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# Two-body strong decay status of the $\rho$ and $\rho_3$ mesons

Xiaoyun Chen and Jialun Ping

*Department of Physics and Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University, Nanjing 210023, People's Republic of China*

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In this work, we study the masses and the two-body strong decay of the $\rho$ and $\rho_3$ states below 2 GeV in the framework of the chiral quark model and the $^3P_0$ models. In the calculations, the wave functions of the particles are obtained numerically by the Gaussian expansion method, and they are applied in the decay calculation. By comparing the results with experimental data, we are trying to identify the exotic states. And it will be helpful to study the structures of $\rho/\rho_3$ states.

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## I. INTRODUCTION

In conventional quark model, meson is described as quark-antiquark bound state. This picture was successfully applied to the bottomonium, charmonium and light mesons [1,2]. However, recent experiments on $XYZ$ particles [3], especially the charged states associated with the heavy quarkonium [4] indicate that there exist mesons beyond the quark-aqntiquark picture. One feature of these exotica is that all states are above the $D\bar{D}$ threshold. For light mesons, the corresponding thresholds are very low. For nonstrange system, the threshold is around 280 MeV, the sum of the masses of two pions. So it is interesting to investigate the light meson states in the nonstrange sector beyond the conventional picture. The task is very difficult because the valence quarks (antiquarks) are the same as the sea quarks (antiquarks), which is different from the heavy mesons where the valence particles are much heavier than the sea particles. The strategy adopted here is treating all the states as ordinary mesons in a well-established quark model, then the states, which cannot be described well in the model, should be the exotica. The $\rho$ mesons are ideal place for this purpose. They are not the Goldstone bosons and they are free of strange valence particles due to the isovector properties. Several $\rho/\rho_3$ particles, we call $\rho$ meson family, have been compiled in the book of Particle Data Group(PDG) [5] and listed in Table I (For several states, $\rho_3(1990)$, $\rho_3(2250)$, the compilation did not make the average, and the information is taken from experiment [6]). The quantum numbers $J^P$ of these states are $1^-$ and $3^-$. For $1^-$ states, the possible orbital angular momenta are $L = 0$ and 2, and the states are denoted as $^3S_1$ and $^3D_1$. Similarly, $^3D_3$ and $^3G_3$ contribute to the states with $3^-$.

The $\rho$ mesons have been investigated extensively before, and a brief review of the research status can be found in a recent systematic study of the $\rho$ family [7]. In Ref. [7], the masses of the $\rho$ mesons were fixed by Regge trajectory and the decay widths were calculated by the $^3P_0$ model. In calculating the decay width, the wave functions of the states involved were approximated by simple harmonic oscillators (SHOs). From their calculations and our previous studies [8], one can see that the decay widths are sensitive to the parameter $R$ of SHOs and the SHOs are not a good approximation of the wave functions for the excited $q\bar{q}$ states. It is expected to have a calculation which minimizes the effect of parameter $R$. In the present work, we first assume that these states are categorized into the meson family made up with $q\bar{q}$ and get the mass spectrum of the $\rho$ family in the framework of the chiral quark model (ChQM) by using the Gaussian expansion method (GEM) [9], a powerful method with high-precision for few-body systems. In this approach, the spin-orbital and tensor interactions are treated exactly. And then the Okubo-Zweig-Iizuka(OZI) allowed two-body strong decay behaviors of the states are studied by using the $^3P_0$ model. In evaluating the transition matrix elements, the wave functions of the states obtained in the mass spectrum calculation are used. In this way, no parameter is introduced into the wave function of mesons and a self-consistent study is achieved. By comparing the theoretical results with the experimental data, the underlying properties of the states reported experimentally can be understood, and the identification of the exotica is expected.

The paper is organized as follows. In the next section the chiral quark model and GEM for $q\bar{q}$ system are presented.

TABLE I. The $\rho$ and $\rho_3$ particles reported by experiments. The average values of the mass and the total width are extracted from PDG [5].

| State | Mass (MeV) | Width (MeV) |
|---|---|---|
| $\rho$ mesons with $1^-$ | | |
| $\rho(770)$ | $775.26 \pm 0.34$ | $147.8 \pm 0.9$ |
| $\rho(1450)$ | $1465 \pm 25$ | $400 \pm 60$ |
| $\rho(1570)$ | $1570 \pm 36 \pm 62$ | $144 \pm 75 \pm 43$ |
| $\rho(1700)$ | $1720 \pm 20$ | $250 \pm 100$ |
| $\rho(1900)$ | $1909 \pm 17 \pm 25$ | $48 \pm 17 \pm 2$ |
| $\rho(2150)$ | $2155 \pm 21$ | $320 \pm 70$ |
| $\rho_3$ mesons with $3^-$ | | |
| $\rho_3(1690)$ | $1688.8 \pm 2.1$ | $161 \pm 10$ |
| $\rho_3(1990)$ [6] | $1982 \pm 14$ | $188 \pm 24$ |
| $\rho_3(2250)$ [6] | $2260 \pm 20$ | $160 \pm 25$ |

 

In Sec. III, we briefly introduce the $^3P_0$ model. The numerical results and discussions are given in Sec. IV. The last section is devoted to the summary of the present work.

## II. CHIRAL QUARK MODEL AND GEM

The most commonly used quark model is the chiral quark model [2]. In this model, the massive constituent quark and antiquark interact with each other through Goldstone boson exchange in addition to the effective one-gluon-exchange. Besides, the phenomenological color confinement and the chiral partner $\sigma$-meson exchange are also introduced. The model has been successfully applied to describe the properties of hadrons and hadron-hadron interactions. In our work, the Hamiltonian of ChQM for light mesons can be written as:

$$H = m_q + m_{\bar{q}} + \frac{p^2}{2\mu} + V^C + V^\pi + V^K + V^\eta + V^\sigma + V^G, \tag{1}$$

$$\begin{aligned}
V^C &= -[-a_c(1-e^{-\mu_c r}) + \Delta]\boldsymbol{\lambda}_q^c \cdot \boldsymbol{\lambda}_{\bar{q}}^{*c} + V_{LS}^C,\\
V_{LS}^C &= \frac{a_c\mu_c e^{-\mu_c r}}{4m_q^2 m_{\bar{q}}^2 r}\boldsymbol{\lambda}_q^c \cdot \boldsymbol{\lambda}_{\bar{q}}^{*c}[((m_q^2+m_{\bar{q}}^2)(1-2a_s) + 4m_q m_{\bar{q}}(1-a_s))\mathbf{S}_+\cdot\mathbf{L} + (m_{\bar{q}}^2 - m_q^2)\mathbf{S}_-\cdot\mathbf{L}],\\
V^\pi &= \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_q m_{\bar{q}}}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi v^\pi \sum_{a=1}^{3}\lambda_q^a \lambda_{\bar{q}}^{*a},\\
V^K &= \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_q m_{\bar{q}}}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K v^K \sum_{a=4}^{7}\lambda_q^a \lambda_{\bar{q}}^{*a},\\
V^\eta &= \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_q m_{\bar{q}}}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta v^\eta\\
v^\chi &= \left[Y(m_\chi r) - \frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r)\right]\boldsymbol{\sigma}_q\cdot\boldsymbol{\sigma}_{\bar{q}} + \left[H(m_\chi r) - \frac{\Lambda_\chi^3}{m_\chi^3}H(\Lambda_\chi r)\right]S_{q\bar{q}}, \qquad \chi = \pi, K, \eta,\\
V^\sigma &= -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma\left\{\left[Y(m_\sigma r) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r)\right] + \frac{m_\sigma^2}{2m_q m_{\bar{q}}}\left[G(m_\sigma r) - \frac{\Lambda_\sigma^3}{m_\sigma^3}G(\Lambda_\sigma r)\right]\mathbf{S}_+\cdot\mathbf{L}\right\},\\
V^G &= -\frac{\alpha_s}{4}\boldsymbol{\lambda}_q^c\cdot\boldsymbol{\lambda}_{\bar{q}}^{*c}\left\{\frac{1}{r} - \frac{\boldsymbol{\sigma}_q\cdot\boldsymbol{\sigma}_{\bar{q}}}{6m_q m_{\bar{q}}}\frac{e^{-r/r_0(\mu)}}{r r_0^2(\mu)} - \frac{1}{4m_q m_{\bar{q}}}\left[\frac{1}{r^3} - \frac{e^{-r/r_g(\mu)}}{r^3}\left(1 + \frac{r^2}{3r_g^2(\mu)} + \frac{r}{r_g(\mu)}\right)S_{q\bar{q}}\right]\right\} + V_{LS}^G,\\
V_{LS}^G &= \frac{\alpha_s}{16}\boldsymbol{\lambda}_q^c\cdot\boldsymbol{\lambda}_{\bar{q}}^{*c}\frac{1}{m_q^2 m_{\bar{q}}^2}\left[\frac{1}{r^3} - \frac{e^{-r/r_g(\mu)}}{r^3}\left(1 + \frac{r}{r_g(\mu)}\right)[((m_q + m_{\bar{q}})^2 + 2m_q m_{\bar{q}})\mathbf{S}_+\cdot\mathbf{L} + (m_{\bar{q}}^2 - m_q^2)\mathbf{S}_-\cdot\mathbf{L}]\right],\\
S_{q\bar{q}} &= \frac{\boldsymbol{\sigma}_q\cdot\mathbf{r}\boldsymbol{\sigma}_{\bar{q}}\cdot\mathbf{r}}{r^2} - \frac{1}{3}\boldsymbol{\sigma}_q\cdot\boldsymbol{\sigma}_{\bar{q}}, \qquad \mathbf{S}_\pm = \mathbf{S}_q \pm \mathbf{S}_{\bar{q}}, \qquad r_0(\mu) = \hat{r}_0/\mu, \qquad r_g(\mu) = \hat{r}_g/\mu.\\
Y(x) &= e^{-x}/x, \qquad H(x) = (1 + 3/x + 3/x^2)Y(x), \qquad G(x) = (1/x + 1/x^2)Y(x),
\end{aligned} \tag{2}$$

where $m_1$, $m_2$ are masses of quark and antiquark, and $\mu$ is their reduced mass. $\mathbf{p}$ is the relative momentum between quark and antiquark. $\boldsymbol{\sigma}, \boldsymbol{\lambda}, \boldsymbol{\lambda}^c$ are the $SU_2$ Pauli, $SU_3$ flavor and $SU_3$ color Gell-Mann matrices, respectively. $g_{ch}^2/4\pi$ is the chiral coupling constant, determined from $\pi$-nucleon coupling constant. $\alpha_s$ is the effective scale-dependent running quark-gluon coupling constant [2],

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln((\mu^2 + \mu_0^2)/\Lambda_0^2)} \tag{3}$$

All other symbols have their usual meanings. All the parameters needed in the present calculation are taken from Ref. [2] and listed in Table II.

The meson spectrum of mesons is obtained by solving the Schrödinger equation

$$H\Psi_{M_I M_J}^{IJ} = E^{IJ}\Psi_{M_I M_J}^{IJ} \tag{4}$$

The meson wave function $\Psi_{M_I M_J}^{IJ}$ with quantum numbers $IJ$ is constructed from the orbital, spin, flavor and color wave functions

$$\Psi_{M_I M_J}^{IJ} = [\psi_{lm}(\mathbf{r})\chi_{sm_s}]_{M_J}^J \phi_{M_I}^I \omega^c, \tag{5}$$

where [] means angular momentum coupling, $\psi_{lm}(\mathbf{r})$, $\chi_{sm_s}$, $\phi_{M_I}^I$, $\omega^c$ are orbit, spin, flavor and color wave functions.

In our work, the GEM is applied to get the wave functions of mesons. In this approach, the orbital wave function is expanded by Gaussians,

$$\psi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n \psi^G_{nlm}(\mathbf{r}), \tag{6}$$

$$\psi^G_{nlm}(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \tag{7}$$

where $n_{\max}$ is the number of Gaussians, and $c_n$ is the expansion coefficient which can be obtained by solving the Schrödinger equation. $N_{nl}$ is the normalization constant of gaussian wave function. Gaussian size parameters are taken as the following geometric progression numbers

$$\nu_n = \frac{1}{r_n^2}, \qquad r_n = r_1 a^{n-1}, \qquad a = \left(\frac{r_{\max}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{8}$$

Substituting Eqs. (5), (6), (7) into Eq. (4), a generalized eigen-equation is obtained:

$$\sum_{n',\alpha'} (H^{IJ}_{n\alpha,n'\alpha'} - E^{IJ} N^{IJ}_{n\alpha,n'\alpha'}) C^{IJ}_{n'\alpha'} = 0, \tag{9}$$

$$\begin{aligned} H^{IJ}_{n\alpha,n'\alpha'} &= \langle \Phi^{IJM}_{n\alpha} | H | \Phi^{IJM}_{n'\alpha'} \rangle, \\ N^{IJ}_{n\alpha,n'\alpha'} &= \langle \Phi^{IJM}_{n\alpha} | 1 | \Phi^{IJM}_{n'\alpha'} \rangle. \end{aligned} \tag{10}$$

## III. $^3P_0$ MODEL

The $^3P_0$ model (quark pair creation model) was originally introduced by Micu [10] and further developed by Le Yaouanc, Ackleh, Roberts *et al.* [11–13]. It can be applied to the OZI rule allowed two-body strong decays of a hadron. The transition operator in the model is,

$$\begin{aligned} T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3+\mathbf{p}_4) \\ \times \mathcal{Y}^m_1\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right) \chi^{34}_{1-m} \phi^{34}_0 \omega^{34}_0 b^\dagger_3(\mathbf{p}_3) d^\dagger_4(\mathbf{p}_4), \end{aligned} \tag{11}$$

TABLE II. The model parameters in our work.

| Quark masses | $m_u = m_d$ (MeV) | 313 |
|---|---|---|
| Goldstone bosons | $m_\pi$ (fm$^{-1}$) | 0.70 |
| | $m_\sigma$ (fm$^{-1}$) | 3.42 |
| | $m_\eta$ (fm$^{-1}$) | 2.77 |
| | $\Lambda_\pi = \Lambda_\sigma$ (fm$^{-1}$) | 4.2 |
| | $\Lambda_\eta$ (fm$^{-1}$) | 5.2 |
| | $g^2_{ch}/(4\pi)$ | 0.54 |
| | $\theta_p(^\circ)$ | −15 |
| Confinement | $a_c$ (MeV) | 430 |
| | $\mu_c$ (fm$^{-1}$) | 0.70 |
| | $\Delta$ (MeV) | 181.10 |
| One-gluon-exchange | $\alpha_0$ | 2.118 |
| | $\Lambda_0$ (fm$^{-1}$) | 0.113 |
| | $\mu_0$ (MeV) | 36.976 |
| | $\hat{r}_0$ (MeV) | 28.170 |

where $\gamma$ represents the probability of the quark-antiquark pair with momentum $\mathbf{p}_3$ and $\mathbf{p}_4$ created from the vacuum with the vacuum quantum numbers $J^{PC} = 0^{++}$. Because the intrinsic parity of the antiquark is negative, the created quark-antiquark pair must be in the state $^3P_0$. $\phi^{34}_0$ and $\omega^{34}_0$ are flavor and color singlet states, respectively (the quark and the antiquark in the original meson are indexed by 1 and 2). The S-matrix element for the process $A \to B + C$ is written as

$$\langle BC|T|A\rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \tag{12}$$

where $\mathbf{P}_B$ and $\mathbf{P}_C$ are the momenta of B and C mesons in the final state, which satisfies $\mathbf{P}_A = \mathbf{P}_B + \mathbf{P}_C = 0$ in the center of mass frame of meson A. $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ is the helicity amplitude of the process $A \to B + C$, which can be obtained as

$$\begin{aligned} \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{P}) = \gamma\sqrt{8E_A E_B E_C} \sum_{\substack{M_{L_A}, M_{S_A}, \\ M_{L_B}, M_{S_B}, \\ M_{L_C}, M_{S_C}, m}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A}\rangle \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B}\rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C}\rangle \\ \times \langle 1m1-m|00\rangle \langle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} | \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m}\rangle [\langle \phi^{14}_B \phi^{32}_C | \phi^{12}_A \phi^{34}_0\rangle \mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3) \\ + (-1)^{1+S_A+S_B+S_C} \langle \phi^{32}_B \phi^{14}_C | \phi^{12}_A \phi^{34}_0\rangle \mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(-\mathbf{P}, m_2, m_1, m_3)], \end{aligned} \tag{13}$$

with the momentum space integral,

$$\mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3) = \int d\mathbf{p} \psi^*_{n_B L_B M_{L_B}}\left(\frac{m_3}{m_1+m_3}\mathbf{P} + \mathbf{p}\right) \psi^*_{n_C L_C M_{L_C}}\left(\frac{m_3}{m_2+m_3}\mathbf{P} + \mathbf{p}\right) \psi_{n_A L_A M_{L_A}}(\mathbf{P}+\mathbf{p}) \mathcal{Y}^m_1(\mathbf{p}), \tag{14}$$

where $\mathbf{P} = \mathbf{P}_B = -\mathbf{P}_C$, and $\mathbf{p} = \mathbf{p}_3$, $m_3$ is the mass of the created quark $q_3$. To compare the theoretical results with experimental data, the partial wave amplitude $\mathcal{M}^{JL}(A \to BC)$ is needed, it relates to the helicity amplitude by the Jacob-Wick formula [14],

$$\mathcal{M}^{JL}(A\to BC)=\frac{\sqrt{2L+1}}{2J_A+1}\sum_{M_{J_B},M_{J_C}}\langle L0JM_{J_A}|J_AM_{J_A}\rangle \times\langle J_BM_{J_B}J_CM_{J_C}|JM_{J_A}\rangle\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}) \tag{15}$$

At last the decay width of the process $A\to B+C$ is calculated by

$$\Gamma=\pi^2\frac{|\mathbf{P}|}{M_A^2}\sum_{JL}|\mathcal{M}^{JL}|^2, \tag{16}$$

with

$$|\mathbf{P}|=\frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A},$$

where $M_A$, $M_B$, and $M_C$ are the masses of the mesons $A$, $B$, and $C$, respectively. In evaluating the momentum space integral Eq. (14), the wave functions of mesons $A$, $B$, $C$, which are obtained in the mass spectrum calculation, are used. Because the wave functions are expanded by a series of Gaussians, the integral can be evaluated analytically.

Additionally, for the parameter $\gamma$, we made an overall fit of the experimental values of light mesons listed in PDG and we take $\gamma=6.65$ for $u\bar{u}$ and $d\bar{d}$ pair creation, and $\gamma=6.65/\sqrt{3}$ is applied for $s\bar{s}$ pair creation [15].

## IV. MASS SPECTRUM OF $\rho/\rho_3$ AND THEIR DECAY WIDTHS

In the present calculation, the GEM is employed to get the mass spectra and the corresponding wave functions of the $\rho/\rho_3$ meson family. For the energy spectrum below 2.0 GeV, the convergent results are obtained with $n_{\max}=20$, $r_1=0.01$ fm and $r_{n_{\max}}=3$ fm. The mass spectra of the $\rho/\rho_3$ states are shown in Table III and Table IV. From Tables III and IV, we can see the obtained masses of mesons are well consistent with the values in Ref. [2], which shows the validity of the present method (the small differences come from the different treating of spin-orbit coupling and the tensor interaction, and the perturbative method is invoked to deal with $L-S$ coupling and tensor interaction in Ref. [2]). By comparing the masses of mesons obtained here with the experimental values of the states which listed in Table I, one can make a primary assignment. However, to justify the assignment, the comparison of the decay widths of these states are needed. The calculated decay widths and branching ratios of $\rho/\rho_3$ states are presented in Tables II–VI. In the following, we make the analysis of the states one by one.

(1) $\rho(770)$

It is a consensus that the ground state of $\rho$ mesons is $\rho(770)$ [16] (the theoretical mass is 766 MeV) which is classified as $1^3S_1$ state naturally.

TABLE III. The partial decay width of $\rho$ mesons whose masses are below 2 GeV (unit: MeV).

| State mass | $2^3S_1$ 1475 | $1^3D_1$ 1519 | $3^3S_1$ 1800 | $2^3D_1$ 1826 | $4^3S_1$ 1927 |
|---|---|---|---|---|---|
| $\pi\pi$ | 6 | 15 | 5 | 4 | 2 |
| $\pi h_1(1170)$ | 3 | 89 | 0.2 | 13 | 0 |
| $\pi\pi(1300)$ | 6 | 7 | 3 | 4 | 2 |
| $\pi\omega(1420)$ | ··· | 0 | 3 | 4 | 0.3 |
| $\pi\omega(1650)$ | ··· | ··· | 0 | 0.1 | 0.1 |
| $\pi\omega$ | 4 | 12 | 0 | 3 | 0 |
| $\pi a_1(1260)$ | 13 | 187 | 2 | 29 | 0.9 |
| $\pi a_2(1320)$ | 0 | 1 | 3 | 4 | 0.4 |
| $\pi\omega_3(1670)$ | ··· | ··· | ··· | 0 | 0 |
| $\eta\rho$ | 117 | 28 | 29 | 4 | 6 |
| $\rho\rho$ | ··· | 6 | 70 | 9 | 22 |
| $\eta b_1(1235)$ | ··· | ··· | 4 | 6 | 1 |
| $\pi\pi_2(1670)$ | ··· | ··· | ··· | 22 | 0.2 |
| $KK$ | 1 | 1 | 1 | 0.3 | 0.4 |
| $KK^*$ | 0 | 0 | 0 | 0 | 0 |
| $K^*K^*$ | ··· | ··· | 1 | 0 | 2.4 |
| $KK_1(1270)$ | ··· | ··· | 0 | 1 | 0 |
| $KK_1(1400)$ | ··· | ··· | ··· | ··· | 0 |
| Total width | 150 | 346 | 121 | 103 | 38 |

(2) $\rho(1450)$

The total decay width and some measured branching ratios are shown in Tables III and V. So far its structure is still in confusion and there are a lot of

TABLE IV. The partial decay width of $\rho_3$ mesons whose masses are below 2 GeV (unit: MeV).

| State mass | $1^3D_3$ 1636 | $2^3D_3$ 1878 | $3^3D_3$ 1948 | $1^3G_3$ 1951 |
|---|---|---|---|---|
| $\pi\pi$ | 26 | 14 | 1 | 1 |
| $\pi h_1(1170)$ | 7 | 2 | 0.2 | 36 |
| $\pi\pi(1300)$ | 2 | 1 | 0 | 5 |
| $\pi\omega(1420)$ | 0 | 2 | 0.2 | 1 |
| $\pi\omega(1650)$ | ··· | 0 | 0 | 0 |
| $\pi\omega$ | 48 | 22 | 2 | 2 |
| $\pi a_1(1260)$ | 28 | 8 | 1 | 69 |
| $\pi a_2(1320)$ | 11 | 4 | 0.3 | 13 |
| $\pi\omega_3(1670)$ | ··· | 0.7 | 0 | 1 |
| $\eta\rho$ | 37 | 13 | 0.9 | 5 |
| $\rho\rho$ | 107 | 24 | 1 | 5 |
| $\eta b_1(1235)$ | ··· | 0.7 | 0 | 33 |
| $\pi\pi_2(1670)$ | ··· | 3 | 0.2 | 28 |
| $\pi\pi(1800)$ | ··· | ··· | 0 | 0 |
| $KK$ | 0.2 | 0.2 | 0 | 0 |
| $KK^*$ | 0 | 0 | 0 | 0 |
| $K^*K^*$ | ··· | 2 | 0.2 | 0 |
| $KK_1(1270)$ | ··· | 0 | 0 | 2 |
| $KK_1(1400)$ | ··· | ··· | 0 | 0.1 |
| $KK^*(1410)$ | ··· | ··· | 0 | 0 |
| $KK_2^*(1430)$ | ··· | ··· | 0 | 0 |
| Total width | 266 | 97 | 7 | 201 |

controversies about it. Godfrey and Isgur had good reason in their potential model for supposing that the $2^3S_1$ state should be around 1450 MeV [17] with decay width $\Gamma_{\rho_S} \approx 500$ MeV, whereas a mass of 1660 MeV was predicted for the $^3D_1$ state with width $\Gamma_{\rho_D} \approx 300$ MeV. The chiral quark model calculation supported this assignment [18]. Recently, by analyzing several ratios $\Gamma_{\pi\pi}/\Gamma_{\pi a_1(1260)}$, $\Gamma_{\pi h_1(1170)}/\Gamma_{\pi a_1(1260)}$ and $\Gamma_{\pi a_1(1260)}/\Gamma_{\mathrm{Total}}$ in $^3P_0$ model, He L. P. *et al.* concluded that it was easy to explain $\rho(1450)$ as a $2^3S_1$ state [7]. In chiral symmetry approach, interpreting $\rho(1450)$ as a $1^3D_1$ state was also suggested [19,20]. Nevertheless, the exotic state explanation of $\rho(1450)$ was proposed. By considering the strong decay mode $a_1(1260)\pi$ of the state, $\rho(1450)$ was taken as a good candidate of vector hybrid in flux-tube model [21]. Close *et al.* indicated that the vector hybrid with mass about 1.5 GeV can strongly decay into $a_1(1260)\pi$ [22] and $a_1(1260)$ can strongly couple with $3\pi$ [5]. In other words, a vector hybrid with the mass about 1.5 GeV can strongly decay into $4\pi$, which is similar with $\rho(1450)$ since many experiments have found that the $\rho(1450)$ dominantly decays into $4\pi$ [23–25]. This interpretation was supported by the further study of Barnes *et al.* [26]. However, the Crystal Barrel measurement found that the ratio of the $4\pi$ relative to the $2\pi$ decay of the $\rho(1450)$ is in contradiction to the interpretation of a pure hybrid state, it suggested that it is not a pure $2^3S_1$ state either [25].

In the present calculation, the masses of $2^3S_1$ and $1^3D_1$ are 1475 MeV and 1519 MeV, respectively, which are both close to that of $\rho(1450)$. Here we give a explanation of the notations used. Due to channel coupling, the eigenstates are the combinations of $S$-wave states and $D$-wave states. We use $n^{2S+1}S_J$ to denote the $n$th eigenstate with dominant $S$-wave one, and $n^{2S+1}D_J$ to denote the $n$th eigenstate with $D$-wave as the main component. From the results, we found that the mixing of the $S$ wave and $D$ wave is very small, so the notation is reasonable. For $2^3S_1$ state, the dominant decay mode is the $\eta\rho$ with $\Gamma_{\eta\rho} = 117$ MeV; and the second strong decay mode is $a_1(1260)\pi$; the partial decay widths to $\pi\pi$, $h_1(1170)\pi$, $\pi(1300)\pi$, $\pi\omega$, $KK$ are small, and the decay modes $a_2(1320)\pi$ and $KK^*$ can be neglected. By summing over all possible partial decay widths, the total width of $2^3S_1$ state is around 150 MeV. Experimentally, PDG gives the educated guess of the decay width of $\rho(1450)$, $400 \pm 6$ MeV [5]. Our results of $2^3S_1$ are different from the results of Ref. [7], where the $\pi\pi$, $a_1(1260)\pi$ are the main decay modes. $h_1(1170)\pi$, $\pi\omega$ and $\eta\rho$ are also the significant decay modes. The $KK$ and $\pi\pi(1300)$ widths are small, and the decay width of $a_2(1320)\pi$ is tiny. The total width of $2^3S_1$ state in Ref. [7] is 310 MeV which is larger than our result. The differences of decay widths between two approaches mainly come from the fact that the different wave functions are used in the two approaches.

For the $1^3D_1$ state, its partial decay widths and the total width are also given in Table III. In our calculation, the dominant decay modes of $1^3D_1$ state are $\pi a_1(1260)$ and $\pi h_1(1170)$. The $\pi\pi$, $\pi\omega$ and $\eta\rho$ decay widths are relatively narrow, and the state couples weakly to $\pi\pi(1300)$, $\pi a_2(1320)$, $\rho\rho$ and $KK$. The total decay width is 346 MeV, which is consistent with the experimental value of $\rho(1450)$, $400 \pm 60$ MeV [5]. For this state, our results are similar to that of Ref. [7].

Since PDG has not given the dominant decay modes and the corresponding decay widths of $\rho(1450)$, it is difficult to identify which state $\rho(1450)$ belongs to. In Table V, we list the partial branching ratios of $2^3S_1$ and $1^3D_1$ state with the experimental data of $\rho(1450)$ [5]. From the table, we can see that a small branching ratio, $\frac{\Gamma_{\eta\rho}}{\Gamma_{\mathrm{Total}}} < 0.04$ of $\rho(1450)$ is obtained by RVUE measurement, which indicates that $\eta\rho$ is not the dominant decay mode of $\rho(1450)$ which is not consistent with the theoretical value of $2^3S_1$ state, 0.77, and the corresponding branching ratio of $1^3D_1$ equals 0.08, which is close to the experimental value of $\rho(1450)$. And for $\frac{\Gamma_{a_1(1260)\pi}}{\Gamma_{h_1(1170)\pi}}$, the theoretical values of $2^3S_1$ and $1^3D_1$ state equal 4 and 2.1, respectively. which are all compatible with the experimental value, 3.4 of $\rho(1450)$. Besides, $\Gamma_{\eta\rho}/\Gamma_{\omega\pi}$ equals 29 for $2^3S_1$ state and 2.3 for $1^3D_1$ state. Experimentally, different branching ratios are obtained by different measurements. The SPEC measurement obtained a rather large branching ratio, $\Gamma_{\eta\rho}/\Gamma_{\omega\pi} > 2$, but the RVUE collaboration reported a small one, $\Gamma_{\eta\rho}/\Gamma_{\omega\pi} \sim 0.24$ [5]. From these branching ratios, if we consider $\rho(1450)$ as a $q\bar{q}$ configuration, the better assignment is $1^3D_1$. However, the branching ratio $\Gamma_{\omega\pi}/\Gamma_{\mathrm{Total}}$ of $2^3S_1$ state is the same with $1^3D_1$ state, $\frac{\Gamma_{\omega\pi}}{\Gamma_{\mathrm{Total}}} = 0.03$ which is different from the experimental value, 0.21

TABLE V. The partial decay branching ratios of $2^3S_1$ and $1^3D_1$ states with experimental data of the $\rho(1450)$ listed in PDG.

| Branching ratios | $2^3S_1$ state | $1^3D_1$ state | $\rho(1450)$ [5] |
|---|---|---|---|
| $\Gamma_{\omega\pi}/\Gamma_{\mathrm{Total}}$ | 0.03 | 0.03 | 0.21 |
| $\Gamma_{a_2(1320)\pi}/\Gamma_{\mathrm{Total}}$ | 0 | 0.002 | Not seen |
| $\Gamma_{KK^*}/\Gamma_{\mathrm{Total}}$ | 0 | 0 | Possibly seen |
| $\Gamma_{\eta\rho}/\Gamma_{\mathrm{Total}}$ | 0.77 | 0.08 | $<0.04$ |
| $\Gamma_{\pi\pi}/\Gamma_{\omega\pi}$ | 1.5 | 1.25 | 0.32 |
| $\Gamma_{a_1(1260)\pi}/\Gamma_{h_1(1170)\pi}$ | 4 | 2.1 | 3.4 |
| $\Gamma_{\eta\rho}/\Gamma_{\omega\pi}$ | 29 | 2.3 | $>2$ (SPEC) $\sim 0.24$ (RVUE) |
| $\Gamma_{\pi(1300)\pi}/\Gamma_{\rho\rho}$ | … | 1.2 | 3.4 |

of $\rho(1450)$. In conclusion, in our theoretical calculation, to assign $\rho(1450)$ as the $q\bar{q}$ $1^3D_1$ state is possible because the branching ratios involved the main decay modes of $1^3D_1$ are compatible with the experimental data. However, there are still some discrepancies between theory and experiments for this assignment, for example, $\Gamma_{\omega\pi}/\Gamma_{\text{Total}}$, $\Gamma_{\eta\rho}/\Gamma_{\text{Total}}$. Other configurations, tetraquark, molecule, hybrid etc. are possible. Further measurements of the partial decay widths of $\rho(1450)$ to $\pi a_1(1260)$ and $\pi h_1(1170)$ are needed to clarify the situation.

(3) $\rho(1570)$

There are relatively less experiments to study the state $\rho(1570)$. *BABAR* Collaboration obtained the $\rho''$ with the mass $1570 \pm 36 \pm 62$ MeV and the width $144 \pm 75 \pm 43$ MeV [27], which is now denoted as $\rho(1570)$ state [5].

If we accept the $\rho(1450)$ is a $1^3D_1$ state, the $\rho(1570)$ may be the $2^3S_1$ state according to the mass spectrum. To check the assignment, the two-body strong decay behavior of $2^3S_1$ state is given in Table III. For $2^3S_1$, the theoretical mass equals 1475 MeV and the total width is about 150 MeV which are both consistent with the $\rho(1570)$. And our calculations show that the $\eta\rho$ mode is the dominant decay mode of $2^3S_1$ state. Experimentally, only the decay of $\rho(1570)$ to $\omega\pi$ was seen and no precise branching ratio is obtained. So more experimental measurements of the branching ratios of $\rho(1570)$ especially $\eta\rho$ decay mode are needed to justify the assignment.

(4) $\rho(1700)$

For $\rho(1700)$, it has been observed in several $4\pi$ decay modes and the results favor the assignment of the $\rho(1700)$ as $^3D_1$ state [25]. The nonrelativistic quark model and $^3P_0$ model analysis of $e^+e^- \to \omega\pi$ process supports the assignment [28]. The recent work of He *et al.* also suggested that $\rho(1700)$ is a candidate of $1^3D_1$ state. To interpret it as $3^3S_1$ state is less plausible.

If one accepts that $\rho(1570)$ is the $2^3S_1$ state and $\rho(1450)$ is the $1^3D_1$ state, $\rho(1700)$ may be a candidate of $3^3S_1$ and $2^3D_1$ state according to the mass spectrum (see Table III) although the theoretical masses of these two states are a little higher than the PDG's value, $1720 \pm 20$ MeV. To identify which $\rho(1700)$ belongs to, the various decay modes of $3^3S_1$ and $2^3D_1$ states are calculated and given in Table III and several branching ratios are also given in Table VI. For the $3^3S_1$ state the mass is 1800 MeV, and the total width is 121 MeV in the $^3P_0$ model which is not far from the experimental value $\Gamma(\rho(1700)) = 250 \pm 100$ MeV. The main two-body decay modes of $3^3S_1$ are $\rho\rho$ and $\eta\rho$, where $\rho\rho$ can contribute largely to the $4\pi$ or $\rho\pi\pi$ final states.

TABLE VI. The partial decay branching ratios of $3^3S_1$ and $2^3D_1$ states with experimental data of the $\rho(1700)$ listed in PDG.

| Branching ratios | $3^3S_1$ state | $2^3D_1$ state | $\rho(1700)$ [5] |
|---|---|---|---|
| $\Gamma_{\pi\pi}/\Gamma_{\text{Total}}$ | 0.04 | 0.04 | $0.15 \sim 0.3$ |
| $\Gamma_{\eta\rho}/\Gamma_{\text{Total}}$ | 0.24 | 0.04 | $<0.04$ |
| $\Gamma_{a_2(1320)\pi}/\Gamma_{\text{Total}}$ | 0.02 | 0.04 | Not seen |
| $\Gamma_{\rho\rho}/\Gamma_{a_2(1260)\pi}$ | 35 | 0.31 | $\sim$0.56 |
| $\Gamma_{\rho\rho}/\Gamma_{h_1(1170)\pi}$ | 350 | 0.69 | $\sim$0.53 |
| $\Gamma_{\rho\rho}/\Gamma_{\pi(1300)\pi}$ | 23 | 2.25 | $\sim$0.30 |

And $\rho(1700)$ also couples strongly to $4\pi$ and $\rho\pi\pi$ [5]. However, many branching ratios of $3^3S_1$ state are not compatible with the experimental data of $\rho(1700)$, for example, the branching ratio $\Gamma_{\eta\rho}/\Gamma_{\text{Total}}$ of $3^3S_1$ state equals 0.24, which is larger than experimental value, $<0.04$ (see Table VI).

For the $2^3D_1$ state, Table III lists its theoretical mass (1826 MeV), all two body strong decay widths and total decay width (103 MeV). The mass and the total decay width are also not far from the experimental values of $\rho(1700)$. It is predicted that the $2^3D_1$ state decays dominantly to $a_1(1260)\pi$ and $\pi\pi_2(1670)$, where $\pi\pi_2(1670)$ can also contribute largely to $4\pi$ final state since $\pi_2(1670)$ can strongly decay into $3\pi$ [5]. In addition, $2^3D_1$ state couples weakly to $\rho\rho$ and $\eta\rho$ which is opposite with the $3^3S_1$ state. From the Table VI, we can see that the branching ratios, $\Gamma_{\eta\rho}/\Gamma_{\text{Total}}$, $\Gamma_{\rho\rho}/\Gamma_{a_2(1260)\pi}$, or $\Gamma_{\rho\rho}/\Gamma_{h_1(1170)\pi}$ of the $2^3D_1$ state are more consistent with those of $\rho(1700)$ than $3^3S_1$ state. So the assignment of $\rho(1700)$ to $2^3D_1$ is preferred in the present calculation. To justify the assignment, the precise measurements of branching ratios $\Gamma_{\rho\rho}/\Gamma_{a_2(1260)\pi}$ and $\Gamma_{\eta\rho}/\Gamma_{a_2(1260)\pi}$ of $\rho(1700)$ are expected.

(5) $\rho(1900)$

The state $\rho(1900)$ was first observed in 1996 by FENICE collaboration with mass 1.87 GeV and width $\sim$10 MeV [29], then the E687 experiment at Fermilab observed a narrow dip in $3\pi^+3\pi^-$ diffractive photoproduction. The mass and the width are $M = 1.911 \pm 0.004 \pm 0.001$ GeV$/c^2$ and $\Gamma = 29 \pm 11 \pm 4$ MeV$/c^2$. If it is interpreted as a resonance, the quantum numbers are $J^{PC}I^G = 1^{--}1^+$ because of the six-pion final state [30]. The observations are confirmed by *BABAR* and CMD3 collaborations in the process $e^+e^- \to 6\pi$ [31,32].

Because the mass of the state is around the threshold of $N\bar{N}$, the recent analysis indicated the structure may be a nonresonant cusp or threshold effect due to the opening of the $N\bar{N}$ channel [33]. However, resonance explanation of $\rho(1900)$ is still possible. In the flux tube model, a nonstrange state with the mass $\sim$1.9 GeV$/c^2$ was expected [21,34].

The lattice calculations also have the similar predictions [35–37]. And the narrow dip structure at $1.9\ \mathrm{GeV}/c^2$ in diffractive photoproduction can be fitted well by a $J^{PC} = 1^{--}$ isovector state [38]. The latest analysis of Ref. [7], the state $\rho(1900)$ was taken as the $3^3S_1$ isovector state.

In the present calculation, the candidate of $\rho(1900)$ may be a $3^3S_1$ or $4^3S_1$ state since their theoretical masses, 1800 MeV and 1927 MeV(see Table III), are both close to the experimental mass of $\rho(1900)$. And the Table III also gives the decay width of the $3^3S_1$ and $4^3S_1$ states. The calculated two-body decay width of the $4^3S_1$ state is around 38 MeV and the main decay mode is $\rho\rho$ with branching fraction $\Gamma_{\rho\rho}/\Gamma_{\rm Total} = 0.59$. The total width of $3^3S_1$ state is 121 MeV much larger than that of $4^3S_1$ state and it strongly couples to $\eta\rho$ and $\rho\rho$. Experimentally, there are no two-body strong decay modes of $\rho(1900)$ observed. Referring to the total decay width, *BABAR* Collaboration has obtained the total width of $\rho(1900)$ with $\Gamma = 130 \pm 30$ MeV in $e^+e^- \to 3\pi^+3\pi^-\gamma$ in 2006 [31] which is consistent with that of $3^3S_1$ state. In 2008, they refreshed the total width $\Gamma = 48 \pm 17 \pm 2$ MeV of $\rho(1900)$ in $e^+e^- \to \phi\pi^0\gamma$ [27], which is corresponding to that of $4^3S_1$ state. So the situation is very confusing. More experimental measurements about the total width of $\rho(1900)$ and the partial decay widths to $\eta\rho$ and $\rho\rho$ are expected to clear the situation. And our results can provide some useful reference.

(6) $\rho(2150)$

The next member of the $\rho$ family which is listed in PDG is $\rho(2150)$. Recently, B. Aubert *et al.* studied the process $e^+e^- \to 2(\pi^+\pi^-)\pi^0\gamma$, and observed a structure, which can be fitted well with a resonance with $M_{\rho(2150)} = 2.15 \pm 0.04 \pm 0.05\ \mathrm{GeV}/c^2$ and $\Gamma_{\rho(2150)} = 0.35 \pm 0.04 \pm 0.05\ \mathrm{GeV}/c^2$ [39]. Because of its high energy, the state is out of the scope of the present calculation.

(7) $\rho_3(1690)$

Now let us turn to discuss the $\rho_3$ family. From the Table I, we can see that there are three $\rho_3$ particles listed in PDG [5]. The three $\rho_3$ particles can be good candidates of the $1^3D_3$, $2^3D_3$ and $33^3D_3$ states in the Regge trajectory analysis [7].

The lowest state is $\rho_3(1690)$ with mass $1688.8 \pm 2.1$ MeV [5]. It was first observed in 1965 by Goldberg [40] and Forino *et al.* [41], and it was once thought as a $\pi^+\pi^-$ resonance or a three-$\rho$ meson molecular state [42]. In our calculation, for the ground state of $\rho_3$ family, the $D$-wave component percentage is 99.9925%, and the $G$-wave just occupied 0.0075%. So we can ignore the weak mixing between $D$ and $G$ wave, and call the ground state of $\rho_3$ $1^3D_3$ state, so does $n3^3D_3$ ($n = 2,3$) state. And for $1^3G_3$ state, it represents the state that the main component is $G$ wave rather than $D$ wave. The masses, the partial and the total decay widths of the states in the $\rho_3$ family whose masses are below 2 GeV are shown in Table IV. For the ground state $1^3D_3$, the mass is 1636 MeV, which is close to the mass of $\rho_3(1690)$, so it may be a good candidate of $\rho_3(1690)$. The total decay width of $1^3D_3$ state is 266 MeV, and the dominant decay mode is $\rho\rho$, which can dominantly decay into $4\pi$. The second main decay mode is $\pi\omega$, and the coupling of the state to $\pi\pi$, $\pi\omega(1420)$, $KK$ and $KK^*$ is relatively weak which is similar with those of Ref. [7]. Experimentally, PDG does not give the average width of $\rho_3(1690)$, and it ranges from $126 \pm 40$ MeV to $204 \pm 18$ MeV which is consistent with our theoretical values. Besides, it dominantly decays into $4\pi$ with branching ratio, $(71.1 \pm 1.9)\%$ which also supports the assignment. Moreover, Table VII gives some theoretical branching ratios of $1^3D_3$ state and the experimental values of $\rho_3(1690)$. From the table, we can see that for the main decay modes $\rho\rho$ and $\pi\omega$, whose branching ratios of $1^3D_3$ state $\Gamma_{\omega\pi}/\Gamma_{\rm Total}$ and $\Gamma_{\omega\pi}/(\Gamma_{\omega\pi} + \Gamma_{\rho\rho})$ are consistent with $\rho_3(1690)$. For the branching ratios of weak decay modes, $\Gamma_{KK}/\Gamma_{\rm Total}$ and $\Gamma_{KK}/\Gamma_{\pi\pi}$, there are some difference between theoretical and experimental values within the error range. So by comparison, it is also safe to assign the state $\rho_3(1690)$ as a $1^3D_3$ state.

(8) $\rho_3(1990)$

The second $\rho_3$ particle is $\rho_3(1990)$. PDG [5] does not give the average mass and the total width of this state. Anisovich *et al.* got the resonance parameters ($3^{--}$) with mass $\sim 1982 \pm 14$ MeV and width $\sim 188 \pm 24$ MeV from a combined fit to $P\bar{P} \to \omega\pi^0$, $\omega\eta\pi^0$ and $\pi^-\pi^+$, using both $\omega \to \pi^0\gamma$ and $\omega \to \pi^-\pi^+\pi^0$ decays in 2002 [6]. And RVUE also gave the mass $\sim 2007$ MeV and width $\sim 287$ MeV of $\rho_3(1990)$, which was observed in the $\pi\pi$ invariant mass spectrum of $p\bar{p} \to \pi\pi$ [43].

The calculated masses and the decay widths of the $2^3D_3$, $3^3D_3$ and $1^3G_3$ eigenstates are given in Table IV. We found that the masses of these states are all close to 1990 MeV. For the $2^3D_3$ state, the $^3P_0$

TABLE VII. The partial decay branching ratios of $1^3D_3$ state with experimental data of $\rho_3(1690)$ listed in PDG.

| Branching ratios | $1^3D_3$ state | $\rho_3(1690)$[5] |
|---|---|---|
| $\Gamma_{\pi\pi}/\Gamma_{\rm Total}$ | 0.09 | 0.236 |
| $\Gamma_{KK}/\Gamma_{\pi\pi}$ | 0.008 | 0.067 |
| $\Gamma_{KK}/\Gamma_{\rm Total}$ | 0.0007 | 0.0158 |
| $\Gamma_{a_2(1320)\pi}/\Gamma_{\rho\eta}$ | 0.3 | 5.5 |
| $\Gamma_{\omega\pi}/\Gamma_{\rm Total}$ | 0.16 | 0.16 |
| $\Gamma_{\omega\pi}/(\Gamma_{\omega\pi} + \Gamma_{\rho\rho})$ | 0.3 | 0.22 |

model predicts that it couples strongly to $\rho\rho$ and $\pi\omega$ and the total width is around 97 MeV, which is smaller than the experimental data of $\rho_3(1990)$ obtained by Anisovich *et al.*, $188 \pm 24$ MeV [6]. The $3^3D_3$ state has the even smaller decay width, 7 MeV. For $1^3G_3$ state, it strongly decays to $\pi a_1(1260)$, $\pi h_1(1170)$, $\eta b_1(1235)$ and $\pi\pi_2(1670)$, and the total decay width is 201 MeV, which is close to the experiment data of $\rho_3(1990)$. By now, it is difficult to make the assignment of the $\rho_3(1990)$ due to the lack of experimental data on the decay properties. To find more information on the partial decay widths of $\rho_3(1990)$ is crucial to identify the state. And our theoretical results will be useful to the future research on $\rho_3(1690)$.

Up to now, many experiments have observed the $\rho_3(2250)$ [44–46]. Just like the state $\rho_3(1990)$, PDG also does not give the average mass, partial decay widths or total width of $\rho_3(2250)$. So more experimental measurements are needed to understand the structure of the state.

## V. SUMMARY

Many $\rho/\rho_3$ mesons have been found in experiment by now. But the nature of them is still not very clear. So classifying these $\rho/\rho_3$ particles into the meson family is an interesting work which can improve our knowledge of light hadron spectrum. In the present work, we have calculated the OZI-allowed two-body strong decay behaviors of $\rho/\rho_3$ mesons whose masses are below 2 GeV by using the wave functions of the states obtained from solving the Schrödinger equation. In our work, the chiral quark model, $^3P_0$ model and the Gaussian expansion method are applied.

Just like the analysis above, the theoretical partial decay branching ratios and the total decay width of the $1^3D_1$ state are more consistent with the experimental values of $\rho(1450)$ than $2^3S_1$ state, so we think $\rho(1450)$ is the $1^3D_1$ state rather than $2^3S_1$ state. For $\rho(1570)$, it may be taken as $2^3S_1$ state temporarily according to the mass spectrum and the total decay width and due to the lack of the experimental information on branching ratios. The measurement of the $\eta\rho$ partial decay width of $\rho(1570)$ is helpful to assure the assignment. Besides, in our present work, we tend to assign $\rho(1700)$ as $2^3D_1$ state, and more precise measurements in experiment are needed to find the state ($3^3S_1$ or $4^3S_1$ state) which $\rho(1900)$ belongs to. With regard to the states in $\rho_3$ family, assigning the state $\rho_3(1690)$ to $1^3D_3$ state is convincing, and it is difficult to make the convincing assignment for other two $\rho_3$ states due to the lack of decay information experimentally. Nevertheless, our calculation will give some useful advice for future experimental work.

In the charmonium sector, most states above the $D\bar{D}$ threshold are not accommodated to the $c\bar{c}$ picture. Whereas in the light $q\bar{q}$ sectors, taking $\rho/\rho_3$ family as an example, we can see that the most states in the $\rho/\rho_3$ family can be described as normal mesons although they are above the $q\bar{q}$ thresholds (Other possibilities are not excluded in the present calculation because of the lack of experimental data). The possible reason is that for the $\rho$ family, all states are above the $\pi\pi$ threshold, the four quark effects have been absorbed in the model parameters which are obtained by fitting the light meson spectra. Another problem of the quark model calculation is the over prediction of the states, and the tetraquark will add more states if they are possible. As a phenomenological approach, the results are model parameter dependent. By invoking the high Fock components, it is possible to change the pattern of the spectrum. Clearly further study is needed.

By employing the GEM, we can obtain reliable wave functions of mesons in the framework of the chiral quark model. Moreover, with the help of obtained wave functions, the calculation of the two-body strong decay width by $^3P_0$ model is self-consistent. This is crucial to extract the reliable information from the meson spectrum. It is expected that the abundant decay information provided by the present work which will benefit further experimental study on light hadron spectrum.

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