.02-0.04}^{+0.11+0.03+0.01+0.03}, \tag{20}$$

$$R_{a_2 \omega/f_2 \rho} \equiv \frac{\text{Br}(B_c \rightarrow a_2^+ \omega)}{\text{Br}(B_c \rightarrow f_2 \rho^+)} = 0.90_{-0.00-0.03-0.01-0.01}^{+0.01+0.01+0.00+0.02}, \tag{21}$$

where the uncertainties arising from the errors of the inputs have been greatly canceled, though these parameters involved in the meson wave functions are not factored out. These ratios and the detectable decay rates could be helpful to further explore the QCD behavior of the a_2 and f_2 states.

- (4) Analogous to the $B_c \rightarrow K_2^* K$ decays, the $B_c \rightarrow K_2^{*+} \bar{K}^{*0}$ and $B_c \rightarrow \bar{K}_2^{*0} K^{*+}$ modes also have branching ratios that are close to each other for the same reason. More interestingly, the ratio arising from $\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^{*0})$ over $\text{Br}(B_c \rightarrow \bar{K}_2^{*0} K^{*+})$ in the pQCD approach is approximately equal to that [see Eq. (9)] obtained in the $B_c \rightarrow K_2^* K$ decays, although these two branching ratios induced by three polarizations are clearly larger than the $B_c \rightarrow K_2^* K$ ones

only from longitudinal polarization. The related branching ratios and ratio are

$$\begin{aligned}\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^{*0}) &= 1.75_{-0.20}^{+0.20} \times 10^{-6}, \\ \text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^{*0}) &= 1.54_{-0.49}^{+0.60} \times 10^{-6},\end{aligned}\quad (22)$$

and

$$\begin{aligned}R_{\bar{K}^{*0}/K^{*+}} &= \frac{\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^{*0})}{\text{Br}(B_c \rightarrow K_2^{*+} K^{*+})} \\ &= 1.14_{-0.32-0.01-0.01-0.03}^{+0.39+0.00+0.00+0.04}.\end{aligned}\quad (23)$$

The conservation law of angular momentum results in the tensor K_2^* state contributing to the $B_c \rightarrow K_2^* K^*$ decays with only three helicities, $\lambda = 0$ and ± 1 , which makes it behave more like a vector meson. Since the smaller decay constants (as shown in Table I) of both longitudinal and transverse polarizations of the K_2^* meson are adopted, the decay rates and polarization fractions of the $B_c \rightarrow K_2^* K^*$ modes are basically consistent with those of the $B_c \rightarrow \bar{K}^{*0} K^{*+}$ one within errors [3], though $m_{K_2^*}$ is nearly 2 times larger than m_{K^*} .

- (5) As reported by the *BABAR* Collaboration, the fact that $f_L/f_T \gg 1$ for $B \rightarrow \phi K_2^*$ decays [48] while $f_L/f_T \sim 1$ for $B \rightarrow \omega K_2^*$ decays [49] make the well-known ‘‘polarization puzzle’’ more confusing, although both of them have the same helicity structure as the $B \rightarrow \phi K^*$ modes with the penguin-dominated contributions. Furthermore, the branching ratio of the $B^+ \rightarrow \omega K_2^{*+}$ channel is much larger than that of the $B^+ \rightarrow \phi K_2^{*+}$ one by a factor of around 2.5, which is contrary to the ratio of the $B^+ \rightarrow \omega K^{*+}$ and $B^+ \rightarrow \phi K^{*+}$ decay rates [1,4]. The current theoretical studies on these anomalous phenomena cannot give satisfactory explanations, which means that more investigations on the light tensor K_2^* meson are demanded. The CP -averaged branching ratios and polarization fractions of $B_c \rightarrow K_2^*(\rho, \omega, \phi)$ channels are given in the pQCD approach at leading order are presented in Table IV. One can find that these four modes are dominated by the longitudinal decay amplitudes, and the $B_c \rightarrow K_2^{*0} \rho^+$ and $B_c \rightarrow K_2^{*+} \phi$ decay rates are on the order of 10^{-7} within errors; these are expected to be tested by the LHC Run-II experiments at CERN in the near future.
- (6) Likewise, some interesting ratios of $B_c \rightarrow TV$ decays can also provide useful hints about the QCD dynamics involved in the light tensor mesons, as well as in the related decay channels. For example, future precise measurements can tell us the mixing information about $f_2(1275)$ and $f_2'(1525)$ states through the ratio $R_{f_2/f_2'}^{K^*}$,

$$R_{f_2/f_2'}^{K^*} \equiv \frac{\text{Br}(B_c \rightarrow f_2 K^{*+})}{\text{Br}(B_c \rightarrow f_2' K^{*+})} = 0.46_{-0.12-0.04-0.01-0.03}^{+0.13+0.02+0.00+0.01},\quad (24)$$

where the $B_c \rightarrow f_2' K^{*+}$ branching ratio reaches 10^{-7} . If the future measured ratios $R_{f_2/f_2'}^{K^*}$ and $R_{f_2/f_2'}^{K^*}$ deviate from those predicted in Eqs. (16) and (24), then the mixture of the f_{2q} and f_{2s} flavor states should be included for the f_2 and f_2' mesons. It is noted that the $R_{f_2/f_2'}^{K^*}$ is a bit larger than the $R_{f_2/f_2'}^{K^*}$ by a factor of around 1.5, since $\text{Br}(B_c \rightarrow f_2 K^+) \sim \text{Br}(B_c \rightarrow f_2 K^{*+})$ while $\text{Br}(B_c \rightarrow f_2' K^{*+}) \sim 1.5 \times \text{Br}(B_c \rightarrow f_2' K^+)$. More data are demanded on the f_2 and f_2' states to further understand these phenomenologies, in particular, the approximately equal decay rates between $B_c \rightarrow f_2 K^+$ and $B_c \rightarrow f_2 K^{*+}$ modes,

$$\begin{aligned}\text{Br}(B_c \rightarrow f_2 K^+) &= 4.56_{-1.17}^{+1.33} \times 10^{-8}, \\ \text{Br}(B_c \rightarrow f_2 K^{*+}) &= 4.38_{-0.69}^{+0.61} \times 10^{-8}.\end{aligned}\quad (25)$$

After all, the latter process receives contributions from three helicities.

- (7) The isospin symmetry can be observed in the pQCD calculations for $B_c \rightarrow K_2^* \rho$ and $B_c \rightarrow a_2 K^*$ modes, that is,

$$\begin{aligned}\text{Br}(B_c \rightarrow K_2^0 \rho^+) &= 2 \cdot \text{Br}(B_c \rightarrow K_2^{*+} \rho^0), \\ \text{Br}(B_c \rightarrow a_2^+ K^{*0}) &= 2 \cdot \text{Br}(B_c \rightarrow a_2^0 K^{*+}).\end{aligned}\quad (26)$$

However, the $SU(3)$ flavor symmetry cannot be easily seen in the $B_c \rightarrow K_2^* K^*$ and $B_c \rightarrow K_2^* \rho$ decays [Eq. (12)], since these two decays have three helicity structures with different decay constants and different Gegenbauer moments of the vector K^* and ρ mesons in longitudinal and transverse polarizations, respectively. Nevertheless, we can still present the ratios between $\text{Br}(B_c \rightarrow K_2^* \rho^+)$ and $\text{Br}(B_c \rightarrow K_2^* K^*)$, which can be used to show the (non)validity of the $SU(3)$ flavor symmetry by combining future precise measurements,

$$\begin{aligned}R_{\rho/\bar{K}^*} &\equiv \frac{\text{Br}(B_c \rightarrow K_2^0 \rho^+)}{\text{Br}(B_c \rightarrow K_2^{*+} \bar{K}^{*0})} \\ &= 0.041_{-0.011-0.000-0.001-0.001}^{+0.015+0.000+0.001+0.000},\end{aligned}\quad (27)$$

$$\begin{aligned}R_{\rho/K^*} &\equiv \frac{\text{Br}(B_c \rightarrow K_2^0 \rho^+)}{\text{Br}(B_c \rightarrow \bar{K}_2^{*0} K^{*+})} \\ &= 0.047_{-0.001-0.001-0.002-0.001}^{+0.000+0.000+0.001+0.000}.\end{aligned}\quad (28)$$

Two more relations can also be written as follows:

$$R_{K^*/\rho}^{a_2} \equiv \frac{\text{Br}(B_c \rightarrow a_2^0 K^{*+})}{\text{Br}(B_c \rightarrow a_2^0 \rho^+)} = 0.036_{-0.008-0.001-0.001-0.002}^{+0.007+0.000+0.000+0.000}, \quad (29)$$

$$R_{K^*/\rho}^{f_2} \equiv \frac{\text{Br}(B_c \rightarrow f_2 K^{*+})}{\text{Br}(B_c \rightarrow f_2 \rho^+)} = 0.045_{-0.004-0.001-0.000-0.001}^{+0.002+0.002+0.000+0.001}, \quad (30)$$

where, by combining the ratios $R_{K/\pi}^{a_2}$ and $R_{K/\pi}^{f_2}$ in Eqs. (14) and (15), $\text{Br}(B_c \rightarrow a_2^0 \rho^+) \sim 2.5 \times \text{Br}(B_c \rightarrow a_2^0 \pi^+)$ but $\text{Br}(B_c \rightarrow a_2^0 K^{*+}) \simeq \text{Br}(B_c \rightarrow a_2^0 K^+)$ result in the relation $R_{K/\pi}^{a_2} > R_{K^*/\rho}^{a_2}$. However, $\text{Br}(B_c \rightarrow f_2 \rho^+) \sim 1.3 \times \text{Br}(B_c \rightarrow f_2 \pi^+)$ but $\text{Br}(B_c \rightarrow f_2 K^{*+}) \simeq \text{Br}(B_c \rightarrow f_2 K^+)$ leads to the relation $R_{K^*/\rho}^{f_2} < R_{K/\pi}^{f_2}$. It is worth mentioning that the $R_{K/\pi}^{a_2}$ is a bit larger than the $R_{K/\pi}^{f_2}$ in the $B_c \rightarrow (a_2^0, f_2)(\pi^+, K^+)$ decays, while the $R_{K^*/\rho}^{a_2}$ is slightly smaller than the $R_{K^*/\rho}^{f_2}$ in the $B_c \rightarrow (a_2^0, f_2) \times (\rho^+, K^{*+})$ modes, which could be tested and further clarified by the related experiments with good precision in the future.

- (8) Recently, the three-body (or quasi-two-body) B meson decays have attracted more and more attention, since the two B factories and the LHC experiments have collected lots of data on the related channels. It is suggested that the considered light tensor states in this work can also be studied through the resonant contributions in the relevant three-body modes [50]; for example, the $f_2(1270)$ and $f_2'(1525)$ mesons can be investigated in the $B_c \rightarrow f_2(1270)(\rightarrow \pi\pi)(\pi, K)$ and $B_c \rightarrow f_2'(1525)(\rightarrow KK) \times (\pi, K)$ channels, respectively, which can play important roles in exploring the QCD dynamics of the light tensor mesons. These studies can also

help to further deepen our understanding of the three-body decay mechanism.

In summary, we have analyzed the nonleptonic charmless $B_c \rightarrow TP, TV$ decays in the pQCD approach. Due to the angular momentum conservation, the light tensor meson can only contribute with one ($\lambda = 0$) or three helicities ($\lambda = 0, \pm 1$). By properly redefining the polarization tensor, the new polarization vector ϵ_T of the light tensor meson can be obtained, which is slightly different than the ϵ_V of the vector meson with coefficients $\sqrt{\frac{2}{3}}$ and $\sqrt{\frac{1}{2}}$ for longitudinal and transverse polarizations, respectively. Therefore, the decay amplitudes can be easily presented with appropriate replacements from the $B_c \rightarrow PV, VV$ decay modes. The CP -averaged branching ratios and polarization fractions for the considered channels have been predicted in the pQCD approach. Most of the CKM-favored processes have decay rates of 10^{-6} , which are expected to be measured soon by the LHC experiments at CERN. Numerically, all of the $B_c \rightarrow TV$ modes are governed by the longitudinal contributions. Many interesting ratios among the branching ratios have been derived, as well as some simple relations that can be used to exhibit the (non)validity of the $SU(3)$ flavor symmetry. The predictions about these concerned $B_c \rightarrow TP, TV$ decays in the pQCD approach can be confronted with measurements in the (near) future, which are expected to shed some light on the annihilation decay mechanism in the related decay channels.

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