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Lorentz violation in Bhabha scattering at finite temperature

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Corrections to the Bhabha scattering cross section, due to Lorentz violation, at finite temperature are calculated. The vertex interaction between fermions and photons is modified by introducing the Lorentz violation, for the Standard Model extension, from CPT odd nonminimal coupling. The finite temperature corrections are calculated using the thermo field dynamics formalism. The Lorentz violation corrections are presented for zero to high temperatures.

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I. INTRODUCTION

Lorentz and CPT symmetry have been the foundation of the Standard Model (SM). The SM allows violation of some symmetries, such as charge conjugation C, parity P, and time reversal T. These symmetries are broken separately or combined. However, the CPT symmetry is preserved. Although successfully confirmed, the SM is not a fundamental theory since gravity is not included. Tiny violations of Lorentz and CPT symmetries could emerge in models unifying gravity with quantum physics such as string theory [1]. Opportunities to experimentally detect these violations will likely arise at the Planck scale, $\sim 10^{19}$ GeV. The search for Lorentz violation has been considered for various subfields of physics [2]. The Standard Model extension (SME) [3,4] contains the Standard Model, General Relativity, and all possible operators that break Lorentz symmetry. There are two versions of the SME: (i) a minimal extension which has operators with dimensions $d \leq 4$ and preserves conventional quantization, Hermiticity, gauge invariance, power counting renormalizability, and positivity of the energy and (ii) a nonminimal version of the SME associated with operators of higher dimensions.

The structure of the SME is one way to investigate the Lorentz violation. Another interesting way is to modify the interaction vertex between fermions and photons, i.e., a new nonminimal coupling term added to the covariant derivative. The nonminimal coupling term may be CPT odd or CPT even. CPT-odd nonminimal coupling has been investigated in a different context; for example, the induction of topological phases on fermion systems has been evaluated [5,6], the contribution to magnetic moment generation of massive neutral particles with spin $1/2$ and spin 1 has been analyzed [7], corrections to the hydrogen spectrum have been investigated [8], modifications to the Aharonov-Bohm-Casher problem have been examined [9], and corrections for Bhabha and Comptom scattering cross section have been calculated [10,11]. The CPT-even nonminimal coupling has some applications, such as its effect on the cross section of the electron-positron scattering having been investigated [12], modification in the Dirac equation in the nonrelativistic regime having been analyzed [13], radiative generation of the CPT-even gauge term of the SME having been constructed [14], and effects induced on the magnetic and electric dipole moments having been investigated [15], among others.

In this paper, Bhabha scattering ($e^+e^- \to e^+e^-$) [16] is considered. It is a process usually used in tests of experiments at high energy accelerators [17–20]. Bhabha scattering in the context of the nonminimal coupling term at zero temperature has been analyzed [10,11]. Lorentz symmetry violation is expected to be small at very high energies, i.e., Planck energies ($\sim 10^{19}$ GeV). But this is not valid in all cases. It is likely that Lorentz violation operators with dimension $d > 4$ will be relevant in searches involving very high energies [21]. Although Bhabha scattering, at high energy in colliders like the Large Electron–Positron Collider (LEP), are still at zero temperature, there is certainly no investigation at extremely high energy with nonzero temperature. One possibility is to consider Bhabha scattering at finite temperature at the surface or interior of a star. It is well known that the temperature at the center of stars is typically about $\sim 2 \times 10^7$ K($\sim$1.5 KeV) while at the surface it is much less, $\sim 6 \times 10^3$ K (a few eV). Even though these are small numbers, it is still important to calculate the role of temperature in Bhabha scattering. These estimates will give us a reasonable idea of the role of SME at finite temperatures. Such a scattering would modify the distribution of particles. How does the cross section for Bhabha scattering at a high temperature change? What are the modifications due to Lorentz violation at high temperature? To understand

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temperature effects in this process, the thermo field dynamics (TFD) formalism is used.

TFD formalism is a thermal quantum field theory [22–26]. Its basic elements are (i) the doubling of the original Fock space and (ii) the Bogoliubov transformation. This doubling consists of Fock space composed of the original S and a fictitious space $\tilde{S}$ (tilde space). The map between the tilde $\tilde{A}_i$ and nontilde A_i operators is defined by the following tilde (or dual) conjugation rules,

$$(A_iA_j)^{\sim} = \tilde{A}_i\tilde{A}_j, \qquad (\tilde{A}_i)^{\sim} = -\xi A_i,$$
$$(A_i^\dagger)^{\sim} = \tilde{A}_i{}^\dagger, \qquad (cA_i + A_j)^{\sim} = c^*\tilde{A}_i + \tilde{A}_j, \quad (1)$$

with $\xi = -1$ for bosons and $\xi = +1$ for fermions. The physical variables are described by nontilde operators. The Bogoliubov transformation is a rotation involving these two spaces. As a consequence, the propagator is written in two parts: $T = 0$ and $T \neq 0$ components.

This paper is organized as follows. In Sec. II, a brief introduction to the TFD formalism is presented. In Sec. III, the nonminimal coupling Lorentz violating is considered. The S-matrix elements at finite temperature are calculated. The cross section for Bhabha scattering with Lorentz violation at finite temperature is determined. The limit at zero temperature is recovered. In Sec. IV, some concluding remarks are presented.

II. THERMO FIELD DYNAMICS

TFD is a real time formalism of quantum field theory at finite temperature. The main feature of TFD is that the thermal average of any operator A is equal to its temperature dependent vacuum expectation value, i.e., $\langle A\rangle = \langle 0(\beta)|A|0(\beta)\rangle$, where $|0(\beta)\rangle$ is a thermal vacuum, $\beta = \frac{1}{k_BT}$ with T being the temperature and k_B being the Boltzmann constant. This affirmation requires doubling the degrees of freedom in a Hilbert space accompanied by the temperature dependent Bogoliubov transformation. This doubling is defined by the tilde ($\sim$) conjugation rules, associating each operator in S to two operators in S_T, where the expanded space is $S_T = S\otimes\tilde{S}$, with S being the standard Fock space and $\tilde{S}$ being the fictitious space. The thermal vacuum, $|0(\beta)\rangle$, belongs to the space S_T. For an arbitrary operator A, the standard doublet notation is

$$A^a = \begin{pmatrix} A \\ \xi\tilde{A}^\dagger \end{pmatrix}. \quad (2)$$

The Bogoliubov transformation introduces a rotation in the tilde and nontilde variables, thus introducing thermal quantities.

For fermions with $c_p^\dagger$ and c_p being creation and annihilation operators, respectively, Bogoliubov transformations are

$$c_p = \mathsf{u}(\beta)c_p(\beta) + \mathsf{v}(\beta)\tilde{c}_p^\dagger(\beta), \quad (3)$$

$$c_p^\dagger = \mathsf{u}(\beta)c_p^\dagger(\beta) + \mathsf{v}(\beta)\tilde{c}_p(\beta), \quad (4)$$

$$\tilde{c}_p = \mathsf{u}(\beta)\tilde{c}_p(\beta) - \mathsf{v}(\beta)c_p^\dagger(\beta), \quad (5)$$

$$\tilde{c}_p^\dagger = \mathsf{u}(\beta)\tilde{c}_p^\dagger(\beta) - \mathsf{v}(\beta)c_p(\beta), \quad (6)$$

where $\mathsf{u}(\beta) = \cos\theta(\beta)$ and $\mathsf{v}(\beta) = \sin\theta(\beta)$. The anticommutation relations for creation and annihilation operators are similar to those at zero temperature,

$$\{c(k,\beta), c^\dagger(p,\beta)\} = \delta^3(k-p),$$
$$\{\tilde{c}(k,\beta), \tilde{c}^\dagger(p,\beta)\} = \delta^3(k-p), \quad (7)$$

and other anticommutation relations are null.

For bosons with $a_p^\dagger$ and a_p being creation and annihilation operators, respectively, the Bogoliubov transformation are

$$a_p = \mathsf{u}'(\beta)a_p(\beta) + \mathsf{v}'(\beta)\tilde{a}_p^\dagger(\beta), \quad (8)$$

$$a_p^\dagger = \mathsf{u}'(\beta)a_p^\dagger(\beta) + \mathsf{v}'(\beta)\tilde{a}_p(\beta), \quad (9)$$

$$\tilde{a}_p = \mathsf{u}'(\beta)\tilde{a}_p(\beta) + \mathsf{v}'(\beta)a_p^\dagger(\beta), \quad (10)$$

$$\tilde{a}_p^\dagger = \mathsf{u}'(\beta)\tilde{a}_p^\dagger(\beta) + \mathsf{v}'(\beta)a_p(\beta), \quad (11)$$

where $\mathsf{u}'(\beta) = \cosh\theta(\beta)$ and $\mathsf{v}'(\beta) = \sinh\theta(\beta)$. Algebraic rules for thermal operators are

$$[a(k,\beta), a^\dagger(p,\beta)] = \delta^3(k-p),$$
$$[\tilde{a}(k,\beta), \tilde{a}^\dagger(p,\beta)] = \delta^3(k-p), \quad (12)$$

and other commutation relations are null.

III. BHABHA SCATTERING

For the SME, there are two types of operators that break Lorentz symmetry, minimal and nonminimal. A lot of possibilities have been investigated [21,27,28]. Besides studies of the structure of the SME, other ideas were proposed to examine Lorentz-violating operators in this broad framework. An alternative procedure is to modify just the SME interaction part via a nonminimal coupling. The new interaction breaks the Lorentz and CPT symmetries. This coupling will modify the standard Lagrangian to

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \quad (13)$$

where the covariant derivative includes a nonminimal coupling term, i.e.,

$$D_\mu = \partial_\mu + ieA_\mu + igb^\nu \tilde{F}_{\mu\nu} \tag{14}$$

with e, g, and b^μ being the electron charge, a coupling constant, and a constant 4-vector, respectively. $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\rho}F^{\alpha\rho}$ is the dual electromagnetic tensor with $\epsilon_{0123} = 1$ and $F^{\alpha\rho} = \partial^\alpha A^\rho - \partial^\rho A^\alpha$. There are other possible nonminimal coupling terms that exhibit Lorentz violation [28]. In this paper, a Lorentz-violating CPT-odd term is chosen to study the Bhabha scattering term at finite temperature. This choice is made since it has been used in other investigations, for example, topological phases [29] and other studies [30]. Our purpose is to indicate details that would be necessary to establish its presence. This nonminimal coupling leads to the new interaction. The interaction Lagrangian is given by

$$\mathcal{L}_I = -e\bar{\psi}\gamma^\mu\psi A_\mu - gb^\nu\bar{\psi}\gamma^\mu\psi\partial^\alpha A^\rho\epsilon_{\mu\nu\alpha\rho}. \tag{15}$$

The first term describes the usual QED vertex, and the second term is a new vertex that implies violation of Lorentz symmetries due to the 4-vector b^ν, which specifies a preferred direction in the space-time. These vertices are represented as

$$\bullet \to V^\mu = -ie\gamma^\mu \tag{16}$$

and

$$\times \to gV_\rho = -gb^\nu\gamma^\mu q^\alpha\epsilon_{\mu\nu\alpha\rho}, \tag{17}$$

where q^α is the momentum operator. The nonminimal, i.e., the derivative coupling, term is used in Eq. (14). The dispersion relation grows with momentum [31] for this coupling. Although it grows with the momentum, still at high energies its contribution remains small relative to the result with the Lorentz invariant coupling.

Our interest is to calculate the cross section for the process, $e^-(p_1)e^+(p_2) \to e^-(p_3)e^+(p_4)$, at finite temperature. This process has the Feynman diagrams given in Fig. 1.

The cross section at finite temperature, taking an average over the spin of the incoming particles and summing over the spin of the outgoing particles, is defined as

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \frac{1}{64\pi^2E_{\mathrm{CM}}^2}\cdot\frac{1}{4}\sum_{\mathrm{spins}}|\mathcal{M}(\beta)|^2, \tag{18}$$

where E_{CM} is the center-of-mass energy and $\mathcal{M}(\beta)$ is the S-matrix element at finite temperature. Some details about the cross section for scattering processes have been considered [31]. Modifications in the propagators due to the Lorentz-violating terms leave the linear momentum and the velocity to be misaligned. This leads to a modification of the cross section. Any possible modifications that will likely arise in the cross section due to the modifications in momentum-velocity relation not been considered in this paper.

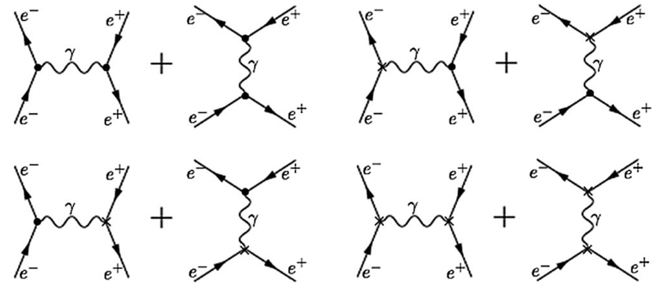

FIG. 1. Exchange and annihilation diagrams with different vertices.

The transition amplitude for Bhabha scattering is calculated as

$$\mathcal{M}(\beta) = \langle f,\beta|\hat{S}^{(2)}|i,\beta\rangle, \tag{19}$$

with $\hat{S}^{(2)}$, the second order term, of the $\hat{S}$-matrix that is defined as

$$\hat{S} = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dx_1dx_2\cdots dx_n\mathbb{T}[\hat{H}_I(x_1)\hat{H}_I(x_2)\cdots\hat{H}_I(x_n)], \tag{20}$$

where $\mathbb{T}$ is the time ordering operator and $\hat{H}_I(x) = H_I(x) - \tilde{H}_I(x)$ describes the interaction. The thermal states are

$$\begin{aligned} |i,\beta\rangle &= c_{p_1}^\dagger(\beta)d_{p_2}^\dagger(\beta)|0(\beta)\rangle, \\ |f,\beta\rangle &= c_{p_3}^\dagger(\beta)d_{p_4}^\dagger(\beta)|0(\beta)\rangle, \end{aligned} \tag{21}$$

with $c_{p_j}^\dagger(\beta)$ and $d_{p_j}^\dagger(\beta)$ being creation operators. The transition amplitude becomes

$$\begin{aligned} \mathcal{M}(\beta) &= \frac{(-i)^2}{2!}\int d^4xd^4y\langle f,\beta|(\mathcal{L}_I\mathcal{L}_I - \tilde{\mathcal{L}}_I\tilde{\mathcal{L}}_I)|i,\beta\rangle \\ &= (\mathcal{M}_0(\beta) + \mathcal{M}_b(\beta) + \mathcal{M}_{bb}(\beta)) \\ &\quad - (\tilde{\mathcal{M}}_0(\beta) + \tilde{\mathcal{M}}_b(\beta) + \tilde{\mathcal{M}}_{bb}(\beta)), \end{aligned} \tag{22}$$

where the matrix element in conventional QED is

$$\begin{aligned} \mathcal{M}_0(\beta) = -\frac{e^2}{2}\int d^4xd^4y\langle f,\beta|\bar{\psi}(x)\gamma^\mu\psi(x)\bar{\psi}(y) \\ \times\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)|i,\beta\rangle. \end{aligned} \tag{23}$$

The linear and quadratic matrix elements for the Lorentz-violating interaction are

$$\begin{aligned} \mathcal{M}_b(\beta) = -egb^\nu\epsilon_{\mu\nu\sigma\rho}\int d^4xd^4y\langle f,\beta|\bar{\psi}(x)\gamma^\omega\psi(x)\bar{\psi}(y) \\ \times\gamma^\mu\psi(y)A_\omega(x)\partial^\sigma A^\rho(y)|i,\beta\rangle \end{aligned} \tag{24}$$

and

$$\mathcal{M}_{bb}(\beta) = -\frac{1}{2} g^2 b^\nu b^\rho \epsilon_{\mu\nu\alpha\sigma}\epsilon_{\omega\rho\delta\gamma} \int d^4x d^4y \langle f,\beta|\bar{\psi}(x)\gamma^\mu\psi(x)\bar{\psi}(y)\gamma^\omega\psi(y)\partial^\alpha A^\sigma(x)\partial^\delta A^\gamma(y)|i,\beta\rangle. \tag{25}$$

There are similar equations for matrix elements that include tilde operators.

The fermion field is written as

$$\psi(x) = \int dp N_p [c_p u(p) e^{-ipx} + d_p^\dagger v(p) e^{ipx}], \tag{26}$$

where N_p is the normalization constant; c_p and d_p are annihilation operators for electrons and positrons, respectively; and $u(p)$ and $v(p)$ are Dirac spinors. The Lorentz invariant transition amplitude becomes

$$\begin{aligned}\mathcal{M}_0(\beta) = -e^2 N \int d^4x d^4y \int d^4p (\mathsf{u}^2 - \mathsf{v}^2)^2 [\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma^\nu v(p_2) e^{-ix(p_1-p_3)} e^{iy(p_4-p_2)} \\ - \bar{u}(p_2)\gamma^\mu v(p_1)\bar{v}(p_3)\gamma^\nu u(p_4) e^{ix(p_1+p_2)} e^{-iy(p_3+p_4)}] \langle 0(\beta)|\mathbb{T} A_\mu(x) A_\nu(y)|0(\beta)\rangle,\end{aligned} \tag{27}$$

where Bogoliubov transformations Eqs. (3)–(6) are used. With $\mathsf{u}(\beta) = \cos\theta(\beta)$ and $\mathsf{v}(\beta) = \sin\theta(\beta)$, we get $(\mathsf{u}^2 - \mathsf{v}^2)^2 = \tanh^2(\frac{\beta|q_0|}{2})$, where $q_0 = \omega$. The photon propagator at finite temperature [23,32] is

$$\langle 0(\beta)|\mathbb{T} A_\mu(x) A_\nu(y)|0(\beta)\rangle = i \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \Delta_{\mu\nu}(q,\beta), \tag{28}$$

with $\Delta'_{\mu\nu}(q) \equiv \Delta_{\mu\nu}(q,\beta) = \Delta^{(0)}_{\mu\nu}(q) + \Delta^{(\beta)}_{\mu\nu}(q)$, where $\Delta^{(0)}_{\mu\nu}(q)$ and $\Delta^{(\beta)}_{\mu\nu}(q)$ are the zero and finite temperature parts, respectively. Explicitly,

$$\begin{aligned}\Delta^{(0)}_{\mu\nu}(q) &= \frac{\eta_{\mu\nu}}{q^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \Delta^{(\beta)}_{\mu\nu}(q) &= -\frac{2\pi i \delta(q^2)}{e^{\beta q_0} - 1} \begin{pmatrix} 1 & e^{\beta q_0/2} \\ e^{\beta q_0/2} & 1 \end{pmatrix} \eta_{\mu\nu}.\end{aligned} \tag{29}$$

Then,

$$\begin{aligned}\mathcal{M}_0(\beta) = -ie^2 N \int d^4x d^4y \int d^4p \Big[\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma^\nu v(p_2) e^{-ix(p_1-p_3)} e^{iy(p_4-p_2)} \mathbb{I}(q_1) \\ \times \Delta'_{\mu\nu}(q_1) \tanh^2\left(\frac{\beta|(q_1)_0|}{2}\right) \bar{u}(p_2)\gamma^\mu v(p_1)\bar{v}(p_3)\gamma^\nu u(p_4) e^{ix(p_1+p_2)} e^{-iy(p_3+p_4)} \mathbb{I}(q_2) \\ \times \Delta'_{\mu\nu}(q_2) \tanh^2\left(\frac{\beta|(q_2)_0|}{2}\right) \Big],\end{aligned} \tag{30}$$

where $\mathbb{I}(q_l) = \int \frac{d^4q_l}{(2\pi)^4} e^{-iq_l(x-y)}$ with $l = 1, 2$. Using the definition of the four-dimensional delta function,

$$\int d^4x d^4y e^{-ix(p_1-p_3+q_l)} e^{-iy(p_2-p_4-q_l)} = \delta^4(p_1 - p_3 + q_l)\delta^4(p_2 - p_4 - q_l), \tag{31}$$

and carrying out the q integral, we get

$$\begin{aligned}\mathcal{M}_0(\beta) = -ie^2 \int \frac{d^4p}{(2\pi)^4} \delta^4(p_1 + p_2 - p_3 - p_4) \Big[\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma^\nu v(p_2) \Delta'_{\mu\nu}(p_3 - p_1) \\ \times \tanh^2\left(\frac{\beta|(p_3 - p_1)_0|}{2}\right) - \bar{u}(p_2)\gamma^\mu v(p_1)\bar{v}(p_3)\gamma^\nu u(p_4) \Delta'_{\mu\nu}(p_1 + p_2) \tanh^2\left(\frac{\beta|(p_1 + p_2)_0|}{2}\right) \Big],\end{aligned} \tag{32}$$

where $N = 1$ was used. The remaining delta function expresses overall four-momentum conservation. The final transition amplitude is written as

$$\mathcal{M}_0(\beta) = -ie^2\Big[\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma_\mu v(p_2)\Delta'(p_3-p_1)\tanh^2\left(\frac{\beta|(p_3-p_1)_0|}{2}\right) - \bar{u}(p_2)\gamma^\nu v(p_1)\bar{v}(p_3)\gamma_\nu u(p_4)\Delta'(p_1+p_2)\tanh^2\left(\frac{\beta|(p_1+p_2)_0|}{2}\right)\Big], \tag{33}$$

where

$$\Delta'_{\mu\nu}(q_l) \equiv \Delta'(q_l)\eta_{\mu\nu} \tag{34}$$

with

$$\Delta'(q_l) = \frac{1}{q_l^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{2\pi i\delta(q_l^2)}{e^{\beta(q_l)_0}-1}\begin{pmatrix} 1 & e^{\beta(q_l)_0/2} \\ e^{\beta(q_l)_0/2} & 1 \end{pmatrix}. \tag{35}$$

Considering the center-of-mass frame,

$$p_1 = (E,\vec{p}), \qquad p_2 = (E,-\vec{p}),$$
$$p_3 = (E,\vec{p'}) \quad \text{and} \quad p_4 = (E,-\vec{p'}), \tag{36}$$

where $|\vec{p}|^2 = |\vec{p'}|^2 = E^2$, $\vec{p}\cdot\vec{p'} = E^2\cos\theta$, and $s = (2E)^2 = E_{CM}^2$, we get $|(p_3-p_1)_0| = |(p_1+p_2)_0| = E_{CM}$; then,

$$\mathcal{M}_0(\beta) = -ie^2[\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma_\mu v(p_2)\Delta'(p_3-p_1) - \bar{u}(p_2)\gamma^\nu v(p_1) \times \bar{v}(p_3)\gamma_\nu u(p_4)\Delta'(p_1+p_2)]\tanh^2\left(\frac{\beta E_{CM}}{2}\right). \tag{37}$$

In a similar way, the linear term in the Lorentz-violating parameter becomes

$$\mathcal{M}_b(\beta) = 2egb^\nu\epsilon_{\mu\nu\sigma\rho}[(p_3-p_1)^\sigma\bar{u}(p_1)\gamma^\rho u(p_3)\bar{v}(p_4)\gamma^\mu v(p_2)\Delta'(p_3-p_1) + (p_1+p_2)^\sigma\bar{u}(p_2)\gamma^\rho v(p_1)\bar{v}(p_3)\gamma^\mu u(p_4)\Delta'(p_1+p_2)]\tanh^2\left(\frac{\beta E_{CM}}{2}\right), \tag{38}$$

and the quadratic term in the Lorentz-violating coefficients is

$$\mathcal{M}_{bb}(\beta) = ig^2b^\nu b^\rho\eta^{\sigma\gamma}\epsilon_{\mu\nu\alpha\sigma}\epsilon_{\omega\rho\delta\gamma}[q_1^\alpha q_1^\delta\bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma^\omega v(p_2) \times \Delta'(p_3-p_1) - q_2^\alpha q_2^\delta\bar{u}(p_2)\gamma^\mu v(p_1)\bar{v}(p_3)\gamma^\omega u(p_4)\Delta'(p_1+p_2)]\tanh^2\left(\frac{\beta E_{CM}}{2}\right), \tag{39}$$

where $q_1 = p_3 - p_1$ and $q_2 = p_1 + p_2$. The results for the transition amplitudes obtained in Ref. [10] are recovered in the limit $T \to 0$, which implies $\tanh^2(\beta E_{CM}/2) \to 1$ and $(e^{\beta E_{CM}} - 1)^{-1} \to 0$.

The unpolarized cross section is obtained by calculating

$$|\mathcal{M}(\beta)|^2 = |\mathcal{M}_0(\beta) + \mathcal{M}_b(\beta) + \mathcal{M}_{bb}(\beta)|^2. \tag{40}$$

This calculation is accomplished using the completeness relations:

$$\sum u(p_1)\bar{u}(p_1) = \not{p}_1 + m,$$
$$\sum v(p_1)\bar{v}(p_1) = \not{p}_1 - m. \tag{41}$$

In addition, the relation

$$\bar{v}(p_2)\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha v(p_2) = \mathrm{tr}[\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha v(p_2)\bar{v}(p_2)] \tag{42}$$

is used. The trace calculations involve the product of up to eight gamma matrices and the Levi-Cività symbol. Henceforth, the electron mass is ignored since all the momenta are large compared to the electron mass.

Considering $b^\nu = (b_0, 0)$, a timelike 4-vector, the differential cross section, Eq. (18), at finite temperature is

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \left[\frac{e^4(\cos 2\theta + 7)^2}{256\pi^2 E_{CM}^2(\cos\theta - 1)^2} + \frac{b_0^2 e^2 g^2 \sin^2(\frac{\theta}{2})}{256\pi^2(\cos\theta - 1)^2}(-65\cos\theta + 6\cos 2\theta + \cos 3\theta + 122)\right]\tanh^4\left(\frac{\beta E_{CM}}{2}\right) + \frac{1}{256\pi^2 E_{CM}^2}[\Pi_1(\beta) + b_0^2\Pi_2(\beta)] \times \tanh^4\left(\frac{\beta E_{CM}}{2}\right), \tag{43}$$

where

$$\Pi_1(\beta) = \frac{64\pi^2 e^4 E^4}{(e^{\beta E_{CM}} - 1)^2}[(11 + 4\cos\theta + \cos 2\theta)\delta^2(-2E^2(1 - \cos\theta)) + (6 + 2\cos 2\theta)\delta^2(4E^2) + 16\cos^4(\theta/2)\delta(-2E^2(1-\cos\theta))\delta(4E^2)],$$
$$\Pi_2(\beta) = \frac{256\pi^2 e^2 g^2 E^6 \sin^2(\theta/2)}{(e^{\beta E_{CM}} - 1)^2}[(\cos 2\theta + 32\sin^2(\theta/2) + 15)\delta^2(-2E^2(1-\cos\theta)) + 8\cos^4(\theta/2)\delta(-2E^2(1-\cos\theta))\delta(4E^2). \tag{44}$$

The corrections due to the Lorentz violation parameter, b_0, are small. Corrections in higher order in b_0 are ignored. The propagator at finite temperature introduces the product of delta functions with identical arguments [33–36]. This problem is avoided by working with the regularized form of delta functions and their derivatives [37]:

$$2\pi i\delta^n(x) = \left(-\frac{1}{x + i\epsilon}\right)^{n+1} - \left(-\frac{1}{x - i\epsilon}\right)^{n+1}. \tag{45}$$

This result shows that corrections for electron-positron scattering due to Lorentz violation are altered at finite temperature. In addition, at very high temperature, $\beta \to 0$, $e^{\beta E_{CM}} - 1 \to 0$; then, the temperature effect is very large.

At zero temperature, $\Pi_1(\beta), \Pi_2(\beta)$ go to zero, and $\tanh^4(\frac{\beta E_{CM}}{2}) \to 1$; then, the cross section becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_{T=0} = \frac{e^4(\cos 2\theta + 7)^2}{256\pi^2 E_{CM}^2(\cos\theta - 1)^2} + \frac{b_0^2 e^2 g^2 \sin^2(\frac{\theta}{2})(-65\cos\theta + 6\cos 2\theta + \cos 3\theta + 122)}{256\pi^2(\cos\theta - 1)^2}. \tag{46}$$

The same result was obtained in Ref. [10].

IV. CONCLUSION

Small violations of Lorentz and CPT symmetries emerge from theories that try to unify SM and general relativity. The SME provides a general theoretical description of violations of Lorentz and CPT invariance. An alternative procedure is to modify just the interaction part using a nonminimal coupling. In this paper, one, among several, nonminimal coupling is used. Then, the effect of Lorentz violation and finite temperature on Bhabha scattering are investigated. Finite temperature effects are introduced using the TFD formalism. This gives us a good estimate of the importance of the Lorentz-violating operators at finite temperature on the cross section. From our results, we expect that these operators play a minor role for processes in the interior of stars. However, if all the minimal and nonminimal Lorentz-violating operators are considered, relevant effects may arise for processes at very high energies, like astrophysical processes. Finally, it is important to conclude that the present results give us a reasonable estimate of the Lorentz-violating operators in the SME at high temperatures on the Bhabha scattering.

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