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Fermion localization in a backreacted warped spacetime

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We consider a five dimensional anti–de Sitter (AdS) warped spacetime in presence of a massive scalar field in the bulk. The scalar field potential fulfills the requirement of modulus stabilization even when the effect of backreaction of the stabilizing field is taken into account. In such a scenario, we explore the role of backreaction on the localization of bulk fermions which in turn determines the effective radion-fermion coupling on the brane. Our result reveals that both the chiral modes of the zeroth Kaluza-Klein (KK) fermions get localized near TeV brane as the backreaction of the scalar field increases. We also show that the profile of massive KK fermions shifts towards the Planck brane with an increasing backreaction parameter. Some implications in the context of LHC physics are discussed.

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I. INTRODUCTION

Ever since the original proposal of Kaluza-Klein (KK) regarding the existence of extra spatial dimension(s), it is often believed that our Universe is a three-brane embedded in a higher dimensional spacetime and is described through a low energy effective theory on the brane carrying the signatures of extra dimensions [1,2]

Til date, the Standard Model (SM) of particle physics is the best suited model for describing the possible interactions between fundamental particles up to TeV scale. However, the Standard Model carries a quadratic divergence in the radiatively corrected Higgs mass which can be set to the desired value 126 GeV only through an unnatural fine-tuning of the parameters of the theory. Among various models proposed to solve this fine-tuning problem [3–9] the Randall-Sundrum (RS) warped geometry model [5] earns a special attention since it resolves the gauge hierarchy problem without introducing any intermediate scale between Planck and the TeV scale. The interbrane separation, known as modulus (or radion), is assumed to be ∼ Planck length in order to generate the required hierarchy between the branes. A suitable potential with a stable minimum is therefore needed for modulus stabilization. Goldberger and Wise (GW) proposed a useful stabilization mechanism [10] by introducing a massive scalar field in the bulk with appropriate boundary values. Not only the stable value of the modulus appear as a parameter in the low energy effective theory on the brane, but it is a fluctuation about that stable value leads to dynamical modulus (or radion) field, which couples to the fields on the observable brane. This resulted into a large volume of work on phenomenological and cosmological implications [11–16] of modulus field in RS warped geometry model. Though the backreaction of the stabilizing scalar field was originally neglected in GW proposal, its implications are subsequently studied in [12,17]. It has been demonstrated in [17] that the modulus of the RS scenario can be stabilized using the GW prescription even by incorporating the backreaction of the stabilizing field. In such a braneworld scenario, several models were proposed by placing the standard model fields inside the bulk. Especially the localization property of a bulk fermion field [18–25] has been a subject of great interest where explanations for observed chiral nature of massless fermion and the hierarchial masses of fermions among different generations have been explored. In this context, it is observed that the overlap of the bulk fermion wave function on our visible brane plays crucial role in determining the effective radion-fermion coupling which in turn determines phenomenology of the radion with brane matter fields.

In view of above, it is worthwhile to address the role of backreaction of the stabilizing field on fermion localization. We aim to address this in the present work.

Our paper is organized as follows: The backreacted RS scenario and its modulus stabilization is described in Sec. II. Section III addresses the localization property of bulk fermion field and its consequences. The paper ends with some concluding remarks.

II. BACKREACTED RS MODEL AND ITS MODULUS STABILIZATION

The action for the RS geometry with a stabilizing scalar field Φ [12] is as follows:

$$S = \int d^5x\sqrt{G}[-M^3R + \Lambda] + \int d^5x\sqrt{G}[(1/2)G^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)] - \int d^4x\sqrt{-g_{\rm hid}}\lambda_{\rm hid}(\Phi) - \int d^4x\sqrt{-g_{\rm vis}}\lambda_{\rm vis}(\Phi), \quad (1)$$

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where M is the five-dimensional Planck scale, Λ is the bulk cosmological constant, G_{MN} is the five-dimensional metric, g_{vis} and g_{vis} are the induced metric on the hidden and visible brane, respectively. λ_{vis}, λ_{vis} are the self interactions of scalar field (including brane tensions) on the Planck and TeV branes. The background metric ansatz is given by

$$ds^2 = \exp\left[-2A(y)\right]\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{2}$$

where $A(y)$ is the warp factor. The bulk scalar field is assumed to depend only on the extra dimensional coordinate (y) [12].

In order to get an analytic solution of the backreacted geometry, the form of the scalar field potential is chosen as [12]

$$V(\Phi) = (1/2)\Phi^2(u^2 + 4uk) - (\kappa^2/6)u^2\Phi^4, \tag{3}$$

where $k = \sqrt{-\kappa^2\Lambda/6}$. The potential contains quadratic as well as quartic self interaction of the scalar field, and the two terms are connected by a common free parameter "u". Using this form of the potential, one obtains a solution of coupled gravitational-scalar field equations as

$$A(y) = k|y| + (\kappa^2/12)\Phi_P^2 \exp(-2u|y|) \tag{4}$$

$$\Phi(\varphi) = \Phi_P \exp(-u|y|), \tag{5}$$

where Φ_P is the value of the scalar field on Planck brane. From Eq. (4), it can be argued that $\kappa\Phi_P$ controls the deviation of the warp factor from RS model, and thus $\kappa\Phi_P$ is known as the scalar field backreaction parameter. Moreover, λ_{vis} and λ_{vis} can be obtained from the boundary conditions on branes as

$$\lambda_{\mathrm{vis}} = 6k/\kappa^2 - u\Phi_P^2 \tag{6}$$

$$\lambda_{\mathrm{vis}} = -6k/\kappa^2 + u\Phi_P^2 \exp(-2u\pi r_c), \tag{7}$$

where r_c is the compactification radius of the extra dimension. Once the solutions of $A(y)$ and $\Phi(y)$ are obtained [see Eqs. (4) and (5)], the modulus can be stabilized using GW prescription. It has been demonstrated in [17] that the interbrane separation in a backreacted RS scenario is stabilized at a value given by

$$k\pi r_c = \frac{k}{u}\ln\left\{\frac{\kappa\Phi_P}{2\sqrt{1+\frac{2k}{u}}}\right\}. \tag{8}$$

In the original Randall-Sundrum (RS) model, all the parameters are connected by a single dimensionful (mass dimension [1]) quantity k (which is related to the bulk cosmological constant). The parameter r_c, namely the modulus, is determined by $kr_c \approx 12$ so that the gauge hierarchy problem can be resolved. Subsequently, to stabilize the value of the modulus to this value, Goldberger and Wise (GW) introduced a bulk scalar field with a mass parameter m, and the resulting stable value of the modulus turned out to be proportional to $\frac{k^2}{m^2}$. By choosing $\frac{k}{m}$ appropriately, GW could stabilize the value of r_c to the desired value, $kr_c \approx 12$. However, in GW approach, the effect of backreaction of the bulk scalar on warp factor was neglected. The goal of the present work is to include the effect of backreaction of the bulk scalar on the warp factor with the chosen form of the scalar potential [see Eq. (3)]. Here, the scalar mass and couplings are determined by the parameters u and k. Just as in GW work, here the parameter u appears in the stable value of r_c [in Eq. (8)]. However, now due to inclusion of backreaction, u (along with k) also explicitly appears in the warp factor which in turn modifies the bulk field profile and the resulting masses and couplings. It may be noted that the present construction would not only help us to formulate a stable braneworld model capable of addressing the gauge hierarchy issue but may lead to a new phenomenological scenario in the context of braneworld physics, which includes the effects of the backreaction of the stabilizing field. We now show how the scalar backreaction affects the localization of fermion field within the five dimensional spacetime.

III. FERMION LOCALIZATION

Consider a bulk massive fermion field propagating on a background geometry model characterized by action in Eq. (1). The Lagrangian for the Dirac fermions is given by

$$L_{\mathrm{Dirac}} = e^{-4A(y)}[\bar{\Psi}i\Gamma^a D_a\Psi - m_5\bar{\Psi}\Psi],$$

where $\Psi = \Psi(x^\mu, y)$ is the fermion field and m_5 is its mass. $\Gamma^a = (e^{A(y)}\gamma^\mu, -i\gamma^5)$ denotes the five-dimensional gamma matrices, where γ^μ and γ^5 represent 4D gamma matrices in chiral representation. Curved gamma matrices obey the Clifford algebra, i.e., $[\Gamma^a, \Gamma^b] = 2G^{ab}$. The covariant derivative D_a can be calculated by using the metric in Eq. (2) and is given by

$$D_\mu = \partial_\mu - \frac{1}{2}\Gamma_\mu\Gamma^4 A'(y)e^{-A(y)}$$
$$D_5 = \partial_y.$$

Using this setup, the Dirac Lagrangian L_{Dirac} turns out to be

$$L_{\mathrm{Dirac}} = e^{-4A(y)}\bar{\Psi}[ie^{A(y)}\gamma^\mu\partial_\mu + \gamma^5(\partial_y - 2A'(y)) - m_5]\Psi. \tag{9}$$

We decompose the five-dimensional spinor via Kaluza-Klein (KK) mode expansion as $\Psi(x^\mu, y) = \sum\chi^n(x^\mu)\xi^n(y)$, where the superscript n denotes the nth KK mode. $\chi^n(x^\mu)$ is the projection of $\Psi(x^\mu, y)$ on the three-brane, and $\xi^n(y)$ is the extra dimensional component of 5D spinor. Left (χ_L) and right (χ_R) states are constructed by $\chi^n_{L,R} = \frac{1}{2}(1 \mp \gamma^5)\chi^n$. Thus, the KK mode expansion can be written in the following way:

$$\Psi(x^\mu, y) = \sum [\chi^n_L(x^\mu)\xi^n_L(y) + \chi^n_R(x^\mu)\xi^n_R(y)]. \quad (10)$$

Substituting the KK mode expansion of $\Psi(x^\mu, y)$ in the Dirac field lagrangian given in Eq. (9), we obtain the following equations of motion for $\xi_{L,R}(y)$ as follows:

$$e^{-A(y)}[\pm(\partial_y - 2A'(y)) + m_5]\xi^n_{R,L}(y) = -m_n \xi^n_{L,R}(y), \quad (11)$$

where m_n is the mass of nth KK mode. The 4D fermions obey the canonical equation of motion $i\gamma^\mu \partial_\mu \chi^n_{L,R} = m_n \chi^n_{L,R}$. Moreover, Eq. (11) is obtained provided the following normalization conditions hold:

$$\int_0^\pi dy e^{-3A(y)} \xi^m_{L,R} \xi^n_{L,R} = \delta_{m,n} \quad (12)$$

$$\int_0^\pi dy e^{-3A(y)} \xi^m_L \xi^n_R = 0. \quad (13)$$

In the next two subsections, we discuss the localization scenario for massless and massive KK modes, respectively.

A. Massless KK mode

For massless mode, the equation of motion of $\xi_{L,R}$ takes the following form (taking $\frac{\kappa \Phi_P}{\sqrt{2}} = l$):

$$\exp\left(-ky - \frac{l^2}{6}e^{-2uy}\right)\left[\pm\left(\partial_y - 2k + \frac{2l^2 u}{3}e^{-2uy}\right) + m_5\right] \times \xi_{R,L}(y) = 0, \quad (14)$$

where we use the form of warp factor, i.e., $A(y) = ky + \frac{l^2}{6}e^{-2uy}$. Solution of Eq. (14) is given by

$$\xi_{L,R}(y) = \sqrt{\frac{k}{e^{2k\pi r_c} - 1}}\left[1 + \exp\left(\frac{2l^2}{3}e^{-u\pi r_c}\right)\right]^{1/2} * \exp\left(\frac{l^2}{6}e^{-2uy}\right)e^{2ky} \quad (15)$$

for $m_5 = 0$, and

$$\xi_{L,R}(y) = \sqrt{\frac{k}{e^{(2k \pm m_5)\pi r_c} - 1}}\left[1 + \exp\left(\frac{2l^2}{3}e^{-u\pi r_c} \mp \frac{2m_5}{k}\right)\right]^{1/2} * \exp\left(\frac{l^2}{6}e^{-2uy}\right)e^{(2k \pm m_5)y} \quad (16)$$

for $m_5 \neq 0$.

The overall normalization constants in Eqs. (15) and (16) are determined by using the normalization condition presented earlier in Eq. (12). It may be noticed that left and right chiral modes have the same solution when $m_5 = 0$, but the degeneracy between the two chiral modes lifted in the presence of nonzero bulk fermionic mass term.

It is worthwhile to study how the localization scenario depends on the backreaction parameter (l) as well as the bulk mass parameter (m_5).

1. Effect of backreaction parameter

From Eq. (15), we obtain the Fig. 1 between $\xi_{L,R}$ and y for various values of backreaction parameter. The constant y hypersurfaces at $y = 0$ and $y = 36$ represent Planck and TeV branes, respectively. We focus into the region near the TeV brane (see Fig. 1) to depict the localization properties of the left and right modes.

Figure 1 clearly demonstrates that for $m_5 = 0$, the two chiral modes get more and more localized on TeV brane as the backreaction parameter comes close to $l = 25$ (needed to solve the gauge hierarchy problem for $\frac{u}{k} = 0.2$) from a lower value. Thus, the solution of a hierarchy problem and the localization of a fermion are linked with each other. The figure also reveals that the larger the backreaction parameter l, the localization of both the chiral modes of massless fermions becomes sharper near the visible brane.

On the other hand, for small values of l, the fermions are clearly localized deep inside the bulk spacetime and without any backreaction (i.e., $l = 0$); we retrieve the RS solution where the fermions are peaked away from the visible brane. Thus, without any bulk mass term, the fermions can be localized at different regions inside the bulk by adjusting the value of the backreaction parameter.

From Eq. (16), we obtain the plots of left and right chiral modes for various l in the presence of a nonzero bulk fermionic mass.

Figures 2 and 3 reveal that as the backreaction parameter increases, the peak of both the left and right chiral wave function get shifted towards the visible brane.

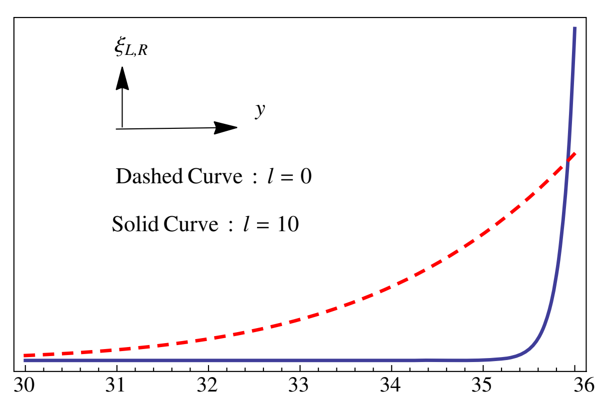

FIG. 1. $\xi_{L,R}$ vs y for $k = 1$, $\frac{u}{k} = 0.01$, and $m_5 = 0$.

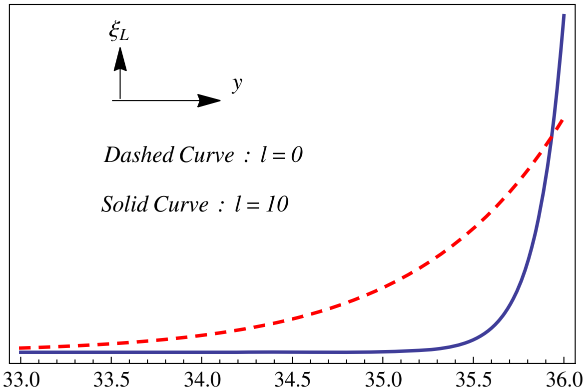

FIG. 2. ξ_L vs y for $k = 1$, $\frac{u}{k} = 0.01$ and $m_5 = 0.5k$.

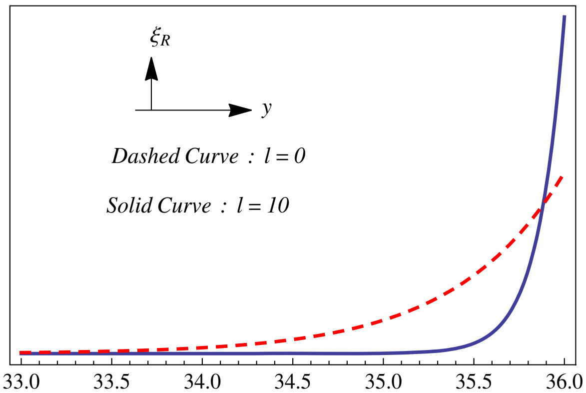

FIG. 3. ξ_R vs y for $k = 1$, $\frac{u}{k} = 0.01$ and $m_5 = 0.5k$.

Moreover, using the solution of $\xi_{L,R}(y)$ [in Eq. (16)], we obtain the effective coupling [26] between the radion and zeroth order fermionic KK mode as follows:

$$\lambda_L = \sqrt{\frac{k}{24M^3}} e^{A(\pi r_c)} \exp\left[\frac{l^2}{3}\exp(-2u\pi r_c)\right] \times \left[1 + \exp\left(\frac{2l^2}{3} e^{-u\pi r_c} - \frac{2m_5}{k}\right)\right] \times \left(\frac{e^{(k+2m_5)\pi r_c}}{e^{(k+2m_5)\pi r_c} - 1}\right) \tag{17}$$

for a left-handed chiral mode and,

$$\lambda_R = \sqrt{\frac{k}{24M^3}} e^{A(\pi r_c)} \exp\left[\frac{l^2}{3}\exp(-2u\pi r_c)\right] \times \left[1 + \exp\left(\frac{2l^2}{3} e^{-u\pi r_c} + \frac{2m_5}{k}\right)\right] \times \left(\frac{e^{(k-2m_5)\pi r_c}}{e^{(k-2m_5)\pi r_c} - 1}\right) \tag{18}$$

for a right-handed mode. It is evident that the effective radion-fermion coupling increases (for both a left and right chiral mode) with the backreaction parameter. It is expected because the peak of both a left and right chiral wave function gets shifted towards the visible brane as the backreaction parameter increases. In the search of an extra dimension in LHC, the physics of radion and its decay channels play an important role. Among different decay channels, it is worthwhile to search for flavor violating radion decay channel through leptonic decay modes. It has been shown that if the radion production cross section is large, one can get a significant decay channel into l^+l^- in comparison to a similar leptonic decay mode in a Standard Model background (see [27] and references therein). Moreover, the $\tau^+\tau^-$ decay channel has been shown to contribute about 5% when the radion mass is 50 GeV with coupling (fermion-radion) of the order of 500 GeV^{-1}. It is therefore considered to be an important channel for a very light radion signal at the LHC. Additionally, if the radion mass is greater than the top quark mass, then it can decay also into a top and charm quark which is shown to be significant for a radion mass ≈250 GeV and a coupling ≈100 GeV^{-1}. For all such processes, the present scenario with an enhanced radion-fermion coupling due to back-reaction effect would allow a larger mass bound for the radion and also should lead to an enhanced top production whose signal from the background can give hint to flavor violating decay of radion.

B. Massive KK mode

In this section, we study the localization of higher Kaluza-Klein modes. For massive KK modes, the equation of motion for a fermionic wave function is given as

$$\exp\left(-ky - \frac{l^2}{6} e^{-2uy}\right)\left[\pm\left(\partial_y - 2k + \frac{2l^2 u}{3} e^{-2uy}\right) + m_5\right] \times \xi^n_{R,L}(y) = -m_n \xi^n_{L,R}(y). \tag{19}$$

Recall that m_n is the mass of nth KK mode. Using the rescaling $\tilde{\xi}_{L,R} = e^{\frac{5}{2}A(y)}\xi_{L,R}$, we find that the two helicity states, ξ_L and ξ_R satisfy the same equation of motion and is given by

$$\tilde{\xi}^{n\prime\prime}(y) + \left[-\frac{k^2}{4}\left(1 + \frac{l^2}{3}\frac{u}{k}\right) + \left(m_n^2 - \frac{k^2}{4}\left(1 + \frac{l^2}{3}\frac{u}{k}\right) - m_5^2\right) \times \exp\left(2ky + \frac{l^2}{3} e^{-2uy}\right)\right]\tilde{\xi}^n_{L,R}(y) = 0. \tag{20}$$

Solution of Eq. (20) is given by Bessel function as follows:

$$\xi^n_{L,R}(y) = \sqrt{k}\exp\left(-\frac{5l^2}{12}e^{-2uy}\right) \times \Gamma\left(1+\frac{\sqrt{3k+l^2u}}{4\sqrt{3k}}\right)\mathrm{BesselI}\left[-\frac{\sqrt{3k+l^2u}}{4\sqrt{3k}}, e^{-ky}\frac{\sqrt{3k^2+12m_n^2+12m_5^2+kl^2u}}{4\sqrt{3k}}\right]. \tag{21}$$

The mass spectrum can be obtained from the requirement that the wave function is well behaved on the brane. Demanding the continuity of $\xi_{L,R}$ at $y=0$ and at $y=\pi r_c$ gives the mass term as follows:

$$m_n^2 = e^{-2A(\pi)}[k^2(n^2+2n+1)+m_5^2], \tag{22}$$

where $n = 1, 2, 3\ldots$ Now from the requirement of solving the gauge hierarchy problem, the warp factor at TeV brane acquires the value as $A(\pi) = 36$, which produces a large suppression in the right-hand side of Eq. (22) through the exponential factor. Since $k, m_5 \sim M$, the mass of KK modes $(n = 1, 2, 3\ldots)$ comes at TeV scale. Using the solution of $\xi^n_{L,R}(y)$ [in Eq. (21)], we determine the coupling between massive KK fermion modes and the radion field, given by

$$\lambda^{(n)} = \sqrt{\frac{k}{24M^3}}e^{A(\pi)}\exp\left(-\frac{5l^2}{6}e^{-2u\pi r_c}\right) \times\left(\Gamma\left(1+\frac{\sqrt{3k+l^2u}}{4\sqrt{3k}}\right)\mathrm{BesselI}\left[-\frac{\sqrt{3k+l^2u}}{4\sqrt{3k}}, e^{-k\pi r_c}\frac{\sqrt{3k^2+12m_n^2+12m_5^2+kl^2u}}{4\sqrt{3k}}\right]\right)^2, \tag{23}$$

where $\lambda^{(n)}$ is the coupling between n th KK fermion mode and the radion field. Equation (23) clearly indicates that $\lambda^{(n)}$ decreases with an increasing backreaction parameter.

Equation (21) indicates the relation between $\xi_{L,R}$ and y for various values of l from which one can find the dependence of localization for massive KK fermion modes on the backreaction parameter. From this, the behavior of the first KK mode ($n = 1$) is described in Fig. 4.

Figure 4 clearly depicts that the wave function for first massive KK mode gets more and more localized near Planck brane with an increasing value of the backreaction parameter. As a result, the coupling parameter decreases near the visible brane as the backreaction parameter increases. Moreover, it can also be shown [from Eq. (21)] that as the order of KK mode increases from $n = 1$, the localization of fermions becomes sharper near Planck brane.

Before concluding, it may be mentioned that the bulk fermion mass term (m_5) also affects the localization of the fermion field. Using the solution of $\xi_{L,R}(y)$ presented in Eq. (16), it can be shown that for a fixed value of the backreaction parameter, the left chiral mode of the zeroth KK fermion has higher peak values on the TeV brane as the bulk fermion mass increases, whereas the right chiral mode shows the reverse nature, which is in agreement with [24].

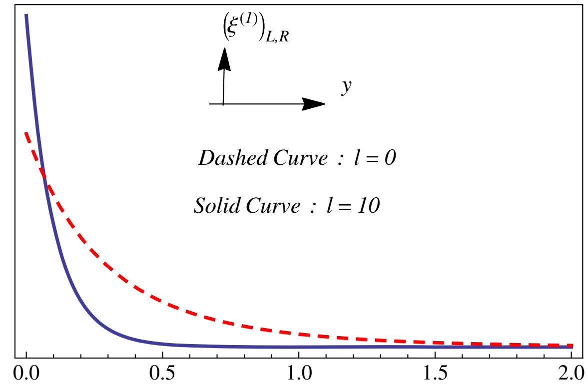

FIG. 4. $\xi^1_{L,R}$ vs y for $k=1$, $\frac{u}{k}=0.01$ and $\frac{m_5}{k}=0.5$.

IV. CONCLUSION

We consider a five-dimensional anti–de Sitter compactified warped geometry model with two three-branes embedded within the spacetime. For the purpose of modulus stabilization, a massive scalar field is invoked in the bulk and its backreaction on spacetime geometry is taken into account. The present construction now addresses the RS phenomenology when both the modulus stabilization and the backreaction are taken into consideration. In this scenario, we study how the backreaction parameter affects the localization of a bulk fermion field within the entire spacetime. Moreover, we also explore how the fermion localization depends on the bulk mass parameter. Our findings are as follows:

(1) For massless KK mode—
  (i) In the absence of a bulk fermion mass, left and right chiral modes can be localized at different regions in the spacetime by adjusting the value of the backreaction parameter (l). However, the localization of both the chiral modes becomes sharper near TeV brane as the value of l increases.
  (ii) In the presence of a nonzero bulk fermion mass, the left as well as right mode get more and more localized as the backreaction parameter becomes larger. Correspondingly the overlap of fermion wave function with the visible brane increases with l, which is depicted in Figs. 2 and 3.
  (iii) The effective coupling between the radion and zeroth order fermionic KK mode is obtained [in Eqs. (17) and (18)]. It is found that the radion-fermion coupling (for both the left and right chiral mode) increases with the increasing value

of the backreaction parameter. This is a direct consequence of the fact that the peak of the left and right chiral mode gets shifted towards the visible brane as the backreaction parameter increases. This in turn enhances the radion to fermion decay amplitude.

(iv) For a fixed value of the backreaction parameter, the left chiral mode has higher peak values on the TeV brane as the bulk fermions become more and more massive, whereas the right chiral mode shows a reverse nature.

(2) For massive KK mode—

(i) The requirement of solving the gauge hierarchy problem confines the mass of higher KK modes at the TeV scale. Moreover, the mass squared gap ($\Delta m_n^2 = m_{n+1}^2 - m_n^2$) depends linearly on n, which is also evident from the mass spectrum in Eq. (22).

(ii) The coupling between the radion and massive KK fermionic mode is determined in Eq. (23). It is found that the coupling parameter decreases with the increasing backreaction parameter.

(iii) From the perspective of a localization scenario, the wave function of massive KK modes are localized near the Planck brane, which increases with the order of the KK mode. As a result, the couplings of the massive KK fermionic modes with the visible brane matter fields become extremely weak and therefore, drastically reduces the possibility of finding the signatures of such massive fermion KK modes on a TeV brane.

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