Heavy baryons and their exotics from instantons in holographic QCD

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We use a variant of the $D4-D8$ construction that includes two chiral and one heavy meson, to describe heavy-light baryons and their exotics as heavy mesons bound to a flavor instanton in bulk. At strong coupling, the heavy meson is shown to *always bind* in the form of a flavor instanton zero mode in the fundamental representation. The ensuing instanton moduli for the heavy baryons exhibits both chiral and heavy quark symmetry. We detail how to quantize it, and derive model independent mass relations for heavy baryons with a single-heavy quark in leading order, in overall agreement with the reported baryonic spectra with one charm or bottom. We also discuss the low-lying masses and quantum assignments for the even and odd parity states, some of which are yet to be observed. We extend our analysis to double-heavy pentaquarks with hidden charm and bottom. In leading order, we find a pair of double-heavy iso-doublets with $IJ^\pi = \frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ assignments for all heavy flavor combinations. We also predict five new Delta-like pentaquark states with $IJ^\pi = \frac{3}{2}\frac{1}{2}^-, \frac{3}{2}\frac{3}{2}^-, \frac{3}{2}\frac{5}{2}^-$ assignments for both charm and bottom.

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I. INTRODUCTION

In QCD the light quark sector (u, d, s) is dominated by the spontaneous breaking of chiral symmetry. The heavy quark sector (c, b, t) is characterized by heavy-quark symmetry [1]. The combination of both symmetries is at the origin of the chiral doubling in heavy-light mesons [2,3] as measured by both the *BABAR* Collaboration [4] and the CLEOII Collaboration [5].

Recently the Belle Collaboration [6] and the BESIII Collaboration [7] have reported many multiquark exotics incommensurate with quarkonia, e.g. the neutral $X(3872)$ and the charged $Z_c(3900)^\pm$ and $Z_b(10610)^\pm$. These exotics have been also confirmed by the DO Collaboration at Fermilab [8], and the LHCb Collaboration at CERN [9]. LHCb has reported new pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ through the decays $\Lambda_b^0 \to J\Psi pK^-, J\Psi p\pi^-$ [10]. More recently, five narrow and neutral excited Ω_c^0 baryon states that decay primarily to $\Theta_c^+K^-$ were also reported by the same collaboration [11]. These flurry of experimental results support new phenomena involving heavy-light multiquark states, *a priori* outside the canonical classification of the quark model.

Some of the tetra-states exotics maybe understood as molecular bound states mediated by one-pion exchange much like deuterons or deusons [12–19]. Nonmolecular heavy exotics were also discussed using constituent quark models [20], heavy solitonic baryons [21,22], instantons [23] and QCD sum rules [24]. The penta-states exotics reported in [10] have been foreseen in [25] and since addressed by many using both molecular and diquark constructions [26], as well as a bound anti-charm to a Skyrmion [27]. String based pictures using string junctions [28] have also been suggested for the description of exotics, including a recent proposal in the context of the holographic inspired string hadron model [29].

The holographic construction offers a framework for addressing both chiral symmetry and confinement in the double limit of large N_c and large t'Hooft coupling $\lambda = g^2 N_c$. A concrete model was proposed by Sakai and Sugimoto [30] using a $D4-D8$ brane construction. The induced gravity on the probe N_f $D8$ branes due to the large stack of N_c $D4$ branes, causes the probe branes to fuse in the holographic direction, providing a geometrical mechanism for the spontaneous breaking of chiral symmetry. The DBI action on the probe branes yields a low-energy effective action for the light pseudoscalars with full global chiral symmetry, where the vectors and axial-vector light mesons are dynamical gauge particles of a hidden chiral symmetry [31]. In the model, light baryons are identified with small size instantons by wrapping $D4$ around S^4, and are dual to Skyrmions on the boundary [32,33]. Remarkably, this identification provides a geometrical description of the baryonic core that is so elusive in most Skyrme models [34]. A first principle description of the baryonic core is paramount to the understanding of heavy hadrons and their exotics since the heavy quarks bind over their small Compton wavelength.

The purpose of this paper is to propose a holographic description of heavy baryons and their exotics that involve light and heavy degrees of freedom through a variant of the $D4-D8$ model that includes a heavy flavor [35] with both chiral and heavy-quark symmetry. The model uses 2 light and 1 heavy branes where the heavy-light mesons are identified with the string low energy modes, and

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approximated by bi-fundamental and local vector fields in the vicinity of the light probe branes. Their masses follow from the vacuum expectation value (VEV) of the moduli span by the dilaton fields in the DBI action. The model allows for the description of the radial spectra of the $(0^\pm, 1^\pm)$ heavy-light multiplets, their pertinent vector and axial correlations, and leads reasonable estimates for the one-pion axial couplings and radiative decays in the heavy-light sector.

In this construction, the heavy baryons will be sought in the form of a bulk instanton in the world volume of $D8$ bound to heavy-light vector mesons, primarily the heavy-light $(0^-, 1^-)$ multiplet. This approach will extend the bound state approach developed in the context of the Skyrme model [27,36] to holography. We note that alternative holographic models for the description of heavy hadrons have been developed in [37,38] without the dual strictures of chiral and heavy quark symmetry.

The organization of the paper is as follows: In Sec. II we briefly outline the geometrical set up for the derivation of the heavy-light effective action through the pertinent bulk DBI and CS actions. In Sec. III, we detail the heavy-meson interactions to the flavor instanton in bulk. In Sec. IV, we show how a vector meson with spin 1 binding to the bulk instanton transmutes to a spin $\frac{1}{2}$. In Sec. V, we identify the moduli of the bound zero mode and quantize it by collectivizing some of the soft modes. The mass spectra for baryons with single- and double-heavy quarks are explicitly derived. Some of our exotics are comparable to those recently reported by several collaborations, while others are new. Our conclusions are in Sec. VI. In the Appendix we briefly review the quantization of the light meson moduli without the heavy mesons.

II. HOLOGRAPHIC EFFECTIVE ACTION

A. D-brane set up

The $D4-D8$ construction proposed by Sakai and Sugimoto [30] for the description of the light hadrons is standard and will not be repeated here. Instead, we follow [35] and consider the variant with $N_f - 1$ light $D8 - \bar{D}8$ (L) and one heavy (H) probe branes in the cigar-shaped geometry that spontaneously breaks chiral symmetry. For simplicity, the light probe branes are always assumed in the antipodal configuration. A schematic description of the set up for $N_f = 3$ is shown in Fig. 1. We assume that the L-brane world volume consists of $R^4 \times S^1 \times S^4$ with [0–9]-dimensions. The light 8-branes are embedded in the $[0-3+5-9]$-dimensions and set at the antipodes of S^1 which lies in the 4-dimension. The warped [5–9]-space is characterized by a finite size R and a horizon at U_{KK}.

B. DBI and CS actions

The lowest open string modes streched between the H- and L-branes are attached to a wrapped S^4 in $D4$ shown

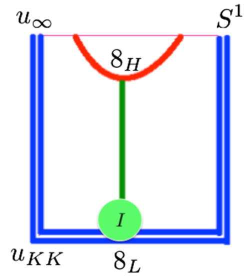

FIG. 1. $N_f - 1 = 2$ antipodal 8_L light branes, and one 8_H heavy brane shown in the τU plane, with a bulk SU(2) instanton embedded in 8_L and a massive HL-string connecting them.

as an instanton in Fig. 1. Near the L brane world volume, these string modes consist of tranverse modes Φ_M and longitudinal modes Ψ, both fundamental with respect to the flavor group $\mathrm{SU}(N_f - 1)$. At nonzero brane separation, these fields acquire a VEV that makes the vector field massive [39]. Strictly speaking these fields are bilocal, but near the L-branes we will approximate them by local vector fields that are described by the standard DBI action in the background of a warped instanton field. In this respect, our construction is distinct from the approaches developed in [37].

With this in mind and to leading order in the $1/\lambda$ expansion, the effective action on the probe L-branes consists of the non-Abelian DBI (D-brane Born-Infeld) and CS (Chern-Simons) action. After integrating over the S^4, the leading contribution to the DBI action is

$$S_{\mathrm{DBI}} \approx -\kappa \int d^4x dz \mathrm{Tr}(\mathbf{f}(z)\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \mathbf{g}(z)\mathbf{F}_{\mu z}\mathbf{F}^{\nu z}). \quad (1)$$

Our conventions are $(-1,1,1,1)$ with $A_M^\dagger = -A_M$. The warping factors are

$$\mathbf{f}(z) = \frac{R^3}{4U_z}, \qquad \mathbf{g}(z) = \frac{9}{8}\frac{U_z^3}{U_{KK}}, \quad (2)$$

with $U_z^3 = U_{KK}^3 + U_{KK}z^2$, $\kappa = \tilde{T}(2\pi\alpha') = a\lambda N_c$ and $a = 1/(216\pi^3)$ [30]. All dimensions are understood in units where the Kaluza-Klein mass $M_{KK} \equiv 1$ unless specified otherwise. The effective fields in the field strengths are $[M, N$ run over $(\mu, z)]$

$$\mathbf{F}_{MN} = \begin{pmatrix} F_{MN} - \Phi_{[M}\Phi_{N]}^\dagger & \partial_{[M}\Phi_{N]} + A_{[M}\Phi_{N]} \\ -\partial_{[M}\Phi_{N]}^\dagger - \Phi_{[M}^\dagger A_{N]} & -\Phi_{[M}^\dagger\Phi_{N]} \end{pmatrix}. \quad (3)$$

The CS contribution to the effective action is (form notation used)

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{R^{4+1}} \mathrm{Tr}\left(\mathbf{A}\mathbf{F}^2 - \frac{1}{2}\mathbf{A}^3\mathbf{F} + \frac{1}{10}\mathbf{A}^5\right), \quad (4)$$

where the normalization to N_c is fixed by integrating the F_4 RR flux over the S^4. The matrix valued 1-form gauge field is

$$\mathbf{A} = \begin{pmatrix} A & \Phi \\ -\Phi^\dagger & 0 \end{pmatrix}. \tag{5}$$

For N_f coincidental branes, the Φ multiplet is massless. However, their brane world-volume supports an adjoint and traceless scalar Ψ in addition to the adjoint gauge field A_M both of which are Hermitian and $N_f \times N_f$ valued, which we have omitted from the DBI action in so far for simplicity. They are characterized by a quartic potential with finite extrema and a VEV v for the diagonal of Ψ [39]. As a result the Φ multiplet acquires a Higgs-like mass of the type

$$\frac{1}{2} m_H^2 \mathrm{Tr}(\Phi_M^\dagger \Phi_M) \sim \frac{1}{2} v^2 \mathrm{Tr}(\Phi_M^\dagger \Phi_M). \tag{6}$$

The VEV is related to the separation between the light and heavy branes [39], which we take it to be the mass following from the length of the stretched HL string, and which we identify as the mass of the heavy-light $(0^-, 1^-)$ multiplet for either charm (D, D^*) or bottom (B, B^*). In the heavy quark limit, the radial spectra, axial and vector correlations, and the one-pion radiative decays of the $(0^-, 1^-)$ multiplet are fairly reproduced by this model [35].

III. HEAVY-LIGHT-INSTANTON INTERACTIONS

In the original two-flavor $D4 - D8$ set up by Sakai and Sugimoto [30] light baryons are first identified with a flavor instanton in bulk [32] and its moduli quantized to yield the nucleon and Delta [33]. This construction holds in our case in the light sector of (1) verbatim and we refer the interested reader to [32,33] for the details of the analysis. The key observations is that the instanton size is small at strong coupling $\rho \sim 1/\sqrt{\lambda}$, as a result of balancing the large and leading attraction due to gravity in bulk (large warpings) and the subleading U(1) Coulomb-like repulsion induced by the Chern-Simons term.

In the geometrical set up described in Fig. 1, the small size instanton translates to a flat space 4-dimensional instanton [32]

$$\begin{aligned} A_M^{cl} &= -\bar{\sigma}_{MN} \frac{x_N}{x^2 + \rho^2}, \\ A_0^{cl} &= \frac{-i}{8\pi^2 a x^2} \left(1 - \frac{\rho^4}{(x^2 + \rho^2)^2}\right) \end{aligned} \tag{7}$$

after using the rescalings

$$\begin{aligned} &x_0 \to x_0, \qquad x_M \to x_M/\sqrt{\lambda}, \qquad \sqrt{\lambda}\rho \to \rho \\ &(A_0, \Phi_0) \to (A_0, \Phi_0), \\ &(A_M, \Phi_M) \to \sqrt{\lambda}(A_M, \Phi_M) \end{aligned} \tag{8}$$

in (1). From here and throughout the rest of the paper, M, N run only over $1, 2, 3, z$. To order λ^0 the rescaled contributions describing the interactions between the light gauge fields A_M and the heavy fields Φ_M to quadratic order split in the form

$$S = aN_c \lambda S_0 + aN_c S_1 + S_{CS}, \tag{9}$$

with each contribution given by

$$\begin{aligned} S_0 &= -(D_M \Phi_N^\dagger - D_N \Phi_M^\dagger)(D_M \Phi_N - D_N \Phi_M) \\ &\quad + 2\Phi_M^\dagger F_{MN} \Phi_N \\ S_1 &= +2(D_0 \Phi_M^\dagger - D_M \Phi_0^\dagger)(D_0 \Phi_M - D_M \Phi_0) \\ &\quad - 2\Phi_0^\dagger F^{0M} \Phi_M - 2\Phi_M^\dagger F^{M0} \Phi_0 \\ &\quad - 2m_H^2 \Phi_M^\dagger \Phi_M + \tilde{S}_1 \\ S_{CS} &= -\frac{iN_c}{24\pi^2}(d\Phi^\dagger A d\Phi + d\Phi^\dagger dA \Phi + \Phi^\dagger dA d\Phi) \\ &\quad - \frac{iN_c}{16\pi^2}(d\Phi^\dagger A^2 \Phi + \Phi^\dagger A^2 d\Phi + \Phi^\dagger (AdA + dAA)\Phi) \\ &\quad - \frac{5iN_c}{48\pi^2} \Phi^\dagger A^3 \Phi + S_C(\Phi^4, A) \end{aligned} \tag{10}$$

and

$$\begin{aligned} \tilde{S}_1 &= +\frac{1}{3} z^2 (D_i \Phi_j - D_j \Phi_i)^\dagger (D_i \Phi_j - D_j \Phi_i) \\ &\quad - 2z^2 (D_i \Phi_z - D_z \Phi_i)^\dagger (D_i \Phi_z - D_z \Phi_i) \\ &\quad - \frac{2}{3} z^2 \Phi_i^\dagger F_{ij} \Phi_j + 2z^2 (\Phi_z^\dagger F_{zi} \Phi_i + \text{c.c.}). \end{aligned} \tag{11}$$

IV. BOUND STATE AS A ZERO-MODE

We now show that in the double limit of large λ followed by large m_Q, a heavy meson in bulk always binds to the flavor instanton in the form of a four-dimensional $(123z)$ flavor zero mode that effectively is a spinor. This holographic zero mode translates equally to either a bound heavy flavor or anti-heavy flavor in our space-time (0123). This is remarkable to holography, as the heavy bound states in the Skyrme-type involve particles but with difficulties anti-particles [36,40]. Indeed, in the Skyrme model, the Wess-Zumino-Witten term which is time odd, carries opposite signs for heavy particles and anti-particles that are magnified by N_c in comparison to the heavy-mesonic action. As a result the particle state is attractive, while the antiparticle state is repulsive.

A. Field equations

We now consider the bound state solution of the heavy meson field Φ_M in the (rescaled) instanton background (7). We note that the field equation for Φ_M is independent of Φ_0 and reads

$$D_M D_M \Phi_N + 2F_{NM}\Phi_M - D_N D_M \Phi_M = 0, \qquad (12)$$

while the constraint field equation (Gauss law) for Φ_0 depends on Φ_M through the Chern-Simons term

$$D_M(D_0\Phi_M - D_M\Phi_0) - F^{0M}\Phi_M - \frac{\epsilon_{MNPQ}}{64\pi^2 a}K_{MNPQ} = 0, \qquad (13)$$

with K_{MNPQ} defined as

$$K_{MNPQ} = +\partial_M A_N \partial_P \Phi_Q + A_M A_N \partial_P \Phi_Q + \partial_M A_N A_P \Phi_Q + \frac{5}{6} A_M A_N A_P \Phi_Q. \qquad (14)$$

In the heavy quark limit it is best to redefine $\Phi_M = \phi_M e^{-im_H x_0}$ for particles. The antiparticle case follows through $m_Q \to -m_H$ with pertinent sign changes. As a result, the preceding field equations remain unchanged for ϕ_M with the substitution $D_0\phi_M \to (D_0 \mp im_H)\phi_M$ understood for particles $(-)$ or antiparticles $(+)$ respectively.

B. Double limit

In the double limit of $\lambda \to \infty$ followed by $m_H \to \infty$, the leading contributions are of order λm_H^0 from the light effective action in (1), and of order $\lambda^0 m_H$ from the heavy-light interaction term S_1 in (10). This double limit is justified if we note that in leading order, the mass of the heavy meson follows from the straight pending string shown in Fig 1, with a value [35]

$$\frac{m_H}{\lambda M_{KK}} = \frac{2}{9\pi}(M_{KK}u_H)^{\frac{2}{3}}, \qquad (15)$$

where u_H is the holographic height of the heavy brane. The double limit requires the ratio in (15) to be parametrically small.

With the above in mind, we have

$$\frac{S_{1,m}}{aN_c} = 4im_H\phi_m^\dagger D_0\phi_m - 2im_H(\phi_0^\dagger D_M\phi_M - \text{c.c.}), \qquad (16)$$

and from the Chern-Simons term in (10) we have

$$\frac{m_H N_c}{16\pi^2}\epsilon_{MNPQ}\phi_M^\dagger F_{NP}\phi_Q = \frac{m_H N_c}{8\pi^2}\phi_M^\dagger F_{MN}\phi_N. \qquad (17)$$

The constraint equation (13) simplifies considerably to order m_Q,

$$D_M\phi_M = 0, \qquad (18)$$

implying that ϕ_M is covariantly transverse in leading order in the double limit.

C. Vector to spinor zero mode

The instanton solution A_M in (7) carries a field strength

$$F_{MN} = \frac{2\bar{\sigma}_{MN}\rho^2}{(x^2+\rho^2)^2}. \qquad (19)$$

We now observe that the heavy field equation (12) in combination with the constraint equation (18) are equivalent to the vector zero-mode equation in the fundamental representation. To show that, we recall that the field strength (19) is self-dual, and S_0 in (10) can be written in the compact form

$$\begin{aligned} S_0 &= -f_{MN}^\dagger f_{MN} + 2\phi_M^\dagger F_{MN}\phi_N \\ &= -f_{MN}^\dagger f_{MN} + 2\epsilon_{MNPQ}\phi_M^\dagger D_M D_Q \phi_N \\ &= -f_{MN}^\dagger f_{MN} + f_{MN}^\dagger \star f_{MN} \\ &= -\frac{1}{2}(f_{MN} - \star f_{MN})^\dagger (f_{MN} - \star f_{MN}) \end{aligned} \qquad (20)$$

after using the Hodge dual $\star$ notation, and defining

$$f_{MN} = \partial_{[M}\phi_{N]} + A_{[M}\phi_{N]}. \qquad (21)$$

Therefore, the second order field equation (12) can be replaced by the anti-self-dual condition (first order) and the transversality condition (18) (first order),

$$\begin{aligned} f_{MN} - \star f_{MN} &= 0 \\ D_M\phi_M &= 0, \end{aligned} \qquad (22)$$

which are equivalent to

$$\sigma_M D_M \psi = D\psi = 0 \quad \text{with} \quad \psi = \bar{\sigma}_M \phi_M. \qquad (23)$$

The spinor zero mode ψ is unique, and its explicit matrix form reads

$$\psi^a_{\alpha\beta} = \epsilon_{\alpha a}\chi_\beta \frac{\rho}{(x^2+\rho^2)^{\frac{3}{2}}}, \qquad (24)$$

which gives the vector zero mode in the form

$$\phi_M^a = \chi_\beta(\sigma_M)_{\beta\alpha}\epsilon_{\alpha a}\frac{\rho}{(x^2+\rho^2)^{\frac{3}{2}}}, \qquad (25)$$

or in equivalent column form

$$\phi_M = \bar{\sigma}_M\chi\frac{\rho}{(x^2+\rho^2)^{\frac{3}{2}}} \equiv \bar{\sigma}_M f(x)\chi. \quad (26)$$

Here χ_α is a constant two-component spinor. It can be checked explicitly that (26) is a solution to the first order equations (22). The interplay between (24) and (25) is remarkable as it shows that in holography a heavy vector meson binds to an instanton in bulk in the form of a vector zero mode that is equally described as a spinor. This duality illustrates the transmutation from a spin 1 to a spin $\frac{1}{2}$ in the instanton field.

V. QUANTIZATION

Part of the classical moduli of the bound instanton-zero-mode breaks rotational and translational symmetry, which will be quantized by slowly rotating or translating the bound state. In addition, it was noted in [32] that while the deformation of the instanton size and holographic location are not collective per say as they incur potentials, they are still soft in comparison to the more massive quantum excitations in bulk and should be quantized as well. The ensuing quantum states are vibrational and identified with the breathing modes (size vibration) and odd parity states (holographic vibration).

A. Collectivization

The leading λN_c contribution is purely instantonic and its quantization is standard and can be found in [33]. For completeness we have summarized it in the Appendix. The quantization of the subleading $\lambda^0 m_H$ contribution involves the zero mode and is new, so we will describe in more detail. For that, we let the zero mode slowly translate, rotate and deform through

$$\Phi \to V(a_I(t))\Phi(X_0(t), Z(t), \rho(t), \chi(t))$$
$$\Phi_0 \to 0 + \delta\phi_0. \quad (27)$$

Here X_0 is the center in the 123 directions and Z is the center in the z directon. a_I is the SU(2) gauge rotation moduli. We denote the moduli by $X_\alpha \equiv (X, Z, \rho)$ with

$$-iV^\dagger\partial_0 V = \Phi = -\partial_t X_N A_N + \chi^a\Phi_a$$
$$\chi^a = -i\mathrm{Tr}(\tau^a a_I^{-1}\partial_t a_I), \quad (28)$$

a_I is the SU(2) rotation which carries the isospin and angular momentum quantum numbers. The constraint equation (13) for ϕ_0 has to be satisfied, which fixes $\delta\phi_0$ at subleading order

$$-D_M^2\delta\phi_0 + D_M\bar{\sigma}_M(\partial_t X_i\partial_{X_i} f\chi + \partial_t\chi)$$
$$+ i(\partial_t X_\alpha\partial_\alpha\Phi_M - D_M\Phi)\bar{\sigma}_M\chi + \delta S_{cs} = 0. \quad (29)$$

The solution to (29) can be inserted back into the action for a general quantization of the ensuing moduli.

B. Leading heavy mass terms

There are three contributions to order $\lambda^0 m_H$, namely

$$16im_H\chi^\dagger\partial_t\chi f^2 + 16im_H\chi^\dagger\chi A_0 f^2 - m_H f^2\chi^\dagger\sigma_\mu\Phi\bar{\sigma}_\mu\chi, \quad (30)$$

with the rescaled U(1) field A_0, and the Chern-Simons term

$$\frac{im_H N_c}{8\pi^2}\phi_M^\dagger F_{MN}\phi_N = \frac{i3m_H N_c}{\pi^2}\frac{f^2\rho^2}{(x^2+1)^2}\chi^\dagger\chi, \quad (31)$$

with the field strength given in (19). Explicit calculations show that the third contribution in (30) vanishes owing to the identity $\sigma_\mu\tau_a\bar{\sigma}_\mu = 0$.

The coupling $\chi^\dagger\chi A_0$ term in (30) induces a Coulomb-like back-reaction. To see this, we set $\psi = iA_0$ and collect all the U(1) Coulomb-like couplings in the rescaled effective action to order $\lambda^0 m_H$

$$\frac{S_C(A_0)}{aN_c} = \int\left(\frac{1}{2}(\nabla\psi)^2 + \psi(\rho_0[A] - 16m_H f^2\chi^\dagger\chi)\right)$$
$$\rho_0[A] = \frac{1}{64\pi^2 a}\epsilon_{MNPQ}F_{MN}F_{PQ}. \quad (32)$$

The static action contribution stemming from the coupling to the U(1) charges ρ_0 and $\chi^\dagger\chi$ is

$$\frac{S_C}{aN_c} \to \frac{S_C[\rho_0]}{aN_c} + 16m_H\chi^\dagger\chi\int f^2(-iA_0^{cl}) - \frac{(16m_H\chi^\dagger\chi)^2}{24\pi^2}. \quad (33)$$

The last contribution is the Coulomb-like self interaction induced by the instanton on the heavy meson through the U(1) Coulomb-like field in bulk. It is repulsive and tantamount of fermion number repulsion in holography.

C. Moduli effective action

Putting all the above contributions together, we obtain the effective action density on the moduli in leading order in the heavy meson mass

$$\mathcal{L} = \mathcal{L}_0[a_I, X_\alpha] + 16aN_c m_H\Bigg(i\chi^\dagger\partial_0\chi^\dagger\int d^4x f^2$$
$$-\chi^\dagger\chi\int d^4x f^2\left(iA_0^{cl} - \frac{3}{16a\pi^2}\frac{\rho^2}{(x^2+\rho^2)^2}\right)\Bigg)$$
$$-aN_c\frac{(16m_H\chi^\dagger\chi)^2}{24\pi^2\rho^2}, \quad (34)$$

with $\mathcal{L}_0$ referring to the effective action density on the moduli stemming from the contribution of the light degrees of freedom in the instanton background. It is identical to the one derived in [32] and to which we refer the reader for further details. In (34) We have made explicit the new contribution due to the bound heavy meson through χ. To this order there is no explicit coupling of the light collective degrees of freedom a_I, to the heavy spinor degree of freedom χ, a general reflection on heavy quark symmetry in leading order. However, there is a coupling to the instanton size ρ through the holographic direction which does not upset this symmetry. After using the normalization $\int d^4 x f^2 = 1$, inserting the explicit form of A_0^{cl} from (7), and rescaling $\chi \to \chi/2\sqrt{aN_c m_H}$, we finally have

$$\mathcal{L} = \mathcal{L}_0[a_I, X_\alpha] + \chi^\dagger i\partial_t \chi + \frac{3}{32\pi^2 a\rho^2}\chi^\dagger\chi - \frac{(\chi^\dagger\chi)^2}{24\pi^2 a\rho^2 N_c}. \tag{35}$$

Remarkably, the bound vector zero mode to the instanton transmutes to a massive spinor with a repulsive Coulomb-like self- interaction. The mass is *negative* which implies that the heavy meson lowers its energy in the presence of the instanton to order λ^0. We note that the preceding arguments carry verbatim to an antiheavy meson in the presence of an instanton, leading (35) with a *positive* mass term. This meson raises its energy in the presence of the instanton to order λ^0. These effects originate from the Chern-Simons action in holography. They are the analogue of the effects due to the Wess-Zumino-Witten term in the standard Skyrme model [36,40]. While they are leading in $1/N_c$ in the latter causing the anti-heavy meson to unbind in general, they are subleading in $1/\lambda$ in the former where to leading order the bound state is always a Bogomol'nyi-Prasad-Sommerfield (BPS) zero mode irrespective of heavy-meson or anti-heavy-meson.

D. Heavy-light spectra

The quantization of (35) follows the same arguments as those presented in [32] for $\mathcal{L}_0[a_I, X_\alpha]$ and to which we refer for further details in general, and the Appendix for the notations in particular. Let H_0 be the Hamiltonian associated to $\mathcal{L}_0[a_I, X_\alpha]$, then the Hamiltonian for (35) follows readily in the form

$$H = H_0[\pi_I, \pi_X, a_I, X_\alpha] - \frac{3}{32\pi^2 a\rho^2}\chi^\dagger\chi + \frac{(\chi^\dagger\chi)^2}{24\pi^2 a\rho^2 N_c}, \tag{36}$$

with the new quantization rule for the spinor

$$\chi_i \chi_j^\dagger \pm \chi_j^\dagger \chi_i = \delta_{ij}. \tag{37}$$

The statistics of χ needs to be carefully determined. For that, we note the symmetry transformation

$$\chi \to U\chi \quad \text{and} \quad \phi_M \to U\Lambda_{MN}\phi_N \tag{38}$$

since $U^{-1}\bar{\sigma}_M U = \Lambda_{MN}\bar{\sigma}_N$. So a rotation of the spinor χ is equivalent to a spatial rotation of the heavy vector meson field ϕ_M which carries spin 1. Since χ is in the spin $\frac{1}{2}$ representation it should be quantized as a fermion. So only the plus sign is to be retained in (37). Also, χ carries opposite parity to ϕ_M, i.e. positive. With this in mind, the spin $\mathbf{J}$ and isospin $\mathbf{I}$ are then related by

$$\vec{\mathbf{J}} = -\vec{\mathbf{I}} + \vec{\mathbf{S}}_\chi \equiv -\vec{\mathbf{I}} + \chi^\dagger \frac{\vec{\tau}}{2}\chi. \tag{39}$$

We note that in the absence of the heavy-light meson $\mathbf{J} + \mathbf{I} = 0$ as expected from the spin-flavor hedgehog character of the bulk instanton (see also the Appendix).

The spectrum of (36) follows from the one discussed in details in [32] with the only modification of Q entering in H_0 as given in the Appendix

$$Q \equiv \frac{N_c}{40a\pi^2} \to \frac{N_c}{40a\pi^2}\left(1 - \frac{15}{4N_c}\chi^\dagger\chi + \frac{5(\chi^\dagger\chi)^2}{3N_c^2}\right). \tag{40}$$

The quantum states with a single bound state $N_Q = \chi^\dagger\chi = 1$ and IJ^π assignments are labeled by

$$|N_Q, JM, lm, n_z, n_\rho\rangle \quad \text{with} \quad IJ^\pi = \frac{l}{2}\left(\frac{l}{2} \pm \frac{1}{2}\right)^\pi, \tag{41}$$

with $n_z = 0, 1, 2, ..$ counting the number of quanta associated to the collective motion in the holographic direction, and $n_\rho = 0, 1, 2, ..$ counting the number of quanta associated to the radial breathing of the instanton core, a sort of Roper-like excitations. Following [32], we identify the parity of the heavy baryon bound state as $(-1)^{n_z}$. Using (40) and the results in [32] as briefly summarized in the Appendix, the mass spectrum for the bound heavy-light states is

$$\begin{aligned} M_{NQ} = &+M_0 + N_Q m_H \\ &+ \left(\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2\left(1 - \frac{15N_Q}{4N_c} + \frac{5N_Q^2}{3N_c^2}\right)\right)^{\frac{1}{2}} M_{KK} \\ &+ \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} M_{KK}, \end{aligned} \tag{42}$$

with M_{KK} the Kaluza-Klein mass and $M_0/M_{KK} = 8\pi^2\kappa$ the bulk instanton mass. The Kaluza-Klein scale is usually set by the light meson spectrum and is fit to reproduce the rho mass with $M_{KK} \sim m_\rho/\sqrt{0.61} \sim 1$ GeV [30]. Whenever possible, we will try to eliminate the uncertainties on the value of M_{KK} through model independent relations for fixed N_Q.

We note that the net effect of the heavy-meson is among other things, an increase in the iso-rotational inertia by expanding (42) in $1/N_c$. The negative N_Q/N_c contribution in (42) reflects on the fact that a heavy meson with a heavy quark mass is attracted to the instanton to order λ^0. As we noted earlier, a heavy meson with a heavy anti-quark will be repelled to order λ^0 hence a similar but positive contribution. The positive N_Q^2/N_c^2 contribution is the repulsive Coulomb-like self interaction. Note that it is of the same order as the rotational contribution which justifies keeping it in our analysis.

(42) is to be contrasted with the mass spectrum for baryons with no heavy quarks or $N_Q = 0$, where the nucleon state is idendified as $N_Q = 0, l = 1, n_z = n_\rho = 0$ and the Delta state as $N_Q = 0, l = 3, n_z = n_\rho = 0$ [32]. The radial excitation with $n_\rho = 1$ can be identified with the radial Roper excitation of the nucleon and Delta, while the holographic excitation with $n_z = 1$ can be interpreted as the odd parity excitation of the nucleon and Delta.

E. Single-heavy baryons

Since the bound zero-mode transmuted to spin $\frac{1}{2}$, the lowest heavy baryons with one heavy quark are characterized by $N_Q = 1, l = \text{even}, N_c = 3$ and $n_z,\ n_\rho = 0,\ 1$, with the mass spectrum

$$M_{X_Q} = +M_0 + m_H + \left(\frac{(l+1)^2}{6} - \frac{7}{90}\right)^{\frac{1}{2}} M_{KK} \tag{43}$$

$$+\frac{2(n_\rho + n_z) + 2}{\sqrt{6}} M_{KK}. \tag{44}$$

1. Heavy baryons

Consider the states with $n_z = n_\rho = 0$. We identify the state with $l = 0$ with the heavy-light iso-singlet Λ_Q with the assignments $IJ^\pi = 0\frac{1}{2}^+$. We identify the state with $l = 2$ with the heavy-light iso-triplet Σ_Q with the assignment $1\frac{1}{2}^+$, and $\Sigma_Q^\star$ with the assignment $1\frac{3}{2}^+$. By subtracting the nucleon mass from (43) we have

$$\begin{aligned} M_{\Lambda_Q} - M_N - m_H &= -1.06 M_{KK} \\ M_{\Sigma_Q} - M_N - m_H &= -0.17 M_{KK} \\ M_{\Sigma_Q^*} - M_N - m_H &= -0.17 M_{KK}. \end{aligned} \tag{45}$$

Hence the holographic and model independent relations

$$\begin{aligned} M_{\Lambda_{Q'}} &= M_{\Lambda_Q} + (m_{H'} - m_H) \\ M_{\Sigma_{Q'}} &= 0.84 m_N + m_{H'} + 0.16(M_{\Lambda_Q} - m_H), \end{aligned} \tag{46}$$

with $Q, Q' = c,\ b$. Using the heavy meson masses $m_D \approx 1870$ Mev, $m_B = 5279$ MeV and $m_{\Lambda_c} = 2286$ Mev we find that $M_{\Lambda_b} = 5655$ MeV in good agreement with the measured value of 5620 MeV. Also we find $M_{\Sigma c} = 2725$ Mev and $M_{\Sigma b} = 6134$ Mev, which are to be compared to the empirical values of $M_{\Sigma c} = 2453$ Mev and $M_{\Sigma b} = 5810$ Mev respectively.

2. Excited heavy baryons

Now, consider the low-lying breathing modes R with $n_\rho = 1$ for the even assignments $0\frac{1}{2}^+, 1\frac{1}{2}^+, 1\frac{3}{2}^+$, and the odd parity excited states O with $n_z = 1$ for the odd assigments $0\frac{1}{2}^-, 1\frac{1}{2}^-, 1\frac{3}{2}^-$. (43) shows that the R-excitations are degenerate with the O-excitations. We obtain ($E = O,\ R$)

$$\begin{aligned} M_{\Lambda_{EQ'}} &= +0.23 M_{\Lambda_Q} + 0.77 m_N - 0.23 m_H + m_{H'} \\ M_{\Sigma_{EQ'}} &= -0.59 M_{\Lambda_Q} + 1.59 m_N + 0.59 m_H + m_{H'}. \end{aligned} \tag{47}$$

We found $M_{\Lambda_{Oc}} = 2686$ MeV which is to be compared to the mass 2595 MeV for the reported charm $0\frac{1}{2}^-$ state, and $M_{\Lambda_{Ob}} = 6095$ MeV which is close to the mass 5912 MeV for the reported bottom $0\frac{1}{2}^-$ state. (47) predicts a mass of $M_{\Sigma_{Oc}} = 3126$ MeV for a possible charm $1\frac{1}{2}^-$ state, and a mass of $M_{\Sigma_{Ob}} = 6535$ MeV for a possible bottom $1\frac{1}{2}^-$ state.

F. Double-heavy baryons

For heavy baryons containing also anti-heavy quarks we note that a rerun of the preceding arguments using instead the reduction $\Phi_M = \phi_M e^{+i m_H x_0}$, amounts to binding an anti-heavy-light meson to the bulk instanton in the form of a zero-mode also in the fundamental representation of spin. Most of the results are unchanged except for pertinent minus signs. For instance, when binding one heavy-light and one anti-heavy-light meson, (35) now reads

$$\begin{aligned} \mathcal{L} = &+\mathcal{L}_0[a_I, X_\alpha] + \chi_Q^\dagger i\partial_t \chi_Q + \frac{3}{32\pi^2 a\rho^2} \chi_Q^\dagger \chi_Q \\ &- \chi_{\bar{Q}}^\dagger i\partial_t \chi_{\bar{Q}} - \frac{3}{32\pi^2 a\rho^2} \chi_{\bar{Q}}^\dagger \chi_{\bar{Q}} \\ &+ \frac{(\chi_Q^\dagger \chi_Q - \chi_{\bar{Q}}^\dagger \chi_{\bar{Q}})^2}{24\pi^2 a\rho^2 N_c}. \end{aligned} \tag{48}$$

As we indicated earlier the mass contributions are opposite for a heavy-light and anti-heavy-light meson. The general mass spectrum for baryons with N_Q heavy-quarks and $N_{\bar{Q}}$ anti-heavy quarks is

$$\begin{aligned} M_{\bar{Q}Q} = &+M_0 + (N_Q + N_{\bar{Q}}) m_H \\ &+ \left(\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2 \left(1 - \frac{15(N_Q - N_{\bar{Q}})}{4N_c} \right.\right. \\ &\left.\left. + \frac{5(N_Q - N_{\bar{Q}})^2}{3N_c^2}\right)\right)^{\frac{1}{2}} M_{KK} \\ &+ \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} M_{KK}. \end{aligned} \tag{49}$$

The double-heavy baryons with QQ content are heavier because of the larger U(1) Coulomb-like repulsion in (49).

1. Pentaquarks

For $N_Q = N_{\bar{Q}} = 1$ we identify the lowest state with $l = 1, n_z = n_\rho = 0$ with pentaquark baryonic states with the IJ^π assignments $\frac{1}{2}\frac{1}{2}^-$ and $\frac{1}{2}\frac{3}{2}^-$, and masses given by

$$M_{\bar{Q}Q} - M_N - 2m_H = 0. \tag{50}$$

Amusingly the spectrum is BPS as both the attraction and repulsion balances, and the two Coulomb-like self repulsions balance against the Coulomb-like pair attraction. Thus we predict a mass of $M_{\bar{c}c} = 4678$ MeV for the $\frac{1}{2}\frac{3}{2}^-$ which is close to the reported $P_c^+(4380)$ and $P_c^+(4450)$. We also predict a mass of $M_{\bar{b}c} = 8087$ MeV and $M_{\bar{b}b} = 11496$ MeV for the yet to be oberved pentaquarks. Perhaps a better estimate for the latters is to trade M_N in (54) for the observed light charmed pentaquark mass $M_{\bar{c}c} = 4678$ MeV using instead

$$M_{\bar{Q}Q} = M_{\bar{Q}'Q'} + 2(m_H - m_{H'}). \tag{51}$$

Using (51) we predict $M_{\bar{b}c} = 7789$ MeV and $M_{\bar{b}b} = 11198$ MeV, which are slightly lighter than the previous estimates. The present holographic construction based on the bulk instanton as a hedgehog in flavor-spin space does not support the $\frac{1}{2}\frac{5}{2}^+$ assignment suggested for the observed $P_c^+(4450)$ through the bound zero mode for the case $N_f = 2$.

2. Excited pentaquarks

For $N_Q = N_{\bar{Q}} = 1$ we now identify the lowest state with $l = 1, n_z = 1, n_\rho = 0$ with the odd parity pentaquarks O with assignments $\frac{1}{2}\frac{1}{2}^+$ and $\frac{1}{2}\frac{3}{2}^+$, and the $l = 1, n_z = 0$, $n_\rho = 1$ with the breathing or Roper R pentaquarks with the same assignments as the ground state. The mass relations for these states are ($E = O$, R)

$$M_{E\bar{Q}Q} - M_N - 2m_H = 0.82M_{KK}, \tag{52}$$

which can be traded for model independent relations

$$M_{E\bar{Q}Q} = 1.51m_N + 2m_H + 0.51(m_{H'} - M_{\lambda_{Q'}}) \tag{53}$$

by eliminating M_{KK} using the first relation in (45). Using (53) we predict $M_{E\bar{c}c} = 4944$ MeV, $M_{E\bar{b}c} = 8353$ MeV, $M_{E\bar{b}b} = 11762$ MeV as the new low lying excitations of heavy pentaquarks with the preceding assignments.

3. Delta-like pentaquarks

For $N_Q = N_{\bar{Q}} = 1$, the present construction allows also for Delta-type pentaquarks which we identify with $l = 3, n_z = n_\rho = 0$. Altogether, we have one $\frac{3}{2}\frac{1}{2}^-$, two $\frac{3}{2}\frac{3}{2}^-$, and one $\frac{3}{2}\frac{5}{2}^-$ states, all degenerate to leading order, with heavy flavor dependent masses

$$M_{\Delta\bar{Q}Q} - M_N - 2m_Q = 0.71M_{KK} \tag{54}$$

Again we can trade M_{KK} using the first relation in (45) to obtain the model independent relation

$$M_{\Delta\bar{Q}Q} = 1.57m_N + 2m_H + 0.57(m_{H'} - M_{\Lambda_{Q'}}). \tag{55}$$

In particular, we predict $M_{\Delta\bar{c}c} = 4976$ MeV, $M_{\Delta\bar{c}b} = 8385$ MeV, and $M_{\Delta\bar{b}b} = 11794$ MeV, which are yet to be observed.

VI. CONCLUSIONS

We have presented a top-down holographic approach to the single- and double-heavy baryons in the variant of $D4 - D8$ we proposed recently [35] (first reference). To order λm_Q^0, the heavy baryons emerge from the zero mode of a reduced (massless) vector meson that transmutes both its spin and negative parity, to a spin $\frac{1}{2}$ with positive parity in the bulk flavor instanton. Heavy mesons and anti-mesons bind on equal footing to the core instanton in holography in leading order in λ even in the presence of the Chern-Simons contribution. This is not the case in non-holographic models where the anti-heavy meson binding is usually depressed by the sign flip in the Wess-Zumino-Witten contribution [36]. Unlike in the Skyrme model, the bulk flavor instanton offers a model independent description of the light baryon core. The binding of the heavy meson over its Compton wavelength is essentially geometrical in the double limit of large λ followed by large m_Q.

We have shown that the bound state moduli yields a rich spectrum after quantization, that involves coupled rotational, translational and vibrational modes. The model-independent mass relations for the low-lying single-heavy baryon spectrum yield masses that are in overall agreement with the reported masses for the corresponding charm and bottom baryons. The spectrum also contains some newly excited states yet to be observed. When extended to double-heavy baryon spectra, the holographic construction yields a pair of degenerate heavy iso-doublets with $IJ^\pi = \frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ assignments. The model gives naturally a charmed pentaquark. It also predicts a number of new pentaquarks with both hidden charm and bottom, and five new Delta-like pentaquarks with hidden charm. The hedgehog flavor instanton when collectively quantized, excludes the $IJ^\pi = \frac{1}{2}\frac{5}{2}^+$ assignment for $N_f = 2$.

The shortcomings of the heavy-light holographic approach stem from the triple limits of large N_c and strong t'Hooft coupling $\lambda = g^2 N_c$, and now large m_H as well. The corrections are clear in principle but laborious in practice. Our simple construct can be improved through a more realistic extension such as improved holographic QCD [41]. Also a simpler, bottom-up formulation following the present general reasoning is also worth formulating for the transparency of the arguments.

Finally, it would be interesting to extend the current analysis for the heavy baryons to the more realistic case of $N_f = 3$ with a realistic mass for the light strange quark as well. Also, the strong decay widths of the heavy baryons and their exotics should be estimated. They follow from $1/N_c$ type corrections using the self-generated Yukawa-type potentials in bulk, much like those studied in the context of the Skyrme model [42]. We expect large widths to develop through S-wave decays, and smaller widths to follow from P-wave decays because of a smaller phase space. Also the hyperfine splitting in the heavy spectra is expected to arise through subleading couplings between the emerging spin degrees of freedom and the collective rotations and vibrations. The pertinent electromagnetic and weak form factors of the holographically bound heavy baryons can also be obtained following standard arguments [32,33]. Some of these issues will be addressed next.

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APPENDIX: QUANTIZATION OF THE INSTANTON MODULI

In this Appendix we summarize some of the essential steps for the quantization of the instanton moduli developed in [32], and fill up for some of the notations used in the main text. In the absence of the heavy mesons, we also take the large λ limit using the same rescaling to re-write the contributions of the light gauge fields as

$$S = aN_c\lambda S_{\rm YM}(A_M, \hat{A}_M) + aN_c S_1(A_0, \hat{A}_0, A_M, \hat{A}_M). \quad \text{(A1)}$$

Here A refers to the SU(2) part of the light gauge field, and $\hat{A}$ to its U(1) part. The equation of motion for A_M, $\hat{A}_M$ are at leading order of λ

$$D_N F_{NM} = 0 \quad \text{and} \quad \partial_N \hat{F}_{NM} = 0. \quad \text{(A2)}$$

They are solved using the flat instanton A_M and 0 for $\hat{A}_M$. The equation of motion for the time components are subleading

$$D_M F_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\hat{F}_{MN}F_{PQ} = 0$$
$$\partial_M \hat{F}_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ} trF_{MN}F_{PQ} = 0. \quad \text{(A3)}$$

They are solved using $A_0 = 0$ and a nonzero $\hat{A}_0$ as defined in the main text.

To obtain the spectrum we promote the moduli of the solution to be time dependent, i.e.

$$(a_I, X_\alpha) \to (a_I(t), X_\alpha(t)). \quad \text{(A4)}$$

Here a_I refers to the moduli of the global SU(2) gauge transformation. In order to satisfy the constraint equation (52) (Gauss's law) we need to impose a further gauge transformation on the field configuration

$$A_M^V = V^\dagger(A_M + \partial_M)V \quad \text{and} \quad A_0^V = V^\dagger\partial_t V. \quad \text{(A5)}$$

Inserting the transformed field configuration in the constraint equation, we find that V is solved by

$$-iV^\dagger\partial_t V = \Phi = -\partial_t X_N A_N + \chi_a \Phi_a, \quad \text{(A6)}$$

with $\chi_a[a_I]$ as defined in the main text. Putting the resulting slowly moving field configuration back in the action, allows for the light collective Hamiltonian [32]

$$\begin{aligned} H_0 &= M_0 + H_Z + H_\rho \\ H_Z &= -\frac{\partial_Z^2}{2m_z} + \frac{m_z\omega_z^2}{2}Z^2 \\ H_\rho &= -\frac{\nabla_y^2}{2m_y} + \frac{m_y\omega_\rho^2}{2}\rho^2 + \frac{Q}{\rho^2} \\ y &= \rho(a_1, a_2, a_3, a_4), \qquad a_I = a_4 + i\vec{a}\cdot\vec{\tau} \\ m_z &= \frac{m_y}{2} = 8\pi^2 aN_c, \qquad \omega_z^2 = \frac{2}{3}, \qquad \omega_\rho^2 = \frac{1}{6} \end{aligned} \quad \text{(A7)}$$

The eigenstates of H_ρ are given by $T^l(a)R_{l,n_\rho}(\rho)$, where T^l are the spherical harmonics on S^3. Under $SO(4) = \mathrm{SU}(2)\times\mathrm{SU}(2)/Z_2$ they are in the $(\frac{l}{2},\frac{l}{2})$ representations, where the two SU(2) factors are defined by the isometry $a_I \to V_L a_I V_R$. The left factor is the isospin rotation, and the right factor is the space rotation. This quantization describes $I = J = \frac{l}{2}$ states. The nucleon is realized as the lowest state with $l = 1$ and $n_\rho = n_z = 0$.

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