Twist-3 fragmentation contribution to polarized hyperon production in unpolarized hadronic collisions

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It has been known for a long time that hyperons produced in hadronic collisions are polarized perpendicular to the production plane of the reaction. This effect cannot be described by using twist-2 collinear parton correlators only. Here we compute the contribution of twist-3 fragmentation functions to the production of transversely polarized hyperons in unpolarized proton-proton collisions in the framework of collinear factorization. By taking into account the relations among the relevant twist-3 fragmentation functions which follow from the QCD equation of motion and the Lorentz invariance property of the correlators, we present the leading-order cross section for this term.

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I. INTRODUCTION

The first observation of transversely polarized hyperons in unpolarized hadronic collisions was already made in the 1970s. Specifically, when colliding protons with a beryllium target and detecting Λ hyperons, it was found that the Λ's show a transverse polarization asymmetry (often denoted as A_N), which is largest (up to 30%) for polarization perpendicular to the reaction plane and vanishes in the reaction plane [1]. This pioneering measurement was followed by a number of corresponding experiments which, in particular, also covered different kinematic ranges [2–11]. Some of the earlier data are reviewed in [12,13]. We also refer to [14] for a list of relevant papers. Generally A_N vanishes for exact midrapidity of the hyperon in a process like $pp \to \Lambda^{\uparrow} X$, and it increases with increasing rapidity. Hyperon polarization was also studied in related reactions such as $\gamma p \to \Lambda^{\uparrow} X$ [15,16], quasireal photoproduction of Λ's in lepton scattering off nucleons and nuclei [17,18], and in electron-positron collisions [19,20].

For high-energy collisions and sufficiently large transverse momentum P_T of the hyperon, A_N can be computed in perturbative quantum chromodynamics (QCD). However, it has been known for a long time that by only using collinear leading-twist (twist-2) parton correlators one cannot describe this type of transverse single-spin asymmetry (SSA) [21]. As A_N is a genuine twist-3 observable, one rather needs the full machinery of collinear higher-twist factorization [22–24]. Already in the early 1980s this approach was used in connection with transverse SSAs [22,25]. Later works further elaborated on these twist-3 calculations, where a main focus was on the transverse target SSA for processes like $p^{\uparrow} p \to \pi X$; see for instance [26–35]. An overview of these calculations can be found in [36].

In collinear factorization, the transverse SSAs receive, *a priori*, twist-3 contributions from two-parton and three-parton correlation functions which are associated with either the initial-state or final-state hadrons. These correlators are parametrized in terms of twist-3 parton distribution functions (PDFs) and fragmentation functions (FFs), respectively. While the complete leading-order (LO) twist-3 cross section for $p^{\uparrow} p \to \pi X$ can be found in the literature [32–35], only part of the twist-3 cross section for $pp \to \Lambda^{\uparrow} X$ is available [37–39]. The present work is a major step towards completing the calculation of all possible terms.

The numerator of the transverse SSA for $pp \to \Lambda^{\uparrow} X$ has two types of contributions. The first one, which contains a twist-3 PDF for one of the unpolarized protons combined with the unpolarized twist-2 PDF for the other proton and the spin-dependent twist-2 "transversity" FF, was derived in Refs. [37–39]. Here we focus on the second contribution, which involves twist-3 FFs and the twist-2 unpolarized PDFs of the protons. (A short version of the present work was presented in [40,41]. We also remind the reader that twist-3 FFs for a polarized Λ were studied for other processes, e.g., $ep \to \Lambda^{\uparrow} X$ [42] and $e^+ e^- \to \Lambda^{\uparrow} X$ [43].) Specifically, we compute all the LO terms that are related to quark-gluon-quark (qgq) fragmentation correlators, while terms given by quark-antiquark-gluon ($q\bar{q}g$) correlators and pure gluon (gg and ggg) correlators will be considered elsewhere.

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Not only is our study important for obtaining a complete analytical result, but it may also be critical for the phenomenology of this observable. In this context we remind the reader that recent work strongly suggests the numerical dominance of the collinear twist-3 fragmentation contribution for A_N in $p^\uparrow p \to \pi X$ [44,45].

The remainder of the paper is organized as follows: In Sec. II, we list the definitions of the twist-3 FFs that are relevant for the present study. In that section we also present relations among the FFs which are based on the QCD equation of motion and Lorentz invariance [46]. These relations are crucial for, in particular, obtaining a frame-independent result for A_N. In Sec. III, we discuss the calculation for the twist-3 fragmentation contribution to the cross section for $pp \to \Lambda^\uparrow X$, while Sec. IV is devoted to a brief summary.

II. TWIST-3 FRAGMENTATION FUNCTIONS AND THEIR RELATIONS

We first recall the definitions of the twist-3 FFs for a transversely polarized spin-$\frac{1}{2}$ hadron. One can identify, *a priori*, three different types of such functions, which in [46] were referred to as *intrinsic*, *kinematical*, and *dynamical* FFs. We start with the intrinsic twist-3 FFs for $\Lambda^\uparrow$. They are defined through a quark-quark (qq) fragmentation correlator according to [46–48]

$$\begin{aligned}\Delta_{ij}(z) &= \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle \\ &\quad\times \langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)|0\rangle \\ &= \left(\gamma_5 \not{S}_\perp \frac{\not{P}_h}{z}\right)_{ij} H_1(z) + M_h \epsilon^{\alpha S_\perp w P_h} (\gamma_\alpha)_{ij} \frac{D_T(z)}{z} \\ &\quad + M_h (\gamma_5 \not{S}_\perp)_{ij} \frac{G_T(z)}{z} + \cdots, \end{aligned} \tag{1}$$

where ψ_i, ψ_j are the quark fields carrying the spinor indices i, j, and a color average is implied in Eq. (1), with $N = 3$ being the number of quark colors. The hadron (Λ) is characterized by its four-momentum P_h and (transverse) spin vector $S_\perp$, while M_h is its mass. The four-vector w^μ is lightlike ($w^2 = 0$) and satisfies $P_h \cdot w = 1$. For simplicity, Wilson lines in the operator (1) are suppressed. Here and below we use the shorthand notation $\epsilon^{\alpha S_\perp w P_h} \equiv \epsilon^{\alpha\beta\gamma\delta} S_{\perp\beta} w_\gamma P_{h\delta}$, where our convention for the Levi-Civita tensor is $\epsilon^{0123} = +1$. The rhs of Eq. (1) contains the twist-2 transversity FF H_1, which describes the probability for a transversely polarized quark to fragment into a transversely polarized hadron, and the intrinsic twist-3 FFs D_T and G_T. These (dimensionless) FFs depend on the fraction z of the quark momentum which is carried by the hadron. Hermiticity implies that they are real valued. From the functions available in (1), it is actually only the naïve time-reversal-odd (T-odd) function D_T which contributes to the piece of the transverse SSA we calculate here.

The kinematical twist-3 FFs parametrize the derivative of the qq correlator [46–48],

$$\begin{aligned}\Delta^\alpha_{\partial ij}(z) &= \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|[\infty w, 0]\psi_i(0)|h(P_h, S_\perp)X\rangle \\ &\quad\times \langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle \overleftarrow{\partial}^\alpha \\ &= -iM_h \epsilon^{\alpha S_\perp w P_h} (\not{P}_h)_{ij} \frac{D_{1T}^{\perp(1)}(z)}{z} \\ &\quad + iM_h S_\perp^\alpha (\gamma_5 \not{P}_h)_{ij} \frac{G_{1T}^{\perp(1)}(z)}{z} + \cdots. \end{aligned} \tag{2}$$

The derivative on the rhs of (2) also acts on the Wilson line, which generally is defined through

$$[0, \lambda\omega] = \mathcal{P} \exp\left\{ig \int_\lambda^0 dt \omega_\mu A^\mu(t\omega)\right\}, \tag{3}$$

where $\mathcal{P}$ indicates path ordering and g is the strong coupling. The FFs $D_{1T}^{\perp(1)}$ and $G_{1T}^{\perp(1)}$ are also real valued. This (T-odd) function is a particular moment of a transverse-momentum-dependent (TMD) FF [47,48],

$$D_{1T}^{\perp(1)}(z) = z^2 \int d^2\vec{p}_\perp \frac{\vec{p}_\perp^{\,2}}{2M_h^2} D_{1T}^\perp(z, z^2 \vec{p}_\perp^{\,2}), \tag{4}$$

with $D_{1T}^\perp$ describing the fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ hadron. Using the so-called generalized parton model, which exclusively works with TMD PDFs and FFs, in Ref. [49] this function was fitted to A_N data for $pp \to \Lambda^\uparrow X$. The result of the fit was then used to estimate transverse SSAs in semi-inclusive deep-inelastic scattering [50], including neutrino-nucleon scattering $\nu N \to \ell^\pm \Lambda^\uparrow X$ for which some data are available [51]. We note that, like with the intrinsic functions, only T-odd kinematical correlators can contribute to our calculation. Namely, only $D_{1T}^{\perp(1)}$ can enter from (2).

Let us finally discuss the dynamical twist-3 FFs. They parametrize the so-called F-type qgq correlator (see, for instance, Refs. [42,46,52]),

$$\begin{aligned}\Delta^\alpha_{Fij}(z, z_1) &= \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z} - \frac{1}{z_1})} \\ &\quad\times \langle 0|\psi_i(0)|h(P_h, S_\perp)X\rangle \\ &\quad\times \langle h(P_h, S_\perp)X|\bar{\psi}_j(\lambda w)\ gF^{\alpha w}(\mu w)|0\rangle \\ &= M_h \epsilon^{\alpha S_\perp w P_h} (\not{P}_h)_{ij} \frac{\hat{D}^*_{FT}(z, z_1)}{z} \\ &\quad - iM_h S_\perp^\alpha (\gamma_5 \not{P}_h)_{ij} \frac{\hat{G}^*_{FT}(z, z_1)}{z} + \cdots, \end{aligned} \tag{5}$$

where $F^{\alpha w} \equiv F^{\alpha\beta} w_\beta$ represents a component of the gluon field strength tensor. The (two-argument) FFs $\hat{D}_{FT}(z, z_1)$ and $\hat{G}_{FT}(z, z_1)$ have support for $1 > z > 0$ and $z_1 > z$ [53–56]. These support properties imply, in particular, that the functions themselves [54] and their z_1-partial derivatives [46] vanish for a vanishing gluon momentum. Therefore, for fragmentation one has no so-called gluonic poles [26,27], which play a very important role for contributions to SSAs caused by twist-3 effects coming from initial-state hadrons. The vanishing of gluonic poles in fragmentation is also intimately connected with the universality of TMD FFs (see, for instance, Refs. [53–55,57,58]). In general, the dynamical twist-3 FFs are complex [34,52,56,59,60], and we use their complex conjugate in Eq. (5). (In this paper we follow the convention of [46] for $\hat{D}_{FT}$ and $\hat{G}_{FT}$.) One can also define D-type twist-3 FFs by replacing $gF^{\alpha w}(\mu w)$ in (5) with the covariant derivative $D^\alpha(\mu w)$. These functions, however, do not represent new independent objects, but they can rather be related to the F-type functions. For the imaginary parts of the FFs, which matter in the present study, one has the relations [30,31,34,52,56,59,60]

$$\mathrm{Im}\hat{D}_{DT}(z, z_1) = P\frac{1}{1/z - 1/z_1}\mathrm{Im}\hat{D}_{FT}(z, z_1) - \delta\left(\frac{1}{z} - \frac{1}{z_1}\right) D_{1T}^{\perp(1)}(z), \qquad (6)$$

$$\mathrm{Im}\hat{G}_{DT}(z, z_1) = P\frac{1}{1/z - 1/z_1}\mathrm{Im}\hat{G}_{FT}(z, z_1), \qquad (7)$$

where here P indicates the principal-value prescription. For the calculation in Sec. III we use the F-type FFs.

Although, *a priori*, all three types of FFs as defined in (1), (2), and (5) appear in the derivation of the twist-3 cross section for $pp \to \Lambda^\uparrow X$, these functions are not independent. There exist relations based on the QCD equation of motion (e.o.m.) and so-called Lorentz invariance relations (LIRs). A comprehensive derivation of these relations as well as a list of references can be found in [46]. The e.o.m. relation which is relevant for the present study takes its simplest form for the D-type functions,

$$\int \frac{dz_1}{z_1^2}\left(\mathrm{Im}\hat{D}_{DT}(z, z_1) - \mathrm{Im}\hat{G}_{DT}(z, z_1)\right) = \frac{D_T(z)}{z}. \qquad (8)$$

Using Eqs. (6) and (7) one finds the e.o.m. relation involving the F-type functions,

$$\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{1/z - 1/z_1}\left(\mathrm{Im}\hat{D}_{FT}(z, z_1) - \mathrm{Im}\hat{G}_{FT}(z, z_1)\right) = \frac{D_T(z)}{z} + D_{1T}^{\perp(1)}(z). \qquad (9)$$

Making use of Lorentz invariance of the parton correlation functions one can further derive the LIR

$$-\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{\mathrm{Im}\hat{D}_{FT}(z, z_1)}{(1/z_1 - 1/z)^2} = \frac{D_T(z)}{z} + \frac{d(D_{1T}^{\perp(1)}(z)/z)}{d(1/z)}. \qquad (10)$$

These relations generally simplify the form of twist-3 cross sections and, in particular, guarantee their frame independence [42,45,46,61–63].

III. TWIST-3 CROSS SECTION FOR $pp \to \Lambda^\uparrow X$

We now sketch the derivation and present the results of the twist-3 spin-dependent cross section for

$$p(p) + p(p') \to \Lambda^\uparrow(P_h, S_\perp) + X. \qquad (11)$$

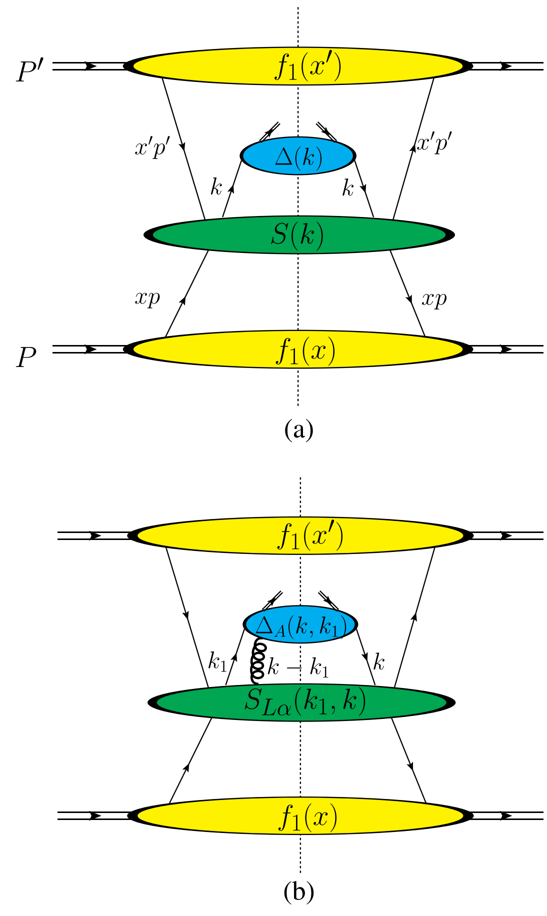

FIG. 1. Generic diagrams giving rise to the twist-3 fragmentation contribution to the polarized cross section for the process in (11). The top blob and bottom blob indicate the unpolarized twist-2 PDFs in the protons. The second blob from the top represents the fragmentation matrix elements for Λ: $\Delta(k)$ in (a) and $\Delta_A(k, k_1)$ in (b). The second blob from the bottom is the partonic hard scattering parts: $S(k)$ in (a) and $S_{L\alpha}(k_1, k)$ in (b). The mirror diagram of (b) also contributes and is included in the third term of (12).

For the derivation of the nonpole contribution to the cross section from twist-3 FFs, we follow the Feynman gauge formalism developed in [56]. Though this reference presented the formalism for semi-inclusive deep inelastic scattering, it can be directly applied to the above process (11). From Eq. (54) of [56], one can read the cross section for (11) as

$$P_h^0 \frac{d\sigma(P_h, S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 s}\int\frac{dx}{x}f_1(x)\int\frac{dx'}{x'}f_1(x')\Bigg[\int\frac{dz}{z^2}\mathrm{Tr}[\Delta(z)S(P_h/z)] - i\int\frac{dz}{z^2}\mathrm{Tr}\left[\Omega^\alpha_\beta\Delta^\beta_\partial(z)\frac{\partial S(k)}{\partial k^\alpha}\bigg|_{k=P_h/z}\right] + 2\mathrm{Re}\left\{(-i)\int\frac{dz}{z^2}\frac{dz_1}{z_1^2}\mathrm{Tr}\left[\Omega^\alpha_\beta\Delta^\beta_F(z,z_1)P\left(\frac{1}{1/z_1-1/z}\right)S_{L\alpha}\left(\frac{P_h}{z_1},\frac{P_h}{z}\right)\right]\right\}\Bigg], \tag{12}$$

where the summation over all channels and parton types is implicit. In Eq. (12), $s=(p+p')^2$ is the square of the center of mass energy and $\Omega^\alpha_\beta = g^\alpha_\beta - P_h^\alpha w_\beta$. The unpolarized twist-2 PDF is denoted by f_1, and the correlators $\Delta(z)$, $\Delta^\beta_\partial(z)$ and $\Delta^\beta_F(z,z_1)$ are defined in (1), (2) and (5). The symbol Tr indicates the trace over color and spinor indices. In deriving Eq. (12), we introduced the partonic hard scattering parts (before collinear expansion), $S(k)$ and $S_{L\alpha}(k_1,k)$, corresponding, respectively, to the fragmentation matrix elements

$$\Delta_{ij}(k) = \frac{1}{N}\sum_X\int d^4\xi e^{-ik\cdot\xi}\langle 0|\psi_i(0)|h(P_h,S_\perp)X\rangle \times\langle h(P_h,S_\perp)X|\bar\psi_j(\xi)|0\rangle, \tag{13}$$

and

$$\Delta_{A,ij}(k,k_1) = \frac{1}{N}\sum_X\int d^4\xi\int d^4\eta e^{-ik_1\cdot\xi}e^{-i(k-k_1)\cdot\eta} \times\langle 0|\psi_i(0)|h(P_h,S_\perp)X\rangle \times\langle h(P_h,S_\perp)X|\bar\psi_j(\xi)gA^\alpha(\eta)|0\rangle, \tag{14}$$

as shown in Figs. 1(a) and 1(b). In Ref. [56], it has been proven that, after the collinear expansion, S and $S_{L\alpha}$ become the partonic hard cross section for the gauge-invariant correlation functions $\Delta(z)$, $\Delta^\beta_\partial(z)$ and $\Delta^\beta_F(z,z_1)$, as shown in (12). Note that $S_{L\alpha}(\frac{P_h}{z_1},\frac{P_h}{z})$ is the hard part for the diagram in which the coherent gluon line from $\Delta^\beta_F(z,z_1)$ [i.e., the gluon line originally from the A^α-field in (14)] is located to the left of the cut, and the effect of the mirror diagram is taken into account by the principal-value prescription and the factor 2 for the third term in (12). Substituting (1), (2) and (5) into Eq. (12), one can cast the cross section in the following form:

$$\begin{aligned}P_h^0\frac{d\sigma(P_h,S_\perp)}{d^3P_h} = {}& \frac{\alpha_s^2 M_h}{s}\sum_{i=1}^2 A^{(i)}(w)\int\frac{dx}{x}f_1(x)\int\frac{dx'}{x'}f_1(x')\int\frac{dz}{z^3}\delta(\hat s+\hat t+\hat u)\\ &\times\Bigg[\frac{D_T(z)}{z}\hat\sigma_T^{(i)} - \frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\hat\sigma_D^{(i)} - D_{1T}^{\perp(1)}(z)\hat\sigma_{ND}^{(i)} + \int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{1}{1/z-1/z_1}\right)\mathrm{Im}\hat D_{FT}(z,z_1)\hat\sigma_{DF1}^{(i)}\\ &+\int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{z_1}{1/z-1/z_1}\right)\mathrm{Im}\hat D_{FT}(z,z_1)\hat\sigma_{DSFP}^{(i)} - \frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{1}{(1/z_1-1/z)^2}\right)\mathrm{Im}\hat D_{FT}(z,z_1)\hat\sigma_{DF2}^{(i)}\\ &+\int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{1}{1/z-1/z_1}\right)\mathrm{Im}\hat G_{FT}(z,z_1)\hat\sigma_{GF1}^{(i)} + \int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{z_1}{1/z-1/z_1}\right)\mathrm{Im}\hat G_{FT}(z,z_1)\hat\sigma_{GSFP}^{(i)}\\ &-\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{1}{(1/z_1-1/z)^2}\right)\mathrm{Im}\hat G_{FT}(z,z_1)\hat\sigma_{GF2}^{(i)}\Bigg],\end{aligned} \tag{15}$$

where $A^{(1)}(w)\equiv\frac{p'\cdot P_h}{p\cdot p'}\epsilon^{P_h p w S_\perp}$, $A^{(2)}(w)\equiv\frac{p\cdot P_h}{p\cdot p'}\epsilon^{P_h w p' S_\perp}$, and each partonic cross section $\hat\sigma_Y^{(i)}$ ($i=1,\ 2,\ Y=T,\ D, ND,\ldots$) is a function of the partonic Mandelstam variables defined as $\hat s=(xp+x'p')^2$, $\hat t=(xp-P_h/z)^2$, $\hat u=(x'p'-P_h/z)^2$. The lowest-order Feynman diagrams for the partonic hard parts S and $S_{L\alpha}$ in each channel are shown in Figs. 2–5. Several comments are in order here: (i) Unlike in the case of the twist-3 PDFs, for twist-3 FFs the nonpole term of the hard scattering coefficients contributes to A_N. [In this context, see also the discussion in the paragraph after Eq. (5).] In particular, the imaginary part of the complex functions $\hat D_{FT}$ and $\hat G_{FT}$ contributes to the spin-dependent cross section, reflecting the naïve T-odd nature of A_N.

(ii) One finds that the z_1 dependence of the hard parts $S_{L\alpha}$ in Eq. (12) has a relatively simple structure that does not

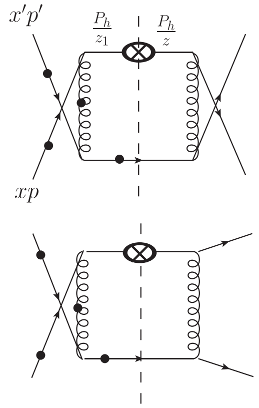

FIG. 2. Feynman diagrams for the hard parts $S(k)$ and $S_{L\alpha}(k_1, k)$ in the $qq \to qq$ (all graphs), $qq' \to qq'$ (top left graph) and $qq' \to q'q$ (top right graph) channels. The circled cross indicates the fragmentation insertion. When ignoring the dots the diagrams determine the hard parts $S(k)$. Graphs for $S_{L\alpha}(k_1, k)$ are obtained by attaching a coherent gluon line from the fragmentation function to one of the four dots in each diagram.

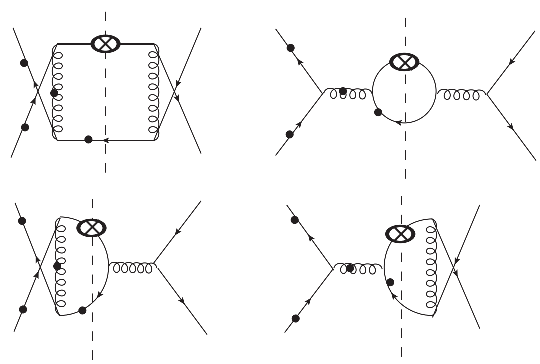

FIG. 3. Same as Fig. 2, but for the $q\bar{q} \to q\bar{q}$ (all graphs), $q\bar{q}' \to q\bar{q}'$ (top left graph) and $q\bar{q} \to q'\bar{q}'$ (top right graph) channels.

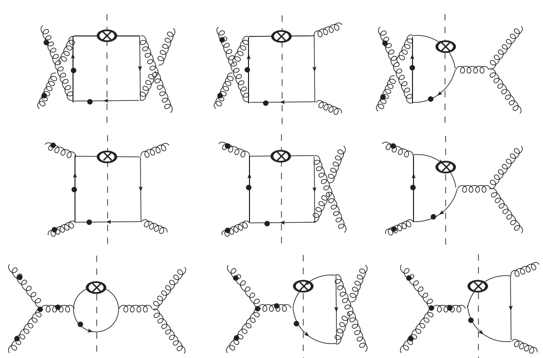

FIG. 4. Same as Fig. 2, but for the $gg \to q\bar{q}$ channel.

"mix" with the partonic Mandelstam variables. Therefore, the contributions from $\mathrm{Im}\hat{D}_{FT}$ and $\mathrm{Im}\hat{G}_{FT}$ can be brought into the form shown in (15), where the hard partonic cross sections $\hat{\sigma}_Y^{(i)}$ are independent of z_1 and only depend on $\hat{s}$, $\hat{t}$, $\hat{u}$.

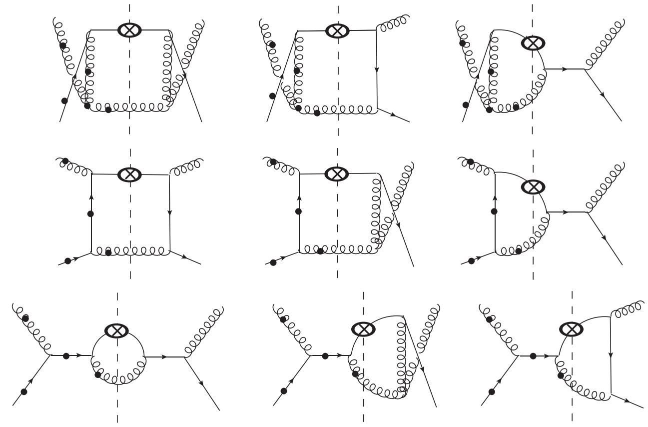

FIG. 5. Same as Fig. 2, but for the $qg \to qg$ channel.

(iii) Calculation of the diagrams provides the relations $\hat{\sigma}_{DF1}^{(i)} = -\hat{\sigma}_{GF1}^{(i)}$, $\hat{\sigma}_{DSFP}^{(i)} = \hat{\sigma}_{GSFP}^{(i)}$, $\hat{\sigma}_{GF2}^{(i)} = 0$ $(i = 1,\ 2)$ for all channels. Using these relations in combination with the e.o.m. relation (9) and the LIR (10) one can rewrite the cross section in (15) in a very compact manner—see Eq. (16) below—by introducing the combinations $\hat{\sigma}_1^{(i)} \equiv \hat{\sigma}_T^{(i)} + \hat{\sigma}_{DF1}^{(i)} + \hat{\sigma}_{DF2}^{(i)}$, $\hat{\sigma}_2^{(i)} \equiv \hat{\sigma}_D^{(i)} - \hat{\sigma}_{DF2}^{(i)}$, $\hat{\sigma}_3^{(i)} \equiv \hat{\sigma}_{ND}^{(i)} - \hat{\sigma}_{DF1}^{(i)}$ and $\hat{\sigma}_4^{(i)} \equiv \hat{\sigma}_{DSFP}^{(i)}$.

(iv) In order to test the frame independence of our result, we have computed the cross section in two different frames: $w_1^\mu = \frac{p'^\mu}{p'\cdot P_h}$ and $w_2^\mu = \frac{p^\mu}{p\cdot P_h}$. In the first frame, $A^{(1)}(w_1)$ is nonzero while $A^{(2)}(w_1)$ vanishes, so the result depends only on $A^{(1)}(w_1)$ and $\hat{\sigma}_{1,2,3,4}^{(1)}(w_1)$. In the second frame, $A^{(2)}(w_2)$ is nonzero while $A^{(1)}(w_2)$ vanishes, so the result depends only on $A^{(2)}(w_2)$ and $\hat{\sigma}_{1,2,3,4}^{(2)}(w_2)$. Since $A^{(1)}(w_1) = A^{(2)}(w_2)$, and we also found $\hat{\sigma}_{1,2,3,4}^{(1)}(w_1) = \hat{\sigma}_{1,2,3,4}^{(2)}(w_2) \equiv \hat{\sigma}_{1,2,3,4}$ for all channels, the result is the same in both frames.

Our final expression for the frame-independent twist-3 cross section reads

$$
\begin{aligned}
P_h^0\frac{d\sigma(P_h,S_\perp)}{d^3P_h} = {}& \frac{2\alpha_s^2 M_h}{s^2}\epsilon^{P_h p p' S_\perp}\int\frac{dx}{x}f_1(x)\int\frac{dx'}{x'}f_1(x') \\
&\times\int\frac{dz}{z^3}\delta(\hat{s}+\hat{t}+\hat{u}) \\
&\times\Bigg[\frac{D_T(z)}{z}\hat{\sigma}_1 - \left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}_2 \\
&-D_{1T}^{\perp(1)}(z)\hat{\sigma}_3 + \int_z^\infty\frac{dz_1}{z_1^2}\left(\frac{z_1}{1/z-1/z_1}\right) \\
&\times(\mathrm{Im}\hat{D}_{FT}(z,z_1)+\mathrm{Im}\hat{G}_{FT}(z,z_1))\hat{\sigma}_4\Bigg]. \qquad (16)
\end{aligned}
$$

This represents the complete result of the cross section caused by twist-3 effects of the qq and qgq fragmentation correlators depicted in Fig. 1. The partonic cross sections for each channel are given as follows:

(1) $qq' \to qq'$:

$$\hat{\sigma}_1 = \frac{\hat{s}(\hat{t}^2 - 2\hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{2(\hat{s}^3 + 2\hat{s}^2\hat{u} + \hat{u}^3)}{\hat{t}^3\hat{u}}, \tag{17}$$

$$\hat{\sigma}_2 = \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2\hat{u}} - \frac{1}{N^2}\frac{(2\hat{t} - \hat{u})(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}}, \tag{18}$$

$$\hat{\sigma}_3 = \left(1 - \frac{1}{N^2}\right)\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3}, \tag{19}$$

$$\hat{\sigma}_4 = -\frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}}. \tag{20}$$

(2) $q'q \to qq'$:

$$\hat{\sigma}_1 = \frac{\hat{s}(2\hat{t}^2 - \hat{u}^2)}{\hat{t}\hat{u}^3} + \frac{1}{N^2}\frac{2(\hat{s}^3 + 2\hat{s}^2\hat{t} + \hat{t}^3)}{\hat{t}\hat{u}^3}, \tag{21}$$

$$\hat{\sigma}_2 = -\frac{\hat{s}^2 + \hat{t}^2}{\hat{t}\hat{u}^2} - \frac{1}{N^2}\frac{(\hat{t} - 2\hat{u})(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^3}, \tag{22}$$

$$\hat{\sigma}_3 = -\left(1 - \frac{1}{N^2}\right)\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3}, \tag{23}$$

$$\hat{\sigma}_4 = \frac{\hat{s}(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^3} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^2}. \tag{24}$$

(3) $qq \to qq$:

$$\begin{aligned}\hat{\sigma}_1 = {}& \frac{\hat{s}(\hat{t}^2 - 2\hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{2(\hat{s}^3 + 2\hat{s}^2\hat{u} + \hat{u}^3)}{N^2\hat{t}^3\hat{u}} + \frac{\hat{s}(2\hat{t}^2 - \hat{u}^2)}{\hat{t}\hat{u}^3} \\ & + \frac{2(\hat{s}^3 + 2\hat{s}^2\hat{t} + \hat{t}^3)}{N^2\hat{t}\hat{u}^3} + \frac{\hat{s}^2(\hat{t} - \hat{u})}{N\hat{t}^2\hat{u}^2} - \frac{2\hat{s}^2(\hat{t} - \hat{u})}{N^3\hat{t}^2\hat{u}^2},\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{\sigma}_2 = {}& \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2\hat{u}} - \frac{1}{N^2}\frac{(2\hat{t} - \hat{u})(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}\hat{u}^2} \\ & - \frac{1}{N^2}\frac{(\hat{t} - 2\hat{u})(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^3} + \frac{1}{N^2}\frac{2\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2},\end{aligned} \tag{26}$$

$$\begin{aligned}\hat{\sigma}_3 = {}& \left(1 - \frac{1}{N^2}\right)\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3} - \left(1 - \frac{1}{N^2}\right)\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3} \\ & + \left(\frac{1}{N} - \frac{1}{N^3}\right)\frac{\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2},\end{aligned} \tag{27}$$

$$\begin{aligned}\hat{\sigma}_4 = {}& -\frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}} + \frac{\hat{s}(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^3} \\ & - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^2} + \left(\frac{1}{N} + \frac{1}{N^3}\right)\frac{\hat{s}^2(\hat{t} - \hat{u})}{\hat{t}^2\hat{u}^2}.\end{aligned} \tag{28}$$

(4) $q\bar{q} \to q'\bar{q}'$:

$$\hat{\sigma}_1 = -\frac{\hat{t}^2 - 2\hat{t}\,\hat{u} - \hat{u}^2}{\hat{s}^2\hat{t}} + \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}, \tag{29}$$

$$\hat{\sigma}_2 = -\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{t}} - \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}, \tag{30}$$

$$\hat{\sigma}_3 = \frac{1}{N^2}\frac{2(\hat{t} - \hat{u})}{\hat{s}^2}, \tag{31}$$

$$\hat{\sigma}_4 = -\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{t}} - \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}. \tag{32}$$

(5) $\bar{q}q \to q'\bar{q}'$:

$$\hat{\sigma}_1 = -\frac{\hat{t}^2 + 2\hat{t}\,\hat{u} + \hat{u}^2}{\hat{s}^2\hat{u}} - \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}, \tag{33}$$

$$\hat{\sigma}_2 = \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{u}} + \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}, \tag{34}$$

$$\hat{\sigma}_3 = \frac{1}{N^2}\frac{2(\hat{t} - \hat{u})}{\hat{s}^2}, \tag{35}$$

$$\hat{\sigma}_4 = \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{u}} + \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}}. \tag{36}$$

(6) $q\bar{q}' \to q\bar{q}'$:

$$\hat{\sigma}_1 = \frac{\hat{s}^2 - 2\hat{s}\,\hat{u} - \hat{u}^2}{\hat{s}^3} + \frac{1}{N^2}\frac{2(\hat{s}^3 + 2\hat{s}\hat{u}^3 + \hat{u}^3)}{\hat{t}^3\hat{u}}, \tag{37}$$

$$\hat{\sigma}_2 = -\frac{1}{N^2}\frac{(2\hat{s} + \hat{u})(\hat{s}^2 + \hat{u}^2)}{\hat{t}^3\hat{u}}, \tag{38}$$

$$\hat{\sigma}_3 = \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3} - \frac{1}{N^2}\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3}, \tag{39}$$

$$\hat{\sigma}_4 = \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3} + \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}}. \tag{40}$$

(7) $\bar{q}'q \to q\bar{q}'$:

$$\hat{\sigma}_1 = -\frac{\hat{s}^2 - 2\hat{s}\,\hat{t} - \hat{t}^2}{\hat{u}^3} - \frac{1}{N^2}\frac{2(\hat{s}^3 + 2\hat{s}\hat{t}^3 + \hat{t}^3)}{\hat{t}\hat{u}^3}, \tag{41}$$

$$\hat{\sigma}_2 = \frac{1}{N^2}\frac{(2\hat{s} + \hat{t})(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^3}, \tag{42}$$

$$\hat{\sigma}_3 = -\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3} + \frac{1}{N^2}\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3}, \tag{43}$$

$$\hat{\sigma}_4 = -\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^2}. \tag{44}$$

(8) $q\bar{q} \to q\bar{q}$:

$$\hat{\sigma}_1 = -\frac{\hat{t}^2 - 2\hat{t}\,\hat{u} - \hat{u}^2}{\hat{s}^2\hat{u}} + \frac{2(\hat{t}^2 + \hat{u}^2)}{N^2\hat{s}\,\hat{t}\,\hat{u}} + \frac{\hat{s}^2 - 2\hat{s}\,\hat{u} - \hat{u}^2}{\hat{s}^3} + \frac{2(\hat{s}^3 + 2\hat{s}\hat{u}^2 + \hat{u}^3)}{N^2\hat{t}^3\hat{u}} + \frac{\hat{u}(\hat{s} - \hat{t})}{N\hat{s}\hat{t}^2} - \frac{3\hat{u}(\hat{s} - \hat{t})}{N^3\hat{s}\hat{t}^2}, \quad (45)$$

$$\hat{\sigma}_2 = -\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{t}} - \frac{2(\hat{t}^2 + \hat{u}^2)}{N^2\hat{s}\,\hat{t}\,\hat{u}} - \frac{(2\hat{s} + \hat{u})(\hat{s}^2 + \hat{u}^2)}{N^2\hat{t}^3\hat{u}} - \frac{1}{N}\frac{2\hat{u}}{\hat{s}\,\hat{t}} + \frac{1}{N^3}\frac{2\hat{u}}{\hat{t}^2}, \quad (46)$$

$$\hat{\sigma}_3 = \frac{1}{N^2}\frac{2(\hat{t} - \hat{u})}{\hat{s}^2} + \left(1 - \frac{1}{N^2}\right)\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3} - \frac{1}{N}\frac{\hat{u}^2}{\hat{s}\hat{t}^2} + \frac{1}{N^3}\frac{\hat{u}^2}{\hat{s}\hat{t}^2}, \quad (47)$$

$$\hat{\sigma}_4 = -\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{t}} - \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}} + \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^3} + \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}} + \left(\frac{1}{N} + \frac{1}{N^3}\right)\frac{\hat{u}(\hat{s} - \hat{t})}{\hat{s}\hat{t}^2}. \quad (48)$$

(9) $\bar{q}q \to q\bar{q}$:

$$\hat{\sigma}_1 = -\frac{\hat{t}^2 + 2\hat{t}\,\hat{u} - \hat{u}^2}{\hat{s}^2\hat{u}} - \frac{2(\hat{t}^2 + \hat{u}^2)}{N^2\hat{s}\,\hat{t}\,\hat{u}} - \frac{\hat{s}^2 - 2\hat{s}\,\hat{t} - \hat{t}^2}{\hat{u}^3} - \frac{2(\hat{s}^3 + 2\hat{s}\hat{t}^2 + \hat{t}^3)}{N^2\hat{t}\hat{u}^3} - \frac{\hat{t}(\hat{s} - \hat{u})}{N\hat{s}\hat{u}^2} + \frac{3\hat{t}(\hat{s} - \hat{u})}{N^3\hat{s}\hat{u}^2}, \quad (49)$$

$$\hat{\sigma}_2 = \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{u}} + \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}} + \frac{(2\hat{s} + \hat{t})(\hat{s}^2 + \hat{t}^2)}{N^2\hat{t}\hat{u}^3} + \frac{1}{N}\frac{2\hat{t}}{\hat{s}\,\hat{u}} - \frac{1}{N^3}\frac{2\hat{t}}{\hat{u}^2}, \quad (50)$$

$$\hat{\sigma}_3 = \frac{2(\hat{t} - \hat{u})}{N^2\hat{s}^2} - \frac{N^2 - 1}{N^2}\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3} + \frac{1}{N}\frac{\hat{t}^2}{\hat{s}\hat{u}^2} - \frac{1}{N^3}\frac{\hat{t}^2}{\hat{s}\hat{u}^2}, \quad (51)$$

$$\hat{\sigma}_4 = \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2\hat{u}} + \frac{1}{N^2}\frac{2(\hat{t}^2 + \hat{u}^2)}{\hat{s}\,\hat{t}\,\hat{u}} - \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^3} - \frac{1}{N^2}\frac{2(\hat{s}^2 + \hat{t}^2)}{\hat{t}\hat{u}^2} - \left(\frac{1}{N} + \frac{1}{N^3}\right)\frac{\hat{t}(\hat{s} - \hat{u})}{\hat{s}\hat{u}^2}. \quad (52)$$

(10) $qg \to qg$:

$$\hat{\sigma}_1 = -\frac{2\hat{s}^5 + 3\hat{s}^4\hat{u} - \hat{s}^3\hat{u}^2 + \hat{s}^2\hat{u}^3 - 3\hat{s}\hat{u}^4 - 2\hat{u}^5}{\hat{s}\hat{t}^3\hat{u}^2} + \frac{\hat{s}^3 + 2\hat{s}^2\hat{u} - 2\hat{s}\hat{u}^2 - \hat{u}^3}{N^2\hat{s}\,\hat{t}\,\hat{u}^2} + \frac{\hat{s}^3 - \hat{s}^2\hat{u} + \hat{s}\hat{u}^2 - \hat{u}^3}{(N^2 - 1)\hat{t}^3\hat{u}}, \quad (53)$$

$$\hat{\sigma}_2 = -\frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2\hat{u}^2} - \frac{\hat{s}^2 + \hat{u}^2}{N^2\hat{s}\,\hat{t}\,\hat{u}} - \frac{\hat{s}^3 - \hat{s}^2\hat{u} + \hat{s}\hat{u}^2 - \hat{u}^3}{(N^2 - 1)\hat{t}^3\hat{u}}, \quad (54)$$

$$\hat{\sigma}_3 = -\frac{(\hat{s}^2 + \hat{u}^2)^2}{\hat{s}\hat{t}^3\hat{u}} - \frac{1}{N^2}\frac{1}{\hat{s}}, \quad (55)$$

$$\hat{\sigma}_4 = \frac{\hat{s}^5 + \hat{s}^3\hat{u}^2 - \hat{s}^2\hat{u}^3 - \hat{u}^5}{\hat{s}\hat{t}^3\hat{u}^2} - \frac{1}{N^2}\frac{\hat{s} - \hat{u}}{\hat{t}\,\hat{u}} - \frac{1}{N^2 - 1}\frac{\hat{s}^3 - \hat{s}^2\hat{u} + \hat{s}\hat{u}^2 - \hat{u}^3}{\hat{t}^3\hat{u}}. \quad (56)$$

(11) $gq \to qg$:

$$\hat{\sigma}_1 = \frac{2\hat{s}^5 + 3\hat{s}^4\hat{t} - \hat{s}^3\hat{t}^2 + \hat{s}^2\hat{t}^3 - 3\hat{s}\hat{t}^4 - 2\hat{t}^5}{\hat{s}\hat{t}^2\hat{u}^3} - \frac{\hat{s}^3 + 2\hat{s}^2\hat{t} - 2\hat{s}\hat{t}^2 - \hat{t}^3}{N^2\hat{s}\hat{t}^2\hat{u}} - \frac{\hat{s}^3 - \hat{s}^2\hat{t} + \hat{s}\hat{t}^2 - \hat{t}^3}{(N^2 - 1)\hat{t}\hat{u}^3}, \quad (57)$$

$$\hat{\sigma}_2 = \frac{\hat{s}(\hat{s}^2 + \hat{t}^2)}{\hat{t}^2\hat{u}^2} + \frac{1}{N^2}\frac{\hat{s}^2 + \hat{t}^2}{\hat{s}\,\hat{t}\,\hat{u}} + \frac{1}{N^2 - 1}\frac{\hat{s}^3 - \hat{s}^2\hat{t} + \hat{s}\hat{t}^2 - \hat{t}^3}{\hat{t}\hat{u}^3}, \quad (58)$$

$$\hat{\sigma}_3 = \frac{(\hat{s}^2 + \hat{t}^2)^2}{\hat{s}\,\hat{t}\,\hat{u}^3} + \frac{1}{N^2}\frac{1}{\hat{s}}, \quad (59)$$

$$\hat{\sigma}_4 = -\frac{\hat{s}^5 + \hat{s}^3\hat{t}^2 - \hat{s}^2\hat{t}^3 - \hat{t}^5}{\hat{s}\hat{t}^2\hat{u}^3} + \frac{1}{N^2}\frac{\hat{s} - \hat{t}}{\hat{t}\,\hat{u}} + \frac{1}{N^2 - 1}\frac{\hat{s}^3 - \hat{s}^2\hat{t} + \hat{s}\hat{t}^2 - \hat{t}^3}{\hat{t}\hat{u}^3}. \quad (60)$$

(12) $gg \to q\bar{q}$:

$$\hat{\sigma}_1 = -\frac{N}{N^2 - 1}\frac{(\hat{t} - \hat{u})(2\hat{t}^4 + 5\hat{t}^3\hat{u} + 4\hat{t}^2\hat{u}^2 + 5\hat{t}\hat{u}^3 + 2\hat{u}^4)}{\hat{s}^2\hat{t}^2\hat{u}^2} + \frac{1}{N(N^2 - 1)}\frac{\hat{t}^3 + 2\hat{t}^2\hat{u} - \hat{t}\hat{u}^2 - \hat{u}^3}{\hat{t}^2\hat{u}^2} + \frac{N}{(N^2 - 1)^2}\frac{\hat{t}^3 - \hat{t}^2\hat{u} + 2\hat{t}\hat{u}^2 - \hat{u}^3}{\hat{s}^2\hat{t}\,\hat{u}}, \quad (61)$$

$$\hat{\sigma}_2 = \frac{N}{N^2 - 1}\frac{(\hat{t}^2 + \hat{u}^2)(\hat{t}^3 - \hat{u}^3)}{\hat{s}^2\hat{t}^2\hat{u}^2} - \frac{N}{(N^2 - 1)^2}\frac{\hat{t}^3 - \hat{t}^2\hat{u} + \hat{t}\hat{u}^2 - \hat{u}^3}{\hat{s}^2\hat{t}\,\hat{u}}, \quad (62)$$

$$\hat{\sigma}_3 = -\frac{1}{N(N^2 - 1)}\frac{\hat{t} - \hat{u}}{\hat{t}\,\hat{u}}, \quad (63)$$

$$\hat{\sigma}_4 = \frac{N}{N^2 - 1}\frac{\hat{t}^5 + \hat{t}^3\hat{u}^2 - \hat{t}^2\hat{u}^3 - \hat{u}^5}{\hat{s}\hat{t}^2\hat{u}^2} - \frac{1}{N^2}\frac{\hat{t} - \hat{u}}{\hat{t}\,\hat{u}} - \frac{1}{N^2 - 1}\frac{\hat{t}^3 - \hat{t}^2\hat{u} + \hat{t}\hat{u}^2 - \hat{u}^3}{\hat{s}^2\hat{t}\,\hat{u}}. \quad (64)$$

Note that for a LO calculation of the fragmentation term, more channels contribute to A_N for $pp \to \Lambda^\uparrow X$ than for $p^\uparrow p \to \pi X$ [34] due to the chiral-odd nature of the parton correlators involved in the latter case.

IV. SUMMARY

We have calculated, to leading order in perturbation theory, the twist-3 fragmentation contribution to the transverse SSA A_N for hyperon production in unpolarized proton-proton collisions. Specifically, we have taken into account all contributions arising from qq and qgq fragmentation correlators. We have verified that the result of the cross section is frame independent when taking into account relations between the twist-3 FFs which are based on the QCD equation of motion and the Lorentz invariance of the parton correlators.

In order to complete the calculation of this spin-dependent twist-3 fragmentation effect, one also needs to include $q\bar{q}g$ correlators (see Fig. 6) as well as gg and ggg correlators. We mention that the $q\bar{q}g$ graphs need the pure gluon ones in order to have a gauge-invariant subset of diagrams. (In the case of A_N for $p^\uparrow p \to \pi X$ the contribution from the former vanishes after summing over all graphs, while the latter do not contribute at all [34].) Let us finally mention that so far the properties of trigluon FFs have only been studied to some extent [52]. In particular, the LIRs in this case are not yet known. We plan to address these topics elsewhere.

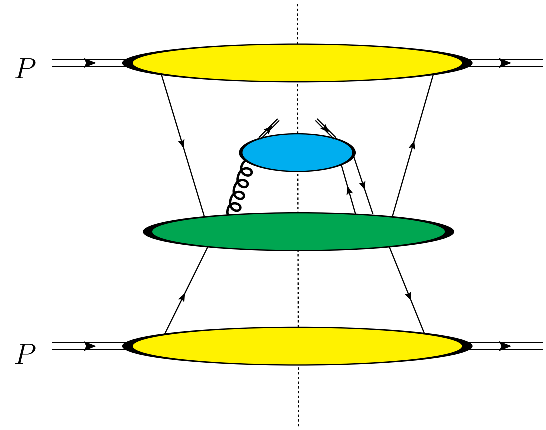

FIG. 6. Additional twist-3 fragmentation contribution to $pp \to \Lambda^\uparrow X$ which is not included in the present study.

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