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Spectroscopy of singly, doubly, and triply bottom baryons

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Recently, some singly bottom baryons have been established experimentally, but none of the doubly or triply bottom baryons have been observed. Under the Regge phenomenology, the mass of an unobserved ground-state doubly or triply bottom baryon is expressed as a function of masses of the well-established light baryons and singly bottom baryons. Then, the values of Regge slopes and Regge intercepts for baryons containing one, two, or three bottom quarks are calculated. After that, the masses of the orbitally excited singly, doubly, and triply bottom baryons are estimated. Our predictions may be useful for the discovery of these baryons and their J^P assignments.

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I. INTRODUCTION

According to the Particle Data Group's latest "Review of Particle Physics" (RPP) [1], some singly bottom baryons [Λ_b, $\Lambda_b(5912)$, $\Lambda_b(5920)$, Σ_b, Σ_b^*, Ξ_b, Ξ_b', Ξ_b^*, and Ω_b] have been established. However, none of the doubly or triply bottom baryons have been observed. Therefore, in the present work we will focus on searching for mass relations which can be used to express the mass of a doubly or triply bottom baryon as a function of masses of the well-established baryons. This is the main motivation of this work.

It is noted that the Gell-Mann–Okubo formula [2] cannot be directly applied to the charmed and bottom hadrons due to higher-order breaking effects. The Regge trajectory ansatz is an effective phenomenological model to study mass relations [3–7] and mass spectra [8–15] for mesons and baryons. In a previous work [4], a way was proposed to express the mass of doubly charmed baryons as a function of masses of the well-established light baryons and singly charmed baryons. In the present work, under Regge phenomenology, we will express the mass of an unobserved ground state (the orbital quantum number $L = 0$) baryon containing one, two, or three bottom quarks as a function of the masses of the well-established light baryons and singly bottom baryons. The mass values will be given and compared with those obtained in many other approaches [7,8,16–63].

The slopes and intercepts of Regge trajectories are useful for many spectral and nonspectral purposes [3,11,64], for example, in the fragmentation [65] and recombination [66] models. Therefore, Regge slopes and Regge intercepts are fundamental constants of hadronic dynamics, perhaps more important than the masses of particular states [67]. Thus, the determination of Regge slopes and intercepts of hadrons is of great importance since this affords opportunities for a better understanding of the dynamics of strong interactions in the production processes of charmed and bottom hadrons at high energies and estimates of their production rates [64]. In our previous work in 2008 [3], the numerical values of Regge slopes and Regge intercepts for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ light and charmed baryon trajectories were extracted. In Ref. [11], Li *et al.* gave the values of Regge slopes and intercepts for all $SU(5)$ mesons (involving the u, d, s, c, b quarks). In Ref. [8], Ebert *et al.* gave the values of Regge slopes for singly charmed and singly bottom baryons. In the present work, we will extract Regge slopes and Regge intercepts for the singly, doubly, and triply bottom baryons and calculate masses of the orbitally excited baryons ($L = 1$, 2, 3, 4) lying on these Regge trajectories.

The remainder of this paper is organized as follows. In Sec. II, under Regge phenomenology, the masses of the unobserved ground-state singly, doubly, and triply bottom baryons Ω_b^*, Ω_{bb}^*, Ω_{bb}, Ξ_{bb}^*, Ξ_{bb}, and Ω_{bbb} will be given. In Sec. III, we will calculate Regge slopes and Regge intercepts for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ singly, doubly, and triply bottom baryon trajectories. After that, the masses of the orbitally excited baryons lying on these trajectories will be estimated. In Sec. IV, a short discussion and conclusion will be given.

II. MASSES OF THE UNOBSERVED Ω_b^*, $\Omega_{bb}^{(*)}$, $\Xi_{bb}^{(*)}$, AND Ω_{bbb} BARYONS

In this section, we will first give a short introduction of Regge phenomenology and express the quadratic masses of the ground-state unobserved Ω_b^*, $\Omega_{bb}^{(*)}$, $\Xi_{bb}^{(*)}$, and Ω_{bbb} baryons as functions of the quadratic masses of the well-established

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light baryons and singly bottom baryons. After that, the mass values will be given and compared.

A. Regge phenomenology

Regge theory is concerned with almost all aspects of strong interactions, including the particle spectra, the high-energy behavior of scattering amplitudes, and the forces between particles [68]. It is known from Regge theory that all mesons and baryons are associated with Regge trajectories (Regge poles which move in the complex angular momentum plane as a function of energy) [69]. Hadrons lying on the same Regge trajectory which have the same internal quantum numbers are classified into the same family [68,70]. Regge trajectories for hadrons can be parametrized as follows:

$$J = \alpha(M) = a(0) + \alpha' M^2, \tag{1}$$

where α' and $a(0)$ are, respectively, the slope and intercept of the Regge trajectory on which the particles lie. For a baryon multiplet, Regge intercepts and Regge slopes for different flavors can be related by the following relations (see Ref. [3] and references therein):

$$a_{iik}(0) + a_{jjk}(0) = 2a_{ijk}(0), \tag{2}$$

$$\frac{1}{\alpha'_{iik}} + \frac{1}{\alpha'_{jjk}} = \frac{2}{\alpha'_{ijk}}, \tag{3}$$

where i, j, and k denote arbitrary light or heavy quarks. Based on Eqs. (2) and (3), one can introduce two parameters γ_x and λ_x,

$$\gamma_x = \frac{1}{\alpha'_{nnx}} - \frac{1}{\alpha'_{nnn}}, \tag{4}$$

$$\lambda_x = a_{nnn}(0) - a_{nnx}(0), \tag{5}$$

where n represents the light unflavored quark u or d, and x denotes i, j, or k. Therefore,

$$a_{ijk}(0) = a_{nnn}(0) - \lambda_i - \lambda_j - \lambda_k, \tag{6}$$

$$\frac{1}{\alpha'_{ijk}} = \frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_k. \tag{7}$$

From Eq. (1), we have

$$J = a_{nnn}(0) + \alpha'_{nnn} M^2_{nnn}, \tag{8}$$

$$J = a_{ijk}(0) + \alpha'_{ijk} M^2_{ijk}. \tag{9}$$

From Eqs. (6)–(9), we have

$$M^2_{ijk} = (\alpha'_{nnn} M^2_{nnn} + \lambda_i + \lambda_j + \lambda_k)\left(\frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_k\right). \tag{10}$$

As demonstrated in Ref. [3], in order to evaluate the high-order effects, we introduce the parameter δ,

$$\delta_{ij,q} \equiv M^2_{iiq} + M^2_{jjq} - 2M^2_{ijq}, \tag{11}$$

where q is an arbitrary light or heavy quark. Combining Eqs. (10) and (11), we can prove that

$$\delta_{ij,q} = M^2_{iiq} + M^2_{jjq} - 2M^2_{ijq} = 2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j). \tag{12}$$

From Eq. (12), one can see that $\delta_{ij,q}$ is independent of the q quark.

B. Mass expressions and masses for the $\frac{3}{2}^+$ Ω_b^*, Ω_{bb}^*, Ξ_{bb}^*, and Ω_{bbb} baryons

For the $\frac{3}{2}^+$ multiplet, noticing that $\delta^{\frac{3}{2}^+}_{ij,q}$ is independent of q in the above relation (12), when $i = n$, $j = s$, $q = s, c, b$, Eq. (12) can be expressed as

$$\begin{aligned}\delta^{\frac{3}{2}^+}_{ns,q} &= M^2_{\Sigma^*} + M^2_{\Omega} - 2M^2_{\Xi^*} = M^2_{\Sigma_c^*} + M^2_{\Omega_c^*} - 2M^2_{\Xi_c^*} \\ &= M^2_{\Sigma_b^*} + M^2_{\Omega_b^*} - 2M^2_{\Xi_b^*}.\end{aligned} \tag{13}$$

When $i = s$, $j = b$, $q = n$, s, b, Eq. (12) can be expressed as

$$\begin{aligned}\delta^{\frac{3}{2}^+}_{sb,q} &= M^2_{\Xi^*} + M^2_{\Xi_{bb}^*} - 2M^2_{\Xi_b^*} = M^2_{\Omega} + M^2_{\Omega_{bb}^*} - 2M^2_{\Omega_b^*} \\ &= M^2_{\Omega_b^*} + M^2_{\Omega_{bbb}} - 2M^2_{\Omega_{bb}^*}.\end{aligned} \tag{14}$$

Using Eqs. (1) and (2), we obtain

$$\alpha'_{iik} M^2_{iik} + \alpha'_{jjk} M^2_{jjk} = 2\alpha'_{ijk} M^2_{ijk}. \tag{15}$$

With Eqs. (3) and (15), when the quark masses satisfy $m_j > m_i$, we can obtain [3]

$$\begin{aligned}\frac{\alpha'_{jjk}}{\alpha'_{iik}} = \frac{1}{2M^2_{jjk}} \times [&(4M^2_{ijk} - M^2_{iik} - M^2_{jjk}) \\ &+ \sqrt{(4M^2_{ijk} - M^2_{iik} - M^2_{jjk})^2 - 4M^2_{iik}M^2_{jjk}}].\end{aligned} \tag{16}$$

Therefore, for the $\frac{3}{2}^+$ multiplet,

$$\frac{\alpha'_{bbs}}{\alpha'_{nns}} = \frac{1}{2M^2_{\Omega^{*-}_{bb}}} \times [(4M^2_{\Xi^{*0}_b} - M^2_{\Sigma^{*+}} - M^2_{\Omega^{*-}_{bb}}) + \sqrt{(4M^2_{\Xi^{*0}_b} - M^2_{\Sigma^{*+}} - M^2_{\Omega^{*-}_{bb}})^2 - 4M^2_{\Sigma^{*+}}M^2_{\Omega^{*-}_{bb}}}],$$
$$\frac{\alpha'_{bbs}}{\alpha'_{sss}} = \frac{1}{2M^2_{\Omega^{*-}_{bb}}} \times [(4M^2_{\Omega^{*-}_b} - M^2_{\Omega^-} - M^2_{\Omega^{*-}_{bb}}) + \sqrt{(4M^2_{\Omega^{*-}_b} - M^2_{\Omega^-} - M^2_{\Omega^{*-}_{bb}})^2 - 4M^2_{\Omega^-}M^2_{\Omega^{*-}_{bb}}}],$$
$$\frac{\alpha'_{sss}}{\alpha'_{nns}} = \frac{1}{2M^2_{\Omega^-}} \times [(4M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}} - M^2_{\Omega^-}) + \sqrt{(4M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}} - M^2_{\Omega^-})^2 - 4M^2_{\Sigma^{*+}}M^2_{\Omega^-}}]. \quad (17)$$

With the identical equation $\frac{\alpha'_{bbs}}{\alpha'_{nns}} \equiv \frac{\alpha'_{bbs}}{\alpha'_{sss}} \times \frac{\alpha'_{sss}}{\alpha'_{nns}}$, we have

$$\begin{aligned}
&\frac{1}{2M^2_{\Omega^*_{bb}}} \times [(4M^2_{\Xi^*_b} - M^2_{\Sigma^*} - M^2_{\Omega^*_{bb}}) + \sqrt{(4M^2_{\Xi^*_b} - M^2_{\Sigma^*} - M^2_{\Omega^*_{bb}})^2 - 4M^2_{\Sigma^*}M^2_{\Omega^*_{bb}}}] \\
&= \frac{1}{2M^2_{\Omega^*_{bb}}} \times [(4M^2_{\Omega^*_b} - M^2_{\Omega} - M^2_{\Omega^*_{bb}}) + \sqrt{(4M^2_{\Omega^*_b} - M^2_{\Omega} - M^2_{\Omega^*_{bb}})^2 - 4M^2_{\Omega}M^2_{\Omega^*_{bb}}}] \\
&\times \frac{1}{2M^2_{\Omega}} \times [(4M^2_{\Xi^*} - M^2_{\Sigma^*} - M^2_{\Omega}) + \sqrt{(4M^2_{\Xi^*} - M^2_{\Sigma^*} - M^2_{\Omega})^2 - 4M^2_{\Sigma^*}M^2_{\Omega}}]. \quad (18)
\end{aligned}$$

In the following, we will give the mass expressions and mass values for the ground-state unobserved bottom baryons. Before this, we will apply the method first to the known Ω_c^* in order to see how well it works. From Eq. (13), we can obtain the squared mass expression for Ω_c^* as a function of the squared masses of Σ^*, Ξ^*, Ω, Σ_c^*, and Ξ_c^*,

$$M^2_{\Omega^*_c} = M^2_{\Sigma^*} + M^2_{\Omega} - 2M^2_{\Xi^*} - M^2_{\Sigma^*_c} + 2M^2_{\Xi^*_c}. \quad (19)$$

From the latest RPP [1], $M_{\Sigma^*} = 1385.0 \pm 2.3$ MeV, $M_{\Xi^*} = 1533.4 \pm 1.7$ MeV, $M_{\Omega} = 1672.45 \pm 0.29$ MeV, $M_{\Sigma^*_c} = 2518.1 \pm 0.9$ MeV, and $M_{\Xi^*_c} = 2645.9 \pm 0.5$ MeV. Inserting these mass values into the relation (19), we get $M_{\Omega^*_c} = 2769.8 \pm 2.6$ MeV. In the latest RPP [1], $M_{\Omega^*_c} = 2765.9 \pm 2.0$ MeV. Our result for $M_{\Omega^*_c}$ is in reasonable agreement with the experimental value. In the following we will apply the method to bottom baryons.

From Eq. (13), we can obtain the squared mass of Ω_b^* as a function of the squared masses of Σ^*, Ξ^*, Ω, Σ_b^*, and Ξ_b^*,

$$M^2_{\Omega^*_b} = M^2_{\Sigma^*} + M^2_{\Omega} - 2M^2_{\Xi^*} - M^2_{\Sigma^*_b} + 2M^2_{\Xi^*_b}. \quad (20)$$

From Eqs. (18) and (20), we can obtain the squared mass of Ω_{bb}^* as a function of the squared masses of Σ^*, Ξ^*, Ω, Σ_b^*, and Ξ_b^*,

$$M^2_{\Omega^*_{bb}} = 5M^2_{\Xi^*_b} - 2M^2_{\Xi^*} - M^2_{\Sigma^*_b} + \frac{M^2_{\Omega}}{2} + \frac{5M^2_{\Sigma^*}}{2} - \frac{2(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})(M^2_{\Xi^*} - M^2_{\Sigma^*})}{M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*}} + \frac{\sqrt{M^4_{\Omega} - 2M^2_{\Omega}(4M^2_{\Xi^*} + M^2_{\Sigma^*}) + (M^2_{\Sigma^*} - 4M^2_{\Xi^*})^2}}{2(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})} \times \sqrt{(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})(M^2_{\Omega} - 4M^2_{\Xi^*_b} - 2M^2_{\Xi^*} - 4M^2_{\Sigma^*_b} + M^2_{\Sigma^*}) + 4(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})^2}. \quad (21)$$

From Eqs. (14), (20), and (21), we can obtain the mass expressions for Ξ_{bb}^* and Ω_{bbb}.

$$M^2_{\Xi^*_{bb}} = 3M^2_{\Xi^*_b} + M^2_{\Xi^*} + M^2_{\Sigma^*_b} + \frac{M^2_{\Sigma^*}}{2} - \frac{M^2_{\Omega}}{2} - \frac{2(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})(M^2_{\Xi^*} - M^2_{\Sigma^*})}{M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*}} + \frac{\sqrt{M^4_{\Omega} - 2M^2_{\Omega}(4M^2_{\Xi^*} + M^2_{\Sigma^*}) + (M^2_{\Sigma^*} - 4M^2_{\Xi^*})^2}}{2(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})} \times \sqrt{(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})(M^2_{\Omega} - 4M^2_{\Xi^*_b} - 2M^2_{\Xi^*} - 4M^2_{\Sigma^*_b} + M^2_{\Sigma^*}) + 4(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})^2}, \quad (22)$$

$$M^2_{\Omega_{bbb}} = 9M^2_{\Xi^*_b} + \frac{9M^2_{\Sigma^*}}{2} - \frac{M^2_{\Omega}}{2} - \frac{6(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})(M^2_{\Xi^*} - M^2_{\Sigma^*})}{M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*}} + \frac{\sqrt{-2M^2_{\Omega}(4M^2_{\Xi^*} + M^2_{\Sigma^*}) + M^4_{\Omega} + (M^2_{\Sigma^*} - 4M^2_{\Xi^*})^2}}{2(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})/3} \times \sqrt{(M^2_{\Omega} - 2M^2_{\Xi^*} + M^2_{\Sigma^*})(M^2_{\Omega} - 4M^2_{\Xi^*_b} - 2M^2_{\Xi^*} - 4M^2_{\Sigma^*_b} + M^2_{\Sigma^*}) + 4(M^2_{\Xi^*_b} - M^2_{\Sigma^*_b})^2}. \quad (23)$$

From the latest RPP [1], $M_{\Sigma^*} = 1385.0 \pm 2.3$ MeV, $M_{\Xi^*} = 1533.4 \pm 1.7$ MeV, $M_{\Omega} = 1672.45 \pm 0.29$ MeV, $M_{\Sigma_b^*} = 5833.6 \pm 2.4$ MeV, and $M_{\Xi_b^*} = 5952.1 \pm 3.3$ MeV. Inserting the corresponding mass values into Eq. (20), we have $M_{\Omega_b^*} = 6069.4 \pm 6.9$ MeV. Therefore, $M_{\Omega_b^*} - M_{\Omega_b} = 21.4 \pm 7.2$ MeV. This value is close to the following experimental data: $M_{\Sigma_b^*} - M_{\Sigma_b} = 21.2 \pm 2.0$ MeV, and $M_{\Xi_b^{*-}} - M_{\Xi_b^{\prime -}} = 20.3 \pm 0.1$ MeV, as it should be. The prediction $M_{\Omega_b^*} - M_{\Omega_b} = 21.4 \pm 7.2$ MeV is in reasonable agreement with the value (24.0 ± 0.7 MeV) given in Ref. [62]. Inserting the above mass values of Σ^*, Ξ^*, Ω, Σ_b^*, and Ξ_b^* into Eq. (21), we get $M_{\Omega_{bb}^*} = 10431 \pm 40$ MeV. From Eq. (23) we have $M_{\Omega_{bbb}} = 14788 \pm 80$ MeV. Similarly, we can obtain the expression for $M_{\Xi_{bb}^*}$ and its value is 10316.3 ± 37.8 MeV (10316 ± 38 MeV when truncated to the 1 MeV digit). A comparison of the masses of Ω_b^*, Ω_{bb}^*, Ω_{bbb}, and Ξ_{bb}^* extracted in the present work and those given in other references is shown in Table I.

TABLE I. The masses of the ground-state unobserved singly, doubly, and triply bottom baryons (in units of MeV). Our results are labeled with "Pre". The isospin splittings are $M_{\Xi_{bb}^-} - M_{\Xi_{bb}^0} = 2.3 \pm 0.7$ MeV and $M_{\Xi_{bb}^{*-}} - M_{\Xi_{bb}^{*0}} = 1.6 \pm 0.6$ MeV.

	Ω_b^*	Ξ_{bb}	Ω_{bb}	Ξ_{bb}^*	Ω_{bb}^*	Ω_{bbb}
Pre	6069.4 ± 6.9	10199 ± 37	10320 ± 37	10316 ± 38	10431 ± 40	14788 ± 80
[16]	6170 ± 150	10170 ± 140	10320 ± 140	10220 ± 150	10380 ± 140	14830 ± 100
[17]	6051	10003	10142	10048	10181	14248
[18]	6102	10130	10422	10144	10432	14569
[19]	6000 ± 160	9780 ± 70	9850 ± 70	10350 ± 80	10280 ± 50	13280 ± 100
[20]	6102	10340	10454	10367	10486	14834
[21]		10093	10133	10180	10200	
[22]	6088	10202	10359	10237	10389	
[23]	6035 ± 60			10250 ± 120	10395 ± 120	14760 ± 180
[24]	6090 ± 50	10340 ± 100	10370 ± 100	10370 ± 100	10400 ± 100	
[25]		10272	10369	10337	10429	
[26]		10000 ± 80	10090 ± 70			
[27]						14370 ± 80
[28]						14300
[29]	$6085 \pm 47 \pm 20$	$10143 \pm 30 \pm 23$	$10273 \pm 27 \pm 20$	$10178 \pm 30 \pm 24$	$10308 \pm 27 \pm 21$	$14366 \pm 9 \pm 20$
[30]	6036 ± 80	10185 ± 53	10250 ± 51	10191 ± 56	10283 ± 51	14370 ± 10
[31]		10162 ± 12		10184 ± 12		
[32]	5986, 6135	10334, 10467	10397, 10606	10431, 10525	10495, 10664	15023, 15175
[33]		9960 ± 90	9970 ± 90	10300 ± 200	10400 ± 200	14300 ± 200
[34]		10080	10100			
[35]		9940 ± 910		10330 ± 1090		
[36]	6028 – 6132	9998 – 10137	10154 – 10269	10053 – 10206	10228 – 10355	14444 – 14688
[37]	6096	10339, 10344	10478	10468, 10473	10607	15118
[38]	6096	10062	10208	10101	10244	14276
[39]		10090	10180	10130	10200	
[40]		9800	9890	9840	9930	
[41]		10185	10271	10216	10289	
[42]	6079	10189	10293	10218	10321	
[45]						14200 – 15670
[46]	6094	10314	10447	10339	10467	
[49]		10194	10267			14398
[50]	$6044 \pm 18^{+20}_{-21}$	$10127 \pm 13^{+12}_{-26}$	$10225 \pm 9^{+12}_{-13}$	$10151 \pm 14^{+16}_{-25}$	$10246 \pm 10^{+18}_{-12}$	
[51]				9858	10088	14440
[52]		10197^{+10}_{-17}	10260^{+14}_{-34}	10236^{+9}_{-17}	10297^{+5}_{-28}	
[53]	$6069 \pm 34(^{-18}_{+30})(^{+35}_{-0})$	$10314 \pm 46^{-10}_{+11}$	$10365 \pm 40(^{-11}_{+12})(^{+16}_{-0})$	$10333 \pm 55^{-7}_{+6}$	$10383 \pm 39(^{-8}_{+8})(^{+12}_{-0})$	
[54]		10230	10320	10280	10360	
[55]		10300	10340	10340	10380	
[56]		10160	10340			
[58]	6142	10440	10620	10451	10628	15129
[59]		10100	10180			
[60]	6090					
[62]	6082.8 ± 5.6					
[71]	6058.9 ± 8.1					

C. Masses of the ground unobserved $\frac{1}{2}^+$ Ω_{bb} and Ξ_{bb} baryons

Based on Eqs. (13) and (14), we have

$$(M^2_{\Omega^*_{bb}} - M^2_{\Xi^*_{bb}}) + (M^2_{\Xi^*} - M^2_{\Sigma^*}) = (M^2_{\Omega^*_b} - M^2_{\Sigma^*_b}). \quad (24)$$

For the $\frac{1}{2}^+$ multiplet, we have a mass relation similar to Eq. (24),

$$(M^2_{\Omega_{bb}} - M^2_{\Xi_{bb}}) + (M^2_{\Xi} - M^2_{\Sigma}) = (M^2_{\Omega_b} - M^2_{\Sigma_b}). \quad (25)$$

The linear forms in Eqs. (24) and (25) can satisfy the instanton model [72] and the SU(8) symmetry [73].

For the $\frac{1}{2}^+$ multiplet, based on Eq. (12), $\delta^{\frac{1}{2}^+}_{nb,s} + \delta^{\frac{1}{2}^+}_{sb,n}$ can be expressed as

$$\delta^{\frac{1}{2}^+}_{nb,s} + \delta^{\frac{1}{2}^+}_{sb,n} = M^2_{\Sigma} + M^2_{\Omega_{bb}} - 2\left(\frac{3M^2_{\Xi_b} + M^2_{\Xi'_b}}{4}\right) + M^2_{\Xi} + M^2_{\Xi_{bb}} - 2\left(\frac{3M^2_{\Xi_b} + M^2_{\Xi'_b}}{4}\right). \quad (26)$$

In Ref. [3], it was pointed out that the values of δ_{ij} are only a little different between different multiplets for the same i and j, although δ_{ij} are very sensitive to different quark flavors i or j. As done in Refs. [3,4], assuming that $\delta^{\frac{1}{2}^+}_{ij} = \delta^{\frac{3}{2}^+}_{ij}$, we have $\delta^{\frac{1}{2}^+}_{nb} + \delta^{\frac{1}{2}^+}_{sb} = \delta^{\frac{3}{2}^+}_{nb} + \delta^{\frac{3}{2}^+}_{sb}$. The expression for $\delta^{\frac{3}{2}^+}_{sb}$ has been given in Eq. (14). Based on Eq. (12), when $i = n$, $j = b$, $q = s$, $\delta^{\frac{3}{2}^+}_{nb,s}$ can be expressed as

$$\delta^{\frac{3}{2}^+}_{nb,s} = M^2_{\Sigma^*} + M^2_{\Omega^*_{bb}} - 2M^2_{\Xi^*_b}. \quad (27)$$

With Eqs. (14), (25), (26), and (27), we can obtain the expressions for $M_{\Omega_{bb}}$ and $M_{\Xi_{bb}}$. Then, inserting the masses of Σ, Ξ, Σ_b, Ξ_b, Ξ'_b, Ω_b, Σ^*, Ω, and Ξ^*_b from RPP [1] and Ω^*_b, Ω^*_{bb} obtained in Sec. II, we can get the following values: $M_{\Omega_{bb}} = 10320 \pm 37$ MeV and $M_{\Xi_{bb}} = 10199 \pm 37$ MeV. A comparison of the masses of Ξ_{bb} and Ω_{bb} extracted in the present work and those given in other references is also shown in Table I.

D. Isospin splitting for the ground Ξ^*_{bb} and Ξ_{bb} baryons

Considering the difference between u and d quarks (which is the difference between the isospin multiplet), for $\frac{3}{2}^+$ baryons, Eq. (24) can be expressed as

$$(M^2_{\Omega^{*-}_{bb}} - M^2_{\Xi^{*0}_{bb}}) + (M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}) = (M^2_{\Omega^{*-}_b} - M^2_{\Sigma^{*+}_b}),$$
$$(M^2_{\Omega^{*-}_{bb}} - M^2_{\Xi^{*-}_{bb}}) + (M^2_{\Xi^{*-}} - M^2_{\Sigma^{*-}}) = (M^2_{\Omega^{*-}_b} - M^2_{\Sigma^{*-}_b}). \quad (28)$$

Similarly, considering the isospin-breaking effects, for $\frac{1}{2}^+$ baryons, Eq. (25) can be expressed as

$$(M^2_{\Omega^-_{bb}} - M^2_{\Xi^0_{bb}}) + (M^2_{\Xi^0} - M^2_{\Sigma^+}) = (M^2_{\Omega^-_b} - M^2_{\Sigma^+_b}),$$
$$(M^2_{\Omega^-_{bb}} - M^2_{\Xi^-_{bb}}) + (M^2_{\Xi^-} - M^2_{\Sigma^-}) = (M^2_{\Omega^-_b} - M^2_{\Sigma^-_b}). \quad (29)$$

From Eq. (28), we can obtain the expression for the isospin splitting $M_{\Xi^{*-}_{bb}} - M_{\Xi^{*0}_{bb}}$ and its value is 1.6 ± 0.6 MeV (where the uncertainties come from the errors of the input data). From Eq. (29), we can also obtain the expression for the isospin splitting $M_{\Xi^-_{bb}} - M_{\Xi^0_{bb}}$ and its value is 2.3 ± 0.7 MeV.

III. REGGE PARAMETERS AND MASSES OF THE ORBITALLY EXCITED SINGLY, DOUBLY, AND TRIPLY BOTTOM BARYONS

In Sec. II, all of the masses of unobserved ground-state $\frac{1}{2}^+$ and $\frac{3}{2}^+$ singly, doubly, and triply bottom baryons were given. In this section, we will calculate the Regge parameters and the masses of the orbitally excited baryons lying on the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ bottom baryon trajectories.

A. Regge parameters and masses of the orbitally excited singly, doubly, and triply bottom baryons lying on the $\frac{3}{2}^+$ trajectories

With the help of $\alpha'_{nns} = 2/(M^2_{\Sigma(2030)} - M^2_{\Sigma^*})$ from Eq. (17), we can obtain the expression for α'_{bbs} and get its value ($\alpha'_{bbs} = 0.175 \pm 0.014$ GeV^{-2}). Similarly, using the masses of baryons presented in Eq. (17), with the aid of Eq. (3), we can obtain expressions for α'_{bnn}, α'_{bsn}, α'_{bss}, α'_{bbn}, and α'_{bbb}. Their values are given as follows: $\alpha'_{bnn} = 0.295 \pm 0.022$ GeV^{-2}, $\alpha'_{bsn} = 0.294 \pm 0.020$ GeV^{-2}, $\alpha'_{bss} = 0.292 \pm 0.019$ GeV^{-2}, $\alpha'_{bbn} = 0.176 \pm 0.015$ GeV^{-2}, and $\alpha'_{bbb} = 0.125 \pm 0.011$ GeV^{-2}. The results are shown in Table II.

From Eq. (1), we have the Regge intercepts

$$a(0) = J - \alpha' M^2. \quad (30)$$

For example, with the mass for Σ^*_b and the value of α'_{bnn} obtained above, from Eq. (30), $a_{bnn}(0)$ is found to be

TABLE II. Regge slopes (in units of GeV^{-2}) and Regge intercepts of the ground-state $\frac{3}{2}^+$ singly, doubly, and triply bottom baryons.

	bnn (Σ^*_b)	bsn (Ξ^*_b)	bss (Ω^*_b)	bbn (Ξ^*_{bb})	bbs (Ω^*_{bb})	bbb (Ω_{bbb})
α'	0.295 ± 0.022	0.294 ± 0.020	0.292 ± 0.019	0.176 ± 0.015	0.175 ± 0.014	0.125 ± 0.011
$a(0)$	-8.55 ± 0.74	-8.91 ± 0.71	-9.26 ± 0.68	-17.22 ± 1.44	-17.58 ± 1.41	-25.89 ± 2.14

TABLE III. The masses of the singly bottom baryons lying on the $\frac{3}{2}^+$ trajectories (in units of MeV). Our results are labeled with "Pre". The numbers in boldface are the experimental values taken as the input.

	Σ_b^*					Ξ_b^*					Ω_b^*				
J^P	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$
Pre	$\mathbf{5833.6 \pm 2.4}$	6117 ± 20	6388 ± 38	6648 ± 55	6898 ± 71	$\mathbf{5952.1 \pm 3.3}$	6232 ± 21	6499 ± 38	6756 ± 54	7003 ± 68	6069.4 ± 6.9	6345 ± 22	6609 ± 38	6863 ± 52	7108 ± 66
[1]	5833.6 ± 2.4					5952.1 ± 3.3									
[8]	5834	6084	6260	6459	6635	5963	6226	6414	6610	6782	6088	6334	6517	6713	6884
[16]	5850 ± 200					6020 ± 170					6170 ± 150				
[17]	5769					5912					6051				
[20]	5858	6172	6333			5980	6201	6395			6102	6311	6497		
[22]	5834	6083	6262			5963	6218	6390			6088	6336	6502		
[24]	5850 ± 40					5980 ± 40					6090 ± 50				
[46]	5845	6144									6094	6345			
[47]		5980 ± 180					6180 ± 160					6430 ± 130			
[60]	5844					5967					6090				
[62]	5832.7 ± 2					5959 ± 4					6082.8 ± 5.6				
[63]	5805	6090	6340	6540											

-8.55 ± 0.74. Similarly, Regge intercepts $a(0)$ for other $\frac{3}{2}^+$ trajectories can be determined from Eq. (30). The results are also shown in Table II. From Eq. (1), the masses of orbitally excited states are

$$M_J^2 = [J - a(0)]/\alpha'. \tag{31}$$

Then, with the values for α'_{bnn} and $a_{bnn}(0)$ obtained above, from Eq. (31), the masses of the orbitally excited baryons ($L = 1, 2, 3, 4$, while $J^P = \frac{5}{2}^-, \frac{7}{2}^+, \frac{9}{2}^-, \frac{11}{2}^+$) lying on the Σ_b^* trajectories can be expressed as functions of the masses of the well-established light baryons and singly bottom baryons. The numerical results are shown in Table III. Similarly, the mass expressions for all of the orbitally excited singly and doubly $\frac{3}{2}^+$ bottom baryons can be extracted. The numerical results are shown in Tables III and IV, respectively. (Here and below, the masses of orbitally excited baryons are truncated to the 1 MeV digit.) The wave function of a baryon is antisymmetric. For the triply bottom baryon Ω_{bbb}, the flavor wave function is symmetric, while the color wave function is antisymmetric. Therefore, its spin-orbital wave function should be symmetric. So, spin symmetry ($S_{3q} = \frac{3}{2}$) requires orbital symmetry. Therefore, the odd-parity Ω_{bbb} baryons ($L = 1, 3$) can only have spin antisymmetry ($S_{3q} = \frac{1}{2}$). This will lead to real masses that are lower than our calculated results for such baryons. The above $J^P = \frac{5}{2}^-, \frac{9}{2}^-$ will be changed to $J^P = \frac{3}{2}^-, \frac{7}{2}^-$ for $L = 1, 3$ Ω_{bbb} baryons, respectively. The numerical results are shown in Table V.

B. Regge parameters and masses of the orbitally excited singly and doubly baryons lying on the $\frac{1}{2}^+$ trajectories

For the $\frac{1}{2}^+$ trajectories, according to the heavy quark symmetry in the heavy quark limit, the Regge slopes of Σ_b, Ξ'_b, Ω_b, Ξ_{bb}, and Ω_{bb} can be considered to be approximately equal to those of Σ_b^*, Ξ_b^*, Ω_b^*, Ξ_{bb}^*, and Ω_{bb}^*, respectively. According to Refs. [7,74], $\alpha'_{\Lambda_b} \simeq \alpha'_{\Sigma_b}$, $\alpha'_{\Xi_b} \simeq \alpha'_{\Xi'_b}$. We take these approximations in this work. Similar to the $\frac{3}{2}^+$ trajectories, from Eq. (30), one has the Regge intercepts $a(0)$ for $\frac{1}{2}^+$ trajectories. The numerical results are shown in Table VI.

With the values for $a(0)$ and α' listed in Table VI, from Eq. (31), the masses of the orbitally excited baryons ($L = 1, 2, 3, 4$, while $J^P = \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, \frac{9}{2}^+$) lying on the $\frac{1}{2}^+$ trajectories can be extracted. The numerical results are shown in Tables VII and VIII.

IV. DISCUSSION AND SUMMARY

In the present work, we focused on studying masses of the unobserved singly, doubly, and triply bottom baryons. The results for the ground states do not depend on unobservable parameters (such as quark masses and Regge slopes) and distrustful resonances. The Regge slopes and Regge intercepts of the singly, doubly, and triply

TABLE IV. The masses of the doubly bottom baryons lying on the $\frac{3}{2}^+ \Xi_{bb}^*$ and Ω_{bb}^* trajectories (in units of MeV). Our results are labeled with "Pre".

J^P	Ξ_{bb}^*					Ω_{bb}^*				
	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$
Pre	10316 ± 38	10588 ± 59	10853 ± 80	11112 ± 99	11365 ± 118	10431 ± 40	10700 ± 60	10964 ± 80	11221 ± 99	11472 ± 117
[20]	10367	10731	10608			10486	10766	10732		
[21]	10133	10580	10510			10257	10670	10627		
[22]	10237	10661				10389	10798			
[46]	10339	10759				10467	10808			
[61]	10352	10695	11011							

TABLE V. The masses of the triply bottom baryons lying on the $\frac{3}{2}^+ \Omega_{bbb}$ trajectory (in units of MeV). Our results are labeled with "Pre".

J^P	Ω_{bbb}				
	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{7}{2}^+$	$\frac{7}{2}^-$	$\frac{11}{2}^+$
Pre	14788 ± 80	15055 ± 101	15318 ± 123	15577 ± 143	15831 ± 163
[16]	14830 ± 100	14950 ± 110			
[33]	14300 ± 200	14900 ± 200			
[20]	14834	14976	15101		
[43]	14372	14620	14800		
[44]	14372	14720	14960		
[48]	$14371 \pm 4 \pm 11 \pm 1$	14714 ± 29	14969 ± 40		

TABLE VI. Regge intercepts and Regge slopes (in units of GeV^{-2}) of the ground-state $\frac{1}{2}^+$ singly and doubly bottom baryons.

	bnn (Λ_b)	bnn (Σ_b)	bsn (Ξ_b)	bsn (Ξ_b')	bss (Ω_b)	bbn (Ξ_{bb})	bbs (Ω_{bb})
α'	0.295 ± 0.022	0.295 ± 0.022	0.294 ± 0.020	0.294 ± 0.020	0.292 ± 0.019	0.176 ± 0.015	0.175 ± 0.014
$a(0)$	-8.82 ± 0.68	-9.48 ± 0.73	-9.36 ± 0.68	-9.85 ± 0.71	-10.19 ± 0.69	-17.80 ± 1.41	-18.17 ± 1.39

bottom baryon trajectories were extracted. After that, the masses of the orbitally excited ($L = 1, 2, 3, 4$) singly, doubly, and triply bottom baryons were calculated. In this work, all of the input masses of baryons used in the calculations were taken from the latest RPP [1]. The uncertainties of the results only come from the errors of the input masses. The Regge slopes and intercepts used in this work were also calculated from masses of light baryons and singly bottom baryons. No systematic error due to any small deviation from the Regge trajectories has been taken into account in this work.

In Table I, we compare the masses of ground-state unobserved singly, doubly, and triply bottom baryons extracted in the present work and those given in other references. In Tables II and VI, the Regge parameters (slopes and intercepts) are given for the $\frac{3}{2}^+$ and $\frac{1}{2}^+$ trajectories, respectively. In Tables III, IV, and V, the masses of singly, doubly, and triply bottom baryons lying on the $\frac{3}{2}^+$ trajectories are shown. In Tables VII and VIII, the masses of singly and doubly bottom baryons lying on the $\frac{1}{2}^+$ trajectories are shown, respectively. Our results are neither too big nor too small.

Experimentally, there are two orbitally excited singly bottom baryons: $\Lambda_b(5912)$ and $\Lambda_b(5920)$ [1], with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ and $M_{\Lambda_b(5912)} = 5912.11 \pm 0.13 \pm 0.23$ MeV, $M_{\Lambda_b(5920)} = 5919.81 \pm 0.23$ MeV, respectively. $\Lambda_b(5912)$ and $\Lambda_b(5920)$ were first reported by the LHCb Collaboration [75]. $\Lambda_b(5920)$ was confirmed by the CDF Collaboration [76]. The mass difference of these two states is small (< 8 MeV). This indicates that the spin-orbital coupling has a small impact on the mass of the singly bottom baryon. In the present work, the mass of the $J^P = \frac{3}{2}^-$ ($L = 1$) singly bottom baryon is determined to be 5913 ± 21 MeV (shown in Table I), which is consistent with the experimental data.

In Ref. [23], Bjorken pointed out that $\frac{M_{\Omega_{bbb}}}{M_\Upsilon} = 1.56 \pm 0.02$ and gave the mass of Ω_{bbb} as 14760 ± 180 MeV, which is consistent with our result, $M_{\Omega_{bbb}} = 14788 \pm 80$ MeV (shown in Table I). In the present work, the central values of mass splittings ($M_{\Omega_{bb}^*} - M_{\Xi_{bb}^*} = 10431 - 10316 = 115$ MeV, $M_{\Omega_{bb}} - M_{\Xi_{bb}} = 10320 - 10199 = 121$ MeV) are reasonable (close to the usual constituent quark difference $m_s - m_{u,d} \approx 120$ MeV). The central values of spin splittings ($M_{\Omega_{bb}^*} - M_{\Omega_{bb}} = 10431 - 10320 = 111$ MeV and $M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 10316 - 10199 = 117$ MeV) are a little big. However, in Refs. [19,33,35], the central values of these spin splittings were even bigger than ours (more than 300 MeV).

TABLE VII. The masses of the singly bottom baryons lying on the $\frac{1}{2}^+$ trajectories (in units of MeV). Our results are labeled with "Pre". The numbers in boldface are the experimental values taken as the input.

J^P	Λ_b $\frac{1}{2}^+$	Λ_b $\frac{3}{2}^-$	Λ_b $\frac{5}{2}^+$	Λ_b $\frac{7}{2}^-$	Λ_b $\frac{9}{2}^+$	Ξ_b $\frac{1}{2}^+$	Ξ_b $\frac{3}{2}^-$	Ξ_b $\frac{5}{2}^+$	Ξ_b $\frac{7}{2}^-$	Ξ_b $\frac{9}{2}^+$
Pre	**5619.51** $\pm$ **0.23**	5913 $\pm$ 21	6193 $\pm$ 40	6461 $\pm$ 58	6718 $\pm$ 74	**5793.1** $\pm$ **1.8**	6080 $\pm$ 19	6354 $\pm$ 37	6616 $\pm$ 53	6869 $\pm$ 68
[1]	5619.51 $\pm$ 0.23	5919.73 $\pm$ 0.32				5793.1 $\pm$ 1.8				
[8]	5620	5942	6196	6411	6599	5803	6130	6373	6581	6762
[16]	5650 $\pm$ 200					5730 $\pm$ 180				
[20]	5612	5941	6183			5806	6093	6300		
[22]	5622	5947	6197	6405		5812	6130	6365	6558	
[46]	5618	5939	6212							
[47]		5880 $\pm$ 110					6070 $\pm$ 130			
[57]	5619	5920	6153	6351	6526	5801	6106	6349	6559	6747
[60]	5624	5890				5825	6076			
[62]	5619.7 $\pm$ 1.7	5940 $\pm$ 2				5790-5800	6115 $\pm$ 4			
[63]	5585	5920	6165	6360	6580					

J^P	Σ_b $\frac{1}{2}^+$	Σ_b $\frac{3}{2}^-$	Σ_b $\frac{5}{2}^+$	Σ_b $\frac{7}{2}^-$	Σ_b $\frac{9}{2}^+$	Ξ'_b $\frac{1}{2}^+$	Ξ'_b $\frac{3}{2}^-$	Ξ'_b $\frac{5}{2}^+$	Ξ'_b $\frac{7}{2}^-$	Ξ'_b $\frac{9}{2}^+$	Ω_b $\frac{1}{2}^+$	Ω_b $\frac{3}{2}^-$	Ω_b $\frac{5}{2}^+$	Ω_b $\frac{7}{2}^-$	Ω_b $\frac{9}{2}^+$
Pre	**5813.4** $\pm$ **2.8**	6098 $\pm$ 20	6369 $\pm$ 39	6630 $\pm$ 56	6881 $\pm$ 72	**5935.0** $\pm$ **0.5**	6215 $\pm$ 19	6486 $\pm$ 34	6745 $\pm$ 49	6994 $\pm$ 63	**6048.0** $\pm$ **1.9**	6325 $\pm$ 18	6590 $\pm$ 34	6844 $\pm$ 48	7090 $\pm$ 62
[1]	5813.4 $\pm$ 2.8					5935.0 $\pm$ 0.5					6048.0 $\pm$ 1.9				
[8]	5808	6096	6284	6500	6687	5936	6234	6432	6641	6821	6064	6340	6529	6736	6915
[16]	5800 $\pm$ 190	6000 $\pm$ 180				5960 $\pm$ 170	6140 $\pm$ 140				6110 $\pm$ 160	6260 $\pm$ 150			
[20]	5833	6101	6325			5970	6190	6393			6081	6304	6492		
[22]	5805	6076	6248			5937	6212	6377			6065	6330	6490		
[46]	5823	6132	6397								6076	6336	6561		
[47]		5960 $\pm$ 180					6170 $\pm$ 170					6430 $\pm$ 130			
[60]	5789	6039				5913	6157				6037	6278			
[62]	5611.5 $\pm$ 2	5940 $\pm$ 2				5930 $\pm$ 5					6052.1 $\pm$ 5.4				
[63]	5795	6070	6325	6525											

TABLE VIII. The masses of the doubly bottom baryons lying on the $\frac{1}{2}^{+}\Xi_{bb}$ and Ω_{bb} trajectories (in units of MeV). Our results are labeled with “Pre”.

	Ξ_{bb}					Ω_{bb}				
J^P	$\frac{1}{2}^{+}$	$\frac{3}{2}^{-}$	$\frac{5}{2}^{+}$	$\frac{7}{2}^{-}$	$\frac{9}{2}^{+}$	$\frac{1}{2}^{+}$	$\frac{3}{2}^{-}$	$\frac{5}{2}^{+}$	$\frac{7}{2}^{-}$	$\frac{9}{2}^{+}$
Pre	10199 ± 37	10474 ± 59	10742 ± 80	11004 ± 99	11259 ± 118	10320 ± 37	10593 ± 58	10858 ± 77	11118 ± 96	11372 ± 115
[16]	10170 ± 140	10390 ± 150				10320 ± 140	10520 ± 150			
[20]	10340	10495	10676			10454	10619	10720		
[21]	10093	10343	10497			10210	10462			
[22]	10202	10408				10359	10566			
[33]	9960 ± 90	10430 ± 150				9970 ± 90	10570 ± 150			
[46]	10314	10476	10592			10447	10608	10729		
[61]	10322	10692	11002							

The isospin splitting for the $\frac{1}{2}^{+}$ doubly bottom baryons in the present work, $M_{\Xi_{bb}^{-}} - M_{\Xi_{bb}^{0}} = 2.3 \pm 0.7$ MeV, is smaller than 5.3 ± 1.1 MeV in Ref. [77] and 6.3 ± 1.7 MeV in Ref. [78]. For the $\frac{3}{2}^{+}$ doubly bottom baryons, we have also calculated the value of the isospin splitting: $M_{\Xi_{bb}^{*-}} - M_{\Xi_{bb}^{*0}} = 1.6 \pm 0.6$ MeV. In Ref. [78] strong and electromagnetic sources of isospin breaking were handled separately. Regge theory appears to be a pure QCD emergent phenomenon. In this work, we did not consider the electromagnetic corrections separately because the electromagnetic corrections cancel out in Eqs. (14) and (29). These can be tested by experiments in the future.

In Ref. [8], Ebert *et al.* gave the values of the Regge slopes for singly bottom baryons, which are a little bigger than the corresponding values in this work. For example, $\alpha'_{\Xi_b} = 0.349 \pm 0.019$ GeV^{-2} in Ref. [8], while $\alpha'_{\Xi_b} = 0.294 \pm 0.020$ GeV^{-2} in this work. The mass of the orbitally excited state decreases with the increase of the value of the Regge slope. From Tables III and VII, one can compare the masses given in Ref. [8] and those given in the present work. The mass differences are small when $L = 1, 2$.

From Tables I, III–V, and VII–VIII, we can see that the masses of some ground-state and orbitally excited singly, doubly, and triply bottom baryons obtained here are in good agreement with the existing experimental data and those given in many other different approaches. However, our predictions for some highly orbitally excited baryons deviate considerably from some predictions in the literature. We expect that our predictions on the masses and Regge parameters in this manuscript can all be tested at LHCb in the near future.

In this work, the squared mass relations were used rather than the linear mass relations taken in Refs. [72,73]. When the mass relations include light baryons, bottom baryons, and doubly bottom baryons [such as Eqs. (14) and (29)], the quadratic mass relations and the linear mass relations lead to significantly different values. Besides the addition of baryon spectra, searching for doubly and triply bottom baryons can numerically check the quadratic mass relations and the linear mass relations. The triply bottom baryons Ω_{bbb} are of considerable theoretical interest [27,79], since they are free of light quark contamination and may serve as a clean probe of the interplay between perturbative and nonperturbative QCD. Therefore, more efforts should be given to study doubly and triply bottom baryons both theoretically and experimentally.

The mass expressions, Regge parameters, and mass values in this work could be useful for the discovery of the unobserved singly, doubly, and triply bottom baryon states and the J^P assignment of these baryon states when they are observed in the near future.

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