Quadrupole stellar oscillations: The impact of gravitational waves from the Galactic Center

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Acoustic quadrupole modes of sunlike stars vibrate when perturbed by a passing gravitational wave generated somewhere in the Universe. Here, we compute the imprint of the gravitational waves on the acoustic spectrum of these stars for gravitational events occurring near the supermassive black hole located at the center of the Milky Way. We found that in most cases the impact of gravitational waves in low-order quadrupole modes is not above the current observational threshold of detectability, although this should be in the reach of the next generation of near infrared observatories and asteroseismology satellite missions. Equally, we found that it is possible to follow the end phase of the coalescence of binaries with large chirp masses, as these phenomena have a unique imprint in the spectra of sunlike stars affecting sequentially several low-order quadrupole modes. Moreover, we discuss the different imprints on the acoustic spectra of the different types of binary systems constituted either by two white dwarfs, two neutron stars, two black holes or a compact star and a massive black hole.

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I. INTRODUCTION

The Milky Way harbors a supermassive black hole at its core. Its location coincides with a well-known compact radio source, Sagittarius A* [1]. With a diameter of 0.3 AU this source is surrounded by one of the most dense stellar populations of the Galaxy. The most compelling proof of the existence of a black hole in Sagittarius A* is the continuous observation of the nearest stars orbiting in a very fast Keplerian motion, which is only possible with a very concentrated massive object located in the Galactic Center [2]. For example, a star of 15 $M_{\odot}$, known as S2, spins around the central object in an elliptical orbit with a period of 15.2 yr and a pericenter of 120 A.U. (smallest distance from the central object). From the motion of S2, Ghez *et al.* [3] made the first estimation of the mass of the central black hole. These authors have determined this mass to be $4.110^6\ M_{\odot}$. More recently, from the measurement of the proper motions of several thousand stars within one parsec from the central black hole, Schödel *et al.* [4] have estimated simultaneously the black hole's mass at $3.6^{+0.2}_{-0.4}\,10^6\ M_{\odot}$ and an additional distributed mass of $1.0 \pm 0.5 \times 10^6\ M_{\odot}$. The latter mass term is due to a local population with a few tens of million of stars. The population consists of metal-rich, M, K, and G old giant stars, main-sequence B stars [5] and compact stars (or stellar remnants).

Equally, like the Milky Way, many other massive galaxies are known to have a core made of a supermassive black hole surrounded by a concentrated and dense stellar environment. Throughout the lifetime of a galaxy, the central black hole grows by capturing many of the neighboring stars and clouds of molecular gas. A complex network of gravitational interactions between stars occurs continuously in the galactic core. During these stellar encounters, a countless number of binaries form between neighboring pairs of white dwarfs (WD-WD), neutron stars (NS-NS), and stellar black holes (BH-BH). The recent discovery of gravitational radiation from the merger of two stellar black-hole binaries, ensures that the last type of binaries must be quite common in the Galaxy [6,7]. Another type of binary that is created with regularity corresponds to the ones that are formed between the supermassive black hole and nearby compact stars. In binaries like these for which the lighter star is 4 orders of magnitude less massive than the companion, the associated gravitational event is classified as an extreme mass ratio inspiral (EMRI). As described above, stellar (compact) binaries and EMRIs should form in large numbers in the core of the Milky Way.

Nonradial oscillations have been discovered in many stars in the Milky Way by the COROT [8] mission and Kepler's main and extended missions [9,10]. More than 18 000 main-sequence subgiants and red giant stars have been shown to have oscillation spectra with identical properties. The combined spectra of these stars spans the frequency range from 10^{-7} to 10^{-2} Hz [11,12]. The excitation and damping of these oscillations is attributed to the turbulent motions of convection in the external layers of these stars. These physical processes that excite stellar oscillations are identical to the ones found for the Sun. For that reason such oscillations are known as sunlike oscillations and the stars as sunlike stars. These stars are found in many regions of the Milky Way. The Kepler mission alone has measured

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oscillations in stars located at distances up to 15 kiloparsecs. In many cases the sunlike stars belong to stellar clusters found in the direction of the Galactic Center, several of which are located above the galactic disk or in the bulge. Furthermore, by making high precision observations in many new directions of the Galaxy, the future PLATO satellite [13] should be able to increase significantly the quantity and quality of sunlike stars discovered in the Milky Way.

This work focuses on the study of the impact of gravitational waves emitted by compact binaries (with special focus on EMRIs events) in the spectra of sunlike stars. The motivation for the project comes from the fact that the core of the Milky Way is densely populated by compact stars, stellar binaries, and possibly multistellar systems, all of which are being attracted by the supermassive black hole. The gravitational radiation emitted by these binaries will excite some of the oscillations of the sunlike stars located at relatively short distances from the Galactic Center.

Specifically, we are interested in studying the impact of the gravitational waves emitted during the binary contraction, a phase known as the *inspiral phase*. For our convenience this phase is split into the following two stages: (1) the inspiral phase begins when the two stars are far-apart rotating in near-perfect circular orbits (*monochromatic emission*) and (2) finishes at the end of the orbits contraction (*chirp emission*), just before the merger of the two stars. There is a post-merger emission known as the ring-down phase, where the shape distortion of the nearly formed object is also dissipated as gravitational radiation, but this stage is not discussed in this work.

All the different types of binary systems in the inspiral phase emit gravitational waves with a characteristic waveform known as "chirp" whose amplitude and frequency increases with time until the coalescence. Even though the observation of strongest events are expected to be more sporadic, the detection of such phenomena would produce very unique and interesting results that can be used to test General Relativity. For instance, in the case of EMRIs, these waves can be used to probe the gravitational field of the central black hole. In addition, as these waves travel large distances through space, equally these can be used to test the wave properties of gravitational radiation. Theoretical models predict gravitational chirps with strains on Earth of the order of 10^{-17}–10^{-22}, and frequencies varying from 10^{-4} to 10^{-1} Hz. Most of these events are expected to be detected by experiments like eLISA [14–16]. Nevertheless, the strain of such gravitational events will be much larger for stars located nearby these binaries.

Oscillating stars are natural receptors of gravitational waves. Like any resonant sphere, they have an isotropic sensitivity to gravitational radiation—able to absorb gravitational waves from any direction of the sky. For a given frequency of acoustic oscillations, stars have sensitivity much larger than current resonant mass detectors, as their integrated scattering cross section and Q quality factor for gravitational waves is 20 and 2 orders of magnitude larger than the usual resonant mass detectors [17–20], respectively. For that reason in the rest of this article we will refer to these as star detectors.

The first studies of the absorption of gravitational waves by astrophysical objects like Earth, Moon, planets, and stars were developed by Dyson [21] Khosroshahi and Sobouti [22], Zimmerman and Hellings [23], among others. Boughn and Kuhn [24] were the first to use helioseismology to constrain the amplitude of gravitational waves. Recently Siegel and Roth [25] and Lopes and Silk [19,26] have updated these calculations. By using the unseen solar gravity modes, Siegel and Roth [27] determined the maximum amplitude of the strain for the stochastic gravitational wave background [28,29]. By focusing on the study of the Sun as a natural gravitational wave detector Lopes and Silk [19,26] have shown the potential of asteroseismology for gravitational wave searches. McKernan *et al.* [30] were the first to propose that stars near massive black-hole binaries could be efficient resonant GW detectors. Moreover, these authors estimated that the gravitational radiation is absorbed by stars and have shown how such spectra could be observed by second generation space-born gravitational wave detectors.

In this article, we study the impact that the gravitational wave emissions coming from binaries located in the core of Milky Way have on the low-order quadrupole acoustic modes of nearby sunlike stars. We found that for the more massive binaries (including EMRIs), it will be possible to follow the end of the inspiral phase, during which a large emission of gravitational radiation is expected in the last stage of the binary contraction (chirp emission), before the coalescence of the two black holes. We show for the first time that the chirp waveform of the gravitational wave has a unique impact on the acoustic spectrum of the sunlike stars by ringing up one quadruple mode after another. Some gravitational chirp events could excite several modes in the same star. Although these gravitational emissions have relatively large strains, their impact on acoustic modes of nearby sunlike stars leads to relatively small amplitude variations, which we can expect to be well within reach of the next generation of near infrared observatories.

In this study the focus is on the impact of gravitational radiation of low-order quadrupole modes, since recent developments in analyzing asteroseismology data and in our understanding of the theory of stellar pulsations have been quite successful in predicting the properties of acoustic modes. Nevertheless, there is a large research potential in studying the impact of gravitational waves in quadrupole gravity modes and mixed modes of sunlike stars in the main sequence or in the red giant branch. This is particularly so as there is a significant amount of data available from recent asteroseismology surveys. In particular, the impact of gravitational waves in gravity modes, more precisely in the Sun, was recently computed by Siegel and Roth [25]. They found the velocity amplitude to be of the order of 10^{-5}–10^{-3} mm s^{-1} for the Sun, but has the potential to be more significant for other stars. Moreover, the same authors have shown that gravity modes can be used as an independent method to put an upper bound to the stochastic background of gravitational radiation [27].

In Sec. II, we discuss a basic description of the formation and evolution of a binary system and the generation of gravitational waves. In Sec. III we compute the imprint of a gravitational wave chirp in the stellar acoustic spectrum. In the last two sections we present a discussion and conclusion about the impact and relevance of these results.

II. THE GRAVITATION WAVE CHIRP WAVEFORM

Binaries of compact stellar objects are the most studied sources of gravitational radiation. Among other binaries these apply to massive black holes, EMRIs, stellar black holes, neutron stars, and white dwarfs. We start by assuming that the two compact stars of the binary system are in a circular orbit. The binary is considered to be sufficiently faraway from the star detector, such that the incoming gravitational radiation is described by a plane wave field, but near enough to ignore the redshift corrections to the frequency spectrum. The gravitational waves emitted in the end of the inspiral phase have a chirp waveform. In this preliminary study, we will compute gravitational radiation as due to the leading quadrupole which results from the use of a multipole expansion, which also means that high-order terms will be of much smaller amplitude. This is valid since $v/c \ll 1$. Hence, the binary system is held together by gravitational forces as per the virial theorem $(v/c)^2 \sim (R_s/d)$ where R_s is the Schwarzschild radius and d a typical length of the self-gravitating system, which leads to $R_s/d \ll 1$ for all the binaries systems in our study. For instance, for a binary like the one formed by the supermassive black hole and the S2 in the center of the Milky Way, $(R_s/d) \sim 0.0810 A.U/12 A.U. \sim 0.0001$. Nevertheless, more precise predictions of the waveform for gravitational radiation during the chirp phase can be found in the literature, such as in [31]. Unlike monochromatic waves, this waveform changes with time. The frequency and amplitude increase as follows:

(i) As the gravitational frequency f is 2 times the orbital frequency, from Kepler's law the results are that f increases with the reduction of the orbital radius of the binary system. The smaller star slowly enters in an adiabatic inspiral process by going through a succession of quasicircular orbits during which it loses energy by gravitational radiation. Consequently, f increases as the time to coalescence τ decreases $f(\tau)/f_c = 0.0728(f_c\tau)^{-3/8}$ where f_c is a characteristic frequency of the binary system. f_c is equal to $[c^3/(GM_c)]$ where c and G are the speed of light and Newton constant, and M_c is the chirp mass of the binary system $M_c \equiv (m_1 m_2)^{3/5}/M_t^{1/5}$ where m_1 and m_2 are the masses of the two stars and $M_t = m_1 + m_2$.

The inspiral phase ends when the radial distance between the two stars is shorter than the last stable circular orbit, also known as the innermost stable circular orbit (ISCO). When this orbit is passed, the two stars merge and coalesce. The ISCO frequency f_{ISCO} is approximately 2.2 kHz$(M_\odot/M_t)$ where $M_\odot$ is the solar mass. Hence, the gravitational wave with the largest frequency emitted by the binary system $f_{\max}$ at coalescence ($\tau = 0$) is equal to twice the $f_{\rm ISCO}$ [32]. The precoalescence phase will be observed in the spectra of the star detectors if the $f_{\max}$ of the binary has a value within the frequency interval of 10^{-7} to 10^{-2} Hz [19], or M_t has a value between 4.4×10^5 and $4.4 \times 10^{10}\ M_\odot$.

(ii) The strain h increases as the binary system approaches the coalescence. The two polarized components of the h [32], h_+ and $h_\times$ are written in a condensed form, for the time interval corresponding to the orbital changes of the stars in the binary system, from the start of the inspiral phase until the coalescence ($-\infty < \tau < 0$):

$$h_k(t) = h_\star \left(\frac{5}{f_c\tau}\right)^{1/4} g_k(\varphi) \mathrm{C}_k[\Phi(\tau)], \tag{1}$$

where k is one of the two possible polarizations $+$ or $\times$, and $h_\star$ is the strain amplitude equal to $c/(d_\star f_c)$, where $d_\star$ is the distance of the binary system to the star detector. g_k and C_k are geometrical and circular functions. The first is related with the direction of the gravitational wave source, and the latter takes into account the stretching of the gravitational wave as the binary approaches the coalescence. The g_k functions are $g_+ \equiv (1 + \cos^2\varphi)/2$ and $g_\times \equiv \cos\varphi$ where φ is a directional angle. The C_k functions are $C_+ \equiv \cos[\Phi(\tau)]$ and $C_\times \equiv \sin[\Phi(\tau)]$. These last functions are dependent of the phase $\Phi(\tau)$, which is equal to $\Phi_o - 2(f_c\tau/5)^{5/8}$ where Φ_o is the value of the phase at coalescence.

The power spectrum of each of the $h_k(t)$ components, during the inspiral phase ($f \le f_{\max}$), is given by

$$P_k(f) = \bar{h}_\star^2 g_k^2(\varphi) \left(\frac{f}{f_c}\right)^{-14/6}, \tag{2}$$

where $\bar{h}_\star = A_s h_\star \tau_c$ with $\tau_c = f_c^{-1}$ and $A_s = 0.2128$. τ_c gives the time scale of the gravitational wave (GW) event, like for a burst or a Gaussian waveform; $P_k(f)$ is proportional to $h_\star^2 \tau_c^2$ [32]. The power spectrum $P_k(f)$ is equal to the square of the Fourier transform of $h_k(t)$ [Eq. (1)], such that $\tilde{h}_k(f) = \bar{h}_\star g_k(\varphi) e^{i\Psi_k(f)} (f/f_c)^{-7/6}$ where the phase $\Psi_k(f)$ is either $\Psi_+(f) = 2\pi f(t_c + d_\star/c) - \Phi_o - \pi/4 + 3/4(8\pi f/f_c)^{-5/3}$ or $\Psi_\times(f) = \Psi_+(f) + \pi/2$, e.g., [32].

Figure 1 shows a gravitational wave with a positive polarization. For large values of $f_c\tau$ the gravitational wave is monochromatic and for small values of $f_c\tau$ the

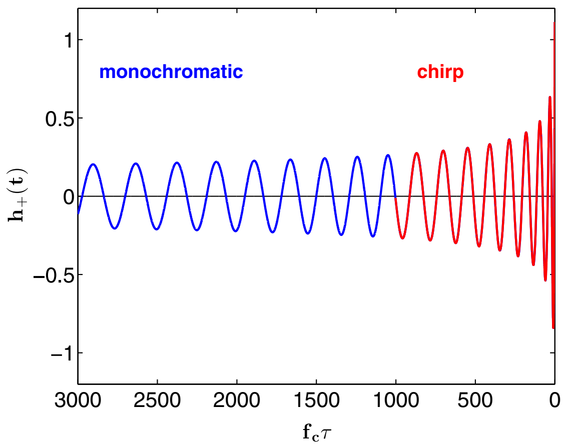

FIG. 1. Strain of the gravitational wave of an inspiral stellar binary system: $h_+(t)$ (with $h_\star = 1$ and $g_k = 1$) as a function of the dimensionless natural unity $f_c\tau$. This illustrates the two basic gravitational waveforms (or gravitational wave phases) of a inspiral binary system. In the case that the time to coalescence is such that $f_c\tau \geq 1000$ the wave is monochromatic. Reversely for $f_c\tau \leq 1000$ the gravitational wave is a chirp.

TABLE I. Inspiral Binary Systems

Binary	$m_1 m_2$ ($M_\odot$)	M_c ($M_\odot$)	f_c (Hz)	$h_\star$[a] ...	$f_{\max}$ (Hz)
Low mass					
WD-WD	0.71–0.13	0.25	8×10^5	1×10^{-17}	5×10^3
NS-NS	1.4–1.4	1.22	2×10^5	6×10^{-17}	2×10^3
BH-BH	5.0–5.0	4.35	5×10^4	2×10^{-16}	4×10^2
BH-BH	14.2–7.5	8.9	2×10^4	4×10^{-16}	200
BH-BH	36.0–29.0	28.1	7×10^3	1×10^{-15}	67
High mass					
EMRI	4×10^6–15	2×10^3	90	1×10^{-13}	1×10^{-3}
EMRI	-10^2	7×10^3	29	3×10^{-13}	1×10^{-3}
EMRI	-10^3	3×10^4	7	1×10^{-12}	1×10^{-3}
BH-BH	4×10^6–10^4	1×10^5	2	5×10^{-12}	1×10^{-3}
BH-BH	10^5–10^5	9×10^4	2.3	4×10^{-12}	2×10^{-2}
BH-BH	5×10^6–5×10^5	4×10^5	0.46	4×10^{-11}	4×10^{-3}
BH-BH	10^6–10^6	9×10^5	0.23	2×10^{-11}	2×10^{-3}

[a]The strain of a gravitational wave at a distance of 1 kiloparsec from the binary.

gravitational wave is a chirp. Equally, Fig. 2 shows the power spectrum $P_+(f)$ and $\tilde{h}_+(f)$ of the same gravitational wave. These results are easily generalized to $h(t)$ waveforms with both polarizations, e.g., [33]. We notice that all gravitational waves produced by the different binary types have this waveform, the difference between them being uniquely related with the value of the characteristic frequency f_c or their chirp mass.

Table I lists the main characteristics of some typical inspiral binary systems. In the table are included the two recently discovered stellar black-hole binaries [6,7]. Equally, we include EMRI binaries formed between the supermassive black hole of the Milky Way and several fiducial (stellar) black holes. Notice that as the chirp and total mass of the binary system increase, f_c and $f_{\max}$ decrease. The chirp waveform is more pronounced for $f \leq 0.005 f_c$ (cf. Fig. 2). In particular, for some binaries the duration of the chirp phase ($\tau f_c \leq 1000$, cf. Fig. 1) varies between a few seconds to several minutes before coalescence, i.e., $\tau_\star$ is of the order of a few seconds to several minutes. This chirp phase occurs in a time scale (or the equivalent frequency scale) that could be detected by sunlike stars, for which the spectral window of stellar oscillations varies from 10^{-7} to 10^{-2} Hz. Therefore, binary systems that have chirp phases with a duration from a few seconds to several minutes, in principle, could affect the oscillations of nearby sunlike stars during this critical phase of their evolution. Binaries of massive black holes should be the first candidates to consider, although these are unlikely to be found in the core of the Milky Way due to their very high masses (cf. Table I). Nevertheless, EMRIs binaries that equally emit gravitational radiation in the same spectral window are much more likely to be found in the core of the Milky Way.

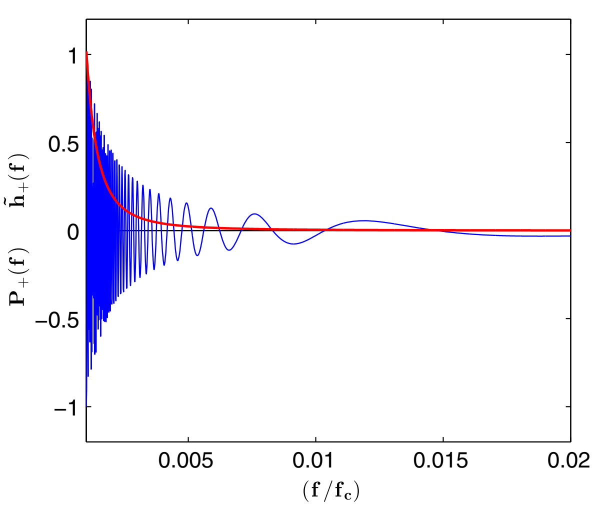

FIG. 2. Inspiral stellar binary system: $P_+(f)$ is the power spectrum of the strain in the chirp phase (red curve) and $\tilde{h}_+(f)$ is the real part of the Fourier transform of $h_+(t)$ (blue curve), as a function of the dimensionless natural quantity f/f_c. Both functions are normalized to their maximum value. Figure 1 shows the related $h_+(t)$.

During the chirp phase several quadrupole modes in the same star will be excited by the passing gravitational wave. Conversely, in the case of monochromatic gravitational waves, at best a single stellar quadrupole mode will be excited (cf. Figure 1). It is worth highlighting that as the inspiral binary approaches the coalescence, $h(t)$ is less accurate and the determination of the exact waveform should take into account the finite structure of the two compact objects in coalescence. Nevertheless the general properties of the gravitational waveform remain.

III. IMPRINT OF GRAVITATIONAL WAVES ON THE ACOUSTIC SPECTRUM

Main-sequence stars are in hydrostatic equilibrium, their transport of energy and their mechanical vibrations occur in quite distinct time scales, the Kelvin-Helmholtz and free-fall time scales. In the Sun these characteristic times are of the order of 30 million years and 40 minutes, respectively. For that reason, the stellar vibrations are represented as a combination of nonradial adiabatic oscillation modes of low amplitude, e.g., [34]. Any perturbed quantity of a mode, like the displacement $\xi_n(\mathbf{r},t)$ is equal to $A(t)\xi_n(\mathbf{r})e^{-i\omega_{\rm n}t-\eta_{\rm n}t}$, where $\omega_{\rm n}$ and $\eta_{\rm n}$ are the frequency and the damping rate, $A(t)$ is the instantaneous amplitude, and $\xi_n(\mathbf{r})$ is the spatial eigenfunction, e.g., [35–38]. The instantaneous amplitude is a solution of

$$\frac{d^2A}{dt^2}+2\eta_n\frac{dA}{dt}+\omega_n^2A=\mathcal{S}_{\rm gw}(t), \tag{3}$$

where $\mathcal{S}_{\rm gw}(t)$ is the source of gravitational radiation, e.g., [39]. Following from the specific properties of gravitational systems as demonstrated in general relativity, a gravitational perturbation only affects modes with degrees equal to or larger than 2. For convenience, we opt to study the leading order of the gravitational perturbation on the quadrupole acoustic modes, see [26] for details. In this study, the mode p_n ($n=0,1,\cdots$) refers to f- and acoustic (p-)quadrupole modes of order n and undefined azimuthal order m, e.g., [32]. $\mathcal{S}_{\rm gw}(t)$ is equal to the $L_n\ddot{h}_m(t)$ where L_n is the modal length of the quadrupole mode and $\ddot{h}_m(t)$ is the m-spherical component ($m\le 2$) of perturbation of the spatial component of the Minkowski metric, e.g., [39]. In this study, we assume that the source of gravitational radiation is at a sufficiently small distance from the star detector, such that we can neglect all redshift corrections. This is justified since all the gravitational sources are located in our own Galaxy. L_n is equal to $1/2R|\chi_n|$ where R is the stellar radius and χ_n reads

$$\chi_n=\frac{3}{4\pi\bar{\rho}_\star}\int_0^1\rho(r)[\xi_{r,n2}(r)+3\xi_{h,n2}(r)]r^3dr, \tag{4}$$

where ρ and $\bar{\rho}_\star$ are the density inside the star and its averaged value. This definition of χ_n is identical to the one used for resonant-mass detectors [26]. In the particular case that $\bar{\rho}_\star$ is constant, Eq. (3) becomes equivalent to the one found for a spherical resonant-mass detector, e.g., [32]. We noticed that there are several χ_n definitions, e.g., [24,25] differing between them only by the normalization condition.

In the computation of the stellar acoustic modes and χ_n, we use an up-to-date solar model, e.g., [40]. The observational frequencies and damping rates used were from Bertello *et al.* [41], Garcia *et al.* [42], Jimenez and Garcia [43], Turck-Chieze *et al.* [44], Baudin *et al.* [45] and Chaplin *et al.* [46]. The theoretical damping rates used were from Belkacem *et al.* [47], Grigahcène *et al.* [48], and Houdek *et al.* [49]. Table II shows the frequencies for the quadrupole acoustic modes of a main-sequence star that we chose to be identical to the Sun. The frequency in Hz in the table corresponds to $\omega_n/2\pi$. A detailed discussion about the properties of low-order acoustic modes can be found in Turck-Chieze and Lopes [50].

The averaged power spectrum P_A ($=\langle|\tilde{A}^2|\rangle$) of an acoustic mode stimulated by a gravitational wave [see Eq. (2)] is computed by taking the Fourier transform of Eq. (3) and neglecting transients terms arising from the initial conditions on A. P_A of an quadrupole mode reads

$$P_{\rm A}(\omega)=\frac{L_n^2\bar{h}_\star^2\omega_c^4g_k^2(\varphi)}{(\omega^2-\omega_n^2)^2+4\eta_n^2\omega^2}\left(\frac{\omega}{\omega_c}\right)^{5/3}, \tag{5}$$

where $\omega_c=2\pi f_c$.

The square of photospheric velocity $V^2(\omega)$ is equal to $(2\pi\tau_{\rm w})^{-1}\int_0^{+\infty}P_{\rm V}(\omega)d\omega$, where $\tau_{\rm w}$ is the duration of the gravitational wave impact on the star's quadrupole mode. $P_{\rm V}$ ($\equiv\omega^2P_{\rm A}$) is the power spectrum of the square of photospheric velocity [45]. Hence the impact of a *chirp gravitational wave emission* [as defined by Eqs. (1) and (2)] with a frequency ω that resonates with the frequency of the stellar mode ω_n reads

TABLE II. Quadrupole Acoustic Modes of One Solar Mass.

Mode	Frequency[a] (Hz)	L_n (cm)	Q_n (no-dim)
	$\times10^{-6}$	$\times10^{7}$	$\times10^{+8}$
f	347	2.347	38
p_1	382	3.841	4.1
p_2	514	0.737	1.0
p_3	664	0.219	0.28
p_4	811	0.074	0.10
	$\times10^{-6}$	$\times10^{5}$	$\times10^{6}$
p_5	959	2.867	3.8
p_6	1104	1.211	1.7
p_7	1249	0.524	0.74
p_8	1394	0.238	0.33
p_9	1535	0.108	0.12
	$\times10^{-6}$	$\times10^{4}$	$\times10^{4}$
p_{10}	1674	1.082	6.7
p_{11}	1810	0.520	4.1
p_{12}	1945	0.272	2.9
p_{13}	2082	0.153	2.1
p_{14}	2217	0.054	1.9
p_{15}	2352	0.033	1.8
p_{16}	2485	0.022	1.7
p_{17}	2619	0.014	1.5
p_{18}	2754.	0.010	1.5

[a]Frequency table of solar acoustic modes obtained from a compilation made by Turck-Chieze and Lopes [50]. The frequencies in italic correspond to theoretical predictions for the current solar model as in reference Lopes and Turck-Chieze [51].

$$V_n^2 = \frac{h_\star^2 L_n^2 \omega_n^4}{\alpha_s^2 \eta_n^2} \mathcal{C}_n, \qquad (6)$$

where V_n^2 is equal to $V^2(\omega_n)$, $\mathcal{C}_n$ is a chirp factor given by $(\tau_c^2 \Delta\omega_n/\tau_{\rm w})(\omega_n/\omega_c)^{-7/3}$, and $\alpha_{\rm s}$ $(\equiv (2\sqrt{2\pi})/(A_s g_k(\varphi)\gamma_s))$ is a multiplicative factor related to stellar observations. $\alpha_{\rm s}$ (with $g_k = 1$) is equal to $24/\gamma_s$ where γ_s is a unity photospheric numerical factor. As the contribution for the $V^2(\omega)$ integral is only significant near each ω_n, in computing the previous equation, we approximate $\omega^{11/3}((\omega^2-\omega_n^2)^2 + 4\eta_n^2\omega^2)^{-1}$ in $P_{\rm V}(\omega)$ by its first term of the Taylor series $\omega_n^{5/3}/4\eta_n^2$, and the integral limits around each ω_n by $\omega_n - \Delta\omega_n/2$ and $\omega_n + \Delta\omega_n/2$ where $\Delta\omega_n$ is the equivalent linewidth. We notice that $V^2(\omega_n)$ is proportional to η_n^{-2}: Eqs. (5) and (6) are related with $V^2(\omega_n) = \omega_n^2(\Delta\omega_n/\tau_{\rm w})/2\pi P_A(\omega_n)$. Thus, if the linewidth $\Delta\omega_n$ relates with the damping time as $\Delta\omega_n \sim \tau_\eta^{-1}$, and $\tau_{\rm w} \sim \tau_c$ then $(\tau_c^2\Delta\omega_n/\tau_{\rm w})$ in $\mathcal{C}_n$ simplifies to τ_c/τ_η.

In the particular case that the excitation of the stellar mode is due to monochromatic gravitational wave emission, it is reasonable to assume that $\omega_n \sim \omega_c$ and $\tau_n \sim \tau_c$ for which $\mathcal{C}_n = 1$. In these circumstances, Eq. (6) simplifies to $V_n = h_\star L_n \omega_n^2/(\alpha_s \eta_n)$. Equally V_n can be expressed as a function of the quality factor of the mode $Q_n \equiv \omega_n/(2\eta_n)$ as $V_n = 2h_\star L_n Q_n \omega_n/\alpha_s$. The values of Q_n are shown in Table II. This result was computed by Lopes and Silk [26], and an identical expression was previously obtained by Siegel and Roth [25] for acoustic and gravity quadrupole modes in the Sun. Moreover, this result shows that the photospheric velocity of modes excited by a gravitational chirp waveform [Eq. (6) with $\mathcal{C}_n \neq 1$] differs only by the simple factor from the excitation due to a monochromatic gravitational wave.

In both gravitational emission cases the excitation of a quadrupole mode occurs when the frequency of the gravitational wave ω matches the frequency of the stellar mode ω_n. Nevertheless, this only occurs for a very short period of time; hence, the exact calculation of the impact of the gravitational waves must take into account the time drift of ω in relation to ω_n. In the case that the time drift is small, ω is approximated by $\omega_n + \dot{\omega}\tau$, where $\dot{\omega}$ is the frequency time variation of the gravitational wave and τ is the time difference to the resonance [19].

The impact of the gravitational wave on an oscillation mode is maximum when the gravitational time drift $\tau_{\rm gw} \equiv 1/\sqrt{\dot{\omega}}$ is larger than the damping time of the mode $\tau_n = 1/(2\eta_n)$; i.e., the ratio $\mathcal{T}_n = \tau_{\rm gw}/\tau_n$ is larger than one. Figure 3 compares these two characteristic times. In this study $\tau_{\rm gw}$ is $0.577(GM_c/c^2)^{-5/6}\omega^{-11/6}$, e.g., [19,32]. Accordingly, a stellar mode for which $\mathcal{T}_n \gg 1$, the excitation is known as the steady-state solution or a *saturated mode of oscillation*. In the calculation of this photospheric velocity $V_{n,s}$, the frequency drift $\dot{\omega}$ is neglected, as such $V_{n,s} = V_n$. Reversely, for the case that $\mathcal{T}_n \ll 1$, the excitation is known as an *undamped mode of oscillation*; the photospheric velocity $V_{n,u}$ is such that the contribution $\dot{\omega}$ is taken into account in the calculation of $V_{n,u}$, following Lopes and Silk [19] and McKernan *et al.* [30] $V_{n,u} = \mathcal{T}_n^{1/2} V_n$. Figure 4 shows the $V_{n,u}$ of quadrupole acoustic

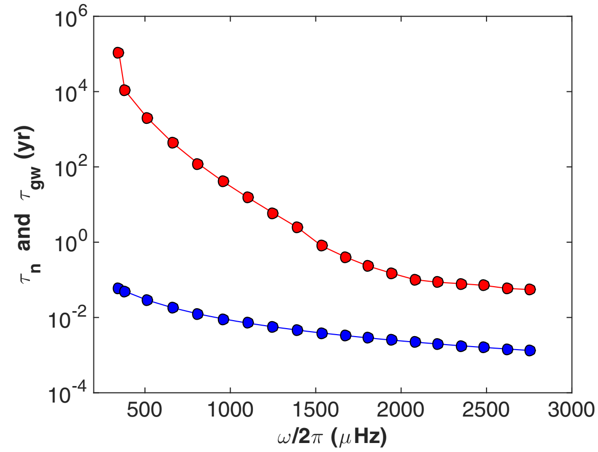

FIG. 3. Comparison between the dumping time of the quadrupole acoustic modes τ_n (red circle) and the characteristic time drift time of the gravitational waves $\tau_{\rm gw}$ (blue circle). The gravitational radiation is assumed to be due to the occurrence of an EMRI (with $m_1 = 4 \times 10^6\ M_\odot$ and $m_2 = 15\ M_\odot$) at the center of the Galaxy (see Table I).

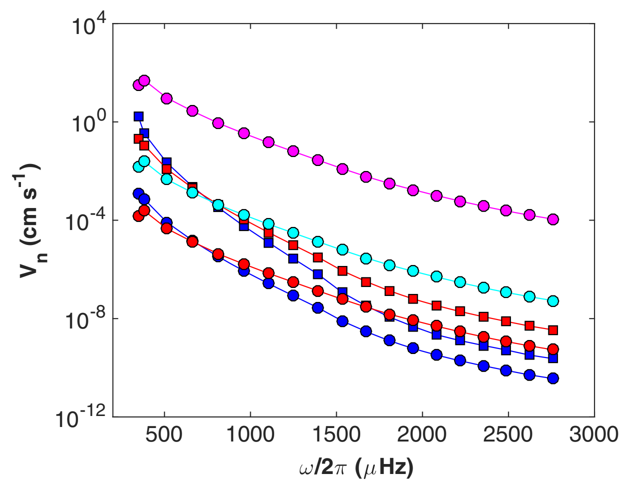

FIG. 4. Photospheric velocity amplitude of the low-order acoustic quadrupole modes $(n = 0, 1, \cdots)$ of a star detector located at a distance of 1 kiloparsec from the Galactic Center for two types of gravitational radiation emission: (a) monochromatic wave, $V_{n,s}$ (blue square) and $V_{n,u}$ (blue circle) as given by Eq. (6) with $\mathcal{C}_n = 1$. (b) chirp wave, $V_{n,s}$ (red square) and $V_{n,u}$ (red circle) as given by Eq. (6) with $\mathcal{C}_n \neq 1$. (c) $V_{n,u}$ is the same used in case (b), but now the star detector is located at a 10 parsec (cyan circle) or 1000 A.U. (magenta circle). The gravitational radiation is assumed to be due to the occurrence of an EMRI (with $m_1 = 4 \times 10^6\ M_\odot$ and $m_2 = 15\ M_\odot$) at the center of the Galaxy (see Table I).

modes excited by a chirp gravitational wave emission and a monochromatic gravitational wave emission. In this case all stellar modes have a $\mathcal{T}_n$ that varies from 10^{-6} to 10^{-2}; therefore, all the modes are in an undamped mode of oscillation since $\mathcal{T}_n \ll 1$. $V_{n,s}$ is also shown in the same figure. In the calculation of $V_{n,s}$ and $V_{n,u}$ we used Eq. (6) for which the gravitational wave radiation was estimated from Eq. (2). The power spectrum of the incoming gravitational radiation is given by Eq. (1).

IV. DISCUSSION

The most important factor affecting the amplitude of V_n is the distance of the star detector to the compact binary. An EMRI merger corresponding to the capture of the S2 star by the supermassive black hole in the Galactic Center will produce a gravitational event with a strain amplitude of $\sim 10^{-13}$ at a distance of 1000 parsecs. This result is obtained from the relation $h_\star = c/(d_\star f_c)$ (see previous section) where $f_c = 90$ Hz (see Table I). In particular, for the case of a binary of black holes that has masses identical to the ones found for the first time by the LIGO Collaboration [6], the strain amplitude at the same distance is $\sim 10^{-15}$ (see Table I). This corresponds to a strain of 3×10^{-21} at a distance of 410 Mpc, which is close to the 10^{-21} strain amplitude measured by the LIGO experiment on Earth.

Figure 4 shows the $V_{n,u}$ for this EMRI event, where the sunlike star is located at a distance of 1000 parsec, 10 parsecs and 1000 A.U. from the Galactic Center. The last distance, although unlikely, gives us an order of magnitude of the phenomena. For illustrative purposes, other quantities are also shown in the same figure. The Sun is the fiducial star detector in this analysis. In the computation of the photospheric velocity, the values of ω_n and η_n correspond to the observed solar frequencies and the theoretical predictions of damping rates for the Sun by Belkacem *et al.* [47] and Houdek *et al.* [49]. The L_n varies from 10^7–10^4 cm as computed by Lopes and Silk [19] for these modes. α has the numerical value 24.0 (with $\gamma_s \sim 1$).

These results can be understood qualitatively. If we neglect numerical factors of the order of unity, the estimation of $V_{n,u}$ is made as follows: for a monochromatic gravitation wave emission [from the expression given by Eq. (6) with $\mathcal{C}_n = 1$], we obtain that $V_{n,u}$ is proportional to $h_\star L_n Q_n \mathcal{T}_n^{1/2}$ where Q_n is the quality factor of the quadrupole mode of order n, such that $Q_n = \omega_n/(2\eta_n)$. If we choose an $\eta_n \sim 10^{-6}$ μHz as a fiducial value of the range of η_n values 10^{-8}–10^{-3} μHz predicted by Belkacem *et al.* [47] and Houdek *et al.* [49] for low-order acoustic modes, for a mode with a frequency 400 μHz we obtain $Q_n \sim 10^9$. Note that a Q_n computed from an observational data set (acoustic modes with much higher frequency), we obtain significantly smaller Q_n values. A typical example from Chaplin *et al.* [46] corresponds to a mode with $\nu \sim 1500$ μHz and $\eta_n \sim 10^{-2}$ μHz for which $Q_n \sim 10^6$. Nevertheless, this result can only be used as a lower value estimation of Q_n, since acoustic modes with these high frequencies (and much higher values of η_n) are not perturbed by incoming gravitational radiation. Therefore, for a fiducial acoustic mode with a frequency of 400 μHz, we estimate $V_n \sim 1$ cm s^{-1} for a $Q_n \sim 10^9$, $L_n \sim 10^7$ cm and $\mathcal{C}_n = 1$ when stimulated by an incoming gravitational wave with a strain $h_\star = 10^{-13}$. Finally, if we take into account that this mode is unsaturated, this value must be multiplied by the $\mathcal{T}_n^{1/2}$ with $\tau_{\rm gw} \sim 10^6$ s and $\tau_n \sim 10^{12}$ s then $V_{n,u} \sim 10^{-3}$ cm s^{-1} (cf. Fig. 4). However, if the star detector is located at a distance of $\sim$1000 AU then $V_n \sim 10$ cm s^{-1}. This study complements the original work of McKernan *et al.* [30], which has found stars to be good resonant absorbers of gravitational radiation. The contribution related with the chirp emission is contained in the term $\mathcal{C}_n$ [Eq. (6) with $\mathcal{C}_n \neq 1$]. This quantity increases with the frequency varying from 10^{-2} up to 10^2. Accordingly, $V_{n,u}$ of low-order modes excited by a chirp gravitational wave ($\mathcal{C}_n \neq 1$) is a factor 10 smaller in comparison to modes excited by a monochromatic wave ($\mathcal{C}_n = 1$), since their velocity ratio is proportional to $\mathcal{C}_n^{1/2}$ (cf. Fig. 4).

In the following, we discuss the two $V_{n,u}$ results given equation (6) with $\mathcal{C}_n = 1$ and $\mathcal{C}_n \neq 1$. These solutions correspond to a monochromatic emission and a chirp emission of gravitational radiation. In both cases, the amplitude of $V(\omega_n)$ decreases with the order mode n, since the modal length L_n (and χ_n) decreases rapidly with increasing n acoustic modes for a main-sequence star. In the following two predictions it is worth highlighting the following:

First, the $V_{n,u}$ for the low-n quadrupole acoustic modes is of the order of 10^{-4}–1 cm s^{-1} (cf. Fig. 4, depending on the distance of the star detector to the binary). These values are below the $V_{n,u}$ currently measured for similar stars in the neighborhood of the Sun by the Kepler mission, for which the excitation of stellar oscillations is well known to be attributed to the convection of the external layers of these stars. As an example, the Procyon A star (F5 IV spectral type star) has $V_n \sim 38$ cm s^{-1}, e.g., [52]. This result is equally valid for monochromatic and chirp emission phases of the inspiraling binary.

Second, the impact of the gravitational waves during the chirp emission phase on $V_{n,u}$ [Eq. (6)] is strongly dependent on the shape of the strain function $h(t)$ [Eq. (1)]. Unlike for the case of excitation of $V_{n,u}$ by a monochromatic gravitational wave for which only a stellar quadrupole mode is excited, during the chirp phase several acoustic modes are excited sequentially by the same gravitational waveform. Figure 5 shows how the global shape of the $P_A(\omega)$ spectrum for quadrupole acoustic modes in the sunlike star is excited by the gravitational radiation coming from the inspiral binary during the chirp emission phase. The $P_A(\omega)$ corresponds to a gravitational event shown in Fig. 5 and the $V_{n,u}$ is given by Eq. (6).

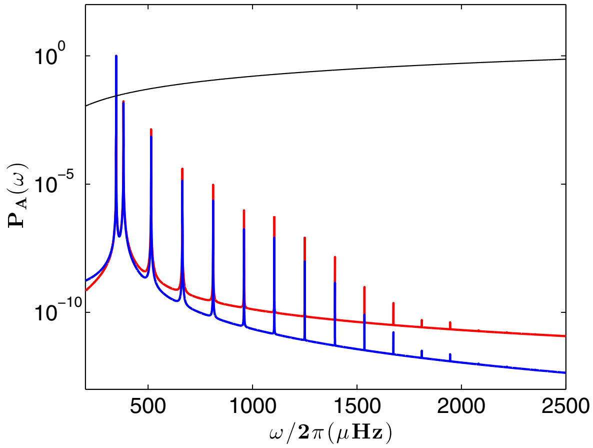

FIG. 5. Amplitude power spectrum of the quadrupole modes of different orders excited by an external GW source with a characteristic frequency $f_c \approx 90$ Hz (identical to a inspiral binary system with $m_1 = 4\times 10^6\ M_\odot$ and $m_2 = 15\ M_\odot$, see Table I): the peaks occur at the location of eigenfrequencies ω_n ($l = 2$ and $n = 0, 1, 2, 3, \cdots$) corresponding to the different acoustic eigenmodes of the Sun. The red curve corresponds to the amplitude power spectrum [Eq. (5)]. The blue curve corresponds to the amplitude power spectrum as given by Eq. (5) with the term $(\omega/\omega_c)^{5/3}$ replaced by one. The black curve corresponds to the term $(\omega/\omega_c)^{5/3}$. All the curves are scaled by their maximum values (in arbitrary units).

These values of $V_{n,u}$ predicted for stars similar to the Sun near the galactic core should be within reach only in a future generation of asteroseismology satellites. Moreover, as $V_{n,u}$ decreases with $d_\star$, it is reasonable to expect that star detectors located in the neighborhood of such binaries (as near as 1000 AU) could have a V_n above the threshold of detectability. However for larger $d_\star$ values, such phenomena will be difficult to observe. In particular, it is unlikely for that measurement to be done by the PLATO mission [13] since, at best, PLATO is expected to measure oscillations in sunlike stars with amplitudes of the order of $\sim 1\ \mathrm{cm\,s^{-1}}$. It will be necessary to wait for an increase of at least 1 or 2 orders of magnitude in the instrumental threshold to be able to measure quadrupole acoustic modes excited by gravitational radiation coming from the Galactic Center. This point is illustrated in Fig. 4 where it is shown how $V_{n,u}$ varies with the distance of the star detector to the source of gravitational waves. For instance, a star detector located at a distance of 10 parsec has $V_{n,u}$ of $10^{-2}\ \mathrm{cm\,s^{-1}}$, while the same star located at a distance of 1000 A.U. has a $V_{n,u}$ of $50\ \mathrm{cm\,s^{-1}}$. Although the last scenario is theoretically possible, it will be very unlikely since the Schwarzschild radius of the supermassive black hole is 0.0810 A.U. and the orbit of the S2 varies between 12 and 2000 AU. It would mean that such star would also be orbiting the supermassive black hole. For comparison, it is worth noticing that in the Sun's case, the precision attained in V_n by the GOLF experiment for a 10 year observational period [43,44,53] varies from $10^{-2}\ \mathrm{cm\,s^{-1}}$ to $3\times 10^{-4}\ \mathrm{cm\,s^{-1}}$. The signal-to-noise ratio of the GOLF experiment is just a few orders of magnitude below the V_n predictions previously mentioned.

V. CONCLUSION

Main-sequence stars like the Sun (with a spectral window of acoustic oscillations $300\ \mu\mathrm{Hz} \le \nu_n \le 5000\ \mu\mathrm{Hz}$) when located at relatively short distances of compact binaries (including massive black-hole and EMRIs binaries) of the Milky Way core have their quadrupole acoustic modes of low order stimulated by the incoming gravitational radiation. This frequency range overlaps the frequency window of gravitational waves emitted by EMRIs $100\ \mathrm{Hz} \le \nu_n \le 10000\ \mu\mathrm{Hz}$. These systems form preferentially in dense stellar regions such as the nucleus of galaxies. The Galaxy nucleus is one of the most dense stellar regions in the Universe with $\sim 10^7$ stars squeezed in spherical regions with a radius of ~ 10 parsec [54]. As in any other galaxy, the nucleus of the Milky Way is one of the preferential locations to look for EMRIs. In particular, these *detector's stars* could follow the end of the binary contraction in the pre-coalescence phase, during which sequentially the low-n quadrupole acoustic modes of the star are stimulated by the incoming gravitational waves. Equally, many other stars, including main-sequence, subgiant, and red giant stars [55] will also be sensitive to the same type of radiation. Hence, all these sunlike stars have a combined spectral window of 0.1 to $10^5\ \mu\mathrm{Hz}$. As such these stars form a network of detectors sensitive to the gravitational radiation coming from the Galactic Center.

A very interesting result of this study is the clear possibility to observe the end phase of the coalescence of binary systems by using sun-like stars as detectors. This is a powerful method to study the gravitational radiation. As the period of the gravitational wave chirp varies within the spectral bandwidth of the star detector, it is certain that different quadrupole modes of the same star will register the same gravitational event. Moreover, as $V_n(\omega)$ is proportional to $(\omega/\omega_c)^{-7/6}$, this relation can be used to look for the gravitational wave signature on low-order quadrupole modes of these stars (cf. Equation (6)). This method is ideal for studying the binaries of massive black holes or EMRIs for which the chirp phase occurs in a time interval varying from a few seconds to several minutes (cf. Table I). This new type of research can complement the gravitational waves experimental detectors like the ELISA instrument. Moreover these star detectors can be used to look for gravitational wave radiation, including the chirp phase of inspiral binary systems in the frequency interval, 10^{-6} to 10^{-4} Hz, which is not currently probed by ground based experiments.

The galactic core is a very efficient machine for converting the gravitational energy of the captured matter into electromagnetic and (possibly) gravitational radiation. Stars like the Sun but near the Galactic Center, for which their spectra of oscillations is well known, form a natural network of detectors for gravitational radiation. In this article, we have shown for the first time that chirp waveforms of gravitational waves have a unique imprint in the spectrum of these sunlike stars. Nevertheless, it is worth highlighting that this study is made for a relatively simple chirp waveform expression although sufficient to make the first prediction of the amplitude stellar modes. A more rigorous calculation must take into account an high-order expression gravitational-wave emission during the inspiral of compact binary systems beyond the quadrupole radiation expression [31].

We could expect that it would be quite difficult to separate the excitation of quadrupole modes in sunlike stars caused by gravitational radiation from the intrinsic excitation and damping of these modes due to turbulent convective motions occurring on the upper layers of the star. Nevertheless, there are three important arguments that could help astronomers to isolate the gravitational wave stimulation from intrinsic excitation: First, current theory of stochastic excitation and damping of nonradial oscillations is very successful in predicting the exactly amplitude of individual acoustic modes, e.g., [56], as well as the global envelope of amplitudes of the acoustic modes in the oscillation spectrum of the star, e.g., [57]. Second, this theory predicts that low-order degree modes (including radial, dipole, and quadrupole modes) with near frequencies have identical amplitudes. Therefore, by taking advantage from the fact that gravitational waves only excite quadrupole modes, the amplitude excess found in these modes above the radial and dipole mode amplitudes can be attributed to excitation due to gravitational radiation. This means that if astronomers found on the oscillation spectrum of a sunlike star one or more quadrupole modes with amplitudes that are well above the amplitudes of neighboring radial and dipole modes, this will be a strong indication that these modes are being stimulated by incoming gravitational radiation, possibly caused by a source located nearby the star. Finally, if the gravitational source is known, it will be possible to compute precisely the amplitude of each mode due to the impact of the gravitational wave, in particular, by taking into account the distance and direction of the gravitational source in relation to the star [26]. Although gravitational waves affect all modes with a degree higher than two, the amplitudes of high-degree modes are very small in comparison to quadrupole modes. Actually, this effect is neglected in high-degree modes since gravitational wave stimulation is insignificant.

As the stars located near the core of the Milky Way have their line of sight obscured by dust, the near infrared band will provide the best option to observe such stars. In principle, a near infrared observatory should be able to observe stars in these dense stellar regions of the galactic nucleus. This option could be a very interesting alternative to optical asteroseismolgy, since the amplitude of stellar oscillations in this band will be only a factor 5 smaller than pulsations in the optical band. Alternatively, an optical mission on the follow-up of the PLATO satellite will be able to observe stars only in regions located well above the galactic disc. In particular, red giant stars could be a very interesting target since these stars can be observed up to distances near the Galactic Center, close to 1000 parsec of the supermassive black hole [58] located in the Galaxy Center. These stars are known to have acoustic, gravity, and mixed quadrupole modes, all of which can be affected by gravitational radiation. Nevertheless the mode amplitude variations on these stars should be very different from the ones computed for sunlike stars, since their internal structures are very different. Another possibility on the optical band is to look for pulsating stars in globular clusters and dwarf galaxies. Additionally, the probability of such detections being achieved successfully would increase significantly if the source of gravitational radiation is located near a population of stars, since in this case the quadrupole modes of several stars are affected simultaneously or contemporaneously within the same field of view.

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