PHYSICAL REVIEW D **95,** 115027 (2017)

Low-scale *D*-term inflation and the relaxion mechanism

Jason L. Evans,[1] Tony Gherghetta,[2] Natsumi Nagata,[3] and Marco Peloso[2]
[1]*School of Physics, KIAS, Seoul 130-722, Korea*
[2]*School of Physics & Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[3]*Department of Physics, University of Tokyo, Tokyo 113–0033, Japan*
(Received 20 April 2017; published 23 June 2017)

We present a dynamical cosmological solution that simultaneously accounts for the early inflationary stage of the Universe and solves the supersymmetric little hierarchy problem via the relaxion mechanism. First, we consider an inflationary potential arising from the D term of a new $U(1)$ gauge symmetry with a Fayet-Iliopolous term that is independent of the relaxion. A technically natural, small $U(1)$ gauge coupling, $g \lesssim 10^{-8}$, allows for a low Hubble scale of inflation, $H_I \lesssim 10^5$ GeV, which is shown to be consistent with Planck data. This feature is then used to realize a supersymmetric two-field relaxion mechanism, where the second field is identified as the inflaton provided that $H_I \lesssim 10$ GeV. The inflaton controls the relaxion barrier height allowing the relaxion to evolve in the early Universe and scan the supersymmetric soft masses. After electroweak symmetry is broken, the relaxion settles at a local supersymmetry-breaking minimum with a range of F-term values that can naturally explain supersymmetric soft mass scales up to 10^6 GeV.

DOI: 10.1103/PhysRevD.95.115027

I. INTRODUCTION

A natural solution to the hierarchy problem in the standard model (SM) has motivated the development of particle physics for decades with predictions of new states near the electroweak scale. However, to date, the experimental results at the Large Hadron Collider (LHC) have begun to call into question of whether naturalness is a relevant guide for physics beyond the standard model. For example, in supersymmetric (SUSY) models, colored superpartner masses need to be heavier than the TeV scale to evade LHC searches [1–3], exacerbating the tuning, while constraints on other models addressing naturalness (such as composite Higgs models [4]) lead to a similar conclusion.

Recently a new approach to naturalness, which evades the LHC constraints, used the idea of cosmological relaxation [5] (for previous studies with a similar idea, see Refs. [6–8]). In this process, an axionlike particle (the relaxion) associated with a shift symmetry, is coupled directly to the Higgs field during a nearly de Sitter phase of the Universe. This coupling contributes to the mass squared of the Higgs field, since initially the relaxion has a very large field value. During the cosmological evolution of the relaxion, caused by an explicit breaking of the shift symmetry, the field value changes and the Higgs mass squared is reduced. Eventually the Higgs mass squared reaches a critical value where it flips sign, triggering electroweak symmetry breaking (EWSB) with the Higgs field developing a vacuum expectation value (VEV). The generation of the Higgs VEV then backreacts on the relaxion potential, causing the relaxion to stop at a local minimum. The slope of the relaxion potential, which is proportional to the shift-symmetry breaking parameter, can then be chosen so that the Higgs VEV is naturally set to be at the weak scale. This provides a technically natural solution to the hierarchy problem.

However, the relaxion process itself is not a completely satisfactory explanation of the hierarchy problem. First, it can only address radiative corrections to the Higgs mass that depend on a cutoff scale that is generally much lower than the Planck scale [5]. The hierarchy problem is therefore only partly alleviated. Second, it requires a very low inflation scale in order to naturally realize a weak scale Higgs VEV. However low-scale inflation models often introduce some new tuning that is not solved by the relaxion process [9]. The first problem is not a concern if the relaxion process is instead embedded into a supersymmetric framework. Supersymmetric models with soft mass scales near the PeV scale (10^6 GeV) are mesotuned, but can easily accommodate a 125 GeV Higgs boson mass [10–17]. Thus if the relaxion actually scans the supersymmetric soft masses, then the cutoff scale in the relaxion mechanism can be identified with the soft mass scale. This naturally provides an explanation for the tuning using just the relaxion [18], or else a two-field relaxion mechanism [19] can be generalized to supersymmetry [20], which preserves the QCD axion solution to the strong CP problem. For other recent studies on the relaxion mechanism, see Refs. [21–33].

In this paper we address the second problem in the context of D-term inflation [34–37] and identify the second field in the two-field relaxion mechanism with the inflaton. In order to realize an inflationary model with a very low Hubble scale [38], the potential must be very flat or else the density perturbations will not satisfy the cosmic microwave

background (CMB) constraints [39]. One way of accomplishing this is to take small-field inflation and tune the initial condition so that the potential is very flat. Although this may work, it is not a very appealing approach to inflation since it is difficult to justify why the initial value of the field is so tuned. Furthermore, when applied to relaxion models, this tuning destroys the naturalness of the relaxion process and the tuning of the Higgs sector has merely been transferred to the inflationary sector.

Large-field inflation, on the other hand, is in general fairly insensitive to the initial field value. However, it is difficult to naturally obtain a sufficiently flat potential to realize the density perturbations if the scale of inflation is too low. A model that combines low-scale inflation with the insensitivity to initial conditions, typical of large-field inflation, can be found in supersymmetry. In D-term inflation [34–37], the Fayet-Iliopoulos (FI) term of some new $U(1)$ gauge symmetry is responsible for inflation, and therefore at tree level the potential is completely flat. Although this flatness is broken at the loop level, it provides the right conditions to obtain low-scale inflation. We show that a successful model of D-term inflation occurs for a $U(1)$ gauge coupling, $g \simeq 7 \times 10^{-9}$, corresponding to a Hubble scale $H_I \simeq 10^5$ GeV. Such a low value of g is technically natural since radiative corrections vanish in the limit $g \to 0$. With this low Hubble scale, CMB modes are produced approximately 39 e-folds before the end of inflation (contrary to the 50–60 e-folds required in typical models). Within D-term inflation this produces a spectral tilt in agreement with observations. However after inflation ends, there is the possibility that cosmic strings will form because the $U(1)$ phase has different values across different patches in the sky. This problem can be evaded if one considers a dynamical generation of the FI term [40], where the $U(1)$ gauge symmetry is broken during inflation, by an amount that negligibly affects the inflationary evolution. This is achieved via a superpotential coupling that induces a spatial alignment of the phase of the $U(1)$ breaking field that prevents the formation of topological defects at the end of inflation. Finally, we show that the fields responsible for this breaking, together with two additional $U(1)$ singlets, allow for a sufficiently fast conversion of the inflationary energy to standard model fields (a decay through the D-term potential is not fast enough, due to the smallness of the gauge coupling g).

With a naturally flat, low-scale inflation model, the inflaton in D-term inflation can be identified with the second field (the "amplitudon") of the supersymmetric two-field relaxion model [20]. In this model the inflaton is coupled to the relaxion and controls the barrier height of the relaxion potential, and also helps to avoid a potential isocurvature problem in the original two-field relaxion model. The slow-roll evolution of the inflaton periodically eliminates the relaxion barrier, allowing the relaxion to move in a stepwise fashion until, after electroweak symmetry is broken, it is eventually trapped at a local supersymmetry-breaking minimum. A quadratic potential with shift-symmetry breaking mass parameter, m_S, controls the slope of the relaxion potential. For a soft mass scale, $m_{\rm SUSY} = 10^5$ GeV, this parameter is constrained to be 10^{-9} GeV $\lesssim m_S \lesssim 10^{-6}$ GeV, provided that the Hubble scale satisfies $H_I \lesssim 10$ GeV. This leads to a model that simultaneously incorporates low-scale D-term inflation consistent with Planck data, and solves the little hierarchy problem in supersymmetric models, while preserving the QCD axion solution to the strong CP problem.

The plan of the paper is as follows. In Sec. II we present a phenomenologically viable D-term inflation model with a low Hubble scale that is consistent with the CMB data, does not form cosmic strings, and has a successful reheating to standard model fields. This low-scale inflation model is then combined with the supersymmetric relaxion mechanism in Sec. III. In Sec. IV, we summarize our results and provide some concluding remarks. The paper ends with two appendixes that contain further details of our model. In Appendix A we present the details of the dynamical generation of the D term, which provides the breaking of the $U(1)$ symmetry during inflation, preventing the formation of cosmic strings, and helps to facilitate reheating. Other details concerning the quantum and thermal corrections arising from the $U(1)$ symmetry breaking are then discussed in Appendix B.

II. D-TERM INFLATION

A. D-term inflation model

Let us begin by reviewing the D-term inflation model [34–37] in light of recent cosmological observations. The basic model of D-term inflation contains three chiral superfields, T, Φ_+, and Φ_-, which have charges of 0, $+1$, and -1 under a $U(1)$ gauge symmetry, respectively. This model takes advantage of the FI term of the $U(1)$ gauge symmetry, with which the auxiliary field of the $U(1)$ gauge field is given by

$$D = g(|\phi_+|^2 - |\phi_-|^2 - \xi), \tag{1}$$

where g is the $U(1)$ gauge coupling, ξ is the FI term, and $\phi_\pm$ are the scalar components of $\Phi_\pm$. We take $\xi > 0$ in what follows. The superpotential for this model is

$$W = \kappa T \Phi_+ \Phi_-, \tag{2}$$

where κ is a dimensionless parameter, which is taken to be real and positive. We write the scalar components of T as

$$T = \frac{1}{\sqrt{2}}(\tau + i\sigma) + \cdots. \tag{3}$$

In the following discussion, we regard σ as the inflaton and consider the case where $|\sigma| \gg |\tau|$.[1] The tree-level scalar potential for this model is then

$$V_{\text{tree}} = \kappa^2 \left[\frac{\tau^2 + \sigma^2}{2} (|\phi_-|^2 + |\phi_+|^2) + |\phi_+ \phi_-|^2 \right] + \frac{g^2}{2} [|\phi_+|^2 - |\phi_-|^2 - \xi]^2. \quad (4)$$

This potential has a SUSY-preserving minimum at $\tau = \sigma = \phi_- = 0$ and $|\phi_+| = \sqrt{\xi}$ with $V_{\text{tree}} = 0$.

If σ has a large field value, however, we can find a local minimum with $V_{\text{tree}} > 0$ at which $\phi_+ = \phi_- = 0$, and the charged fields $\phi_\pm$ have a mass squared

$$m_\pm^2 = \frac{\kappa^2 \sigma^2}{2} \mp g^2 \xi. \quad (5)$$

This local minimum becomes unstable when $|\sigma|$ is below the critical field value

$$\sigma_c \equiv \frac{g}{\kappa} \sqrt{2\xi}. \quad (6)$$

Thus, the initial field value of σ must satisfy $\sigma \gg \sigma_c$. As long as this condition is satisfied, the initial value of the inflaton field is unimportant, like all large-scale inflation models. At this local minimum, the tree-level potential for σ is completely flat: $V_{\text{tree}} = g^2 \xi^2 / 2$.

However, in order to realize a phenomenologically acceptable model, the slope of the inflaton potential must be nonzero. Fortunately, the correct slope is provided by the quantum corrections encoded in the Coleman-Weinberg term [41]. Since we consider the case $\sigma \gg \sigma_c$, we can expand the Coleman-Weinberg potential keeping the leading order term in $g^2 \xi / (\kappa^2 \sigma^2)$. In this limit, we find that the potential is well approximated by

$$V \simeq \frac{g^2 \xi^2}{2} \left[1 + \frac{g^2}{8\pi^2} \ln \left(\frac{\kappa^2 \sigma^2}{2Q^2} \right) \right], \quad (7)$$

where Q is a renormalization scale. Using this potential, the slow-roll parameters are found to be

$$\epsilon \equiv \frac{M_P^2}{2V^2} \left(\frac{\partial V}{\partial \sigma} \right)^2 \simeq \frac{g^4}{32\pi^4} \left(\frac{M_P}{\sigma} \right)^2, \quad (8)$$

$$\eta \equiv \frac{M_P^2}{V} \frac{\partial^2 V}{\partial \sigma^2} \simeq -\frac{g^2}{4\pi^2} \left(\frac{M_P}{\sigma} \right)^2, \quad (9)$$

where $M_P = 2.4 \times 10^{18}$ GeV denotes the reduced Planck mass. At this point, it is already clear that $\epsilon \ll |\eta|$, which we later see is important for realizing an acceptable inflation model in this context.

B. CMB constraints on D-term inflation

To determine the value of ϵ and η, which are constrained by the CMB data, we need to obtain the value of σ at the time when the CMB modes left the horizon. This value can be determined in terms of the number of e-folds of inflation that occurred after the CMB was set,

$$N_{\text{CMB}} = \int H_I dt = \int_{\sigma_c}^{\sigma_{\text{CMB}}} \frac{d\sigma}{M_P \sqrt{2\epsilon}} = \frac{2\pi^2}{g^2 M_P^2} (\sigma_{\text{CMB}}^2 - \sigma_c^2), \quad (10)$$

where H_I is the Hubble parameter during inflation, and σ_{CMB} and σ_c are the field values when the CMB was set and inflation ends, respectively. We find that for $\kappa \ll 10^{-2}$, η is suppressed and $n_s - 1 \equiv d \ln P / d \ln k$ is too large (where P is the scalar power spectrum and k is the wave number). We therefore assume that κ is large enough so that σ_{CMB} is not near the critical point σ_c. In this case, $\sigma_{\text{CMB}} \gg \sigma_c$, and thus the expression (10) can be solved to give

$$\sigma_{\text{CMB}} \simeq \frac{g M_P}{\pi} \sqrt{\frac{N_{\text{CMB}}}{2}}. \quad (11)$$

Notice that if $g \ll 1$, we do not need super-Planckian excursions for inflation to work. As we show below, low-scale inflation requires g to be very small and inflation occurs for field values well below M_P.

Equation (11) allows us to determine the slow-roll parameters in terms of N_{CMB},

$$\epsilon_{\text{CMB}} = \frac{g^2}{16\pi^2} \frac{1}{N_{\text{CMB}}}, \qquad \eta_{\text{CMB}} = -\frac{1}{2N_{\text{CMB}}}. \quad (12)$$

Using these expressions, the spectral tilt is then found to be

$$n_s - 1 = 2\eta_{\text{CMB}} - 6\epsilon_{\text{CMB}} \simeq 2\eta_{\text{CMB}} = -\frac{1}{N_{\text{CMB}}}, \quad (13)$$

where we can neglect ϵ_{CMB} since it is loop suppressed relative to η_{CMB}. As can be seen from this expression, η_{CMB}, and therefore n_s, only depends on N_{CMB}. Typically for large-scale inflation the number of e-folds is $\simeq$50–60. For this model, this would give a spectral tilt $n_s \gtrsim 0.98$, which is already excluded by current Planck results [42]. However, since we now consider low-scale inflation, N_{CMB} is modified. In fact, the number of e-folds of inflation after the CMB is set is significantly altered if the scale of inflation is much lower than that assumed in ordinary large-scale inflation models.

[1]The imaginary part is chosen to be the inflaton so that when the relaxion mechanism is discussed, it can more readily be identified with the amplitudon [20], *i.e.*, the second field in the two-field relaxion scenario [19].

Let us determine the number of e-foldings after the CMB is set for low-scale inflation. The number of e-foldings $N_e(k)$ that corresponds to a wave number k is defined by

$$e^{N_e(k)} \equiv \frac{a_{\rm end}}{a_k}, \tag{14}$$

where $a_{\rm end}$ is the value of the scale factor at the end of inflation, and $a_k \equiv k/H_I$. Then, we obtain [43–45]

$$\begin{aligned} N_e(k) &= -\ln k + \ln H_I + \ln\left(\frac{a_{\rm end}}{a_{\rm reh}}\right) + \ln\left(\frac{a_{\rm reh}}{a_{\rm eq}}\right) + \ln\left(\frac{a_{\rm eq}}{a_0}\right) \\ &= -\ln k + \ln H_I + \frac{1}{3}\ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right) + \frac{1}{4}\ln\left(\frac{\rho_{\rm eq}}{\rho_{\rm reh}}\right) + \ln\left(\frac{a_{\rm eq}}{a_0}\right), \end{aligned} \tag{15}$$

where $\rho_{\rm end}$, $\rho_{\rm reh}$, $\rho_{\rm eq}$ are the energy densities at the end of inflation, at the end of reheating, and at the time of matter-radiation equality, respectively; $a_{\rm eq}$ and a_0 are the scale factors at the time of matter-radiation equality and the present Universe, respectively. In this derivation, we have assumed instantaneous reheating with a sudden transition from matter to radiation domination [where the matter domination is due to the coherent oscillations of the field ϕ_+ before it decays[2]; changes in the thermalization during reheating modify the value of $N_e(k)$, as discussed in [46]]. This radiation-dominated Universe persists until the time of matter-radiation equality. Note that this estimation suffers from uncertainty that originates from the assumption on the cosmological history; for instance, we obtain a smaller $N_e(k)$ if there is an additional matter-dominated period between reheating and big bang nucleosynthesis.

Now we set k equal to the default pivot scale taken by the Planck collaboration [42], $k = 0.05\ {\rm Mpc}^{-1}$. We then obtain

$$\begin{aligned} N_{\rm CMB} &\equiv N_e(k = 0.05\,{\rm Mpc}^{-1}) \\ &\simeq 38.9 + \frac{1}{3}\ln\left(\frac{H_I}{10^5\,{\rm GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{\rm reh}^{1/4}}{100\,{\rm GeV}}\right), \end{aligned} \tag{16}$$

where $\rho_{\rm reh}$ should be understood as the energy density at the time at which the equation of state of the plasma formed at reheating becomes $w = 1/3$. We also note that we can disregard the very small variation of the Hubble parameter, and thus fix $H_I \simeq \rho_{\rm end}^{1/2}/(\sqrt{3}M_P)$ at $N_e = N_{\rm CMB}$.

Using the expression (16) for the number of e-folds after the CMB is set and the expression for the spectral tilt in Eq. (13), we show in Fig. 1 the contours of the spectral tilt as a function of the reheating energy density and Hubble parameter. The blue area in this figure depicts the Planck + BICEP2 + Keck Array combined 1σ range for the spectral tilt [47]. The limit obtained from only the Planck TT + lowP [42] data extends the allowed 1σ region into the pink area. The entire parameter space shown in Fig. 1 falls within the 2σ error bands of both results. The gray shaded region is theoretically excluded since $\rho_{\rm reh}$ exceeds the energy density of the inflation potential. We thus find that the D-term inflation model, with a low Hubble scale $H_I \lesssim 10^5$ GeV, can actually explain the observed value of n_s with a sufficiently high ($\gtrsim$100 GeV) reheating temperature for baryogenesis.

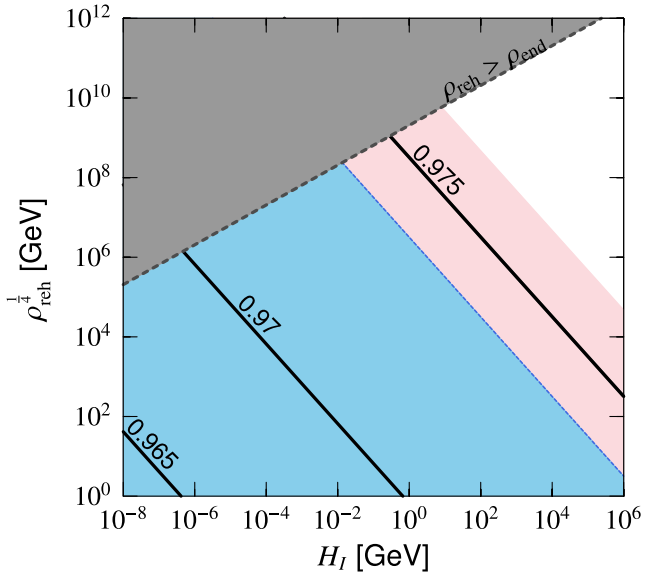

FIG. 1. A plot of the reheating temperature (more precisely, $\rho_{\rm reh}^{1/4}$) as a function of the Hubble parameter, H_I, for various contours of n_s (0.975, 0.97, and 0.965 from top to bottom, which correspond to $N_{\rm CMB} = 40$, 33.3, and 28.6, respectively). The blue area shows the 1σ range given by the Planck + BICEP2 + Keck Array combined results [47]. If one considers only the Planck TT + lowP result [42], also the pink area is included at 1σ. The gray shaded region is theoretically excluded since $\rho_{\rm reh}$ exceeds the energy density of the inflation potential.

Now that we have seen that an acceptable spectral tilt can be realized for low-scale inflation, we next need to verify that this model can generate cosmological perturbations of the right amplitude. The size of the cosmological perturbations is determined by the power spectrum, which is related to the inflation scale through

$$A_s \simeq \frac{V}{24\pi^2 M_P^4 \epsilon_{\rm CMB}} \simeq \frac{\xi^2}{3(1-n_s)M_P^4}, \tag{17}$$

where we have used the expression for $\epsilon_{\rm CMB}$ in Eq. (12). Therefore, the observational value of A_s determines $\sqrt{\xi}$,

$$\sqrt{\xi} \simeq 9.0 \times 10^{15}\ {\rm GeV} \times \left(\frac{1-n_s}{0.03}\right)^{\frac{1}{4}} \left(\frac{A_s}{2.1\times 10^{-9}}\right)^{\frac{1}{4}}. \tag{18}$$

[2]As we discuss later, this field carries most of the inflationary energy right after inflation.

The gauge coupling g is determined from the Hubble parameter during inflation via the relation

$$3M_P^2H_I^2 \simeq \frac{g^2\xi^2}{2}, \tag{19}$$

which, using (18), becomes

$$g \simeq \sqrt{6}\frac{M_P H_I}{\xi} \simeq 7.4\times 10^{-9} \times \left(\frac{H_I}{10^5\ \text{GeV}}\right)\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{2}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{2}}. \tag{20}$$

Note that this very small value for g is technically natural since the gauge coupling quantum corrections vanish in the limit $g \to 0$. This means that a small gauge coupling at a high energy scale remains small at low energies, and there is no need to tune the coupling against radiative corrections.

Finally, we study the mass spectrum of this model after the $U(1)$ gauge symmetry is spontaneously broken. After inflation, ϕ_+ develops a VEV of $\langle\phi_+\rangle = \sqrt{\xi}$. This causes T and Φ_- in the superpotential (2) to form a vectorlike mass term with a mass, $\kappa\sqrt{\xi}$. Notice that since $\langle\Phi_+\rangle$ does not break supersymmetry, the superfield description still holds. As a consequence, the scalar and fermionic components of T and Φ_- have an identical mass of $\kappa\sqrt{\xi}$. On the other hand, Φ_+ is absorbed by the $U(1)$ gauge vector superfield to form a massive vector superfield with a mass of

$$m_{Z'} = g\sqrt{2\xi} = 9.4\times 10^7\ \text{GeV} \times \left(\frac{H_I}{10^5\ \text{GeV}}\right)\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{4}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{4}}. \tag{21}$$

More specifically, a massless Nambu-Goldstone boson that originates from ϕ_+ is absorbed by the $U(1)$ gauge boson via the gauge interaction, while the massless fermionic component of Φ_+ combines with the $U(1)$ gaugino via the gaugino interaction to form a massive Dirac fermion. The radial component of ϕ_+ acquires a mass, $g\sqrt{2\xi}$, which is required by supersymmetry to form a massive vector superfield. As a result, after inflation, we have a vectorlike chiral superfield with a mass of $\kappa\sqrt{\xi}$ and a massive vector superfield with a mass of $m_{Z'} = g\sqrt{2\xi}$.

C. Cosmic strings

1. Cosmic string problem

One complication of D-term inflation is that the generation of cosmic strings after inflation ends. When the $U(1)$ symmetry is broken, the phase of the $U(1)$ breaking field takes different values in different patches of the sky. This leads to the formation of cosmic strings [48,49]. Since the $U(1)$ symmetry is broken at the end of inflation, these cosmic strings contribute to the CMB anisotropies, and thus are stringently constrained by the CMB data [50–55]. The contribution of cosmic strings to the CMB angular power spectrum $C_\ell^{(\text{str})}$ is approximately given by

$$\ell(\ell+1)C_\ell^{(\text{str})} = \mathcal{O}(100)\times T_{\text{CMB}}^2(G\mu)^2, \tag{22}$$

where T_{CMB} is the CMB temperature, G is the gravitational constant, and μ is the mass per unit length of the string, which is given by[3]

$$\mu = 2\pi\langle\phi_+\rangle^2 = 2\pi\xi. \tag{23}$$

Therefore, in our model, the size of $G\mu$ is predicted from Eq. (18) to be

$$G\mu \simeq 3.4\times 10^{-6}\times\left(\frac{1-n_s}{0.03}\right)^{\frac{1}{2}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{\frac{1}{2}}. \tag{24}$$

On the other hand, the Planck 2015 data [42] give a severe bound on this quantity: $G\mu < 3.3\times 10^{-7}$. This clearly shows that the minimal D-term inflation model is disfavored due to the formation of cosmic strings.[4]

There are several proposed ways to solve this problem; however, it is difficult to implement many of them in the context of low-scale inflation. One possible solution [50] is to assume that the cosmological fluctuations are due to some curvaton. This mechanism, however, will not end up working for the relaxion process, since there is also F-term SUSY breaking during inflation that generically gives too large of a mass to the curvaton. Another possible solution is to take a nonminimal Kähler potential [51,56,57]. In these scenarios, either n_s is too large [51,56] or the power spectrum scales down with g [57], and so the low-scale implementation is ruled out by CMB measurements. Another solution is to consider D-term inflation on the part of the potential below the critical point [58]. In this regime, inflation occurs after the $U(1)$ charged field, ϕ_+, obtains a VEV and so no cosmic strings can form. However, this does not work for low-scale inflation since ϵ is much too large. A more elaborate solution is to supplement the $U(1)$ gauge symmetry by a global $SU(2)$ symmetry so that the vacuum manifold is simply connected. In this case,

[3]As discussed in Sec. II B, the masses of the scalar boson and the $U(1)$ gauge boson are equal, and thus the cosmic strings that are generated after the $U(1)$ symmetry breaking are Bogomol'nyi-Prasad-Sommerfield (BPS) strings.

[4]It was previously argued [50] that this constraint may be evaded by taking a very small κ. In this case, inflation occurs in the vicinity of the critical value σ_c given in Eq. (6), and thus $\sigma_{\text{CMB}} \simeq \sigma_c$. On the other hand, Eq. (17) shows that if n_s is very close to 1, we can obtain a sufficiently small ξ to evade the cosmic string bound. Such a value of n_s can be obtained by taking a very small κ, which makes $\sigma_{\text{CMB}} \simeq \sigma_c$ very large and thus $|\eta|$ very small. Nevertheless, this possibility is now excluded by the Planck result, as it restricts the value of n_s and thus ξ cannot become sufficiently small.

instead of topologically stable cosmic strings, semilocal strings are produced when the symmetry is broken [59], which are in general less dangerous compared with stable strings. It turns out, however, that CMB measurements can restrict even semilocal strings [60], and in fact this solution is disfavored by the Planck result [42]. In addition, the presence of a global $SU(2)$ symmetry leads to the formation of textures, which are again severely constrained by the Planck data.

Instead in the next subsection we present one solution that works for our parameter choices, and that we are later able to use in the context of the relaxion mechanism.

2. Dynamical D terms

Cosmic strings form because the $U(1)$ gauge symmetry breaks after inflation is over and each patch of the sky has a different phase for the $U(1)$ breaking field. This can be remedied by breaking the $U(1)$ symmetry before the CMB modes exit the horizon during inflation. This can occur in models where the D term is dynamically generated [40], due to a hidden sector breaking of the $U(1)$ that generates the FI term. Note that the breaking of this symmetry must be sufficiently large, so that Hubble fluctuations do not restore the symmetry in the inflationary sector. This will be accomplished by adding a marginal coupling in the superpotential. Dynamically generated D terms are appealing, since they can more naturally explain a FI term much smaller then the Planck scale. If the hidden $U(1)$ breaking is appropriately coupled to the visible sector, it prevents string formation. The details of how this works are given in Appendix A; however, we summarize the main features below.

Let us add the following superpotential terms to Eq. (2),

$$\Delta W = \kappa_+ T M_+ \Phi_- + \kappa_- T M_- \Phi_+, \tag{25}$$

where $M_\pm$ are the hidden sector fields, which develop VEVs of $\mathcal{O}(\sqrt{\xi})$ with $\langle M_+\rangle \neq \langle M_-\rangle$ to generate the FI term dynamically. In order not to deform the D-term potential considerably, we take $|\kappa_\pm|$ to be much smaller than the $U(1)$ gauge coupling g.[5] Then, using Eq. (A5), we have a linear term for ϕ_+ during inflation,

$$V \supset \frac{\sigma^2}{2}(\kappa\kappa_+^* \phi_+ M_+^* + \text{H.c.}). \tag{26}$$

Because of this linear term, ϕ_+ has a nonzero VEV during inflation, with the minimum of the potential occurring for a particular phase. Since this is the only minimum of the potential until inflation ends, $\kappa^2\sigma^2/2 \simeq g^2\xi$, the VEV in all patches of the sky is driven to the same phase, as shown in Appendix B 1 b. This prevents the formation of CMB-size cosmic strings, provided that the fluctuations in the phase direction are sufficiently small. As discussed in Appendix B 1 c, this requirement gives an upper bound, (B15), on the Hubble parameter during inflation, which becomes

$$H_I < 1\times 10^9 \text{ TeV} \times \left(\frac{|\kappa_+|}{10^{-12}}\right)\left(\frac{|M_+|}{10^{16}\text{ GeV}}\right) \times \left(\frac{\kappa}{10^{-2}}\right)^{-\frac{1}{2}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{4}}\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{2}}. \tag{27}$$

Thus for the values of H_I satisfying the CMB constraints (see Fig. 1), we can always find a value[6] of $|\kappa_+| \ll g$, which satisfies this condition for $M_+ \simeq \sqrt{\xi}$.

Once inflation ends, the Universe is reheated. If the reheat temperature is large enough, thermal fluctuations could generate strings that are much smaller than the CMB size. Although these do not appear in the power spectrum, they could form stable energy configurations, which could overclose the Universe. However for this model, as the end of inflation nears, the VEV of ϕ_+ grows. By the time inflation ends, it is large enough that the VEV of ϕ_+ is always larger than the maximum reheat temperature. Since the temperature sets the size of the thermal fluctuations of the VEV, no cosmic strings form from thermal fluctuations. See Appendix B 2 for the relevant details.

D. Reheating

With knowledge of the mass spectrum of the fields in the inflaton sector obtained in Sec. II B, we can discuss reheating after inflation. In Refs. [61,62], kinetic mixing between the gauge fields associated with the SM hypercharge and the $U(1)$ symmetry driving inflation was used to reheat to SM fields. However, since the $U(1)$ gauge coupling is very small, this kinetic mixing is too small to reheat the SM above the weak scale.[7] In Ref. [63], inflation is driven by the quadratic part of the D-term potential. Inflation models driven by a mass term are no longer compatible with experimental results. Furthermore, the method of reheating used there depends on the gauge coupling g, and in our case it would lead to a reheat temperature lower than the weak scale.

Since our low-scale inflation model is a hybrid inflation model, the energy after inflation is divided between the

[5]Since the couplings $\kappa_\pm$ explicitly break the shift symmetry for the T field, a mass term for the σ field is induced. However, this contribution can be sufficiently small compared with the Coleman-Weinberg effects (7) for $|\kappa_\pm| \ll g$. See Eq. (A15) for a more detailed condition.

[6]The actual constraint on the relative size of these couplings can be found in Eq. (A15) and the discussion that follows. As g, and thus H_I, increases this constraint becomes weaker, and therefore CMB-sized cosmic strings can always be prevented by choosing κ_+ appropriately.

[7]We assume that the reheating temperature is above the electroweak scale to facilitate baryogenesis.

inflaton, σ, and the radial part of ϕ_+. This second component is by far the dominant contribution for $g \ll \kappa$. We thus neglect the energy associated with the inflaton oscillations in this analysis.[8] Therefore, in order to transfer the vacuum energy of inflation into radiation energy, the field ϕ_+ needs to decay to lighter states. Because the mass of ϕ_+ is much smaller than the scale of its VEV, it is difficult to find viable decay modes.[9] In fact, generic decay of ϕ_+ to fields that couple with a strength greater than g will be kinematically forbidden. On the other hand, couplings of ϕ_+ smaller than g would be kinematically allowed but would give a reheating temperature smaller than the weak scale.

Nonrenormalizable couplings do not help. These nonrenormalizable operators arise from integrating out some heavier fields. The mass of these heavier fields, in general, is large since they couple directly to ϕ_+, which has a large VEV. A nonrenormalizable operator with a mass scale of the order of the VEV of ϕ_+ leads to a suppression of $(m_{\phi_+}/\langle\phi_+\rangle)^{2(n-4)}$, where n is the dimension of the operator, in the decay width. Even for $n = 5$, this gives too much suppression to obtain a reheating temperature above the weak scale. If the field couples weakly to ϕ_+ it could lead to a smaller mass scale when the particle is integrated out. However, since it couples weakly to ϕ_+ this nonrenormalizable interaction gets additional suppression from the small coupling it has with ϕ_+, making it difficult to obtain a reheating temperature larger then the weak scale.

The problem persists when we consider couplings in the D-term potential. In the D term, other fields couple with $|\phi_+|^2 - \xi$, and so do not receive a large mass from the VEV of ϕ_+. However, these couplings are proportional to g, and, due to the smallness of this parameter, they lead to a reheating temperature smaller than the weak scale. Nevertheless, this holds the key to reheating for our model. If we couple particles to ϕ_+ in a combination where the VEV cancels, it is possible to reheat above the weak scale.

A simple example of this method of solving this rather difficult problem can be found if we again use the fields, $M_\pm$, which generate the dynamical D term. Using these fields we can couple ϕ_+ to a singlet in the following way,

$$\Delta W = \kappa_1 R\Phi_+ M_- + \kappa_2 R H_u H_d + m_R R\bar{R}, \tag{28}$$

where both R and $\bar{R}$ are singlets and $H_{u,d}$ are the minimal supersymmetric standard model (MSSM) Higgs superfields. These new couplings modify the F term of ϕ_+ and M_- as can be seen in Appendix A.[10] These effects are small because R is stabilized quite close to the origin. However, they also give new contributions to the potential[11]

$$\Delta V_F = |\kappa_1\phi_+ M_- + \kappa_2 H_u H_d + m_R\bar{R}|^2 + |m_R R|^2. \tag{29}$$

The cross terms of the above equation give an interaction for ϕ_+ of the form

$$-\mathcal{L} \supset \kappa_2(\kappa_1\langle M_-\rangle\phi_+ + m_R\bar{R})H_u^\dagger H_d^\dagger + \text{H.c.} \tag{30}$$

The potential in Eq. (29) gives an additional contribution to the mass of ϕ_+ plus a mixing mass for ϕ_+ and $\bar{R}$. If $\kappa_1\langle M_-\rangle, m_R \lesssim g\sqrt{2\xi}$, the mass eigenstates discussed in Sec. II B are fairly unchanged. In this case, the contribution from $\bar{R}$ can be removed from the interaction in Eq. (30) since ϕ_+ and $\bar{R}$ are approximately orthogonal fields. If either $\kappa_1\langle M_-\rangle$ or m_R is larger than $g\sqrt{2\xi}$ decays coming from this interaction become suppressed. For $\kappa_1\langle M_-\rangle \gtrsim g\sqrt{2\xi}$, the mass of the lightest mass eigenstate coming from R and ϕ_+ becomes quite light and so cannot decay to Higgs bosons. If m_R is large, R decouples and all interactions in the superpotential become suppressed by m_R^{-1}, again leading to suppression of this decay mode.

Given that $\kappa_1\langle M_-\rangle \lesssim m_{\phi_+}$, we obtain the constraint $\kappa_1 \lesssim g$. Although this means κ_1 is a very small coupling, its smallness is offset by a large mass scale $\langle M_-\rangle$, and therefore the trilinear coupling of ϕ_+ can be as large as m_{ϕ_+}.

Since the only state in $H_{u,d}$ that is light enough for ϕ_+ to decay to is the SM-like Higgs boson, this interaction becomes

$$-\mathcal{L} \supset \frac{1}{2}\kappa_1\kappa_2 \sin 2\beta\langle M_-\rangle\phi_+ h^2, \tag{31}$$

where h is the SM-like Higgs boson, $\langle H_{u,d}\rangle = v_{u,d}$, and $\tan\beta = v_u/v_d$. The interaction (31) then gives a decay rate for the radial part of ϕ_+,

$$\Gamma_{\phi_+} = \frac{|\kappa_2|^2}{64\pi}\sin^2(2\beta)\left|\frac{\kappa_1\langle M_-\rangle}{m_{\phi_+}}\right|^2 m_{\phi_+}. \tag{32}$$

[8]The energy stored in the inflaton can be easily dissipated by adding an inflaton coupling to the right-handed neutrinos, if necessary. Since the inflaton is quite heavy after inflation, this decay mode can easily thermalize the remaining energy.

[9]In this context, we use the VEV to mean the time-evolving homogeneous background value of the field. Because the VEV of ϕ_+ begins small, some vacuum decay of ϕ_+ is possible for couplings larger than g. However, this decay channel shuts off once the VEV becomes large enough, leaving the majority of the energy left in $\langle\phi_+\rangle$.

[10]We assume that additional superpotential couplings among these fields are negligibly small. This still maintains technical naturalness.

[11]During inflation, R has a nonzero VEV. However, this VEV is less than about a GeV for the parameters we consider. This, plus the fact that κ_2 generally is quite small in order to prevent overclosing the Universe, leads to a very small correction to the Higgs bilinear mass. This small correction to μ has a negligible effect on the relaxion process.

Recall that the radial part of ϕ_+ holds the remaining energy of inflation, and therefore the decay produces the reheating temperature

$$T_R = 485\ \text{GeV} \times \left(\frac{106.75}{g_\rho}\right)^{1/4} \left(\frac{\langle M_- \rangle}{10^{16}\ \text{GeV}}\right) \times \left(\frac{10^8\ \text{GeV}}{m_{\phi_+}}\right)^{1/2} \left(\frac{\kappa_1}{10^{-9}}\right) \left(\frac{\kappa_2}{10^{-8}}\right), \tag{33}$$

where g_ρ is the number of relativistic degrees of freedom and we have taken $\sin 2\beta = 1$.

Now, if the mass of ϕ_+ is lighter then $2m_h$, ϕ_+ can no longer decay to Higgs bosons. The ϕ_+ mass is also given by (21), where it is clear that if $H_I \lesssim 0.1$ GeV, the decay mode to Higgs bosons shuts off. In this case, depending on its mass, ϕ_+ decays into ZZ, WW, $b\bar{b}$, etc., at tree level via the mixing with the Higgs boson. Although these decay modes are suppressed by a small mixing angle, it can still be sufficiently large to allow a reheat temperature above the weak scale.

If $\tilde{R}$, the fermionic component of R, is lighter than the Higgsinos, then $\tilde{R}$ could be produced via the Higgsino decay, in addition to the annihilation of the Higgs fields. Since $\tilde{R}$ is stable in this case, it may overclose the Universe. There are two ways this can be avoided. Since, as we discussed above, $\tilde{R}$ can be as heavy as ϕ_+, it will have a mass as heavy as that in Eq. (21). Experimental constraints allow a bino mass that is lighter than this, especially if H_I is pushed beyond the weak scale to make ϕ_+, and thus $\tilde{R}$ as well, be heavy enough. In this case, $\tilde{R}$ can decay into the bino through the Higgsino exchange. In the relaxion model below, this type of spectrum is only realized for some of the parameter space where the bino mass can be as light as 10^2 GeV.

The other way to prevent overclosure of the Universe from Higgsinos decaying to $\tilde{R}$ is to suppress the reheat temperature below the Higgsino mass. In this case, the Universe never produces Higgsinos and so there would be no $\tilde{R}$ produced from Higgsino decays. If the reheat temperature is larger than the bino mass, $\tilde{R}$ could still be produced from bino decays due to bino-Higgsino mixing. Since the bino can be produced in processes like $hh \to \tilde{B}\tilde{B}$, its production cannot be suppressed if the SM reheats to a temperature above the bino mass. The simplest way to avoid these problems is to just reheat below the bino mass, which requires $\kappa_2 \lesssim 10^{-9}$ for $H_I = 10^5$ GeV.[12] However, it may be possible to reheat above the bino mass in this scenario if the decay of the bino to $\tilde{R}$ is suppressed so that it happens after the bino freezes out. In this case the relic density of the bino could be suppressed during freeze-out by some process such as coannihilation. Since this effectively reduces the number of $\tilde{R}$ produced from bino decays, it may be possible to get a relic density of $\tilde{R}$ that does not overclose the Universe and may even be the dark matter candidate.[13] In the relaxion model we discuss below, only gauginos can be much lighter than the SUSY-breaking scale. Thus, candidates for the coannihilation partner of the bino are the gluino or wino. For the bino-gluino coannihilation case, the bino abundance falls into a desirable value if the mass difference between the bino and gluino is $\lesssim$100 GeV and squark masses are $\lesssim\mathcal{O}(100)$ TeV [64–66]. In the case of the bino-wino coannihilation, on the other hand, the bino-wino mass difference should be $\lesssim\mathcal{O}(10)$ GeV [67]. These coannihilation scenarios may be probed at the LHC by searching for displaced vertex signals [65,67,68]. For detailed discussions on these coannihilation scenarios, see Refs. [64–67] and references therein. Another option is to assume that there is a wino with a mass of $\gtrsim$500 GeV [69,70] or a gluino[14] with a mass of $\gtrsim$2 TeV [1–3], and the bino is heavier than these particles. In this case, the bino mainly decays into these particles, while the abundance of these particles is sufficiently suppressed. A few TeV gluino can be probed at the LHC in the multijets plus missing energy channel [71], while an $\mathcal{O}(100)$ GeV wino can be probed in the disappearing-track searches [72,73]. Even in these cases, we need to take κ_2 to be a small value to suppress the direct $\tilde{R}$ production process $hh \to \tilde{R}\tilde{R}$.

III. THE INFLATON AS AN AMPLITUDON

The above D-term inflation model provides a technically natural realization of low-scale inflation that is consistent with the current Planck results. Since low-scale inflation is needed for the relaxion process and requires a very flat potential, it suggests that the inflaton of this D-term inflation model can be identified with the second (amplitudon) field in the supersymmetric two-field relaxion model discussed in Ref. [20]. We next present a relaxion model that combines these two ideas, thereby relating inflation with solving the supersymmetric little hierarchy problem. In fact, regarding the amplitudon as the inflaton is also desirable from the phenomenological point of view; in the minimal setup discussed in Ref. [20], the light amplitudon field may cause an isocurvature problem, but we can evade this once we identify it with the inflaton.

In the supersymmetric relaxion mechanism, supersymmetry breaking in the visible sector is determined by the F term of the relaxion superfield. Because of this, the determinant of the Higgs mass matrix is dependent on the relaxion field value. Initially, the relaxion field value is

[12]Such a small κ_2 also suppresses the $hh \to \tilde{R}\tilde{R}$ process.

[13]This is only possible when the gravitino is heavier than $\tilde{R}$, which is not always the case.

[14]If the gluino is lighter than the bino, the wino or gravitino needs to be lighter than the gluino in order to make it decay into these particles.

large and the determinant of the Higgs mass matrix is positive. As the relaxion field rolls, the determinant of the Higgs mass matrix eventually becomes negative and electroweak symmetry breaking occurs. Electroweak symmetry breaking generates an additional contribution to the relaxion potential that stops the relaxion from rolling. For properly chosen parameters, the relaxion stops in a local minimum that corresponds to a weak scale Higgs VEV.

A. The inflaton-relaxion model

In Ref. [20], a two-field relaxion model was considered with an additional field coined the amplitudon. This field was responsible for controlling the relaxion barrier height and allowing the relaxion to roll. If we now identify the inflaton of the previous section (contained in T) with this amplitudon, the superpotential for this scenario becomes

$$W = \kappa T \Phi_+ \Phi_- + \frac{1}{2} m_T T^2 + \frac{1}{2} m_S S^2 + \left(m_N + i g_S S + i g_T T + \frac{\lambda}{M_L} H_u H_d \right) N \bar{N}, \quad (34)$$

where the imaginary scalar component of the superfield S is the relaxion, $N, \bar{N}$ are superfields charged under a strongly coupled gauge group [$SU(N)$], and $H_{u,d}$ are the Higgs superfields. The couplings $\lambda, \kappa, g_{S,T}$ are dimensionless [where κ was already introduced in Eq. (2)] and $m_{N,S,T}, M_L$ are mass parameters. Note that m_S is a shift-symmetry breaking parameter that causes the relaxion to roll, and $\Phi_\pm$ are again charged under some additional $U(1)$ so that inflation proceeds as it did in the previous section. In addition m_T is a shift-symmetry breaking parameter that controls the inflaton evolution during the relaxion epoch. We also consider an identical D term to the one in Eq. (1). Comparing this to the model in Ref. [20], the only difference in the superpotential is the addition of the coupling of the amplitudon with two scalar fields, $\phi_\pm$. This is the same interaction that we studied in the previous section for the inflaton.

In addition to these superpotential interactions, the relaxion superfield, S, is coupled to the gauge kinetic function,

$$\mathcal{L} \supset \int d^2\theta \left(\frac{1}{2 g_a^2} - i \frac{\Theta_a}{16\pi^2} - \frac{c_a S}{16\pi^2 f_\phi} \right) \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a) + \mathrm{H.c.}, \quad (35)$$

in a similar way to the QCD axion,[15] where f_ϕ is the global symmetry-breaking scale, c_a is an order one constant, and a runs over the SM gauge symmetries as well as an additional confining $SU(N)$. When this $SU(N)$ confines, the fermionic components of N and $\bar{N}$ condense and generate a $\cos(\phi/f_\phi)$ potential, which is the backreaction that stops the relaxion.

Writing the scalar field components as $S = \frac{s+i\phi}{\sqrt{2}}$ and $T = \frac{\tau+i\sigma}{\sqrt{2}}$, the relevant parts of these superfields for our discussion are the relaxion ϕ, and the amplitudon (inflaton), σ. The relaxion and amplitudon correspond to the Nambu-Goldstone boson of some broken symmetry, and therefore transform under a shift symmetry. If these shift symmetries are exact, the potential for these fields is completely flat. This flatness is lifted by the explicit breaking of the shift symmetry[16] due to the couplings m_S, m_T, and κ in Eq. (34). The scalar potential is then[17]

$$V_{\mathrm{explicit}} = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \frac{g^4\xi^2}{16\pi^2} \ln\left[\frac{|\kappa|^2\sigma^2}{2Q^2}\right]. \quad (36)$$

The explicit breaking of the shift symmetry for the amplitudon arises from the mass term and from integrating out the $\phi_\pm$, which are heavy during both the relaxion and inflation epochs. As we see below, the shift-symmetry breaking mass terms $|m_S|$ and $|m_T|$ are taken to be very small; such small shift-symmetry breaking effects may be explained by means of the "clockwork" mechanism [74–76].[18]

To combine these two theories, we need the explicit mass term for the inflaton to dominate during the relaxion epoch and the loop-induced mass to dominate during inflation, as schematically depicted in Fig. 2. The ratio of these two masses is

$$R_m = \frac{g^4\xi^2}{8\pi^2\sigma^2} \frac{1}{|m_T|^2}. \quad (37)$$

During the relaxion epoch the ratio is

$$R_m = \frac{3}{4\pi^2} \frac{1}{A_s(1-n_s)} \frac{H_I^4}{m_{\mathrm{SUSY}}^2 f_\phi^2} \simeq 10^{-11} \times \left(\frac{A_s}{2.1\times10^{-9}}\right)^{-1} \left(\frac{1-n_s}{0.03}\right)^{-1} \times \left(\frac{H_I}{1\ \mathrm{GeV}}\right)^4 \left(\frac{10^5\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^4 \left(\frac{r_{\mathrm{SUSY}}}{1}\right)^2, \quad (38)$$

[15]The theta term, Θ_a, can be neglected since it is subdominant compared to the effective value obtained in the early Universe from the large ($\gg f_\phi$) field value of ϕ.

[16]This shift symmetry preserves the very flat potential for the inflaton, σ. Higher-order shift-symmetric terms in the Kähler potential, $K = K(S+S^\dagger, T+T^\dagger)$, stabilize s and τ near the origin.

[17]For parameters that require super-Planckian excursions, we consider a no-scale structure to prevent problematic unbounded from below directions arising from the $-3|W|^2/M_P^2$ contribution to the potential. For a more detailed discussion, see [20].

[18]A similar idea was first considered in the context of inflation model building [77].

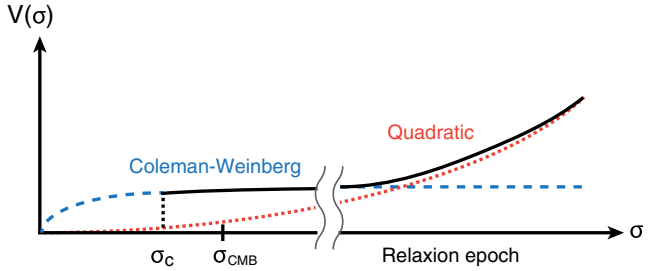

FIG. 2. Schematic description of the scalar potential as a function of the inflaton/amplitudon field, σ. The dashed (dotted) lines represent the pure Coleman-Weinberg (quadratic) potential, while the solid line is the sum of the two potentials.

where we have used $m_T^2\sigma^2 \sim m_S^2\phi^2 \sim m_{\rm SUSY}^2 f_\phi^2$, which comes from the constraints[19] on the relaxion mechanism in Sec. III B, and Eqs. (17) and (20) for $g^4\xi^2$. From this expression, it is clear that m_T dominates in this regime. Note that we have changed our normalization of H_I in this section since $H_I \sim 10^5$ GeV is no longer compatible with the relaxion process [see Eq. (58)]. During the CMB epoch, on the other hand, the ratio is

$$R_m = \frac{3}{4}(1-n_s)\left(\frac{H_I}{|m_T|}\right)^2 \simeq 2\times 10^{12}\times\left(\frac{1-n_s}{0.03}\right)\left(\frac{H_I}{1\,\text{GeV}}\right)^2\left(\frac{10^{-7}\,\text{GeV}}{|m_T|}\right)^2, \quad (39)$$

where we have used Eq. (11), or $\sigma_{\rm CMB} = H_I/(\pi(1-n_s)A_s^{\frac{1}{2}})$. For this regime of the potential, the loop-induced mass dominates. This is just the correct behavior that is needed to use the σ field as both the amplitudon and the inflaton. Therefore, we can ignore the Coleman-Weinberg contribution to the mass during the relaxion epoch and the constraints reduce to those found in Ref. [20], which are summarized in Sec. III B.

The backreaction potential for the relaxion is generated by the fields N, $\bar{N}$ in Eq. (34). The N, $\bar{N}$ fields are charged under the same $SU(N)$ gauge theory that S is coupled to in Eq. (35). When the fermionic components of N, $\bar{N}$ confine at the scale Λ_N, they give a contribution to the scalar potential of the form

$$\mathcal{A}(\phi,\sigma,H_uH_d) = \left[m_N - \frac{1}{\sqrt{2}}(g_S\phi + g_T\sigma) + \frac{\lambda}{M_L}H_uH_d\right],$$
$$V_{\rm period} = \mathcal{A}(\phi,\sigma,H_uH_d)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right), \quad (40)$$

where we have assumed $c_a = \sqrt{2}$. For the model we consider, we take $g_S > 0$ and $g_T < 0$. When the σ field value is very large, the above potential (40) provides a large barrier for the relaxion and therefore the relaxion is initially stabilized at some very large field value. However the inflaton, σ, is free to roll. As σ rolls, the barrier height is reduced until the mass term in Eq. (36) dominates and ϕ begins to roll, tracking σ. This evolution continues until the determinant of the Higgs mass matrix becomes negative and electroweak symmetry is broken. As the Higgs VEV increases, a new barrier develops in the relaxion potential [from the λ term in Eq. (40)] eventually stopping the relaxion at a local minimum. The explicit symmetry-breaking parameter, m_S, is chosen so that this minimum corresponds to a Higgs field with a weak scale VEV.

B. The constraints on the cosmological evolution

Next we examine the constraints on the cosmological evolution that limit the parameter space of the relaxion. To determine these constraints, the relevant part of the scalar potential is given as

$$V = V_{\rm explicit} + V_{\rm period}, \quad (41)$$

which are given, respectively, in Eqs. (36) and (40). However, as we argued above, we can ignore the Coleman-Weinberg contribution during the relaxion epoch. The main constraints on the parameter space are as follows.[20]

(i) *Inflaton/amplitudon slow roll*: In order for the relaxion process to work, we first need the slow roll of the inflaton, σ, to proceed unimpeded, with little effect from the coupling to the relaxion. The equations of motion for σ in the slow-roll regime are

$$\frac{d\sigma}{dt} = -\frac{1}{3H_I}\frac{\partial V}{\partial\sigma} = -\frac{1}{3H_I}\left[m_T^2\sigma - \frac{g_T}{\sqrt{2}}\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)\right]. \quad (42)$$

Since we need the inflaton rolling to be unaffected by the periodic potential piece in Eq. (42), we require that

$$m_T^2\sigma \gg \frac{g_T}{\sqrt{2}}\Lambda_N^3. \quad (43)$$

We can remove the σ field dependence in this expression by using the fact that right before EWSB, $g_S\phi_* \sim -g_T\sigma_*$ [which follows from taking the expression in the square brackets of Eq. (40) to be 0] and $\mu_0 \sim m_{\rm SUSY} \sim m_S\phi_*/f_\phi$ (which follows from

[19]Note that we have introduced the scale $m_{\rm SUSY} \sim F/f_\phi$ where $F \sim m_S\phi$ is the dominant supersymmetry-breaking contribution. See Ref. [20] for further details.

[20]For the discussion of other conditions that lead to weaker constraints, see Ref. [20].

having a negative determinant of the Higgs mass matrix; see Ref. [20]), with ϕ_* and σ_* being the field values when the relaxion stops rolling. Using these relationships, the condition (43) becomes

$$\frac{g_T^2}{g_S} \ll \frac{m_{\rm SUSY} f_\phi}{\Lambda_N^3} \frac{|m_T|^2}{|m_S|}. \tag{44}$$

(ii) *Relaxion initial condition*: Next, we examine the initial condition for the relaxion, ϕ, which we require to be trapped at a local minimum. This requires that the contribution to the mass of ϕ coming from $V_{\rm explicit}$ be subdominant compared to the contribution coming from $V_{\rm period}$. This results in the following constraint,

$$|m_S|^2 \ll g_S \frac{\Lambda_N^3}{f_\phi}, \tag{45}$$

where we have again used the fact that right before EWSB, $g_S \phi_* \sim -g_T \sigma_*$.

(iii) Stability of the relaxion minimum: The next constraint we consider comes from requiring that the Higgs VEV does indeed provide a barrier to eventually stop ϕ from rolling, and stabilize the relaxion at a local minimum. This expression is found from the minimization condition with the following inequality arising from taking $\sin(\frac{\phi}{f_\phi}) = 1$,

$$|m_S| \lesssim \frac{|\lambda| \sin 2\beta}{4 M_L} \frac{v^2 \Lambda_N^3}{m_{\rm SUSY} f_\phi^2}, \tag{46}$$

where $v = \sqrt{v_u^2 + v_d^2}$ is the electroweak VEV. In addition, the term in $V_{\rm period}$ proportional to λ generates a contribution to the soft SUSY breaking B_μ term, which causes the determinant of the Higgs mass matrix to oscillate. Requiring that the amplitude of this oscillation be smaller than the electroweak scale gives the constraint

$$|\lambda| \lesssim \frac{4 M_L v^2}{\Lambda_N^3 \sin 2\beta}. \tag{47}$$

Combining this with Eq. (46), we find

$$|m_S| \lesssim \frac{v^4}{m_{\rm SUSY} f_\phi^2}. \tag{48}$$

(iv) *Classical rolling condition*: Another relevant constraint to this scenario comes from requiring that the relaxion, ϕ, and the inflaton, σ, undergo classical rolling. The classical rolling conditions are determined from $\dot{\sigma}/H_I > H_I$, leading to the constraint

$$\frac{|m_T|^2}{|m_S|} \frac{g_S}{|g_T|} m_{\rm SUSY} f_\phi \gg 3 H_I^3. \tag{49}$$

(v) *ϕ tracks σ after EWSB*: In order for ϕ to settle in its minimum with the Higgs VEV of the order of the weak scale, $\mathcal{A}(\phi, \sigma, H_u H_d)$ needs to grow quickly enough with the Higgs VEV so that ϕ can stop tracking σ. By examining the evolution of $\mathcal{A}(\phi, \sigma, H_u H_d)$ as the Higgs VEV develops, we find that as along as[21]

$$\frac{g_S}{\sin 2\beta} \frac{m_h^2}{m_{\rm SUSY}^2} \frac{\Lambda_N^3}{f_\phi} \frac{v^2}{f_\phi^2} \lesssim \frac{|m_S|^2}{1 - \frac{|m_T|^2}{|m_S|^2}}, \tag{50}$$

is satisfied, ϕ will discontinue its tracking of σ with the Higgs VEV of the order of the weak scale.

(vi) *Loop corrections to inflaton mass*: A new constraint, which is only present when the amplitudon is identified as the inflaton, comes from loop corrections to the mass of σ. First, because the coupling κ breaks the shift symmetry, the Kähler potential is affected by this shift-symmetry breaking at the loop level,

$$\Delta K \simeq \frac{\kappa^2}{16\pi^2} |T|^2. \tag{51}$$

Since κ is the order parameter of this shift-symmetry breaking, it controls the size of all shift-symmetry breaking in the Kähler potential. Because of the inflation constraints discussed above, this parameter must satisfy $\kappa \gtrsim 10^{-2}$. If we include this loop-corrected Kähler contribution in the supergravity scalar potential,

$$V_{\rm SUGRA} = e^{\frac{\kappa}{M_P^2}} \left(D_i W K^{i\bar{j}} D_{\bar{j}} \bar{W} - 3 \frac{|W|^2}{M_P^2} \right), \tag{52}$$

we see that there can be important affects on the amplitudon. With the vacuum energy being nonzero during the relaxion process, the exponential $\exp(K/M_P^2)$, which now depends on σ because of the shift-symmetry breaking, generates a mass for

[21]The additional factors in Eq. (50) as compared to the corresponding expression in Ref. [20] come from considering the contribution to B_μ originating in the Higgs dependent part of $\mathcal{A}(\phi, \sigma, H_u H_d)$. This oscillatory contribution to B_μ gives the dominant contribution to $\phi \frac{d\mathcal{D}(\phi)}{d\phi} \sim m_{\rm SUSY}^4 \frac{f_\phi^2}{v^2 \sin 2\beta}$. Following the same calculation as in Ref. [20], with this single change, gives the constraint in Eq. (50).

the inflaton. The exponential piece can be important because the vacuum energy during the relaxion process changes at least by an amount of the order of

$$\Delta V = m_S^2 \phi_*^2 = m_{\rm SUSY}^2 f_\phi^2. \tag{53}$$

Expanding the exponential in Eq. (52), and using Eq. (53) for the vacuum energy, we obtain an inflaton mass of order

$$\Delta m_\sigma \simeq \frac{\kappa}{4\pi}\frac{m_{\rm SUSY} f_\phi}{M_P} = 3.3\times 10^{-12}\ \text{GeV} \times \left(\frac{\kappa}{10^{-2}}\right)\left(\frac{m_{\rm SUSY}}{10^5\ \text{GeV}}\right)\left(\frac{f_\phi}{10^5\ \text{GeV}}\right). \tag{54}$$

Now in order for the relaxion process to be viable, this correction to the σ mass must be smaller than m_T in the superpotential.

Second, the soft SUSY-breaking effects in the $\phi_\pm$ fields can induce the σ mass term via the Coleman-Weinberg potential of the order of

$$\Delta m_\sigma \simeq \frac{\kappa}{4\pi}\tilde{m}_{\phi_\pm}, \tag{55}$$

where $\tilde{m}_{\phi_\pm}$ denote the soft masses of $\phi_\pm$. If $\tilde{m}_{\phi_\pm}$ is induced by the Planck-suppressed $\phi_\pm$-relaxion operators, then we expect $\tilde{m}_{\phi_\pm} \sim m_{\rm SUSY} f_\phi / M_P$ and thus the contribution (55) is of the same order as (54). If, on the other hand, there is another source of SUSY breaking and it gives a larger contribution to $\tilde{m}_{\phi_\pm}$, then this gives a more severe constraint, as we see in Appendix A.

1. Combined constraints

The relevant constraints can now be combined to restrict the parameter space of the inflaton-relaxion model. However, to simplify the parameter space, we redefine the parameters in a similar manner as was done in Ref. [20],

$$g_S = \zeta\frac{m_S}{f_\phi}, \quad g_S = \zeta\frac{m_T}{f_\sigma}, \quad f \equiv f_\phi = f_\sigma,$$
$$r_{TS} \equiv \frac{m_T}{m_S}, \quad r_\Lambda \equiv \frac{\Lambda_N}{f}, \quad r_{\rm SUSY} \equiv \frac{m_{\rm SUSY}}{f}, \quad M_L = m_{\rm SUSY}, \tag{56}$$

where ζ is a dimensionless parameter. Using this parametrization, we display the constraints in the $m_{\rm SUSY} - m_S$ plane. Recall that the parameter $m_{\rm SUSY}$ represents the "cutoff scale" of the model while m_S is the explicit shift-symmetry breaking parameter. In Fig. 3, we have taken $\zeta = 10^{-8}$, $r_{TS} = 0.1$, $r_\Lambda = 1$, $r_{\rm SUSY} = 1$, and $\kappa = 10^{-2}$. The gray shaded region is excluded because the periodic barrier formed when the Higgs VEV develops

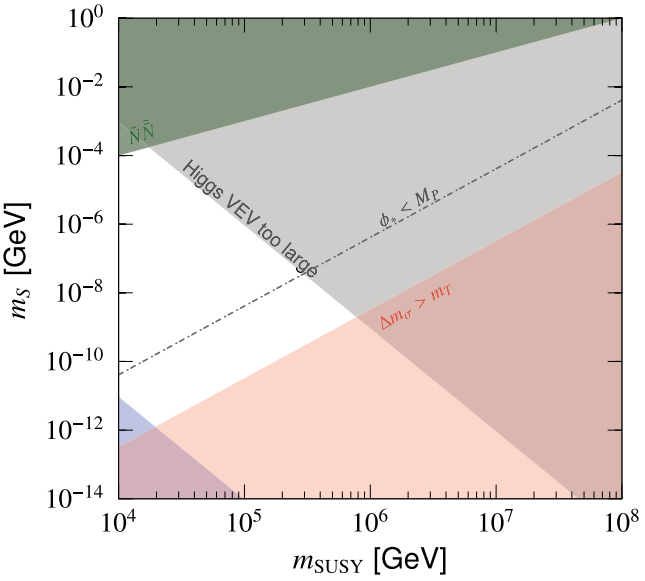

FIG. 3. The allowed parameter region in the $m_{\rm SUSY}$–m_S plane, where $\zeta = 10^{-8}$, $r_{TS} = 0.1$, $r_\Lambda = 1$, $r_{\rm SUSY} = 1$, and $\kappa = 10^{-2}$.

cannot stop the relaxion rolling [Eq. (48)]. The blue shaded region is excluded because ϕ never decouples from σ [Eq. (50)]. In the red region, the shift-symmetry breaking correction to the Kähler potential generates an inflaton mass larger than m_T [(54)]. The green-shaded region is disfavored since the scalar potential may become unstable in the direction of $N\bar{N}$, as discussed in Ref. [20]. Above the dash-dotted line, $\phi_* < M_P$, and thus sub-Planckian field values may be realized. The figure shows that supersymmetric soft masses up to 3×10^5 GeV can be obtained for the range $10^{-10}\ \text{GeV} \lesssim m_S \lesssim 10^{-4}\ \text{GeV}$. We see that the PeV-scale SUSY region is now constrained by the condition $\Delta m_\sigma < |m_T|$, which is a consequence of combining the low-scale D-term inflation model with the two-field relaxion model.

In Fig. 4, we take $\zeta = 10^{-14}$, $r_{TS} = 0.1$, $r_\Lambda = 1$, $r_{\rm SUSY} = 1$, and $\kappa = 10^{-2}$. The color coding of the excluded regions in Fig. 4 is the same as in Fig. 3. For these parameter choices the allowed region now corresponds to supersymmetric soft mass scales up to 10^6 GeV and $10^{-12}\ \text{GeV} \lesssim m_S \lesssim 10^{-8}\ \text{GeV}$. It is found that the new condition $\Delta m_\sigma < |m_T|$ gives a very severe limit on the parameter space in this case.

Finally, in the allowed region, we can always find a value of H_I that satisfies the above constraints. The lower bound on H_I is given by

$$H_I > \max\left\{|m_S|, 4\times 10^{-9}\ \text{GeV} \times \left(\frac{m_{\rm SUSY}}{10^5\ \text{GeV}}\right)^2\left(\frac{1}{r_{\rm SUSY}}\right)\right\}, \tag{57}$$

which comes from requiring that the slow roll of ϕ, and the relaxion vacuum energy, is subdominant compared

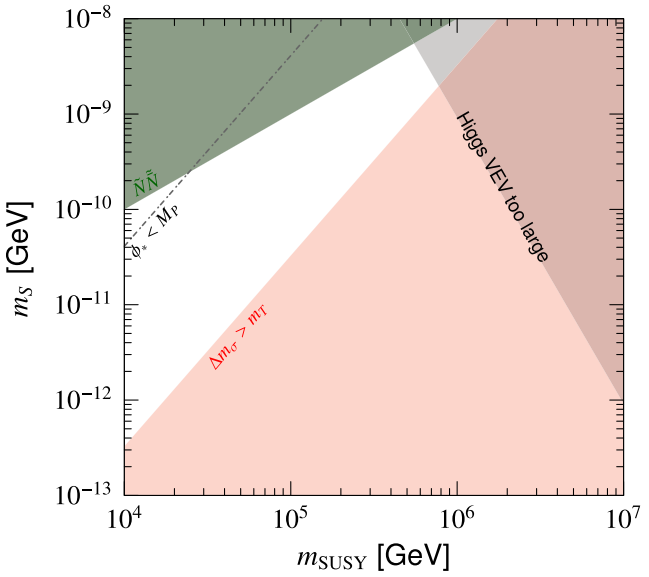

FIG. 4. The allowed parameter region in the $m_{\rm SUSY} - m_S$ plane, where $\zeta = 10^{-14}$, $r_{TS} = 0.1$, $r_\Lambda = 1$, $r_{\rm SUSY} = 1$, and $\kappa = 10^{-2}$.

to that of the inflaton. The upper bound on the Hubble scale is

$$H_I < 4.6\ \text{GeV} \times \left(\frac{r_{TS}}{0.1}\right)^{\frac{1}{3}} \left(\frac{1}{r_{\rm SUSY}}\right)^{\frac{1}{3}} \times \left(\frac{|m_S|}{10^{-7}\ \text{GeV}}\right)^{\frac{1}{3}} \left(\frac{m_{\rm SUSY}}{10^5\ \text{GeV}}\right)^{\frac{2}{3}}, \quad (58)$$

which comes from Eq. (49). In addition, we have upper limits on H_I to evade the cosmic string problem as discussed in Appendix B 1 c.

IV. CONCLUSION

In this paper, we have presented a low-scale inflationary model embedded in a supersymmetric framework that seeks to address the hierarchy problem and be consistent with experimental data. Specifically, we consider a D-term inflationary model, characterized by a new $U(1)$ symmetry with a FI term. There are three parameters of the model that are relevant for the CMB phenomenology: the $U(1)$ gauge coupling, g, the FI scale, $\sqrt{\xi}$, and the energy density $\rho_{\rm reh}$ at reheating (assuming an instantaneous transition between matter domination and radiation domination). To determine the constraints on these parameters we trade the FI scale for the Hubble scale, H_I, at the moment at which the CMB modes were produced. The measured values of the amplitude and the spectral tilt of the primordial scalar perturbations can then be used to obtain g and H_I as a function of $\rho_{\rm reh}$. By requiring $\rho_{\rm reh}^{1/4}$ to be above the electroweak scale (in order to facilitate baryogenesis), we find that a value of n_s compatible with the experimental limits (namely, a sufficiently red scalar spectrum) can be achieved provided $g \lesssim 10^{-8}$ and $H_I \lesssim 10^5$ GeV. For this low scale of inflation, the CMB modes are produced approximately $N_{\rm CMB} \simeq 39e$-folds before the end of inflation (contrary to the 50–60 e-folds typically required in high scale models of inflation). In D-term inflation, this relatively low value of $N_{\rm CMB}$ is used to match the observed value of n_s, since deviations from scale invariance are inversely proportional to $N_{\rm CMB}$.

Another issue typically associated with D-term inflation is the formation of cosmic strings due to the spontaneous breaking of the $U(1)$ symmetry at the end of inflation. We prevent this from occurring by introducing a tiny breaking of the $U(1)$ symmetry throughout the entire inflationary epoch, due to a dynamical D-term mechanism. This mechanism also allows the generation of a FI scale much below the Planck scale. Finally, a low value of g is typically problematic for reheating. For such a value, most of the energy density after inflation is actually stored in the field that spontaneously breaks the $U(1)$ symmetry (leading to the end of inflation). This field obtains a VEV much greater than its mass, and therefore typically gives a large effective mass to any field that it is coupled to with a strength greater than g, preventing its decay into these fields. A way to avoid this kinematic barrier is to introduce superpotential couplings that cancel the VEV, so as to allow the decay into the MSSM Higgs fields, and the eventual reheating into standard model fields. Thus, with the technically natural superpotential couplings and the $U(1)$ gauge coupling g, a low-scale model of supersymmetric inflation can be made to be consistent with Planck data.

This low-scale D-term inflation model leads to an interesting application. It can be combined with the relaxion mechanism in order to identify the inflaton with the second field (amplitudon) of a supersymmetric two-field relaxion model that preserves the QCD axion solution to the strong CP problem. The inflaton now also controls the barrier height of the relaxion periodic potential. As the inflaton rolls, it periodically reduces the barrier height causing the relaxion to move and scan the supersymmetric soft masses. Eventually electroweak symmetry breaking occurs, which produces a new contribution to the relaxion barrier height and traps the relaxion in a supersymmetry-breaking local minimum. The correct electroweak VEV can be obtained for supersymmetric soft masses up to the PeV scale, provided the explicit shift-symmetry breaking parameter $m_S \lesssim 10^{-4}$ GeV, and the Hubble scale satisfies $H_I \lesssim 10$ GeV. This dynamics takes place well before the production of the CMB, at a time in which the energy density of the inflaton is dominated by a quadratic (mass) term, rather than by the Coleman-Weinberg term that instead controls the motion of the inflaton at $N_{\rm CMB}$. The switchover between these two potential terms is a natural consequence of the flatness associated with the logarithmic Coleman-Weinberg term, and it distinguishes our model

from other implementations of the relaxion mechanism. Also by identifying the amplitudon as the inflaton, a potential isocurvature problem in the original two-field relaxion model is avoided. Therefore, the supersymmetric inflaton-relaxion model successfully combines low-scale D-term inflation, which is technically natural, with a solution to the supersymmetric little hierarchy problem. This intriguing connection between the inflaton and the relaxion provides a new way to address the hierarchy problem and deserves further study.

ACKNOWLEDGMENTS

We are grateful to M. Hindmarsh and A. R. Liddle for helpful correspondence, and thank Z. Thomas for initially collaborating on the project. The work of T. G. and M. P. is supported by the U.S. Department of Energy Grant No. DE-SC0011842 at the University of Minnesota. The work of N. N. is supported by Grant-in-Aid for Scientific Research (No. 17K14270).

APPENDIX A: DYNAMICAL D TERMS

Here, we review the dynamical generation of D terms. We basically follow the arguments in Refs. [40,78] where the dynamical generation of D terms is discussed based on the IYIT model [79,80]. We focus on the case of the $SP(1) \cong SU(2)$ strongly interacting gauge theory with $N_f = 2$ quark flavors. For more generic cases, see Ref. [40]. In this case, we have four chiral quark superfields Q^i $(i = 1, \ldots, 4)$, which are in the fundamental representation of $SP(1)$, and six singlet chiral superfields $Z_{ij} = -Z_{ji}$ $(i, j = 1, \ldots, 4)$. We assign the $U(1)$ gauge charge $+1/2$ $(-1/2)$ to $Q^{1,2}$ $(Q^{3,4})$, -1 to $Z_- \equiv Z_{12}$, $+1$ to $Z_+ \equiv Z_{34}$, and 0 to Z_{13}, Z_{14}, Z_{23}, Z_{24}, respectively. The superpotential terms for these fundamental fields are then given by

$$W_{\rm fund} = \frac{1}{2}\sum_{i,j}\lambda_{ij} Z_{ij} Q^i Q^j, \tag{A1}$$

with $\lambda_{ij} = -\lambda_{ji}$ dimensionless Yukawa couplings. We also couple T and $\Phi_\pm$ to this sector via the higher-dimensional operators $T\Phi_- Q^1 Q^2$ and $T\Phi_+ Q^3 Q^4$. In order to facilitate reheating, we include another shift-symmetric singlet, R, and couple it to this strongly coupled sector through the higher-dimensional operator $R\Phi_+ Q^1 Q^2$ as well. There are other renormalizable couplings allowed by the gauge symmetries, such as $TZ_+ Z_-$, $Z_\pm \Phi_\mp$, $\Phi_- Q^1 Q^2$, etc.—we simply assume that all of these unwanted terms are negligible in the following discussion. Such a situation may be realized by geometrically separating the $SP(1)$ sector from the inflation/relaxion sector by means of, say, branes in extra dimensions.

Below the confinement scale of the $SP(1)$ gauge interaction, Λ, the low-energy dynamical degrees of freedom are given by the meson fields $M^{ij} = -M^{ji} \sim Q^i Q^j/\Lambda$. The $U(1)$ charge assignment for these meson fields follows from those for the constituent quark fields; $M_+ \equiv M^{12}$ has $+1$, $M_- \equiv M^{34}$ has -1, and the other meson fields are neutral. The meson fields are subject to the constraint [81]

$$\mathrm{Pf}(M^{ij}) = M^{12}M^{34} - M^{13}M^{24} + M^{14}M^{23} = \Lambda^2. \tag{A2}$$

As in Ref. [40], we assume that λ_{13}, λ_{14}, λ_{23}, and λ_{24} are much larger than $\lambda_+ \equiv \lambda_{12}$ and $\lambda_- \equiv \lambda_{34}$ in order to make sure that all of the neutral fields except T remain at the origin. In this case, the condition (A2) leads to

$$M_+ M_- = \Lambda^2, \tag{A3}$$

and the relevant part of the low-energy effective superpotential is given by

$$\begin{aligned} W_{\rm eff} = {} & \kappa T \Phi_+ \Phi_- + \frac{m_T}{2} T^2 + \kappa_+ T M_+ \Phi_- + \kappa_- T M_- \Phi_+ \\ & + \lambda_+ \Lambda M_+ Z_- + \lambda_- \Lambda M_- Z_+ + \kappa_1 R \Phi_+ M_- \\ & + \kappa_2 R H_u H_d + m_R R \bar{R}, \end{aligned} \tag{A4}$$

where the third, fourth, and seventh terms in the right-hand side of this equation come from the higher-dimensional operators introduced above. Since these terms are generated by nonrenormalizable interactions and/or break the shift symmetry with respect to T or R, the couplings $\kappa_\pm$ and $\kappa_{1,2}$ can be parametrically small.

From the superpotential (A4), we obtain the F-term scalar potential as

$$\begin{aligned} V_F = {} & \left|\kappa\phi_+\phi_- + \frac{i}{\sqrt{2}} m_T \sigma + \kappa_+ M_+ \phi_- + \kappa_- M_- \phi_+\right|^2 \\ & + \left|i\frac{\sigma}{\sqrt{2}}(\kappa\phi_- + \kappa_- M_-) + \kappa_1 R M_-\right|^2 \\ & + \left|i\frac{\sigma}{\sqrt{2}}(\kappa\phi_+ + \kappa_+ M_+)\right|^2 \\ & + \left|\frac{i\kappa_+}{\sqrt{2}}\sigma\phi_- + \lambda_+\Lambda Z_-\right|^2 + \left|\frac{i\kappa_-}{\sqrt{2}}\sigma\phi_+ + \lambda_-\Lambda Z_+ + \kappa_1 R\phi_+\right|^2 \\ & + \left|\kappa_1\phi_+ M_- + \kappa_2 H_u H_d + m_R\bar{R}\right|^2 + |m_R R|^2 \\ & + |\lambda_+\Lambda M_+|^2 + |\lambda_-\Lambda M_-|^2, \end{aligned} \tag{A5}$$

where we have assumed $|\sigma| \gg |\tau|$ as in Sec. II A. There is also a D-term contribution to the scalar potential

$$\begin{aligned} V_D = \frac{g^2}{2}(& |\phi_+|^2 - |\phi_-|^2 + |Z_+|^2 - |Z_-|^2 + |M_+|^2 \\ & - |M_-|^2 - \xi_{\rm tree})^2, \end{aligned} \tag{A6}$$

where $\xi_{\rm tree}$ denotes the tree-level FI term, which can be taken to be 0 when the dynamical sector generates a large enough contribution for D-term inflation to work. This amounts to the difference of the VEVs of $M_\pm$ being large enough.

Now to leading order[22] in $\kappa_\pm$, κ_1, and g, the F terms vanish in the vacuum except for $Z_\pm$ and T with the fields having the following VEVs,[23]

$$\langle M_\pm\rangle = \sqrt{\frac{\lambda_\mp}{\lambda_\pm}}\Lambda, \tag{A7}$$

$$\langle \phi_\pm\rangle = -\frac{\kappa_\pm}{\kappa}\sqrt{\frac{\lambda_\mp}{\lambda_\pm}}\Lambda, \tag{A8}$$

$$\langle Z_\pm\rangle = \frac{i\kappa_+\kappa_-}{\kappa\sqrt{2\lambda_+\lambda_-}}\sigma, \tag{A9}$$

$$\langle R\rangle = -i\frac{\kappa_1\kappa_-\lambda_+\sigma\Lambda^2}{\kappa_1^2\lambda_+\Lambda^2+\lambda_- m_R^2}, \tag{A10}$$

and $\bar{R}$ can be found by solving F_R. We have assumed $g \ll \lambda_\pm$ as in Ref. [40]. This condition can easily be satisfied in the case of low-scale D-term inflation as can be seen from Eq. (20). The details of the calculation for the VEVs of $M_\pm$ can be found in Ref. [40]. By using Eq. (A7), we then obtain the dynamically generated FI term,

$$\xi_{\rm dyn} = \left(\frac{\lambda_+}{\lambda_-}-\frac{\lambda_-}{\lambda_+}\right)\Lambda^2. \tag{A11}$$

This can explain the required value shown in Eq. (18) if $\Lambda \simeq 10^{16}$ GeV. Such a dynamically generated FI term has several advantages. First, this can naturally explain why the FI term is much smaller than the fundamental scale, such as the Planck scale. In addition, this allows the model to couple with supergravity in a consistent manner, which is very difficult if the theory possesses a constant FI term.[24] It also provides a means to suppress cosmic strings and facilitate reheating through additional couplings of the dynamical sector to the inflaton sector.

As mentioned above, the fields $Z_\pm$ develop F terms

$$F_{Z_\pm} = -\sqrt{\lambda_+\lambda_-}\Lambda^2. \tag{A12}$$

Note that since we have assumed $g \ll \lambda_\pm$, this F-term VEV is much larger than the dynamically generated D term $g\xi_{\rm dyn} \sim g\Lambda^2$. The size of $F_{Z_\pm}$ can, however, be much smaller than Λ^2 if one takes $\lambda_\pm$ to be very small.

Since this setup introduces another source of SUSY breaking as well as the shift-symmetry breaking, this sector may give rise to a sizable shift-symmetry breaking effect on the inflaton/amplitudon field. For example, if $F_{Z_\pm}$ induces the gravity-mediated soft masses of $\phi_\pm$, this generates a mass for σ via the Coleman-Weinberg potential as in Eq. (55) of the order of

$$\Delta m_\sigma \simeq \frac{\kappa}{4\pi}\frac{|F_{Z_\pm}|}{M_P}, \tag{A13}$$

and thus the requirement[25] of $\Delta m_\sigma < m_T$ restricts $\sqrt{\lambda_+\lambda_-}$ as[26]

$$\sqrt{\lambda_+\lambda_-} < 3\times 10^{-18}\times\left(\frac{|m_T|}{10^{-7}\ \text{GeV}}\right)\times\left(\frac{\kappa}{10^{-2}}\right)^{-1}\left(\frac{\Lambda}{10^{16}\ \text{GeV}}\right)^{-2}. \tag{A14}$$

This may be in contradiction with the condition $g \ll \lambda_\pm$. However, if we consider the no-scale Kähler terms for $\phi_\pm$, the gravity-mediated mass terms may vanish and thus the dominant contribution comes from anomaly mediation [85,86]. In this case, Δm_σ is suppressed by another factor of $\kappa^2/(16\pi^2)$, which allows $\lambda_\pm$ to be larger than the gauge coupling constant.

If $Z_\pm$ were to couple to T too strongly, their F terms, $F_{Z_\pm}$, would generate a large mass for σ. However, Z does not interact with T even at one-loop level, and thus $F_{Z_\pm}$ do not give sizable effects on σ through radiative corrections. The F terms for $M_\pm$ vanish at the leading order with respect to the small couplings g, $\lambda_\pm$, and $\kappa_\pm$, and thus their effects are very tiny and can be completely neglected. On the other hand, there are nonzero contributions to the F terms of $\phi_\pm$, which can induce a mass term for the σ field through the terms in the second line of Eq. (A5). From a straightforward calculation, we find that this effect is smaller than the Coleman-Weinberg effect if

[22]We have checked this perturbatively in the limit where g, $\kappa_\pm$, and κ_1 are small.

[23]The leading order contributions to the VEVs of all fields except ϕ_+ and $M_\pm$ can be taken to be 0 in the limit $\kappa_- \to 0$. This has no adverse effect on the model we consider.

[24]As pointed out in Refs. [82,83], the Ferrara-Zumino current multiplet [84], which contains the energy-momentum tensor and the supersymmetry current, becomes gauge variant in the presence of a constant FI term, and thus cannot be well defined. This prevents the theory from coupling to minimal supergravity.

[25]Note, this constraint is only important if we wish to identify the inflaton as the amplitudon. However, there is still a restriction on the couplings $\lambda_\pm$ but it is much weaker.

[26]Although $F_{Z_\pm}$ may be larger than F_S, $F_{Z_\pm}$ effects are only communicated gravitationally and so give a subdominant contribution to the MSSM soft masses.

$$[|\kappa_+ M_+|^2 + |\kappa_- M_-|^2]^{\frac{1}{2}} < \frac{\kappa g \sqrt{\bar{\xi}}}{4\pi}. \tag{A15}$$

Because $\langle M_\pm \rangle \sim \sqrt{\bar{\xi}}$, this roughly places a constraint of $\sqrt{|\kappa_+|^2 + |\kappa_-|^2} \lesssim \frac{\kappa g}{4\pi}$. Because g grows with the inflation scale, this constraint on $\kappa_\pm$ becomes weaker for larger inflation scales. Thus, we find that there is a sufficient range of parameter space where both (27) and (A15) are satisfied.

APPENDIX B: COSMIC STRINGS AND INFLATION

In this section, we discuss the effects of quantum and thermal corrections on the $U(1)$ symmetry breaking during inflation. This breaking can prevent the generation of cosmic strings after inflation.

1. Quantum fluctuations and cosmic strings

a. General model of cosmic strings

First, we discuss the effects of quantum fluctuations on the $U(1)$-breaking scalar field during inflation. To that end, we consider a simple toy model that is described by the following Lagrangian,

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi), \tag{B1}$$

with

$$V(\phi) = \frac{\bar{g}^2}{2}(|\phi|^2 - \bar{\xi})^2 - \phi C - \phi^* C^* + |\bar{\kappa}|^2 |I|^2 |\phi|^2, \tag{B2}$$

where I is the inflaton. In the limit that $C = 0$, the theory has a global $U(1)$ symmetry, and the vacuum corresponds to $\phi = 0$ for $|\bar{\kappa} I|^2 > \bar{g}^2 \bar{\xi}$. Instead, when $|\bar{\kappa} I|^2 < \bar{g}^2 \bar{\xi}$ we obtain

$$|\phi|^2 = \bar{\xi}. \tag{B3}$$

In this case, the global $U(1)$ symmetry is spontaneously broken, and the vacuum manifold is $U(1) \cong S^1$ as seen in Eq. (B3). Since the first homotopy group of this manifold is $\pi_1(U(1)) \cong \mathbb{Z}$, vortices and strings can form in three and four spacetime dimensions, respectively. Here, we consider four spacetime dimensions and take the z axis parallel to the string. We use polar coordinates (r, θ) in the x–y plane, with the origin located at the center of the cosmic string.[27]

Let us explicitly see how strings form after the $U(1)$ symmetry is broken. In order for the energy per unit length of the string, or string tension, to be finite, it is necessary that $|\phi| \to \sqrt{\bar{\xi}}$ as $r \to \infty$. However, the phase of ϕ at infinity is not necessarily the same in different directions; for instance, we may have

$$\phi(x) \to \sqrt{\bar{\xi}} e^{in\theta} \qquad (r \to \infty), \tag{B4}$$

with n being an integer. Such nontrivial field configurations (for $n \neq 0$) correspond to the formation of strings in the system.

So far, we have considered global strings. It turns out, however, that the string tension in this case diverges if the spatial volume is infinite. On the other hand, if the $U(1)$ symmetry is a gauge symmetry, then the tension becomes finite thanks to nontrivial field configurations of the $U(1)$ gauge field. In this case, the winding number n corresponds to the magnetic flux in the string core. If the $U(1)$ charge times the gauge coupling of ϕ is given by $\bar{g}$, then the masses of the $U(1)$ gauge boson and ϕ in the broken phase are equal; in this case, we have BPS strings whose string tension is given by

$$\mu = 2\pi \bar{\xi} n. \tag{B5}$$

The model discussed in Sec. II C assumes a $U(1)$ gauge symmetry that gives rise to BPS strings. The behavior of the $U(1)$ symmetry breaking itself can, however, be captured with the simplified model in Eq. (B1), and thus we focus on this in what follows.

b. Effects of the linear term

Here we examine in detail what happens if $C \neq 0$, which explicitly breaks the $U(1)$ symmetry. To that end, let us take

$$\phi = v_r e^{i\alpha}. \tag{B6}$$

Then, Eq. (B2) leads to

$$V(\phi) = \frac{\bar{g}^2}{2}(v_r^2 - \bar{\xi})^2 - v_r e^{i\alpha} C - v_r e^{-i\alpha} C^* + |\bar{\kappa}|^2 |I|^2 v_r^2, \tag{B7}$$

with the vacuum conditions

$$2\bar{g}^2 v_r (v_r^2 - \bar{\xi}) + 2|\bar{\kappa}|^2 |I|^2 v_r = e^{i\alpha} C + e^{-i\alpha} C^*, \tag{B8}$$

$$e^{2i\alpha} = \frac{C^*}{C}. \tag{B9}$$

From Eq. (B9), we see that the phase of $\langle \phi \rangle$ is uniquely selected by the phase of the $U(1)$ symmetry breaking term such that

$$\alpha \equiv -\arg(C) \pmod{2\pi}, \tag{B10}$$

and therefore strings never form. We use this mechanism to evade the formation of cosmic strings.

[27]The translational invariance in the theory is also spontaneously broken.

With the condition (B10), Eq. (B8) leads to

$$2\bar{g}^2 v_r(v_r^2 - \bar{\xi}) + 2|\bar{\kappa}|^2|I|^2 v_r = 2|C|. \tag{B11}$$

Assuming that $\bar{g}$ is very small so that we can neglect the first term, as justified in our D-term inflation model, we obtain v_r as

$$v_r \simeq \frac{|C|}{|\bar{\kappa} I|^2}. \tag{B12}$$

In order for strings not to form, we require that this VEV is larger than quantum fluctuations in v_r induced by inflation and thermal fluctuations. This would guarantee that all patches of the sky have the same phase of $\langle\phi\rangle$ in the end, so no strings could form.

c. Cosmic strings for our model

If the quantum fluctuations around the time inflation ends are large enough, cosmic strings could still form since there could be fluctuations of ϕ which spoil the phase alignment imposed by the $U(1)$-breaking term.[28] Here we evaluate the size of the quantum fluctuations in α from inflation; if the fluctuation $\delta\alpha$ can be as large as π, then strings can form after inflation. During inflation, the size of the fluctuations depends on the size of the field mass in the α direction, m_α, relative to the Hubble parameter during inflation, H_I. The mass m_α can be obtained from Eq. (B7) as[29]

$$m_\alpha^2 = \frac{1}{2v_r^2}\frac{\partial^2 V}{\partial\alpha^2}\bigg|_{\alpha=-\arg(C)} = \frac{|C|}{v_r} \simeq |\bar{\kappa} I|^2, \tag{B13}$$

where we have used Eq. (B12). Since the cosmic strings that can be see in the CMB have lengths that are of the order of the current horizon size, the variations in the phases must be in place when the inflaton has the field value corresponding to $N_{\rm CMB}$. The fluctuations in the α direction at this time are then estimated as [87]

$$\langle\delta\alpha^2\rangle = \frac{H_I^3}{24\pi^2 m_\alpha v_r^2} \simeq \frac{H_I^3|\bar{\kappa} I_{\rm CMB}|^3}{24\pi^2|C|^2}, \tag{B14}$$

where $I_{\rm CMB}$ denotes the field value of I at the time when the CMB is set. The condition $\sqrt{\langle\delta\alpha^2\rangle} < \pi$ imposes a lower bound on $|C|$, given an inflation model.

[28]This may not be a necessary condition for no strings, but it is sufficient. As long as there is only one minimum of the potential for some time after the CMB is set, the quantum fluctuations are no longer correlated on superhorizon scales and large strings do not form. Smaller strings may still form, but they lead to different problems, such as overclosure, and are not constrained by the CMB.

[29]To obtain a mass term for the canonically normalized field, we need to rescale by $\sqrt{2}v_r$.

We now apply this result to the model discussed in Appendix A. This model can be mapped on to the simplified model we considered above by setting $\bar{g} = g$, $\bar{\xi} = \xi$, $\bar{\kappa} = \kappa$, $\phi = \phi_+$, $I = i\sigma/\sqrt{2}$, and $C = \kappa\kappa_+^* M_+^* \sigma^2/2$. Note that from Eqs. (11), (13), (17), (18), and (20), we have $\sigma_{\rm CMB} = H_I/(\pi A_s^{1/2}(1 - n_s))$. The limit $\sqrt{\langle\delta\alpha^2\rangle} < \pi$ then reads

$$H_I < \left[\frac{24\pi^2|\kappa_+|^2|M_+|^2}{\sqrt{2}\kappa A_s^{\frac{1}{2}}(1 - n_s)}\right]^{\frac{1}{2}}. \tag{B15}$$

We can easily find a set of parameters that satisfy this condition as well as Eq. (A15).

Since the mass of ϕ_+ approaches 0 at the end of inflation, it is possible that strings with size much smaller than the current horizon could have formed if the fluctuations during this period are too large. To verify that this is not a problem, we examine the same constraint but in the case that $m_\alpha = 0$, which leads to fluctuations of order $\langle\delta\alpha^2\rangle = H_I^2/(8\pi^2 v_r^2)$. In this case the constraint becomes

$$H_I < 2^{3/2}\pi^2\frac{|\kappa_+|}{\kappa}|M_+| = 2.8\times 10^7\ {\rm GeV} \times\left(\frac{|\kappa_+|}{10^{-12}}\right)\left(\frac{|M_+|}{10^{16}\ {\rm GeV}}\right)\left(\frac{\kappa}{10^{-2}}\right)^{-1}. \tag{B16}$$

This is again compatible with the condition (A15). Since the constraints on strings with sizes much smaller than the CMB scale may not be problematic, we only cite the constraint in Eq. (B15) in the main text. However, since (B16) scales with κ_+, just like the constraint in Eq. (B15), this constraint can be satisfied for any H_I by choosing an appropriate κ_+ that is consistent with Eq. (A15).

2. Thermal fluctuations after inflation

Finally, we need to consider the effect that thermal fluctuations can have on the formation of cosmic strings. Since we are only interested in an order of magnitude estimate, we use

$$\langle\delta\alpha^2\rangle_{\rm therm} \simeq T^2/v_r^2, \tag{B17}$$

where the fluctuations are probably a bit more mild than this. By requiring $\sqrt{\langle\delta\alpha^2\rangle}_{\rm therm} < \pi$, we then obtain an upper bound on the reheating temperature as $T_R < \pi v_r$. This corresponds to $T_R < \pi|\langle\phi_+\rangle|$ in the D-term inflation model.

During most of inflation, the nonzero VEV for ϕ_+ is determined by the linear term and the mass term as we have seen above. However, towards the end of inflation,

$$|\kappa T_c|^2 = \frac{\kappa^2}{2}\sigma_c^2 = g^2\xi, \tag{B18}$$

and so the ϕ_+ mass approaches 0. When Eq. (B18) is satisfied, the mass term is 0, and the VEV is set by the quartic term and the linear term, which can be read from Eq. (B8) as

$$|\langle\phi_+\rangle| = \left|\frac{\kappa_+ M_+ \xi}{\kappa}\right|^{\frac{1}{3}}. \tag{B19}$$

Thus, the upper bound on T_R is given by

$$\begin{aligned} T_R &< \pi\left|\frac{\kappa_+ M_+ \xi}{\kappa}\right|^{\frac{1}{3}} \\ &= 6.5\times 10^{14}\ \text{GeV}\times\left(\frac{|\kappa_+|}{10^{-12}}\right)^{\frac{1}{3}}\left(\frac{|M_+|}{10^{16}\ \text{GeV}}\right)^{\frac{1}{3}} \\ &\quad\times\left(\frac{\kappa}{10^{-2}}\right)^{-\frac{1}{3}}\left(\frac{1-n_s}{0.03}\right)^{\frac{1}{6}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{\frac{1}{6}}, \end{aligned} \tag{B20}$$

where we have used Eq. (18). For the values of H_I we are considering, the reheating temperature can easily satisfy this constraint. However, above we have assumed that the field is not displaced from its minimum as the location of the minimum moves near the end of inflation. If the field can indeed track its minimum, there are no cosmic strings from thermal fluctuations.

Now we need to verify that ϕ_+ can track the minimum as the inflaton approaches the critical value, or at least track it sufficiently long such that thermal fluctuations do not cause strings to form. The minimum begins to move once $|\kappa\sigma|^2 \sim g^2\xi$. To approximate the actual size of the VEV of ϕ_+ once σ hits its critical value, we estimate how far ϕ_+ tracks its minimum. The field ϕ_+ tracks its minimum until its mass m_{ϕ_+} becomes equal to $3H_I/2$, a well-known relationship. During inflation, the VEV of ϕ_+ has plenty of time to settle into its minimum of $|\kappa_+ M_+/\kappa|$, where the mass of ϕ_+ (both the radial and phase directions) is given by $m_{\phi_+}^2 = \frac{\kappa^2\sigma^2}{2} - g^2\xi$. The value of σ where ϕ_+ ceases to track its minimum can then be found from $m_{\phi_+} = 3H_I/2$. At this point, the ϕ_+ VEV is [see Eq. (B12)] given by

$$\begin{aligned} |\langle\phi_+\rangle| &\simeq \frac{|C|}{m_{\phi_+}^2} = \frac{2|\kappa\kappa_+ M_+ \sigma^2|}{9H_I^2} \simeq \frac{4g^2\xi|\kappa_+ M_+|}{9\kappa H_I^2} \\ &\simeq \frac{8}{3\sqrt{3A_s(1-n_s)}}\left|\frac{\kappa_+ M_+}{\kappa}\right|, \end{aligned} \tag{B21}$$

where we have used $\frac{\kappa^2\sigma^2}{2} \simeq g^2\xi$, Eq. (17), and Eq. (19). The upper bound on T_R is then given by

$$\begin{aligned} T_R &< \frac{8\pi}{3\sqrt{3A_s(1-n_s)}}\left|\frac{\kappa_+ M_+}{\kappa}\right| \\ &= 1.5\times 10^{19}\ \text{GeV}\times\left(\frac{|\kappa_+|}{10^{-12}}\right)\left(\frac{|M_+|}{10^{16}\ \text{GeV}}\right) \\ &\quad\times\left(\frac{\kappa}{10^{-2}}\right)^{-1}\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{2}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{2}}. \end{aligned} \tag{B22}$$

This bound is satisfied for the entire parameter space that is compatible with the CMB observation. Indeed, we can show that the maximum possible reheat temperature is always smaller than the VEV of ϕ_+.

[1] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2017-022, 2017.
[2] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2017-021, 2017.
[3] CMS Collaboration, CERN Report No. CMS-PAS-SUS-16-036, 2017.
[4] G. Panico and A. Wulzer, The composite Nambu-Goldstone Higgs, Lect. Notes Phys. **913**, 1 (2016).
[5] P. W. Graham, D. E. Kaplan, and S. Rajendran, Cosmological Relaxation of the Electroweak Scale, Phys. Rev. Lett. **115**, 221801 (2015).
[6] L. F. Abbott, A mechanism for reducing the value of the cosmological constant, Phys. Lett. **150B**, 427 (1985).
[7] G. Dvali and A. Vilenkin, Cosmic attractors and gauge hierarchy, Phys. Rev. D **70**, 063501 (2004).
[8] G. Dvali, Large hierarchies from attractor vacua, Phys. Rev. D **74**, 025018 (2006).
[9] S. Di Chiara, K. Kannike, L. Marzola, A. Racioppi, M. Raidal, and C. Spethmann, Relaxion cosmology and the price of fine-tuning, Phys. Rev. D **93**, 103527 (2016).
[10] L. J. Hall and Y. Nomura, Spread supersymmetry, J. High Energy Phys. 01 (2012) 082.
[11] L. J. Hall, Y. Nomura, and S. Shirai, Spread supersymmetry with wino LSP: Gluino and dark matter signals, J. High Energy Phys. 01 (2013) 036.
[12] M. Ibe and T. T. Yanagida, The lightest Higgs boson mass in pure gravity mediation model, Phys. Lett. B **709**, 374 (2012).
[13] M. Ibe, S. Matsumoto, and T. T. Yanagida, Pure gravity mediation with $m_{3/2} = 10$–100 TeV, Phys. Rev. D **85**, 095011 (2012).
[14] A. Arvanitaki, N. Craig, S. Dimopoulos, and G. Villadoro, Minisplit, J. High Energy Phys. 02 (2013) 126.

[15] N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner, and T. Zorawski, Simply unnatural supersymmetry, arXiv:1212.6971.
[16] J. L. Evans, M. Ibe, K. A. Olive, and T. T. Yanagida, Universality in pure gravity mediation, Eur. Phys. J. C **73**, 2468 (2013).
[17] J. L. Evans, K. A. Olive, M. Ibe, and T. T. Yanagida, Nonuniversalities in pure gravity mediation, Eur. Phys. J. C **73**, 2611 (2013).
[18] B. Batell, G. F. Giudice, and M. McCullough, Natural heavy supersymmetry, J. High Energy Phys. 12 (2015) 162.
[19] J. R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, and G. Servant, Cosmological Higgs-Axion Interplay for a Naturally Small Electroweak Scale, Phys. Rev. Lett. **115**, 251803 (2015).
[20] J. L. Evans, T. Gherghetta, N. Nagata, and Z. Thomas, Naturalizing supersymmetry with a two-field relaxion mechanism, J. High Energy Phys. 09 (2016) 150.
[21] E. Hardy, Electroweak relaxation from finite temperature, J. High Energy Phys. 11 (2015) 077.
[22] F. P. Huang, Y. Cai, H. Li, and X. Zhang, A possible interpretation of the Higgs mass by the cosmological attractive relaxion, Chin. Phys. C **40**, 113103 (2016).
[23] T. Kobayashi, O. Seto, T. Shimomura, and Y. Urakawa, Relaxion window, arXiv:1605.06908.
[24] A. Hook and G. Marques-Tavares, Relaxation from particle production, J. High Energy Phys. 12 (2016) 101.
[25] T. Higaki, N. Takeda, and Y. Yamada, Cosmological relaxation and high scale inflation, Phys. Rev. D **95**, 015009 (2017).
[26] K. Choi and S. H. Im, Constraints on relaxion windows, J. High Energy Phys. 12 (2016) 093.
[27] T. Flacke, C. Frugiuele, E. Fuchs, R. S. Gupta, and G. Perez, Phenomenology of relaxion-Higgs mixing, arXiv:1610.02025.
[28] L. McAllister, P. Schwaller, G. Servant, J. Stout, and A. Westphal, Runaway relaxion monodromy, arXiv:1610.05320.
[29] K. Choi, H. Kim, and T. Sekiguchi, Dynamics of cosmological relaxation after reheating, Phys. Rev. D **95**, 075008 (2017).
[30] Z. Lalak and A. Markiewicz, Dynamical relaxation in 2HDM models, arXiv:1612.09128.
[31] T. You, A dynamical weak scale from inflation, arXiv:1701.09167.
[32] L. Marzola and M. Raidal, Natural relaxation, Mod. Phys. Lett. A **31**, 1650215 (2016).
[33] A. Fowlie, C. Balazs, G. White, L. Marzola, and M. Raidal, Naturalness of the relaxion mechanism, J. High Energy Phys. 08 (2016) 100.
[34] E. D. Stewart, Inflation, supergravity, and superstrings, Phys. Rev. D **51**, 6847 (1995).
[35] P. Binetruy and G. R. Dvali, D-term inflation, Phys. Lett. B **388**, 241 (1996).
[36] E. Halyo, Hybrid inflation from supergravity D terms, Phys. Lett. B **387**, 43 (1996).
[37] P. Binetruy, G. Dvali, R. Kallosh, and A. Van Proeyen, Fayet-Iliopoulos terms in supergravity and cosmology, Classical Quantum Gravity **21**, 3137 (2004).
[38] G. German, G. G. Ross, and S. Sarkar, Low-scale inflation, Nucl. Phys. **B608**, 423 (2001).
[39] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions, Phys. Rep. **215**, 203 (1992).
[40] V. Domcke, K. Schmitz, and T. T. Yanagida, Dynamical D terms in supergravity, Nucl. Phys. **B891**, 230 (2015).
[41] S. R. Coleman and E. J. Weinberg, Radiative corrections as the origin of spontaneous symmetry breaking, Phys. Rev. D **7**, 1888 (1973).
[42] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. **594**, A13 (2016).
[43] A. R. Liddle and D. H. Lyth, The cold dark matter density perturbation, Phys. Rep. **231**, 1 (1993).
[44] A. R. Liddle and S. M. Leach, How long before the end of inflation were observable perturbations produced?, Phys. Rev. D **68**, 103503 (2003).
[45] S. Dodelson and L. Hui, A Horizon Ratio Bound for Inflationary Fluctuations, Phys. Rev. Lett. **91**, 131301 (2003).
[46] D. I. Podolsky, G. N. Felder, L. Kofman, and M. Peloso, Equation of state and beginning of thermalization after preheating, Phys. Rev. D **73**, 023501 (2006).
[47] P. A. R. Ade *et al.* (BICEP2 and Keck Array Collaborations), Improved Constraints on Cosmology and Foregrounds from BICEP2 and Keck Array Cosmic Microwave Background Data with Inclusion of 95 GHz Band, Phys. Rev. Lett. **116**, 031302 (2016).
[48] R. Jeannerot, Inflation in supersymmetric unified theories, Phys. Rev. D **56**, 6205 (1997).
[49] D. H. Lyth and A. Riotto, Comments on D-term inflation, Phys. Lett. B **412**, 28 (1997).
[50] M. Endo, M. Kawasaki, and T. Moroi, Cosmic string from D-term inflation and curvaton, Phys. Lett. B **569**, 73 (2003).
[51] J. Rocher and M. Sakellariadou, Supersymmetric grand unified theories and cosmology, J. Cosmol. Astropart. Phys. 03 (2005) 004.
[52] J. Rocher and M. Sakellariadou, D-Term Inflation, Cosmic Strings, and Consistency with Cosmic Microwave Background Measurement, Phys. Rev. Lett. **94**, 011303 (2005).
[53] M. Wyman, L. Pogosian, and I. Wasserman, Bounds on cosmic strings from WMAP and SDSS, Phys. Rev. D **72**, 023513 (2005); Erratum, Phys. Rev. D **73**, 089905(E) (2006).
[54] R. Battye, B. Garbrecht, and A. Moss, Tight constraints on F- and D-term hybrid inflation scenarios, Phys. Rev. D **81**, 123512 (2010).
[55] R. Battye and A. Moss, Updated constraints on the cosmic string tension, Phys. Rev. D **82**, 023521 (2010).
[56] J. Rocher and M. Sakellariadou, D-term inflation in nonminimal supergravity, J. Cosmol. Astropart. Phys. 11 (2006) 001.
[57] O. Seto and J. Yokoyama, Hiding cosmic strings in supergravity D-term inflation, Phys. Rev. D **73**, 023508 (2006).
[58] W. Buchmuller and K. Ishiwata, Grand unification and subcritical hybrid inflation, Phys. Rev. D **91**, 081302 (2015).
[59] J. Urrestilla, A. Achucarro, and A. C. Davis, D-Term Inflation without Cosmic Strings, Phys. Rev. Lett. **92**, 251302 (2004).

[60] J. Urrestilla, N. Bevis, M. Hindmarsh, M. Kunz, and A. R. Liddle, Cosmic microwave anisotropies from BPS semilocal strings, J. Cosmol. Astropart. Phys. 07 (2008) 010.
[61] C. F. Kolda and J. March-Russell, Supersymmetric D-term inflation, reheating, and Affleck-Dine baryogenesis, Phys. Rev. D **60**, 023504 (1999).
[62] M. Kawasaki and M. Yamada, Affleck-Dine baryogenesis after D-term inflation and solutions to the baryon-dark matter coincidence problem, Phys. Rev. D **91**, 083512 (2015).
[63] T. Kawano and M. Yamaguchi, Reheating in chaotic D-term inflation, Phys. Rev. D **78**, 123511 (2008).
[64] J. Ellis, F. Luo, and K. A. Olive, Gluino coannihilation revisited, J. High Energy Phys. 09 (2015) 127.
[65] N. Nagata, H. Otono, and S. Shirai, Probing bino-gluino coannihilation at the LHC, Phys. Lett. B **748**, 24 (2015).
[66] J. Ellis, J. L. Evans, F. Luo, and K. A. Olive, Scenarios for gluino coannihilation, J. High Energy Phys. 02 (2016) 071.
[67] N. Nagata, H. Otono, and S. Shirai, Probing bino-wino coannihilation at the LHC, J. High Energy Phys. 10 (2015) 086.
[68] K. Rolbiecki and K. Sakurai, Long-lived bino and wino in supersymmetry with heavy scalars and higgsinos, J. High Energy Phys. 11 (2015) 091.
[69] T. Kaji, https://indico.in2p3.fr/event/13763/session/7/contribution/75/material/slides/0.pdf, 2017.
[70] ATLAS, Report No. ATLAS-CONF-2017-017, 2017.
[71] ATLAS Collaboration, CERN Report No. ATL-PHYS-PUB-2014-010, 2014.
[72] M. Ibe, T. Moroi, and T. Yanagida, Possible signals of wino LSP at the Large Hadron Collider, Phys. Lett. B **644**, 355 (2007).
[73] H. Fukuda, N. Nagata, H. Otono, and S. Shirai, Higgsino dark matter or not: Role of disappearing track searches at the LHC and future colliders, arXiv:1703.09675.
[74] K. Choi and S. H. Im, Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry, J. High Energy Phys. 01 (2016) 149.
[75] D. E. Kaplan and R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, Phys. Rev. D **93**, 085007 (2016).
[76] G. F. Giudice and M. McCullough, A clockwork theory, J. High Energy Phys. 02 (2017) 036.
[77] J. E. Kim, H. P. Nilles, and M. Peloso, Completing natural inflation, J. Cosmol. Astropart. Phys. 01 (2005) 005.
[78] V. Domcke and K. Schmitz, A unified model of D-term inflation, Phys. Rev. D **95**, 075020 (2017).
[79] K.-I. Izawa and T. Yanagida, Dynamical supersymmetry breaking in vectorlike gauge theories, Prog. Theor. Phys. **95**, 829 (1996).
[80] K. A. Intriligator and S. D. Thomas, Dynamical supersymmetry breaking on quantum moduli spaces, Nucl. Phys. **B473**, 121 (1996).
[81] N. Seiberg, Exact results on the space of vacua of four-dimensional SUSY gauge theories, Phys. Rev. D **49**, 6857 (1994).
[82] Z. Komargodski and N. Seiberg, Comments on the Fayet-Iliopoulos term in field theory and supergravity, J. High Energy Phys. 06 (2009) 007.
[83] Z. Komargodski and N. Seiberg, Comments on supercurrent multiplets, supersymmetric field theories and supergravity, J. High Energy Phys. 07 (2010) 017.
[84] S. Ferrara and B. Zumino, Transformation properties of the supercurrent, Nucl. Phys. **B87**, 207 (1975).
[85] L. Randall and R. Sundrum, Out of this world supersymmetry breaking, Nucl. Phys. **B557**, 79 (1999).
[86] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, Gaugino mass without singlets, J. High Energy Phys. 12 (1998) 027.
[87] K. A. Olive, Inflation, Phys. Rep. **190**, 307 (1990).