Constraints on axionlike particles and non-Newtonian gravity from measuring the difference of Casimir forces

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We derive constraints on the coupling constants of axionlike particles to nucleons and on the Yukawa-type corrections to Newton's gravitational law from the results of recent experiment on measuring the difference of Casimir forces between a Ni-coated sphere and Au and Ni sectors of a structured disc. Over the wide range of axion masses from 2.61 meV to 0.9 eV the obtained constraints on the axion-to-nucleon coupling are up to a factor of 14.6 stronger than all previously known constraints following from experiments on measuring the Casimir interaction. The constraints on non-Newtonian gravity found here are also stronger than all that following from the Casimir- and Cavendish-type experiments over the interaction range from 30 nm to 5.4 μm. They are up to a factor of 177 stronger than the constraints derived recently from measuring the difference of lateral forces. Our constraints confirm previous somewhat stronger limits obtained from the isoelectronic experiment, where the contribution of the Casimir force was nullified.

I. INTRODUCTION

Both the scalar and pseudoscalar particles are predicted in many extensions of the standard model [1]. The light pseudoscalar particles, axions, and different kinds of axion-like particles play an important role by explaining the absence of both the large electric dipole moment of a neutron and strong *CP* violation in QCD [2–4]. In astrophysics and cosmology axions are considered as the most probable constituents of dark matter [5–13]. An exchange of light scalar particles between atoms of two closely spaced macrobodies leads to the Yukawa-type corrections to Newton's gravitational law [14]. Similar corrections have been predicted by the extradimensional unification schemes with a low-energy compactification scale [15,16]. It is important to remember that at separations below a few micrometers the corrections of Yukawa type, which far exceed the Newtonian gravitation, are not excluded experimentally. Of special interest is the hypothetical scalar particle called the chameleon whose mass depends on the matter density of an environment [17,18]. This particle may be considered as a constituent of dark energy and is discussed in connection with the observed late-time acceleration of the Universe's expansion [19,20].

In spite of many attempts, none of the predicted light pseudoscalar and scalar particles has been discovered so far. Specifically, axionlike particles have been searched for in many laboratory experiments using their interactions with photons, electrons and nucleons (see, e.g., reviews in Refs. [5,21–23]), in astrophysical observations [22–25], and in gravitational experiments [26,27]. The gravitational experiments of Eötvos and Cavendish type were also used to constrain the Yukawa-type corrections to Newtonian gravity mediated by the scalar particles (see Refs. [14,28] for a review and one more recent experiment in Ref. [29]).

As was proposed long ago [30,31], measurements of the van der Waals and Casimir forces can be used for constraining different corrections to Newton's gravitational law. During the last few years these forces have been under an active study both experimentally and theoretically (see Refs. [32,33] for a review). The strongest constraints on the Yukawa-type corrections to Newtonian gravity, following from the most precise measurements of the Casimir interaction, have been obtained in Refs. [34–38] in the interaction range below a micrometer. Recently, the major strengthening was achieved in the isoelectronic experiment, where the Casimir force, acting perpendicular to the test surfaces, was nullified [39] (see also the version of the isoelectronic experiment based on measuring the difference of lateral forces [40]).

Furthermore, it was shown that precise experiments on measuring the Casimir-Polder and Casimir forces place strong limits on the coupling constants of axionlike particles to nucleons [41–44]. For the axionlike particles, which are lighter than 1 eV, even stronger constraints have been derived [45] from the isoelectronic experiment of Ref. [39]. According to the proposal of Ref. [46], the model-independent constraints on an axion are obtainable from measuring the Casimir force between two test bodies with aligned nuclear spins. It was shown also [47,48] that the best laboratory constraints on the parameters of a

chameleon can be obtained from precise measurements of the Casimir force. Experiments of this kind have been proposed in Refs. [49,50].

In this paper, we derive the constraints on the coupling constants of axionlike particles to nucleons and on the Yukawa-type corrections to Newtonian gravity from the recent experiment on measuring the difference of Casimir forces [51] between a Ni-coated sphere and Au and Ni sectors of the structured disc. This disc consisted of alternating Ni and Au sectors deposited on a Si substrate. It was covered by two sufficiently thin homogeneous overlayers made of Ti and Au.

The differential measurements of the Casimir force between metallic test bodies in similar configurations have been proposed in Refs. [52–54] in order to perform a conclusive test on the account of dissipation in the Lifshitz theory of dispersion forces [55,56]. The point is that the measurement data of several precise experiments (see a review in Ref. [32] and more recent results in Refs. [57–59]) have been found to exclude theoretical predictions of the Lifshitz theory combined with the dielectric permittivity of the Drude model taking into account the relaxation properties of free electrons. The same data turned out to be in a very good agreement with theory if the dielectric permittivity of the lossless plasma model is used at low frequencies. It should be noted, however, that within the distance range of all precise experiments below a micrometer the differences in theoretical predictions of both approaches do not exceed a few percent. Because of this, the obtained results have been considered by some authors as not convincing enough; see, e.g., Refs. [60,61] for a discussion).

The situation was changed recently after the experiment on measuring the difference of Casimir forces [51] was performed. In this experiment, the alternative theoretical predictions using the Drude and the plasma models differ by up to a factor of several thousands of percent. That is why an unequivocal exclusion of the Lifshitz theory combined with the Drude model and good agreement of the same theory using the plasma model, demonstrated in the experiment [51], can be considered as conclusive.

Below, we use the differential measurement data of Ref. [51] to derive the constraints on an axion and non-Newtonian gravity and compare them with those following from the previously performed individual measurements of the Casimir interaction. It is shown that over a wide interaction range the obtained constraints are much stronger than all other constraints derived from the Casimir experiments. The constraints on an axion, found here, are complementary (up to a factor of 2 differences) to those of Ref. [45] following from the isoelectronic experiment of Ref. [39], where the Casimir force was nullified. Our present constraints on the corrections to Newtonian gravity are weaker by up to a factor of 10.5 than those derived in Ref. [39], but stronger by up to a factor of 177 than the

constraints derived in recent Ref. [40] exploiting measurements of the lateral force.

The paper is organized as follows. In Sec. II we obtain constraints on the coupling constants of axionlike particles to neutrons and protons from measuring the difference of Casimir forces. In Sec. III the same measurement data are used to derive constraints on the Yukawa-type corrections to Newton's gravitational law. Section IV contains our conclusions and discussion.

Throughout the paper we use units in which $\hbar = c = 1$.

II. CONSTRAINTS ON THE COUPLING CONSTANTS OF AXIONLIKE PARTICLES TO NUCLEONS

We consider the axionlike particles interacting with nucleons (protons and neutrons) via the pseudoscalar Lagrangian [5]. Then, the effective interaction potential between two nucleons situated at the points r_1 and r_2 belonging to two test bodies (a sphere and a disc in our case) arises due to the process of two-axion exchange [26,62,63]

$$V_{kl}(|r_1 - r_2|) = -\frac{g_{ak}^2 g_{al}^2}{32\pi^3 m^2} \frac{m_a}{|r_{12}|^2} K_1(2m_a|r_{12}|). \quad (1)$$

Here, g_{ak} and g_{al} are the axion-proton ($k, l = p$) or axion-neutron ($k, l = n$) dimensionless coupling constants, $m = (m_n + m_p)/2$ is the mean mass of a nucleon, m_a is the mass of an axion, $r_{12} = r_1 - r_2$, and $K_1(z)$ is the modified Bessel function of the second kind. Note that Eq. (1) is derived under the condition $|r_{12}| \gg 1/m$ which is satisfied in all experiments on measuring the Casimir interaction.

In the experiment [51] on measuring the difference of Casimir forces the first test body was a sapphire (Al_2O_3) sphere (with a density $\rho_s = 4.1 \times 10^3$ kg/m^3) covered with the thermally evaporated layers of Cr of thickness $\Delta_{Cr} = 10$ nm ($\rho_{Cr} = 7.15 \times 10^3$ kg/m^3) and Ni of thickness $\Delta_{Ni} = 250$ nm ($\rho_{Ni} = 8.9 \times 10^3$ kg/m^3). The Ni-covered sphere had a radius of $R = 150.8$ μm. The second test body was the structured disc consisting of alternating Au and Ni sectors. It was covered by the homogeneous Ti and Au overlayers with thicknesses $\Delta_{Ti} = 10$ nm and $\Delta_{Au} = 21$ nm, respectively. These overlayers effectively enhance the variation in the difference of Casimir forces between a Ni-coated sphere and sectors of the disc made of Au and Ni when the Drude and plasma models are used in calculations. At the same time, the homogeneous overlayers do not contribute to the difference of additional forces, originating from either two-axion exchange or from the Yukawa-type corrections to Newtonian gravity. To finish with a description of the second test body, we note that the thickness of both Au ($\rho_{Au} = 19.31 \times 10^3$ kg/m^3) and Ni sectors was $D = 2.1$ μm. The structured disc covered with two overlayers was placed on the top of a homogeneous Si wafer of

thickness $\Delta_{Si} = 100 \ \mu$m. This wafer also does not contribute to the difference of additional forces.

The difference of additional forces between a sphere and Au and Ni sectors of the structured disc due to two-axion exchange can be calculated using the potential (1). With appropriately replaced materials of the layers, the result can be found in Ref. [45],

$$|\Delta F_{\text{diff}}^{\text{add}}(a)| = \frac{\pi}{2m_a m^2 m_H H^2}(C_{\text{Au}} - C_{\text{Ni}})$$
$$\times \int_1^\infty du \frac{\sqrt{u^2 - 1}}{u^3} e^{-2m_a u a}(1 - e^{-2m_a u D})X(m_a u),$$
$$\text{(2)}$$

where m_H is the mass of atomic hydrogen, a is the distance between a sphere and the sectors of a disc, and the following notation is introduced,

$$X(z) \equiv C_{\text{Ni}}\Phi(R, z) + (C_{\text{Cr}} - C_{\text{Ni}})e^{-2z\Delta_{\text{Ni}}}\Phi(R - \Delta_{\text{Ni}}, z)$$
$$+ (C_s - C_{\text{Cr}})e^{-2z(\Delta_{\text{Ni}} + \Delta_{\text{Cr}})}\Phi(R - \Delta_{\text{Ni}} - \Delta_{\text{Cr}}, z),$$
$$\text{(3)}$$

with the function Φ defined as

$$\Phi(r, z) = r - \frac{1}{2z} + e^{-2rz}\left(r + \frac{1}{2z}\right). \quad \text{(4)}$$

Here, the coefficients C_M with an index M = Au, Cr, s, and Ni, for gold, chromium, sapphire and nickel, respectively, are defined as

$$C_M = \rho_M\left(\frac{g_{ap}^2}{4\pi}\frac{Z_M}{\mu_M} + \frac{g_{an}^2}{4\pi}\frac{N_M}{\mu_M}\right), \quad \text{(5)}$$

where ρ_M is the density, Z_M and N_M are the number of protons and the mean number of neutrons in an atom or a molecule of the respective material, and $\mu_M = m_M/m_H$, m_M being the mean atomic (molecular) mass of the material M. Note that the values of Z/μ and N/μ for many elements with an account of their isotopic composition are contained in Ref. [14]. In our calculations below we use $Z_M/\mu_M = 0.40422, 0.46518, 0.49422$, and 0.48069 and $N_M/\mu_M = 0.60378, 0.54379, 0.51412$, and 0.52827 for Au, Cr, sapphire, and Ni, respectively.

Now we obtain constraints on the coupling constants g_{an} and g_{ap} from the experimental results of Ref. [51]. For this purpose, we use the measurement set which was found in agreement with theoretical results for the difference in Casimir forces predicted by the Lifshitz theory and the plasma model within the limits of $\Delta F = 1$ fN error over the separation range from 250 to 400 nm (see Fig. 12 of Ref. [51]). This means that the difference of additional forces (2) arising due to two-axion exchange satisfies the condition
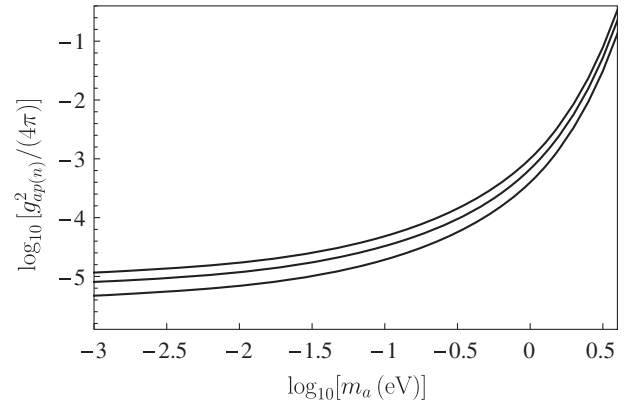

FIG. 1. The lines from top to bottom show the constraints on the coupling constants of axionlike particles to a proton and a neutron as functions of the axion mass, which follow from measuring the difference of Casimir forces under the assumptions $g_{ap}^2 \gg g_{an}^2$, $g_{an}^2 \gg g_{ap}^2$, and $g_{ap}^2 = g_{an}^2$, respectively. The regions of the plane above each line are excluded and those below each line are allowed.

$$|\Delta F_{\text{diff}}^{\text{add}}(a)| < \Delta F. \quad \text{(6)}$$

Note that the distances a between the sphere and the sectors of a rotating disc are connected with the experimental separations z by

$$a = z + \Delta_{\text{Ti}} + \Delta_{\text{Au}} = z + 31 \text{ nm}; \quad \text{(7)}$$

i.e., a and z differ by the combined thickness of Ti and Au overlayers. We have substituted Eqs. (2)–(5) in Eq. (6) and found numerically the values of g_{an}, g_{ap} and m_a satisfying the inequality (6) at different separations a. In so doing, the strongest constraints have been obtained at $a = 291$ nm ($z = 260$ nm).

In Fig. 1, we present the computational results for allowed and excluded regions of the plane $(m_a, g_{ap(n)}^2/4\pi)$ which lie below and above each of the lines, respectively. The three lines from top to bottom are plotted under the respective assumptions $g_{ap}^2 \gg g_{an}^2$, $g_{an}^2 \gg g_{ap}^2$, and $g_{ap}^2 = g_{an}^2$.

In Fig. 2, we compare the constraints of Fig. 1 with the strongest laboratory constraints on the coupling constants of axionlike particles to nucleons obtained so far in the same region of axion masses. The comparison is made under the plausible condition $g_{ap} = g_{an}$ [26]. The line 1 shows the constraints obtained [44] from measurements of the lateral Casimir force between sinusoidally corrugated surfaces [64,65]. By the line 2 we present the constraints found [43] from measuring the effective Casimir pressure by means of a micromechanical torsional oscillator [36,37]. The line 3 is obtained in this work using the experiment [51] on measuring the difference of Casimir forces. It reproduces the bottom line in Fig. 1. The constraints derived [45] from the isoelectronic experiment of Ref. [39], where the Casimir force was nullified, are shown by the line 4. Finally, the line 5
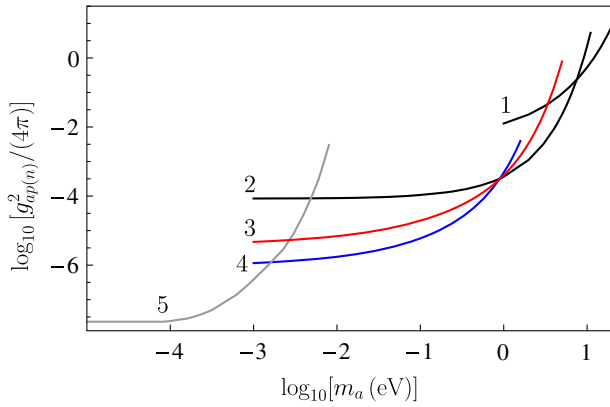
FIG. 2. The line 3 shows the constraints on the coupling constants of axionlike particles to a proton and a neutron as a function of the axion mass obtained here from measurements of the difference of Casimir forces. The other lines show previous constraints derived from measuring the lateral Casimir force (line 1), the effective Casimir pressure (line 2), from the isoelectronic and Cavendish-type experiments (lines 4 and 5). See the text for further discussion. The regions of the plane above each line are excluded and below each line are allowed.

demonstrates the constraints obtained [27] from the Cavendish-type experiment of Ref. [66]. The regions of the plane $(m_a, g_{ap(n)}^2/4\pi)$ above each line are experimentally excluded.

As seen in Fig. 2, the constraints of the line 3, derived here from measuring the difference of Casimir forces, are stronger than the gravitational constraints and all the other constraints obtained from measurements of the Casimir force within the wide range of axion masses m_a from 2.61 meV to 0.9 eV. The maximum strengthening by a factor of 14.6 is achieved for $m_a = 4.88$ meV. Up to a factor of 2 stronger constraints are given by the line 4 obtained from an experiment [39], where the Casimir force was nullified. Thus, one can say that the constraints of the line 3 confirm previous somewhat stronger constraints of the line 4.

III. CONSTRAINTS ON THE YUKAWA-TYPE CORRECTIONS TO NEWTONIAN GRAVITY

Now we consider two atoms with masses m_1 and m_2 situated at the points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ of the test bodies in the same experiment on measuring the difference of Casimir forces. An exchange of one light scalar particle of mass $M = 1/\lambda$ results in the Yukawa-type effective potential, which is usually considered as a correction to the Newtonian gravitational potential [14]

$$V(|\boldsymbol{r}_{12}|) = -\frac{Gm_1 m_2}{|\boldsymbol{r}_{12}|}(1 + \alpha e^{-|\boldsymbol{r}_{12}|/\lambda}). \quad (8)$$

Here, G is the Newtonian gravitational constant and α is a dimensionless constant of the strength of the Yukawa

interaction. As mentioned in Sec. I, the potential (8) also arises in extradimensional unification schemes with a low-energy compactification scale [15,16].

A difference of the additional forces between a sphere and Au and Ni sectors of the structured disc arising due to the potential (8) can be easily calculated as described in Ref. [67]:

$$|F_{\text{diff}}^{\text{Yu}}(a)| = 4\pi^2 G|\alpha|\lambda^3 Re^{-a/\lambda}(\rho_{\text{Au}} - \rho_{\text{Ni}})(1 - e^{-D/\lambda})$$
$$\times [\rho_{\text{Ni}} + (\rho_{\text{Cr}} - \rho_{\text{Ni}})e^{-\frac{\Delta_{\text{Ni}}}{\lambda}}$$
$$+ (\rho_s - \rho_{\text{Cr}})e^{-\frac{\Delta_{\text{Ni}} + \Delta_{\text{Cr}}}{\lambda}}]. \quad (9)$$

The constraints on the parameters α and λ of the potential (8) are obtained from the measurement set of Ref. [51] specified in Sec. II. For this purpose, a difference of the Yukawa-type additional forces (9) was substituted in Eq. (6) in place of $F_{\text{diff}}^{\text{add}}$ and the numerical analysis of the obtained inequality has been performed. The strongest constraints on the parameters α, λ were obtained at the same separation distance $a = 291$ nm, as in Sec. II.

In Fig. 3, our constraints on the Yukawa-type corrections to Newtonian gravity, following from measuring the
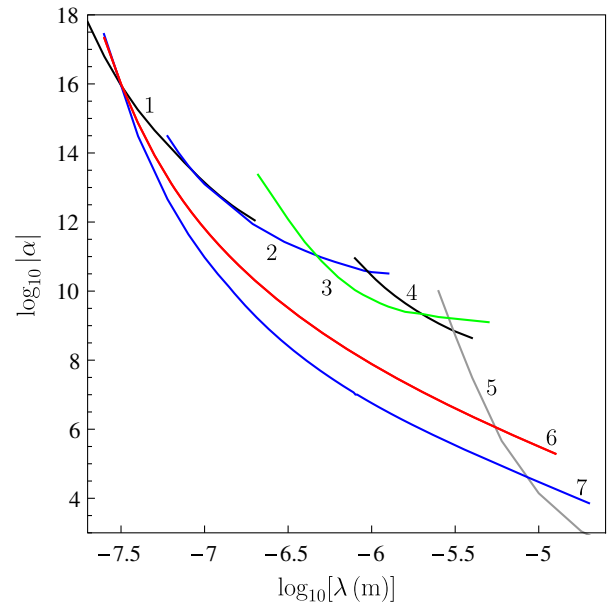

FIG. 3. The line 6 shows the constraints on the strength of a Yukawa-type correction to Newton's gravitational law as a function of the interaction length obtained here from the experiment on measuring the difference of Casimir forces. The other lines show previous constraints derived from measuring the effective Casimir pressure (line 1), from a previous isoelectronic (Casimir-less) experiment (line 2), from an experiment on measuring the difference of lateral forces (line 3), from measuring the Casimir force by means of torsion pendulum (line 4), from the Cavendish-type experiments (line 5), and from the recent isoelectronic experiment (line 7). The regions of the plane above each line are excluded and those below each line are allowed.

difference of Casimir forces in Ref. [51], are shown by the line 6. For comparison purposes, in Fig. 3 the other strongest laboratory constraints are shown in the same interaction range. The line 1 indicates the constraints following from measurements of the effective Casimir pressure [36,37]. The constraints of the line 2 were obtained from the previous isoelectronic (Casimir-less) experiment [68]. The line 3 demonstrates the constraints derived very recently from measuring the difference in lateral forces [40]. The line 4 shows the constraints obtained from measuring the Casimir force by means of a torsion pendulum [38]. The constraints of the line 5 have been obtained from the short-separation Cavendish-type experiment [69–71]. Finally, the line 7 represents the constraints derived from the results of the recent isoelectronic experiment of Ref. [39]. In all cases the regions of the plane $(\lambda, |\alpha|)$ above each line are excluded by the results of respective experiment, and the regions below each line are allowed.

As can be seen in Fig. 3, the constraints of the line 6 obtained here from the experiment [51] on measuring the difference of Casimir forces are quite competitive over the wide interaction region from $\lambda = 30$ nm to $\lambda = 5.4 \ \mu$m. In this region they are much stronger than all the constraints obtained from other measurements of the Casimir force. Specifically, the constraints of line 6 are up to a factor of 16 stronger than that of line 1 derived from measurements of the Casimir pressure. The maximum strengthening holds at $\lambda = 80$ nm. The constraints of line 6 are also stronger than that following from the previous isoelectronic (Casimir-less) experiment (line 2) and recent experiment on measuring the difference of lateral forces (line 3). The maximum strengthenings by the factors of 122 and 177 hold at $\lambda = 435$ nm and 2 μm, respectively. At $\lambda = 3.1 \ \mu$m the constraints of line 6 are stronger by a factor of 100 than that of lines 4 and 5. Our constraints turn out to be stronger than the ones obtained from the Cavendish-type experiment (line 5) in the interaction region $\lambda < 5.4 \ \mu$m.

At $\lambda = 30$ nm the constraints obtained here are of the same strength as those found from the improved isoelectronic experiment of Ref. [39]. At larger λ the latter become stronger than the constraints of line 6 by up to a factor of 10.5. This is explained by the fact that the force sensitivity in the experiment [39] is up to an order of magnitude higher than the measure of agreement between the experimental force differences and theoretical predictions in Ref. [51]. Therefore, both the isoelectronic experiment [39] and the experiment on measuring the difference of Casimir forces can be considered as two independent confirmations for the obtained stronger constraints on the Yukawa-type corrections to Newtonian gravity.

IV. CONCLUSIONS AND DISCUSSION

In the foregoing, we have derived the constraints on the coupling constants of axionlike particles to nucleons and on the non-Newtonian gravity of Yukawa type from the results of a recent experiment on measuring the difference of Casimir forces [51]. This experiment occupies a highly important place among numerous experiments on measuring the Casimir interaction because the predicted force difference varies by thousands of percent depending on which model of dissipation of free electrons is used. An important feature of the employed differential measurement scheme is also that the role of possible background effects, such as the electrostatic patches, surface roughness, and variation of optical properties of material boundaries is largely suppressed [51]. All this allowed an unequivocal exclusion for theoretical predictions of the Lifshitz theory using the Drude model and a conclusive demonstration of an agreement of the same theory using the plasma model with the experimental data. The measure of this agreement was used here to obtain stronger constraints on the axions and non-Newtonian gravity than those obtained from all previous measurements of the Casimir force.

According to our results, the derived constraints on the coupling constant of the axion to nucleons over a wide range of axion masses from 2.61 meV to 0.9 eV are stronger by up to a factor of 16 than all previously known constraints following from the Casimir experiments. They confirm by a factor of 2 stronger constraints obtained previously from the isoelectronic experiment [39] where the Casimir force was nullified.

The constraints on the Yukawa-type corrections to Newton's law of gravitation, derived here from the same experiment, are stronger than all those found from the previously performed measurements of Casimir and gravitational interactions in the range from 30 nm to 5.4 μm. Strengthening is achieved by up to the factors of 16 and 122 as compared to the experiment on measuring the effective Casimir pressure [36,37] and the previous Casimir-less isoelectronic measurement [68], respectively. Our present constraints are stronger by up to the factor of 177 than the results obtained very recently from measuring the difference of lateral forces [40], but weaker by up to a factor 10.5 than the constraints following from the latest version of the isoelectronic experiment [39]. Thus, at the moment the experiment on measuring the difference of Casimir forces and the isoelectronic experiment lead to the strongest constraints on both the coupling constants of axions to nucleons and on the Yukawa-type corrections to Newtonian gravity over the respective regions of axion masses m_a and interaction lengths λ indicated above. Some possibilities on how to further strengthen these constraints using the same experimental setup are discussed in Ref. [72].

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