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Decay of the dimuonium into a photon and a neutral pion

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We compute the decay rate of dimuonium into a neutral pion and a photon. We find that approximately one in 10^5 orthodimuonia decays into this channel. We also determine the contribution of the virtual photon-pion loop to the hyperfine splitting in dimuonium and reproduce its leading effect in the anomalous magnetic moment of the muon.

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I. INTRODUCTION

Dimuonium Dm (also known as true muonium) is a bound state of a muon and an antimuon, analogous to positronium but about 207 times heavier [1,2]. While positronium was discovered already 65 years ago [3], dimuonium has not been observed yet. Recently, however, the prospect for its discovery has become brighter: it may be produced in experiments searching for exotic light bosons [4]. Production of Dm at rest is under consideration at the e^+e^- collider at Budker Institute of Nuclear Physics (BINP) in Novosibirsk [5]. On the theory side, Dm production [6–9], spectrum [10–14], and decays [8,12,13] have been studied. The name "dimuonium" was first introduced in [15].

Although Dm is a purely leptonic system, it is affected by hadrons through higher-order effects. It is the lightest pure leptonic system with hadronic decay channels. Here we present the rate of the dimuonium decay into a neutral pion π^0 and a photon γ, shown in Fig. 1(a), a decay channel that has not been considered so far. It is interesting because it is a new two-body decay of the spin-triplet dimuonium (orthodimuonium, o-Dm), with a clean signature: a monochromatic photon. Such hadronic final states are not accessible to positronium because of its small mass. There is also another hadronic decay channel, with a charged pion, shown in Fig. 1(b), and an analogous channel with the opposite-sign pion. However, these processes are additionally suppressed by inverse powers of the W boson mass and are extremely rare.

II. DECAY RATE OF Dm → $\pi^0\gamma$

We use the amplitude of the $\pi^0\gamma\gamma$ coupling (throughout this paper we use $\hbar = c = 1$),

$$\mathcal{M} = \frac{\alpha F_{\pi^0\gamma\gamma}(q_1^2, q_2^2)}{\pi F_\pi} \epsilon^{\mu\nu\alpha\beta} \epsilon^\star_{1\mu} \epsilon^\star_{2\nu} q_{1\alpha} q_{2\beta}, \quad (1)$$

where $\alpha \simeq 1/137$ is the fine structure constant, $F_\pi \simeq 92$ MeV is the pion decay constant, and $F_{\pi^0\gamma\gamma}(q_1^2, q_2^2)$ is a form factor that we need only with one of the photons on shell; we use a simple vector meson dominance approach to model this form factor,

$$F_{\pi^0\gamma\gamma}(q^2, 0) \simeq \frac{1}{1 - \frac{q^2}{M^2}}, \quad (2)$$

where the mass M is on the order of the ρ-meson mass, $M \simeq m_\rho \simeq 769$ MeV. We find the width

$$\Gamma(\text{o-Dm} \to \pi^0\gamma) = \frac{\alpha^6 E^3}{48\pi^3 F_\pi^2} \left(\frac{1}{1 - \frac{4m_\mu^2}{m_\rho^2}} \right)^2, \quad (3)$$

where o-Dm refers to orthodimuonium, m_μ is the muon mass, and E is the energy of the photon in the final state,

$$E \simeq \frac{4m_\mu^2 - m_\pi^2}{4m_\mu} \simeq 63 \text{ MeV}. \quad (4)$$

The dominant decay channel of orthodimuonium is into an e^+e^- pair, with the rate

$$\Gamma(\text{o-Dm} \to e^+e^-) = \frac{\alpha^5 m_\mu}{6}. \quad (5)$$

The ratio of the decay rates into $\pi^0\gamma$ and the e^+e^- is

$$\frac{\Gamma(\text{o-Dm} \to \pi^0\gamma)}{\Gamma(\text{o-Dm} \to e^+e^-)} = \frac{\alpha(4m_\mu^2 - m_\pi^2)^3}{512\pi^3 F_\pi^2 m_\mu^4} \left(\frac{1}{1 - \frac{4m_\mu^2}{m_\rho^2}} \right)^2 \simeq 0.9 \times 10^{-5}. \quad (6)$$

This branching ratio is small because the coupling of the neutral pion to photons can be interpreted as a quantum

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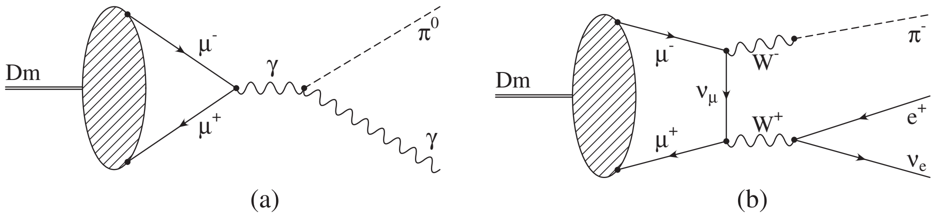

FIG. 1. Decays of dimuonium with pions in the final state. (a) Dm → $\pi^0\gamma$ considered in this work. (b) Decay with a charged pion. It is strongly suppressed because of the large mass of the W boson and neglected here.

(loop-induced) effect [16]. There are additional suppression factors such as the reduced phase space volume. However, there does not seem to be another two-body decay channel accessible to orthodimuonium and not to the spin singlet (paradimuonium), p-Dm (the dominant final state for o-Dm, e^+e^-, can be reached from p-Dm via a two-photon annihilation). Thus in principle the peak of 63 MeV photons (in the rest frame of the decaying o-Dm) can be used to establish the presence of orthodimuonium.

The decay rate into $\pi^0\gamma$ is also much smaller than into three photons,

$$\frac{\Gamma(\mathrm{Dm}\to\pi^0\gamma)}{\Gamma(\mathrm{Dm}\to\gamma\gamma\gamma)}=\frac{3\left(1-\frac{m_\pi^2}{4m_\mu^2}\right)^3 m_\mu^2}{32\pi^2(\pi^2-9)\left(1-\frac{4m_\mu^2}{m_\rho^2}\right)^2 F_\pi^2}\simeq 0.3\%. \qquad (7)$$

III. CONTRIBUTION TO THE HYPERFINE SPLITTING

Since the virtual one-photon annihilation is possible only for orthodimuonium, the $\pi^0\gamma$ loop contributes to the hyperfine splitting (HFS) through the diagram shown in Fig. 2. This is an additional hadronic vacuum polarization contribution to the HFS, not considered in previous calculations, such as for example [12].

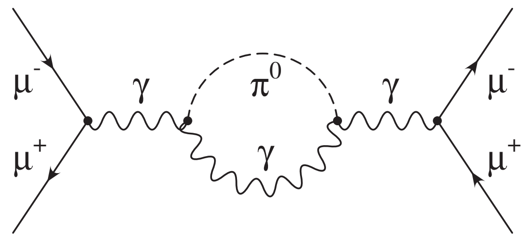

FIG. 2. Virtual annihilation into $\pi^0\gamma$ is possible only for orthodimuonium and thus contributes to the hyperfine splitting of Dm.

We find this contribution to slightly modify the one-photon virtual annihilation,

$$\Delta E=\frac{\alpha^4 m_\mu}{4}\left\{1+\frac{\alpha^2 m_\rho^2}{32\pi^4F_\pi^2}\left[\frac{2\int_0^1 x[\epsilon-(1-x)\delta]\ln\left|1+\frac{1-x}{x[\epsilon-(1-x)\delta]}\right|\mathrm{d}x-1-\frac{i\pi}{3}(1-\frac{\epsilon}{\delta})^3\delta}{(1-\delta)^2}+\frac{1-\epsilon+\epsilon\ln\epsilon}{(1-\epsilon)^2}\right]\right\} \qquad (8)$$

where $\delta=\frac{4m_\mu^2}{m_\rho^2}\simeq 0.08$ and $\epsilon=\frac{m_\pi^2}{m_\rho^2}\simeq 0.03$. The imaginary part reproduces the decay rate, $\Gamma(\text{o-Dm}\to\pi^0\gamma)=-2\mathrm{Im}\Delta E$, in agreement with Eq. (3). The real part gives a correction to the HFS; it is well approximated by dropping $x(1-x)\delta$ in the argument of the log and retaining only terms of first order in ϵ and δ in the remaining result,

$$\mathrm{Re}\Delta E\simeq\frac{\alpha^4 m_\mu}{4}\left[1-\frac{\alpha^2 m_\rho^2}{32\pi^4F_\pi^2}\left(\epsilon\ln\frac{1}{\epsilon}+\frac{\epsilon}{2}+2\delta\right)\right] \qquad (9)$$

$$\simeq\frac{\alpha^4 m_\mu}{4}[1-3.\times 10^{-7}]. \qquad (10)$$

The numerical value of this 0.3 part per million shift is about −6 MHz.

IV. CONTRIBUTION TO THE MUON ANOMALOUS MAGNETIC MOMENT

The $\pi^0\gamma$ loop contribution to the vacuum polarization modifies also the anomalous magnetic moment of the muon, $a_\mu=\frac{g_\mu-2}{2}$. This is realized by closing the antimuon line in Fig. 2 and connecting it to an external magnetic field, as shown in Fig. 3. This effect was determined in [17] using an expansion in two small parameters, the mass ratio m_μ/m_ρ and the normalized difference of the pion and the muon masses squared, $\frac{m_\pi^2-m_\mu^2}{m_\mu^2}$ (see also [18–20]). We can now check that result numerically. To this end, we consider the $\pi^0\gamma$ contribution to the polarization,

$$\Pi_R^{\mu\nu}(p)=i(p^2g^{\mu\nu}-p^\mu p^\nu)\Pi_R(p^2) \qquad (11)$$

where, with the on-shell charge renormalization and to the leading order in $1/M^2$ (we use M again for the vector meson mass in the form factor, to emphasize its role as the ultraviolet cutoff),

$$\Pi_R(p^2) = \frac{\alpha^2}{16\pi^4 F_\pi^2}\int_0^1 \left\{ \frac{1}{\left(1-\frac{p^2}{M^2}\right)^2}\left[A(0,p^2)\ln\frac{A(M^2,p^2)}{A(0,p^2)} - \frac{M^2}{2}\right] + \frac{M^2}{2} - A(0,0)\ln\frac{A(M^2,0)}{A(0,0)}\right\}\mathrm{d}x \tag{12}$$

$$\equiv \frac{\alpha^2}{16\pi^4 F_\pi^2} p^2 J(p^2) \tag{13}$$

$$A(M^2,p^2) \equiv x m_\pi^2 + (1-x)M^2 - x(1-x)p^2 - i0. \tag{14}$$

It is useful to isolate p^2 as an explicit overall factor by rewriting Π_R. In the leading order in $1/M^2$, the integral in Π_R is

$$p^2 J(p^2) = \int_0^1 \left[-p^2 - x(1-x)p^2 \ln\frac{A(M^2,p^2)}{A(0,p^2)} + A(0,0)\ln\frac{A(M^2,p^2)}{A(0,p^2)}\frac{A(0,0)}{A(M^2,0)}\right]\mathrm{d}x. \tag{15}$$

Integrating by parts, we find

$$\int_0^1 \left[-x(1-x)p^2\ln\frac{A(M^2,p^2)}{A(0,p^2)} + A(0,0)\ln\frac{A(M^2,p^2)}{A(M^2,0)}\frac{A(0,0)}{A(0,p^2)}\right]\mathrm{d}x \tag{16}$$

$$= \frac{p^2}{6}\int_0^1 \left[\frac{[(1-x)^2M^2 - x^2m_\pi^2][(3-2x)(1-x)M^2 + x^2m_\pi^2]}{[(1-x)M^2 + xm_\pi^2]\left(p^2 - \frac{m_\pi^2}{1-x} - \frac{M^2}{x}\right)} + \frac{m_\pi^2 x^3}{p^2 - \frac{m_\pi^2}{1-x}}\right]\frac{\mathrm{d}x}{(1-x)^2}. \tag{17}$$

We recognize in the p^2-dependent denominators a similarity to propagators of massive particles. A vector boson of mass m, with a photonlike coupling to the muon, contributes to its anomalous magnetic moment the amount [21]

$$\Delta a_\mu = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}y \frac{y^2(1-y)}{y^2 + (1-y)\frac{m^2}{m_\mu^2}}, \tag{18}$$

so the effect of the $\pi^0\gamma$ loop is

$$\Delta a_\mu(\pi^0\gamma) = \frac{\alpha^3}{96\pi^5 F_\pi^2}\left[2m_\mu^2 + \int_0^1 \frac{\mathrm{d}x}{(1-x)^2}\int_0^1 \mathrm{d}y y^2(1-y)\right. \tag{19}$$

$$\left.\times\left\{\frac{[(1-x)^2M^2 - x^2m_\pi^2][(3-2x)(1-x)M^2 + x^2m_\pi^2]}{[(1-x)M^2 + xm_\pi^2]\left[y^2 + (1-y)\frac{xm_\pi^2 + (1-x)M^2}{x(1-x)m_\mu^2}\right]} + \frac{m_\pi^2 x^3}{y^2 + \frac{1-y}{1-x}\frac{m_\pi^2}{m_\mu^2}}\right\}\right]. \tag{20}$$

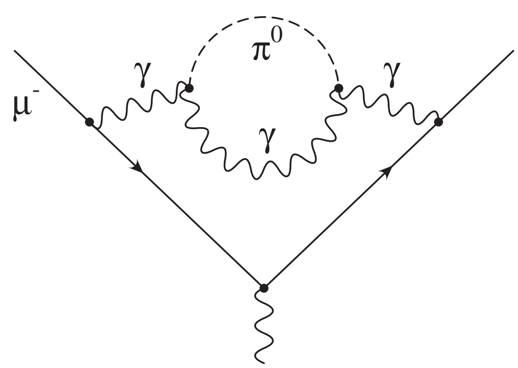

FIG. 3. $\pi^0\gamma$ contribution to the muon anomalous magnetic moment.

Numerically, in the limit $m_\pi = m_\mu$ and $M = m_\rho$, this gives $\Delta a_\mu(\pi^0\gamma) = 4.8\times 10^{-11}$, in agreement with Eq. (10) in [17]. We have also reproduced analytically the leading logarithm $\sim \ln\frac{M^2}{m_\pi^2}$ given in that work.

V. CONCLUSIONS

In Eq. (3) we have given the decay rate of a new decay channel of orthodimuonium, o-Dm $\to \pi^0\gamma$. This channel occurs approximately once every 10^5 decays. As a by-product of this study, we have determined the correction of a virtual $\pi^0\gamma$ loop to the hyperfine splitting of dimuonium. Finally, as a check of our results, we reproduced the leading contribution of this loop to the anomalous magnetic moment of muon and found agreement with the original determination [17].

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[1] S. M. Bilenky, N. Van Hieu, L. L. Nemenov, and F. G. Tkebuchava, Yad. Fiz. **10**, 812 (1969) [Sov. J. Nucl. Phys. **10**, 469 (1969)].
[2] V. W. Hughes and B. Maglic, Bull. Am. Phys. Soc. **16**, 65 (1971).
[3] M. Deutsch, Phys. Rev. **82**, 455 (1951).
[4] A. Banburski and P. Schuster, Phys. Rev. D **86**, 093007 (2012).
[5] S. Eidelman (private communication).
[6] K. Sakimoto, Eur. Phys. J. D **69**, 276 (2015).
[7] S. J. Brodsky and R. F. Lebed, Phys. Rev. Lett. **102**, 213401 (2009).
[8] I. F. Ginzburg, U. D. Jentschura, S. G. Karshenboim, F. Krauss, V. G. Serbo, and G. Soff, Phys. Rev. C **58**, 3565 (1998).
[9] L. L. Nemenov, Yad. Fiz. **15**, 1047 (1972) [Sov. J. Nucl. Phys. **15**, 582 (1972)].
[10] Y. Ji and H. Lamm, Phys. Rev. A **94**, 032507 (2016).
[11] H. Lamm, in *Proceedings, 12th Conference on the Intersections of Particle and Nuclear Physics (CIPANP 2015), Vail, Colorado, 2015*, http://www.slac.stanford.edu/econf/C150519.
[12] U. D. Jentschura, G. Soff, V. G. Ivanov, and S. G. Karshenboim, Phys. Rev. A **56**, 4483 (1997).
[13] S. G. Karshenboim, U. D. Jentschura, V. G. Ivanov, and G. Soff, Phys. Lett. B **424**, 397 (1998).
[14] S. G. Karshenboim, U. Jentschura, V. G. Ivanov, and G. Soff, Eur. Phys. J. D **2**, 209 (1998).
[15] J. Malenfant, Phys. Rev. D **36**, 863 (1987).
[16] J. Steinberger, Phys. Rev. **76**, 1180 (1949).
[17] I. R. Blokland, A. Czarnecki, and K. Melnikov, Phys. Rev. Lett. **88**, 071803 (2002).
[18] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **65**, 097302 (2002).
[19] N. N. Achasov and A. V. Kiselev, Pis'ma Zh. Eksp. Teor. Fiz. **75**, 643 (2002) [JETP Lett. **75**, 527 (2002)].
[20] W. J. Marciano, A. Masiero, P. Paradisi, and M. Passera, Phys. Rev. D **94**, 115033 (2016).
[21] S. J. Brodsky and E. De Rafael, Phys. Rev. **168**, 1620 (1968).