Double parton scattering in the color glass condensate:
Hanbury-Brown-Twiss correlations in double inclusive photon production

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We introduce a technique to study double parton scattering (DPS) in the color-glass-condensate (CGC) approach. We show that the cross section of the DPS in the CGC approach is calculable in terms of new nonperturbative objects, generalized double transverse momentum-dependent parton distribution (2GTMD) functions. We investigate the production of pairs of prompt photons from two partons in the projectile hadron in high-energy proton-nucleus collisions. We show that even for independent partons in the projectile, the prompt photon correlation function exhibits Hanbury Brown and Twiss (HBT) correlations. The width of the HBT peak is controlled by the transverse distance between the partons of the pair, which is of the order of the proton size. Thus, the HBT measurements in two-particle production such as prompt photon pairs provide useful information about the nonperturbative 2GTMDs.

I. INTRODUCTION

The nature of long range rapidity correlations in hadron production observed in proton-proton (p-p) and proton (deuteron)-nucleus (p-A) collisions at the LHC and RHIC has been a subject of intense investigation during the past several years [1–5]. The big question to be answered is whether these correlations arise due to strong collective effects in final state interactions, or due to quasicollectivity present in the initial state wave function which is imprinted on the spectrum of produced particles.

Since first principle analysis of hadron production in a dense environment is very hard, it makes sense to look at simpler probes of this system. Prompt photons have been one such probe that has been used to probe the putative quark-gluon-plasma state created at the early stages of heavy ion collisions [6–16]. Since photon interactions are weak, the correlations between emitted photons, if such exist, would most certainly probe the structure of the initial state alone, and it is interesting to see what can be learned from it. This was the motivation of our previous papers on the subject [17]. In Ref. [17] we considered the production of two photons from the same quark in reaction of the type (shown in Fig. 1)

$$q + A \rightarrow \gamma(k_1) + \gamma(k_2) + \text{jet}(q) + X \qquad (1)$$

and have found an interesting correlated structure albeit short range in rapidity compared to the dihadron correlations. Here, we consider the production of two photons from two valence quarks, i.e. the process of the type (shown in Fig. 2)

$$q + q + A \rightarrow \gamma(k_1) + \gamma(k_2) + \text{jet}(q) + \text{jet}(q') + X. \qquad (2)$$

Naively one might think that such an independent emission process does not lead to correlations in double photon production. However, this is not necessarily the case. Since photons are bosons, upon further reflection one expects to see the Hanbury Brown and Twiss (HBT) correlations between photons emitted from two independent sources. Such correlations involving gluons were discussed in the context of hadron production in p-A scattering in the color-glass-condensate (CGC) approach in [18]. In principle they result in a peak for production of same sign transverse momentum pairs, with the radius of correlation in momentum space given by the inverse gluonic radius of the proton. The hadron HBT signal is, however, rather fragile and is easily masked by final state effects. One expects the photon HBT to be much more resilient. One of the main purposes in this paper is to qualitatively study this effect.
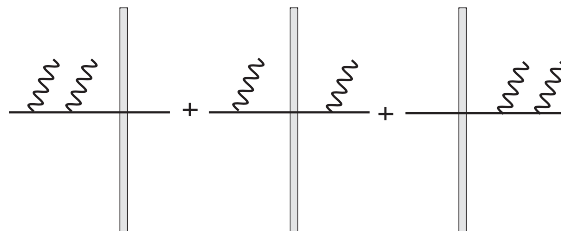

FIG. 1. The diagrams contributing to two prompt photons production from one quark in the background of the CGC field. The shaded box (the CGC shock wave) denotes the interaction of a quark to all orders with the background field via multiple gluon exchanges.
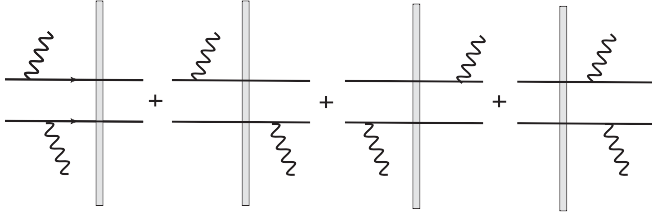
FIG. 2. The diagrams contributing to production of two prompt photons from two quarks in the background of the CGC field. The notation is the same as in Fig. 1.

Although our main interest and explicit calculations are geared toward diphoton production, the approach itself is more general and can be applied to any process of the double parton scattering (DPS) type [given in Eq. (4) below].

The topic of multiparton interactions is one of the most important focal points of studying the multiparticle correlations in perturbative quantum chromodynamics (pQCD). In the CGC approach, at leading order, the so-called single parton scattering (SPS) processes in the pQCD framework correspond to a single parton scattering to the CGC shock wave. The CGC shock wave includes the interaction of a parton to all orders with the background color field of the target. Therefore, in the CGC approach, the corresponding SPS contribution for two-particle production is obtained by considering the following process:

$$\text{parton} + A \rightarrow \text{particle}(k_1) + \text{particle}(k_2) + X. \quad (3)$$

Effects of saturation on dihadron correlations originating from SPS have been studied in [19]. Potentially "richer" sources of correlations are processes where two partons in one projectile hadron collide with the CGC shock wave. Such a process in the language of pQCD is the so-called DPS,

$$\text{parton} + \text{parton} + A \rightarrow \text{particle}(k_1) + \text{particle}(k_2) + X.$$
$$(4)$$

In the CGC framework processes where two observed particles originate from different sources of the color field were studied in the soft limit in Refs. [18,20–27]. While for SPS, the CGC and pQCD approaches are conveniently bridged with the help of the hybrid formalism [28], such a connection has not been made for DPS so far.

In the present paper we extend the hybrid formalism to include the DPS in the CGC approach. We show that the cross section of the DPS in the CGC is calculable in terms of new nonperturbative objects, the generalized double transverse momentum-dependent parton distribution (2GTMD) functions. In the context of the collinear factorization, a similar object, the so-called generalized double parton distribution (2GPD), appears in studies of the DPS [29]. We propose that the properties of the 2GTMDs

can be studied in the small-x kinematics within this hybrid CGC approach. In particular, we show that the diphoton HBT correlations are naturally expressed in terms of the diquark 2GTMDs. We also point out that dihadron correlations at high energy (in the forward direction) should be sensitive to digluon 2GTMD, and such processes should be included as corrections to the calculations of Ref. [19].

We will be working within a variant of the "hybrid" approximation [28] which is appropriate for forward photon production. In the hybrid CGC approach, we assume that the small-x gluon modes of the nucleus have a large occupation number so that the target nucleus can be described in terms of a classical color field. This should be a good approximation for a large enough nucleus at high energy.[1] This color field emerges from the classical Yang-Mills equation with a source term provided by faster partons. The renormalization group equations which govern the separation between the soft and the hard models are then given by the nonlinear Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (JIMWLK) evolution equations [45] (see below). We further assume that the projectile proton is in the dilute regime and can be described in an ordinary perturbative approach. The process Eq. (4) involves double parton scattering, and therefore standard collinear parton distribution functions are not sufficient to characterize the incoming proton state. We will therefore need to model the proton structure in a slightly more refined way.

In the following we will derive the diphoton cross section starting from some simple and intuitive assumptions about the wave function of the two incoming quarks inside the projectile hadron. We show that at large N_c within the CGC approach, the cross section is determined by the dipole scattering amplitude. We also show that within the standard collinear factorization approach the HBT peak has zero width. This is not at all surprising, since the width is expected to be of the order of inverse proton size, while in collinear factorization this size is effectively infinite. Thus any realistic study requires us to go beyond the standard collinear factorization. Since the basic process we consider involves double parton scattering, our final expressions require double parton distributions. These objects are not well determined experimentally, and therefore we do not attempt detailed quantitative predictions. Instead we limit ourselves to qualitative analysis based on a simple model of the initial wave function. We show that the width of the HBT peak is indeed given by the inverse size of this distribution in coordinate space. Therefore, the HBT measurements for two-particle production such as prompt photon pairs provide useful information about the nonperturbative 2GTMDs.

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[1]Note that there is growing evidence that supports the idea that a proton at very high energy and especially at very forward rapidity can be considered as a dense system as well, and therefore in principle the same approximation also applies to high energy p-p scatterings; see for example Refs. [30–44].

This paper is organized as follows: In Sec. II, we first provide a concise description of the theoretical framework for calculating the DPS contribution in the CGC approach. As an example, we focus on calculating the cross section of a pair of prompt photons and a pair of jets in high-energy p-A collisions. In Sec. III, we present our results for inclusive prompt diphoton production obtained from the DPS contribution in the CGC approach. We will also discuss Hanbury Brown and Twiss correlations for diphoton production in high-energy p-p and p-A collisions. We summarize our main results in Sec. IV.

II. SEMI-INCLUSIVE DIPHOTON + DIJET PRODUCTION IN PROTON-NUCLEUS COLLISIONS

In this section, we present the basics of computation of the cross section for the process given in Eq. (4). Although our formulation here is valid for the general production given in Eq. (4), in the following we focus on a case where two produced particles are prompt photons,

$$q(p_1) + q(p_2) + A$$
$$\rightarrow \gamma(k_1) + \gamma(k_2) + \mathrm{jet}(q) + \mathrm{jet}(q') + X. \quad (5)$$

We consider the leading order approximation in dilute-dense collisions at forward rapidity, for example in proton-proton or/and in proton-nucleus collisions. In this setup, the two valence quarks from the projectile wave function emit two photons via Bremsstrahlung. The *two* photon + jet systems are put on shell by interacting coherently over the whole longitudinal extent of the target; see Fig. 2. Although the scattering of the two quarks is independent, the production of two photon + jet systems is not independent due to the interference diagrams (shown in Fig. 2).

In the following, two-dimensional vectors in transverse space are written in boldface.

The cross section for the production of two quarks with momentum q and q' and two prompt photons with momenta k_1 and k_2 in the scattering of two on-shell quarks with momentum p_1 and p_2 off a hadronic target (either a proton or a nucleus), given in Eq. (5), can be written in the following general form:

$$d\sigma^{qq \rightarrow \gamma\gamma qq} = \frac{d^3 k_1}{(2\pi)^3 2k_1^-} \frac{d^3 k_2}{(2\pi)^3 2k_2^-} \frac{d^3 q}{(2\pi)^3 2q^-} \frac{d^3 q'}{(2\pi)^3 2q'^-} \frac{1}{4p_1^- p_2^-} \langle |\langle \mathrm{jet}(q), \mathrm{jet}(q'), \gamma(k_1), \gamma(k_2) | \mathrm{Proton} \rangle|^2 \rangle_{\text{color sources}}. \quad (6)$$

For explicit calculations one needs the two quarks distribution in the proton wave function. In full generality we can write

$$|\mathrm{Proton}\rangle = \frac{1}{2N_c} \sum_{s_1,s_2,c_1,c_2} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \sum_X \tilde{\mathcal{A}}(p_1, p_2, s_1, s_2, c_1, c_2; X)|p_1, s_1, c_1; p_2, s_2, c_2, X\rangle, \quad (7)$$

where (s_1, s_2) and (c_1, c_2) are the spin and the color indices of two quarks (in the projectile proton), respectively, and for simplicity we have assumed that the two quarks have the same flavor. The generalization to include u and d quarks is straightforward and would result in promoting $\tilde{\mathcal{A}}$ to a matrix in flavor space. In this expression X stands for all the "spectator" degrees of freedom in the proton wave function which are integrated over inclusively in the process Eq. (5). These include the occupation numbers of the spectator quarks and gluons as well as the momentum, spin, and color index of these spectators. The factor $\frac{1}{2N_c}$ was introduced for future convenience.

To calculate the cross section we require the reduced two quark density matrix

$$\sum_X |\mathrm{Proton}\rangle\langle\mathrm{Proton}| = \int_{p_1,p_2,p_1',p_2'} \tilde{\mathcal{R}}(p_1, p_2, p_1', p_2', s_1, s_2, s_1', s_2', c_1, c_2, c_1', c_2')|p_1, s_1, c_1; p_2, s_2, c_2\rangle\langle p_1', s_1', c_1'; p_2', s_2', c_2'|, \quad (8)$$

where

$$\tilde{\mathcal{R}}(p_1, p_2, p_1', p_2', s_1, s_2, s_1', s_2', c_1, c_2, c_1', c_2') = \sum_X \tilde{\mathcal{A}}(p_1, p_2, s_1, s_2, c_1, c_2; X)\tilde{\mathcal{A}}^*(p_1', p_2', s_1', s_2', c_1', c_2'; X). \quad (9)$$

While it is possible to perform the calculations with the general density matrix Eq. (8), for simplicity we will assume that the integration over the spectator partons leads to decorrelation of spin and color of the two active quarks in the density matrix. We will also take a simple product ansatz for the density matrix in momentum space. In other words we take the following simple form:

$$\tilde{\mathcal{R}}(p_1, p_2, p'_1, p'_2, s_1, s_2, s'_1, s'_2, c_1, c_2, c'_1, c'_2) = \left(\frac{1}{2N_c}\right)^2 \tilde{\mathcal{P}}(p_1, p_2)\tilde{\mathcal{P}}^*(p'_1, p'_2). \tag{10}$$

The function $\tilde{\mathcal{P}}(p_1, p_2)$ now determines the distribution of the two quarks in the proton on the amplitude level.

We stress that in practical terms the product ansatz makes very little difference since we are not going to assume that the momentum of the pair $p_1 + p_2$ is equal to the total momentum of the proton. The most important feature of Eq. (10) is that the two quarks in Eq. (10) are taken to be totally uncorrelated in spin and color. One can check explicitly that taking an analogous factorized form for a single quark density matrix reproduces exactly the standard expressions for cross section from a single quark where one averages over spin and color on the cross-section level, and the parton distribution function (pdf) given by $\int_{p_T} |\tilde{\mathcal{P}}(p_T, x)|^2$. We will quote the result obtained for a general density matrix later; see Eq. (38).

With this simple form of the reduced density matrix our calculation amounts to replacing the proton wave function in Eq. (6) by

$$|\text{Proton}\rangle \to |\text{Two quarks}\rangle$$

$$= \frac{1}{2N_c} \sum_{s_1, s_2, c_1, c_2} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \tilde{\mathcal{P}}(p_1, p_2)$$

$$\times |p_1, s_1, c_1; p_2, s_2, c_2\rangle. \tag{11}$$

In the following, the spin and the color indices of two quarks in the conjugate amplitude are denoted by (s'_1, s'_2) and (c'_1, c'_2), respectively; see Fig. 3. The indices (s, s') and (c, c') denote spin and color indices of the produced two quarks in the final state. The matrix element of the scattering amplitude in Eq. (6) is given by

$$\langle\text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle$$

$$= \frac{1}{2N_c} \sum_{s, s', s_1, s_2, c, c', c_1, c_2} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \tilde{\mathcal{P}}(p_1, p_2) \times [\langle q, s, c; k_1|p_1, s_1, c_1\rangle\langle q', s', c', k_2|p_2, s_2, c_2\rangle$$

$$+ \langle q, s, c; k_2|p_1, s_1, c_1\rangle\langle q', s', c'; k_1|p_2, s_2, c_2\rangle + \langle q', s', c'; k_1|p_1, s_1, c_1\rangle\langle q, s, c; k_2|p_2, s_2, c_2\rangle$$

$$+ \langle q', s', c'; k_2|p_1, s_1, c_1\rangle\langle q, s, c; k_1|p_2, s_2, c_2\rangle]. \tag{12}$$

In Eq. (12), we perform the sum over the spin and the color of produced quarks. Here $\langle q, s, c; k_1|p_1, s_1, c_1\rangle$ is the perturbative production amplitude for the process,

$$q(p_1, s_1, c_1) + A \to \gamma(k_1) + q(q, s, c) + \text{jet}(q') + X. \tag{13}$$

For brevity of notation, the photon polarization indices and summation over photon polarization are implicit in

Eq. (12) and throughout this paper. The expression in Eq. (12) can be simplified by rearranging terms,

$$\langle\text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle$$

$$= \frac{1}{2N_c} \sum_{s, s', s_1, s_2, c, c', c_1, c_2} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \mathcal{P}(p_1, p_2)$$

$$\times [\langle q, s, c; k_1|p_1, s_1, c_1\rangle\langle q', s', c', k_2|p_2, s_2, c_2\rangle$$

$$+ \langle q, s, c; k_2|p_1, s_1, c_1\rangle\langle q', s', c', k_1|p_2, s_2, c_2\rangle], \tag{14}$$

where the function $\mathcal{P}$ is related to $\tilde{\mathcal{P}}$ via

$$\mathcal{P}(p_1, p_2) = \tilde{\mathcal{P}}(p_1, p_2) + \tilde{\mathcal{P}}(p_2, p_1). \tag{15}$$

Note that while the amplitude $\tilde{\mathcal{P}}$ is not necessarily symmetric under the interchange of the two quarks, the function $\mathcal{P}$ is symmetric by construction. It depends on longitudinal and transverse momentum of two quarks[2]
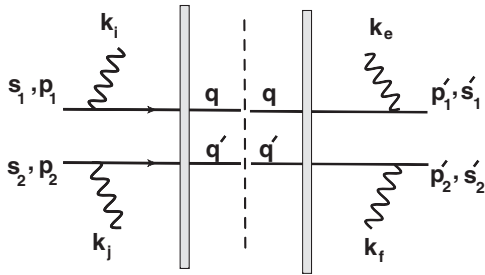

FIG. 3. A typical diagram contributing to two prompt photons production from two quarks in the background of the CGC field. The diagrams on the left and right sides of the dashed line correspond to the amplitude and the complex conjugate amplitude. The cross section at leading-order is given by the sum of four diagrams of this type shown in Fig. 2.

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[2]With a mild abuse of notation we are using the same symbol $\mathcal{P}$ to denote the amplitude as a function of three momenta as well as the function of transverse momenta and the longitudinal momentum fraction.

$$\mathcal{P}(p_1, p_2) \equiv \mathcal{P}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2), \tag{16}$$

where x_1 and x_2 are the longitudinal light-cone fractions of the incoming quarks in the projectile nucleon wave function. The exact values of x_1 and x_2 are given later in Eq. (34). Using Eq. (14), we obtain

$$|\langle \text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle|^2$$

$$= \frac{1}{4N_c^2} \sum_{\text{spin,color}} \int \int \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} \mathcal{P}(p_1, p_2) \mathcal{P}^*(p'_1, p'_2)$$

$$\times [\langle q, s, c; k_1|p_1, s_1, c_1\rangle\langle q', s', c', k_2|p_2, s_2, c_2\rangle\langle p'_2, s'_2, c'_2|q', s', c', k_2\rangle\langle p'_1, s'_1, c'_1|q, s, c; k_1\rangle$$

$$+ \langle q, s, c; k_1|p_1, s_1, c_1\rangle\langle q', s', c', k_2|p_2, s_2, c_2\rangle\langle p'_2, s'_2, c'_2|q', s', c', k_1\rangle\langle p'_1, s'_1, c'_1|q, s, c; k_2\rangle$$

$$+ \langle q, s, c; k_2|p_1, s_1, c_1\rangle\langle q', s', c', k_1|p_2, s_2, c_2\rangle\langle p'_2, s'_2, c'_2|q', s', c', k_2\rangle\langle p'_1, s'_1, c'_1|q, s, c; k_1\rangle$$

$$+ \langle q, s, c; k_2|p_1, s_1, c_1\rangle\langle q', s', c', k_1|p_2, s_2, c_2\rangle\langle p'_2, s'_2, c'_2|q', s', c', k_1\rangle\langle p'_1, s'_1, c'_1|q, s, c; k_2\rangle]. \tag{17}$$

In the lowest order in the electromagnetic α_{em} and the strong α_s coupling constants the $q \to q\gamma$ amplitude can be written in the following formal form:

$$\langle q(q), \gamma(k_1)|q(p)\rangle$$
$$= -e_q \bar{u}(q)[\mathcal{F}(q; p - k_1)G_F^0(p - k_1)\epsilon(k_1)$$
$$+ \epsilon(k_1)G_F^0(q + k_1)\mathcal{F}(q + k_1, p)]u(p), \tag{18}$$

where e_q is the fractional electric charge of the projectile quark, and G_F^0 is the free Feynman propagator of a quark with mass m. In the above u and ϵ_μ denote the quark free spinor and the photon polarization vector, respectively. In the above, the operator matrix $\mathcal{F}$ contains the interaction between the quark and the colored glass condensate target, which resums multiple interactions with the background CGC field [46,47].

Assuming that the target is moving in the positive z direction, we have [7]

$$\mathcal{F}(q; p) = 2\pi\delta(q^- - p^-)\gamma^- \text{sign}(p^-)$$
$$\times \int d^2\mathbf{x}[U(\mathbf{x}) - 1]e^{i(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}}, \tag{19}$$

where $U(\mathbf{x})$ is a unitary matrix in fundamental representation of $SU(N_c)$—the scattering matrix of a quark on the colored glass condensate target,

$$U(x) = T\exp\left(-ig^2 \int dx^- \frac{1}{\mathbf{\nabla}^2}\rho_a(x^-, \mathbf{x})t^a\right). \tag{20}$$

Here ρ is the density of the color sources in the target, and t^a is the generator of $SU(N_c)$ in the fundamental representation. Using the definition of $\mathcal{F}$ in Eq. (19), one can rewrite the amplitude as

$$\langle q(q), \gamma(k_1)|q(p)\rangle = -ie_q\bar{u}(\mathbf{q})\left[\frac{\gamma^-(\not{p} - \not{k}_1 + m)\epsilon(k_1)}{(p - k_1)^2 - m^2} + \frac{\epsilon(k_1)(\not{q} + \not{k}_1 + m)\gamma^-}{(q + k_1)^2 - m^2}\right]u(\mathbf{p})$$

$$\times 2\pi\delta(q^- + k_1^- - p^-)\int d^2\mathbf{x}[U(\mathbf{x}) - 1]e^{i(\mathbf{q}+\mathbf{k}_1-\mathbf{p}_T)\cdot\mathbf{x}},$$

$$\approx -ie_q\bar{u}(\mathbf{q})\gamma^- u(\mathbf{p})\left[\frac{q \cdot \epsilon}{q \cdot k_1} - \frac{p \cdot \epsilon}{p \cdot k_1}\right]2\pi\delta(q^- + k_1^- - p^-)\int d^2\mathbf{x}[U(\mathbf{x}) - 1]e^{i(\mathbf{q}+\mathbf{k}_1-\mathbf{p}_T)\cdot\mathbf{x}}, \tag{21}$$

where in the last line we employed the soft approximation, namely assuming that $|k_1| < |p - q|$. In order to calculate the cross section Eq. (6) we first need to perform the color charge averaging of the expression Eq. (17) over the target CGC field. This is usually done either using the McLerran-Vengopalan model [48] defined by the weight function,

$$W[\rho] = T\exp\left(-\int dx^- d^2\mathbf{x_T}\frac{\rho_a(x^-, \mathbf{x})\rho^a(x^-, \mathbf{x})}{2\mu^2(x^-)}\right), \tag{22}$$

or using numerical solutions of the Balitsky-Kovchegov equation [49]. For our present purposes the exact weight function does not matter. It is important though that any

high energy/density averaging procedure does not affect the spin dependence in Eq. (6).

A. Tracing over color

Using Eq. (21) we can perform the summation over color indices and averaging over the target color field in Eq. (17). A generic term in the integrand of Eq. (17) has the following structure:

$$
\begin{aligned}
\mathcal{I} &= \Bigg\langle \sum_{\text{spin}} \sum_{\text{color}} \langle q,s,c;k_i|p_1,s_1,c_1\rangle\langle p'_1,s'_1,c'_1|q,s,c;k_e\rangle \\
&\quad \times \langle q',s',c';k_j|p_2,s_2,c_2\rangle\langle p'_2,s'_2,c'_2|q',s',c';k_f\rangle \Bigg\rangle_\rho, \\
&= (-ie_q)^4 \delta_{\text{tot}} \times \mathcal{M}_{\text{spin}} \times \Bigg\langle \sum_{c_1,c_2,c'_1,c'_2} (\cdots)_{c_1 c'_1}(\cdots)_{c_2 c'_2} \Bigg\rangle_\rho \\
&= (-ie_q)^4 \delta_{\text{tot}} \mathcal{M}_{\text{spin}} \frac{N_c^2}{N_c+1}(N_c N_F^{(2)} \times N_F^{(2)} + N_F^{(4)}),
\end{aligned}
$$

(23)

where indices $(i,j=1,2)$ and $(e,f=1,2)$ denote the two produced photons (note that $i \neq j$ and $e \neq f$) in the amplitude and conjugate amplitude, respectively; see Fig. 3. The factor $\mathcal{M}_{\text{spin}}$ contains the spin summation [as defined in Eq. (31)] and δ_{tot} is given by

$$
\begin{aligned}
\delta_{\text{tot}} &= (2\pi)^4 \delta(q^- + k_i^- - p_1^-)\delta(q^- + k_e^- - p_1'^-) \\
&\quad \times \delta(q'^- + k_j^- - p_2^-)\delta(q'^- + k_f^- - p_2'^-).
\end{aligned}
$$

(24)

In Eq. (23), $N_F^{(2)}$ and $N_F^{(4)}$ are the traces of two (dipole) and four (quadrupole) lightlike fundamental Wilson lines in the background of the color fields of the target nucleus (or proton), respectively,

$$
\begin{aligned}
N_F^{(2)}(\mathbf{b},\mathbf{r},x_g) &= \frac{1}{N_c}\langle \text{Tr}[1 - U^\dagger(\mathbf{x})U(\mathbf{y})]\rangle_{x_g}, \\
N_F^{(4)}(\mathbf{b},\mathbf{r},\mathbf{b}',\mathbf{r}',x_g) &= \frac{1}{N_c}\langle \text{Tr}[1 - U^\dagger(\mathbf{x})U(\mathbf{y})U^\dagger(\mathbf{x}')U(\mathbf{y}')]\rangle_{x_g},
\end{aligned}
$$

(25)

where the vector $\mathbf{b} \equiv (\mathbf{x}+\mathbf{y})/2$ is the impact parameter of the dipole relative to the target and $\mathbf{r} \equiv \mathbf{x}-\mathbf{y}$ is the dipole transverse vector. Note that the expectation values on the right-hand side are calculated over the ensemble of target fields evolved up to rapidity $y_g = \ln 1/x_g$. The target in principle is evolved by the JIMWLK [45] or Balitsky–Kovchegov [49] equations. The parameter x_g can be related to the rapidities and transverse momenta of the prompt photons and final-state quarks via energy-momentum conservation [17]. In the following for notational simplicity we drop the explicit label x_g on the dipole and quadrupole amplitudes.

The explicit expression for $\mathcal{I}$ in Eq. (23) is given by

$$
\begin{aligned}
\mathcal{I} &= (-ie_q)^4 \delta_{\text{tot}} \mathcal{M}_{\text{spin}} \frac{N_c}{N_c+1} \\
&\quad \times [\langle \text{Tr}[(U(q+k_i-p_1)-1)\cdot(U^\dagger(p'_1-q-k_e)-1)]\rangle_\rho \langle \text{Tr}[(U(q'+k_j-p_2)-1)\cdot(U^\dagger(p'_2-q'-k_f)-1)]\rangle_\rho \\
&\quad + \langle \text{Tr}[(U(q+k_i-p_1)-1)\cdot(U^\dagger(p'_1-q-k_e)-1)\cdot(U(q'+k_j-p_2)-1)\cdot(U^\dagger(p'_2-q'-k_f)-1)]\rangle_\rho].
\end{aligned}
$$

(26)

The first and the second terms in Eq. (26) correspond to the Fourier transformed dipole and quadrupole scattering amplitudes defined in Eq. (25), respectively. It is useful to rewrite the above expression in terms of the dipole transverse separation vector $\mathbf{r}$ (and $\mathbf{r}'$) and the impact parameter $\mathbf{b}$ (and $\mathbf{b}'$),

$$
\begin{aligned}
\frac{\mathcal{I}\times(N_c+1)}{e_q^4 \delta_{\text{tot}}\mathcal{M}_{\text{spin}}N_c} &= \int d^2\mathbf{r}d^2\mathbf{b}\, e^{i\mathbf{r}\cdot(\mathbf{q}+\frac{1}{2}(\mathbf{k}_i+\mathbf{k}_e-\mathbf{p}_1-\mathbf{p}'_1))}e^{i\mathbf{b}\cdot(\mathbf{k}_i-\mathbf{k}_e+\mathbf{p}'_1-\mathbf{p}_1)}\langle \text{Tr}[(U(\mathbf{b}+\mathbf{r}/2)-1)\cdot(U^\dagger(\mathbf{b}-\mathbf{r}/2)-1)]\rangle_\rho \\
&\quad \times \int d^2\mathbf{r}'d^2\mathbf{b}'\, e^{i\mathbf{r}'\cdot(\mathbf{q}'+\frac{1}{2}(\mathbf{k}_j+\mathbf{k}_f-\mathbf{p}_2-\mathbf{p}'_2))}e^{i\mathbf{b}'\cdot(\mathbf{k}_j-\mathbf{k}_f+\mathbf{p}'_2-\mathbf{p}_2)}\langle \text{Tr}[(U(\mathbf{b}'+\mathbf{r}'/2)-1)\cdot(U^\dagger(\mathbf{b}'-\mathbf{r}'/2)-1)]\rangle_\rho \\
&\quad + \int d^2\mathbf{r}d^2\mathbf{b}d^2\mathbf{r}'d^2\mathbf{b}'\, e^{i\mathbf{r}\cdot(\mathbf{q}+\frac{1}{2}(\mathbf{k}_i+\mathbf{k}_e-\mathbf{p}_1-\mathbf{p}'_1))}e^{i\mathbf{r}'\cdot(\mathbf{q}'+\frac{1}{2}(\mathbf{k}_j+\mathbf{k}_f-\mathbf{p}_2-\mathbf{p}'_2))}e^{i\mathbf{b}\cdot(\mathbf{k}_i-\mathbf{k}_e+\mathbf{p}'_1-\mathbf{p}_1)}e^{i\mathbf{b}'\cdot(\mathbf{k}_j-\mathbf{k}_f+\mathbf{p}'_2-\mathbf{p}_2)} \\
&\quad \times \langle \text{Tr}[(U(\mathbf{b}+\mathbf{r}/2)-1)\cdot(U^\dagger(\mathbf{b}-\mathbf{r}/2)-1)\cdot(U(\mathbf{b}'+\mathbf{r}'/2)-1)\cdot(U^\dagger(\mathbf{b}'-\mathbf{r}'/2)-1)]\rangle_\rho.
\end{aligned}
$$

(27)

At large N_c, the term containing the quadrupole amplitude is suppressed relative to the one containing dipoles by a factor $1/N_c$. At leading order in $1/N_c$ one may therefore ignore the quadrupole contribution.

Another simplification arises if we assume that the target is uniform in the impact parameter space. This approximation may be appropriate for p-A scatterings and is almost always employed in CGC based calculations. Under this assumption one can ignore the b dependence in $N_F^{(2)}$, and the integrals over $\mathbf{b}$ and $\mathbf{b}'$ in Eq. (27) lead to delta functions. Therefore the cross section has the structure

$$\mathcal{I} \propto \delta^2(\mathbf{k}_i - \mathbf{k}_e - \mathbf{\Delta}_1)\delta^2(\mathbf{k}_j - \mathbf{k}_f - \mathbf{\Delta}_2), \qquad (28)$$

where we have defined

$$\mathbf{\Delta}_1 = \mathbf{p}_1 - \mathbf{p}_1',$$
$$\mathbf{\Delta}_2 = \mathbf{p}_2 - \mathbf{p}_2', \qquad (29)$$

with $(\mathbf{p}_1, \mathbf{p}_2)$ and $(\mathbf{p}_1', \mathbf{p}_2')$ being the transverse momenta of two projectile quarks in the amplitude and its conjugate amplitude, respectively. The momentum $\mathbf{\Delta}_{1,2}$ is the difference of the momenta of two partons from the wave function of the colliding hadron in the amplitude and the amplitude conjugated. Note that the difference of parton transverse momenta within the parton pair is not conserved. Thus at large N_c we have

$$\mathcal{I} = (-ie_q)^4(2\pi)^4 N_c^2 \delta_{\text{tot}} \mathcal{M}_{\text{spin}} \delta^2(\mathbf{k}_i - \mathbf{k}_e - \mathbf{\Delta}_1)\delta^2(\mathbf{k}_j - \mathbf{k}_f - \mathbf{\Delta}_2) N_F^{(2)}(\mathbf{q} + \mathbf{k}_i - \mathbf{p}_1) N_F^{(2)}(\mathbf{q}' + \mathbf{k}_j - \mathbf{p}_2). \qquad (30)$$

B. Tracing over spin

Now we turn to the spin summation in the expression Eq. (17). The matrix element $\mathcal{M}_{\text{spin}}$ in Eq. (23) is given by

$$\mathcal{M}_{\text{spin}} = \sum_{\alpha,\beta,\gamma,\eta} \sum_{s_1,s_2',s_2,s_2'} \bar{u}_\alpha^{s_1'}(p_1') \mathcal{A}_{\alpha\beta}(q,k_i,k_e,p_1,p_1') u_\beta^{s_1}(p_1) \bar{u}_\gamma^{s_2'}(p_2') \mathcal{A}_{\gamma\eta}(q',k_j,k_f,p_2,p_2') u_\eta^{s_2}(p_2), \qquad (31)$$

where the matrix functions $\mathcal{A}$ is defined as follows:

$$\mathcal{A}(q,k_i,k_e,p,p') = \gamma^-(\slashed{q}+m)\gamma^- \left[\frac{q \cdot \epsilon_i}{q \cdot k_i} - \frac{p \cdot \epsilon_i}{p \cdot k_i}\right]\left[\frac{q \cdot \epsilon_e^*}{q \cdot k_e} - \frac{p' \cdot \epsilon_e^*}{p' \cdot k_e}\right]. \qquad (32)$$

The summation over photon polarization is implicit in Eq. (31). The mass term in the spin matrix element in the above equation is inherited from the quark propagator in Eq. (21). However, in the high-energy limit employed here, the mass term in fact is irrelevant, since $(\gamma^-)^2 = 0$, and it thus disappears from all the following formulas. All the terms in Eq. (17) have a similar structure to Eq. (23) and can be written out explicitly using Eqs. (24), (27), (30), and (31). Therefore, at large N_c we obtain

$$|\langle \text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle|^2 = e_q^4 \pi^2 \sum_{\text{spin}} \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} N^{(2)}(\mathbf{q} + \mathbf{k}_1 - \mathbf{p}_1) N^{(2)}(\mathbf{q}' + \mathbf{k}_2 - \mathbf{p}_2)$$

$$\times \delta(p_1^- - q^- - k_1^-)\delta(p_2^- - q'^- - k_2^-)\left\{ \mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\right.$$

$$\times \bar{u}(p_1)\gamma^- \slashed{q}\gamma^- u(p_1)\bar{u}(p_2)\gamma^- \slashed{q}'\gamma^- u(p_2)\left|\frac{q \cdot \epsilon_1}{q \cdot k_1} - \frac{p_1 \cdot \epsilon_1}{p_1 \cdot k_1}\right|^2 \left|\frac{q' \cdot \epsilon_2}{q' \cdot k_2} - \frac{p_2 \cdot \epsilon_2}{p_2 \cdot k_2}\right|^2$$

$$+ \mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1',x_2',\mathbf{p}_1 + \mathbf{k}_2 - \mathbf{k}_1, \mathbf{p}_2 + \mathbf{k}_1 - \mathbf{k}_2)$$

$$\times \bar{u}(p_1 + k_2 - k_1)\gamma^- \slashed{q}\gamma^- u(p_1)\bar{u}(p_2 + k_1 - k_2)\gamma^- \slashed{q}'\gamma^- u(p_2)$$

$$\times \left[\frac{q \cdot \epsilon_1}{q \cdot k_1} - \frac{p_1 \cdot \epsilon_1}{p_1 \cdot k_1}\right]\left[\frac{q \cdot \epsilon_2^*}{q \cdot k_2} - \frac{(p_1 + k_2 - k_1) \cdot \epsilon_2^*}{(p_1 + k_2 - k_1) \cdot k_2}\right]\left[\frac{q' \cdot \epsilon_2}{q' \cdot k_2} - \frac{p_2 \cdot \epsilon_2}{p_2 \cdot k_2}\right]$$

$$\left. \times \left[\frac{q' \cdot \epsilon_1^*}{q^* \cdot k_1} - \frac{(p_2 + k_1 - k_2) \cdot \epsilon_1^*}{(p_2 + k_1 - k_2) \cdot k_1}\right]\right\} + (k_1 \leftrightarrow k_2), \qquad (33)$$

where we performed the integrals over p_1' and p_2' in Eq. (17) using the delta functions in Eqs. (24) and (30). In the above, the light-cone parameters x_1, x_2, x_1', x_2' in $\mathcal{P}$ and $\mathcal{P}^*$ are given by

$$x_1 = \frac{q^- + k_1^-}{\sqrt{s/2}}, \qquad x_2 = \frac{q'^- + k_2^-}{\sqrt{s/2}},$$

$$x_1' = \frac{q^- + k_2^-}{\sqrt{s/2}} = x_1 + \frac{k_2^- - k_1^-}{\sqrt{s/2}}, \qquad x_2' = \frac{q'^- + k_1^-}{\sqrt{s/2}} = x_2 + \frac{k_1^- - k_2^-}{\sqrt{s/2}}, \qquad (34)$$

with $\sqrt{s}$ being the nucleon-nucleon center-of-mass energy, and we have

$$x_1 + x_2 \leq 1. \qquad (35)$$

We now further simplify the expression in Eq. (33). First, we commute one of the γ^- through $\not{q}$ and use the fact that $\gamma^-\gamma^- = 0$. We also perform the sum over the photon polarization using the completeness of the photon polarization vectors, and neglecting the quark mass. Therefore, we obtain

$$|\langle \text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle|^2$$

$$= e_q^4 16\pi^2 \sum_{\text{spin}} \int \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} N^{(2)}(\mathbf{q}+\mathbf{k}_1-\mathbf{p}_1)N^{(2)}(\mathbf{q}'+\mathbf{k}_2-\mathbf{p}_2)$$

$$\times \delta(p_1^- - q^- - k_1^-)\delta(p_2^- - q'^- - k_2^-)q^- q'^- \{\mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\bar{u}(p_1)\gamma^- u(p_1)\bar{u}(p_2)\gamma^- u(p_2)$$

$$\times \frac{q\cdot p_1}{q\cdot k_1 p_1 \cdot k_1}\frac{q'\cdot p_2}{q'\cdot k_2 p_2 \cdot k_2}$$

$$+ \mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1',x_2',\mathbf{p}_1+\mathbf{k}_2-\mathbf{k}_1,\mathbf{p}_2+\mathbf{k}_1-\mathbf{k}_2)\bar{u}(p_1+k_2-k_1)\gamma^- u(p_1)\bar{u}(p_2+k_1-k_2)\gamma^- u(p_2)$$

$$\times \left[\frac{q\cdot q'}{q\cdot k_1 q'\cdot k_1} + \frac{p_1\cdot(p_2+k_1-k_2)}{p_1\cdot k_1 (p_2+k_1-k_2)\cdot k_1} - \frac{q\cdot(p_2+k_1-k_2)}{q\cdot k_1 (p_2+k_1-k_2)\cdot k_1} - \frac{p_1\cdot q'}{p_1\cdot k_1 q'\cdot k_1}\right]$$

$$\times \left[\frac{q\cdot q'}{q\cdot k_2 q'\cdot k_2} + \frac{p_2\cdot(p_1+k_2-k_1)}{p_2\cdot k_2 (p_1+k_2-k_1)\cdot k_2} - \frac{q'\cdot(p_1+k_2-k_1)}{q'\cdot k_2 (p_1+k_2-k_1)\cdot k_2} - \frac{p_2\cdot q}{p_2\cdot k_2 q\cdot k_2}\right]\} + (k_1 \leftrightarrow k_2). \quad (36)$$

Note that in the above expression each Dirac spinor carries an index s, and these indices are summed over completely independently. However, the spin structure can be simplified further in the high energy limit. At high energy the incoming quark, outgoing quark, and photon are practically collinear, since the scattering angle of the quark is very small at finite transverse momentum transfer. In such kinematics angular momentum conservation requires that the helicities of the incoming and outgoing quarks are opposite. This in turn means that $s_1 = s_1'$ and $s_2 = s_2'$; see Fig. 3. Using the high-energy properties of Dirac spinors, one can perform the remaining spin summation in Eq. (36) and obtain

$$|\langle \text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2)|\text{two quarks}\rangle|^2$$

$$= e_q^4 (16\pi)^2 \int \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} N^{(2)}(\mathbf{q}+\mathbf{k}_1-\mathbf{p}_1)N^{(2)}(\mathbf{q}'+\mathbf{k}_2-\mathbf{p}_2)$$

$$\times \delta(p_1^- - q^- - k_1^-)\delta(p_2^- - q'^- - k_2^-)q^- q'^- \{\mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)p_1^- p_2^- \frac{q\cdot p_1}{q\cdot k_1 p_1 \cdot k_1}\frac{q'\cdot p_2}{q'\cdot k_2 p_2 \cdot k_2}$$

$$+ \mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1',x_2',\mathbf{p}_1+\mathbf{k}_2-\mathbf{k}_1,\mathbf{p}_2+\mathbf{k}_1-\mathbf{k}_2)\sqrt{p_1^-(p_1^-+k_2^--k_1^-)p_2^-(p_2^-+k_1^--k_2^-)}$$

$$\times \left[\frac{q\cdot q'}{q\cdot k_1 q'\cdot k_1} + \frac{p_1\cdot(p_2+k_1-k_2)}{p_1\cdot k_1 (p_2+k_1-k_2)\cdot k_1} - \frac{q\cdot(p_2+k_1-k_2)}{q\cdot k_1 (p_2+k_1-k_2)\cdot k_1} - \frac{p_1\cdot q'}{p_1\cdot k_1 q'\cdot k_1}\right]$$

$$\times \left[\frac{q\cdot q'}{q\cdot k_2 q'\cdot k_2} + \frac{p_2\cdot(p_1+k_2-k_1)}{p_2\cdot k_2 (p_1+k_2-k_1)\cdot k_2} - \frac{q'\cdot(p_1+k_2-k_1)}{q'\cdot k_2 (p_1+k_2-k_1)\cdot k_2} - \frac{p_2\cdot q}{p_2\cdot k_2 q\cdot k_2}\right]\} + (k_1 \leftrightarrow k_2). \quad (37)$$

The products $\mathcal{P}\mathcal{P}^*$ that appear in our final expression Eq. (37) can be interpreted in terms of 2GTMD of the projectile hadron [denoted by $\mathcal{T}$; see Eq. (39)]. First off note that if we discard the simplifying assumption about the factorizability of the reduced density matrix Eq. (10), our final formulas would remain the same apart from the substitution

$$\mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1',x_2',\mathbf{p}_1',\mathbf{p}_2')$$
$$\to \text{tr}[\tilde{\mathcal{R}}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2,x_1',x_2',\mathbf{p}_1',\mathbf{p}_2')], \quad (38)$$

where on the right-hand side the density matrix is traced over the spin and color.

Our approximation of the translational invariance of the nuclear wave function in the impact parameter space [which led to the delta functions in Eq. (28)] means that the total

transverse momenta carried by quarks in the amplitude and complex conjugate amplitude are equal. As a result for the first term in Eq. (37), we have $\mathbf{\Delta}_1 = \mathbf{\Delta}_2 = 0$ while in the second term (the correlated part), we have $\mathbf{\Delta}_1 = \mathbf{\Delta}_2 = \mathbf{\Delta} \neq 0$, where $\mathbf{\Delta} \equiv \mathbf{k}_2 - \mathbf{k}_1$. Additionally note that in the soft approximation which we are employing throughout, $x_1' \approx x_1; x_2' \approx x_2$. Thus the basic quantity that appears in Eq. (36) is

$$\mathcal{T}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2,\mathbf{\Delta})$$
$$\equiv \text{tr}[\tilde{\mathcal{R}}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2,x_1,x_2,\mathbf{p}_1+\mathbf{\Delta},\mathbf{p}_2-\mathbf{\Delta})],$$
$$= \mathcal{P}(x_1,x_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{P}^*(x_1,x_2,\mathbf{p}_1+\mathbf{\Delta},\mathbf{p}_2-\mathbf{\Delta}). \quad (39)$$

In terms of the wave function of the hadron it is defined as (suppressing spin and color indices)

$$T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta) = \sum_{n=3}^{\infty} \int \prod_{i \neq 1,2} \frac{d^2 \mathbf{p}_i}{(2\pi)^2} \int_0^1 \prod_{i \neq 1,2} dx_i$$

$$\times \Psi_n(x_1, x_2, ..., \mathbf{p}_1, \mathbf{p}_2, ...)$$

$$\times \Psi_n^+(x_1, x_2, ..., \mathbf{p}_1 + \Delta, \mathbf{p}_2 - \Delta, ...)$$

$$\times (2\pi)^3 \delta\left(\sum_{i=1}^n x_i - 1\right)\delta\left(\sum_{i=1}^n \mathbf{p}_i\right),$$

$$(40)$$

where Ψ_n is the normalized *n*-parton wave function. It is related to the generalized double parton distribution (2GPD) [29] in a simple way,

$$D(x_1, x_2, \mu_1^2, \mu_2^2, \Delta) = \int \frac{d^2 \mathbf{p}_1}{(2\pi)^2} \frac{d^2 \mathbf{p}_2}{(2\pi)^2} \theta(\mu_1^2 - \mathbf{p}_1^2)\theta(\mu_2^2 - \mathbf{p}_2^2)$$

$$\times T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta), \quad (41)$$

where μ_1^2 and μ_2^2 are the virtualities of the two quarks. This nonperturbative object, 2GPD denoted by D, appears in the calculations involving DPS in pQCD in the collinear factorization framework, for example the four-jet production in proton-proton collisions [29]; see also Refs. [50–53].

Using the above definitions and Eqs. (6) and (37), we can rewrite the cross section of double photon-quark pair production in the following general form:

$$d\sigma^{qq+A \to \gamma\gamma + qq} = \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} [T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, 0)d\sigma^{q(p_1)+A \to \gamma(k_1)+q(q)} \times d\sigma^{q(p_2)+A \to \gamma(k_2)+q(q')}$$

$$+ T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta)d\sigma^{\text{Interference}}],$$

$$(42)$$

where $d\sigma^{q+A \to \gamma+q}$ is the cross section of the single prompt photon-quark production in $q + A$ collisions calculable via diagrams in Fig. 1 and can be immediately extracted from our final expression in Eq. (37). The cross section of $d\sigma^{q+A \to \gamma+q}$ obtained here is consistent with the soft approximation in Ref. [17]. In the approximation of uncorrelated partons, we have

$$T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta) \approx G_{\text{GTMD}}(x_1, \mathbf{p}_1, \Delta)G_{\text{GTMD}}(x_2, \mathbf{p}_2, \Delta),$$

$$(43)$$

where $G_{\text{GTMD}}(x_1, \mathbf{p}_1, \Delta)$ is the one-particle generalized transverse momentum-dependent parton distribution (GTMD) [54–59]. Under the assumption of uncorrelated partons in the projectile hadron, the first term in Eqs. (37) and (42) can be factorized into two independent cross sections for prompt photon-quark production. Hence, the first part in Eq. (37) [and in Eq. (42)] contains the contributions of independent production. The second term

in Eq. (37) [and in Eq. (42)] cannot be factorized into two independent terms even if the partons in the projectile wave function are uncorrelated. This term leads to nontrivial correlations between two produced photons, whose nature we discuss in the next section. Note that the correlated part (second term) corresponds to the interference diagrams where the produced photons in the amplitude and its conjugated amplitude have different momenta; see Fig. 3. It is remarkable that the correlations of two produced photons are explicitly related to the fact that $\Delta \neq 0$ in the second term.

III. HANBURY BROWN AND TWISS CORRELATIONS IN INCLUSIVE DIPHOTON CROSS SECTION

The inclusive diphoton cross section is obtained by substituting the expression in Eq. (37) into Eq. (6), and integrating over q and q'. Thus we obtain

$$\frac{d\sigma^{qq+A \to \gamma\gamma+X}}{d^3 k_1 d^3 k_2} = \frac{e_q^4}{(2\pi)^6} \frac{1}{k_1^- k_2^-} \int \int \int \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{d^2 \mathbf{q}'}{(2\pi)^2} N^{(2)}(\mathbf{q} + \mathbf{k}_1 - \mathbf{p}_1)N^{(2)}(\mathbf{q}' + \mathbf{k}_2 - \mathbf{p}_2)$$

$$\times \left\{ T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, 0) \frac{q \cdot p_1}{q \cdot k_1 p_1 \cdot k_1} \frac{q' \cdot p_2}{q' \cdot k_2 p_2 \cdot k_2} + T(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta) \frac{\sqrt{p_1^-(p_1^- + \Delta^-)p_2^-(p_2^- - \Delta^-)}}{p_1^- p_2^-} \right.$$

$$\times \left[\frac{q \cdot q'}{q \cdot k_1 q' \cdot k_1} + \frac{p_1 \cdot (p_2 - \Delta)}{p_1 \cdot k_1 (p_2 - \Delta) \cdot k_1} - \frac{q \cdot (p_2 - \Delta)}{q \cdot k_1 (p_2 - \Delta) \cdot k_1} - \frac{p_1 \cdot q'}{p_1 \cdot k_1 q' \cdot k_1} \right]$$

$$\left. \times \left[\frac{q \cdot q'}{q \cdot k_2 q' \cdot k_2} + \frac{p_2 \cdot (p_1 + \Delta)}{p_2 \cdot k_2 (p_1 + \Delta) \cdot k_2} - \frac{q' \cdot (p_1 + \Delta)}{q' \cdot k_2 (p_1 + \Delta) \cdot k_2} - \frac{p_2 \cdot q}{p_2 \cdot k_2 q \cdot k_2} \right] \right\},$$

$$(44)$$

where

$$q^- \equiv p_1^- - k_1^-; \quad q'^- \equiv p_2^- - k_2^-; \quad \Delta = k_2 - k_1. \quad (45)$$

The second term in this expression, the interference term expresses the HBT correlations. We note that under the naive parton model assumption the transverse momenta of all incoming partons vanish, and we have $\mathcal{T}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta) \propto \delta(\Delta)$; thus the HBT peak has zero width.

To understand the qualitative features of this expression beyond this naive approximation we consider the following

special kinematics. First we take the two photons to be soft, $k_{1(2)}^- \ll p_1^-, p_2^-$. We also assume that the transverse momentum of the two photons are large, but are not too different from each other $|\mathbf{k}_1 + \mathbf{k}_2| \gg |\mathbf{k}_1 - \mathbf{k}_2|$. In the spirit of the parton model, the intrinsic transverse momentum in the proton wave function is small, and thus the integration over $\mathbf{p}_1$ and $\mathbf{p}_2$ is dominated by the region $|\mathbf{p}_1|, |\mathbf{p}_2| \ll |\mathbf{k}_1| \approx |\mathbf{k}_2|$. Additionally we assume that the dipole scattering amplitude is saturated, and thus the momentum transfer is strongly peaked at $Q_s \ll k$. In the following we use the notation $k = |\mathbf{k}_1 + \mathbf{k}_2|/2$. In this kinematics we obtain

$$\left[\frac{q \cdot q'}{q \cdot k_1 q' \cdot k_1} + \frac{p_1 \cdot (p_2 - \Delta)}{p_1 \cdot k_1 (p_2 - \Delta) \cdot k_1} - \frac{q \cdot (p_2 - \Delta)}{q \cdot k_1 (p_2 - \Delta) \cdot k_1} - \frac{p_1 \cdot q'}{p_1 \cdot k_1 q' \cdot k_1} \right] \approx -\frac{4(k_1^-)^2}{s/2} \frac{1}{x_1 x_2} \frac{\mathbf{q}' \cdot \mathbf{q}}{k^4}, \quad (46)$$

$$\left[\frac{q \cdot q'}{q \cdot k_2 q' \cdot k_2} + \frac{p_2 \cdot (p_1 + \Delta)}{p_2 \cdot k_2 (p_1 + \Delta) \cdot k_2} - \frac{q' \cdot (p_1 + \Delta)}{q' \cdot k_2 (p_1 + \Delta) \cdot k_2} - \frac{p_2 \cdot q}{p_2 \cdot k_2 q \cdot k_2} \right] \approx -\frac{4(k_2^-)^2}{s/2} \frac{1}{x_1 x_2} \frac{\mathbf{q}' \cdot \mathbf{q}}{k^4}. \quad (47)$$

For the interference contribution to the cross section we obtain

$$\frac{d\sigma^{qq+A\to\gamma\gamma+X}}{d^3 k_1 d^3 k_2} \bigg|_{\text{interference}} \approx \frac{e_q^4}{(2\pi)^6} k_1^- k_2^- \frac{16}{(2\pi)^2 k^8} \int \frac{d^2 \mathbf{p}_1}{(2\pi)^2} \frac{d^2 \mathbf{p}_2}{(2\pi)^2} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{d^2 \mathbf{q}'}{(2\pi)^2} \frac{(\mathbf{q}' \cdot \mathbf{q})^2}{p_1^- p_2^-} N^{(2)}(\mathbf{q} + \mathbf{k}_1 - \mathbf{p}_1)$$
$$\times N^{(2)}(\mathbf{q}' + \mathbf{k}_2 - \mathbf{p}_2) \mathcal{T}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta). \quad (48)$$

Assuming rotational invariance of the dipole scattering amplitude we can estimate the average value of momentum as

$$\int \frac{d^2 \mathbf{q}}{(2\pi)^2} \mathbf{q}_i \mathbf{q}_j N^{(2)}(\mathbf{q} + \mathbf{k}_1 - \mathbf{p}_1) = \int \frac{d^2 \mathbf{q}}{(2\pi)^2} (\mathbf{q} - \mathbf{k}_1 + \mathbf{p}_1)_i (\mathbf{q} - k_1 + p_1)_j N^{(2)}(\mathbf{q})$$
$$= \frac{1}{2} \delta_{ij} \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \mathbf{q}^2 N^{(2)}(\mathbf{q}) = \frac{1}{2} \delta_{ij} Q_s^2 S_{\text{eff}}, \quad (49)$$

where S_{eff} is an effective interaction area, Q_s is the saturation scale of the system, and we have used the fact that

$$\int \frac{d^2 \mathbf{q}}{(2\pi)^2} N^{(2)}(\mathbf{q}) = 0, \qquad \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \mathbf{q}_j N^{(2)}(\mathbf{q}) = 0. \quad (50)$$

Therefore we obtain

$$\frac{d\sigma^{qq+A\to\gamma\gamma+X}}{d^3 k_1 d^3 k_2} \bigg|_{\text{interference}} \approx \frac{2e_q^4}{(2\pi)^6} \frac{k_1^- k_2^-}{s} \frac{16 Q_s^4 S_{\text{eff}}^2}{(2\pi)^2 k^8} \int \frac{d^2 \mathbf{p}_1}{(2\pi)^2} \frac{d^2 \mathbf{p}_2}{(2\pi)^2} dx_1 dx_2 \mathcal{T}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta). \quad (51)$$

We did not indicate the virtuality of the double parton distribution in the above, but it is clearly given by the large momentum scale in the problem, which is the transverse momentum of the individual photons k_T.

The form of the 2GTMD in Eq. (51) is not known experimentally. Nevertheless, the physics of the correlation present in Eq. (51) is clearly that of the HBT effect. We find correlation between the bosons (photons) emitted from uncorrelated sources (quarks). Indeed the behavior of this interference term is precisely a typical HBT behavior.

The easiest way to see this is in the approximation where the two incoming partons are taken to be uncorrelated in the proton wave function in the high energy limit. In this case we assume

$$\mathcal{P}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2) = \mathcal{P}(x_1, \mathbf{p}_1) \mathcal{P}(x_2, \mathbf{p}_2). \quad (52)$$

The exact shape of $\mathcal{P}(x, \mathbf{p})$ does not matter much. The only important aspect of it is that it should reflect the existence of the nonperturbative distance scale R. This

scale determines the physical size of the quark cloud in the proton and is thus naturally associated with the proton radius. In momentum space this means that the transverse momentum dependent distribution (TMD) should decrease beyond $p \sim R^{-1}$. For illustrative purposes here we assume a simple Gaussian distribution for the intrinsic momentum dependence[3]

$$\mathcal{P}(x, \mathbf{p}) \propto e^{-\frac{1}{2}R^2|\mathbf{p}|^2}. \tag{53}$$

We then have

$$\int \frac{d^2\mathbf{p}_1}{(2\pi)^2} \frac{d^2\mathbf{p}_2}{(2\pi)^2} \mathcal{T}(x_1, x_2, \mathbf{p}_1, \mathbf{p}_2, \Delta) = f_q(x_1)f_q(x_2)e^{-\frac{1}{2}R^2\Delta^2}, \tag{54}$$

where the function $f_q(x_1)$ is the usual quark pdf (the virtuality is implicit). The interference thus leads to enhancement of the cross section for $|\mathbf{k}_1 - \mathbf{k}_2| < 1/R$—a typical HBT correlation behavior. The nonperturbative scale R can therefore be directly measured by measuring photon correlations. The magnitude of the effect drops pretty fast at the large transverse momentum of the photons, but presumably at $k_T \sim Q_s$ the interference piece should not be significantly suppressed relative to the independent production piece.

Another popular assumption in the literature is to approximate the GPD by $G_{\text{GPD}}(x_1, \mathbf{p}_1^2, \Delta) \approx G(x_1, \mathbf{p}_1^2)\mathcal{F}(\Delta)$ where G is the conventional parton (quark) distribution of the nucleon and $\mathcal{F}(\Delta)$ is the nonperturbative proton form factor [29,62]. If this factorization is assumed to hold at any virtuality p_1^2, it is equivalent to a similar factorizable approximation for GTMD: $G_{\text{GTMD}}(x_1, \mathbf{p}_1, \Delta) \approx G(x_1, \mathbf{p}_1)\mathcal{F}(\Delta)$ where $G(x_1, \mathbf{p}_1)$ is the TMD. This, via the use of Eq. (43) again leads to Eq. (54) with the Gaussian factor replaced by $\mathcal{F}(\Delta)$. The form factor is maximal at $\Delta = 0$ and decreases on the momentum scale μ, which has the same physical meaning as the scale R^{-1} introduced in Eq. (53).

We note that a similar form factor for gluons was discussed in the literature and the functional form $\mathcal{F}_g(\Delta) = \frac{1}{(\Delta^2/\mu^2+1)^2}$ was extracted from exclusive vector meson production with $\mu^2 \approx 1$ GeV^2 [29,62]. The value of the proton size extracted from $\mathcal{F}_g$ is rather small, which is consistent with many other experimental indications of a small gluonic radius of the proton [63]. For diphoton HBT we expect a different and larger transverse distance scale to dominate the HBT correlations.

IV. CONCLUSIONS

In this paper we have developed the hybrid calculational approach to forward particle production to include DPS

processes in the saturated environment. The main technical ingredient that appears in this approach is the 2GTMD function. In the "hybrid" approach, the DPS means two partons from the projectile hadron coherently colliding with the CGC shock wave. Thus on the target side all multiple scattering interactions are resummed in our calculation. In this sense we do not distinguish between interactions of a single, double, or higher number of target partons. For that reason the nuclear 2GTMD (or 2GPD) does not appear as a distinct object in our calculation, and only the 2GTMD (or 2GPD) of the projectile proton is relevant. This is in contrast to the study of Ref. [64] where because the target was considered in the standard pQCD approach, the nuclear 2GPD contribution is separately identified and can be studied in multiple-jet production in *p-A* collisions.

We studied in detail the diphoton correlations that arise due to the DPS process. We found that these correlations reflect the Hanbury Brown and Twiss effect, and lead to enhanced double photon production when the transverse momentum of the two photons are within the inverse proton radius of each other. At high momentum of produced photons the correlated piece decreases quite fast (as $1/k^8$), but it should give a significant enhancement when the photon momenta are not much larger than the target saturation scale. We showed that the width of the HBT peak probes the transverse distance between the parton of the pair in the 2GTMDs. Therefore, the HBT measurements in two-particle production such as prompt photon pairs provide useful information about the nonperturbative 2GTMDs.

It would be interesting to compare the magnitude of the correlated cross section we find here with the correlations generated through SPS [17]. We did not attempt a quantitative comparison, since the 2GTMDs are not known with significant accuracy. It is interesting to note, however, that parametrically the DPS contribution can be competitive with the SPS one, especially for intermediate transverse momentum photons with $|\mathbf{k}|$ not much larger than Q_s. Although one requires two quarks to scatter, in the saturated regime where the quark scattering amplitude is of the order one, this is not suppressed by powers of α_s. Additionally, the DPS is enhanced by a factor roughly equal to the number of quarks in the proton. Thus all in all the DPS contribution can be comparable to the SPS one. It would be very interesting if such correlations could be observed experimentally.

Finally, we note that the DPS HBT correlations are not limited to photon production. In particular these effects were not included in the CGC calculation of dihadron production at forward rapidities [19]. Quite clearly a calculation similar to ours can be performed for double gluon inclusive production, and it should also lead to HBT correlations. Indeed we expect that for gluons the suppression of the correlated part at high k_T will be significantly

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[3]Such a Gaussian distribution is supported by various phenomenological studies (see for example Refs. [60,61]). In this paper, however, we are using it merely as an illustration.

smaller than for photons, since it involves production of only two high-k_T particles in the final state, rather than four as in the present case.

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