PHYSICAL REVIEW D 95, 114012 (2017)

Very narrow excited Ω_c baryons

Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

[1]School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

[2]Enrico Fermi Institute and Department of Physics, University of Chicago, 5620 South Ellis Avenue, Chicago, Illinois 60637, USA

(Received 19 April 2017; published 14 June 2017)

Recently, LHCb reported the discovery of five extremely narrow excited Ω_c baryons decaying into $\Xi_c^+ K^-$. We interpret these baryons as bound states of a c quark and a P-wave ss diquark. For such a system, there are exactly five possible combinations of spin and orbital angular momentum. The narrowness of the states could be a signal that it is hard to pull apart the two s quarks in a diquark. We predict two of spin $1/2$, two of spin $3/2$, and one of spin $5/2$, all with negative parity. Of the five states, two can decay in S-wave, and three can decay in D-wave. Some of the D-wave states might be narrower than the S-wave states. We discuss the relations among the five masses expected in the quark model and the likely spin assignments, and we compare them with the data. A similar pattern is expected for negative-parity excited Ω_b states. An alternative interpretation is noted in which the heaviest two states are $2S$ excitations with $J^P = 1/2^+$ and $3/2^+$, while the lightest three are those with $J^P = 3/2^-, 3/2^-, 5/2^-$, expected to decay via D-waves. In this case, we expect $J^P = 1/2^-$ Ω_c states around 2904 and 2978 MeV.

DOI: 10.1103/PhysRevD.95.114012

I. INTRODUCTION

Very recently, LHCb reported the discovery of five extremely narrow excited Ω_c baryons decaying into $\Xi_c^+ K^-$ [1], with the masses and widths shown in Table I. We quote also our favored spin-parity assignment for these states, which we shall choose among the $5! = 120$ possible permutations if all five states are P-wave excitations of the ss diquark with respect to the charmed quark. Some more recent calculations [2–6] reach the same conclusion. In parentheses, we note an alternative assignment if the two heaviest states are $2S$ excitations.

This discovery raises some immediate questions, which we address in detail:

(i) Why five states? Are there more in this css system?
(ii) Why are they so narrow?
(iii) What are their spin-parity assignments?
(iv) Can one understand the mass pattern?
(v) Are there other similar states with different quark content—in particular, very narrow excited Ω_b baryons?

In Sec. II, we comment on P-wave css baryons. We then analyze spin-dependent forces for the css system in Sec. III, building upon similar results [7] obtained previously for the negative-parity Σ_c states. We evaluate the energy cost for a P-wave css excitation in Sec. IV, carry our results over to the Ω_b system in Sec. V, discuss alternative interpretations of the spectrum in Sec. VI, and conclude in Sec. VII. Details of calculating the spin-dependent mass shifts are presented in Appendix A, with a linearized approximation in Appendix B.

II. P-WAVE $c(ss)$ SYSTEM

Consider the (ss) in $c(ss)$ to be an S-wave color $\bar{\mathbf{3}}_c$ diquark. Then it must have spin $S_{ss} = 1$. This spin can be combined with the spin $1/2$ of the charm quark c for a total spin $S = 1/2$ or $3/2$. Consider states with relative orbital angular momentum $L = 1$ between the spin-1 diquark and the charm quark. Combining $L = 1$ with $S = 1/2$, we get states with total spin $J = 1/2, 3/2$, while combining $L = 1$ with $S = 3/2$ produces states with $J = 1/2, 3/2, 5/2$. All five states have negative parity P. Those with $J^P = 1/2^-$ decay to $\Xi_c^+ K^-$ in an S-wave, while those with $J^P = 3/2^-$, $5/2^-$ decay to $\Xi_c^+ K^-$ in a D-wave.

TABLE I. Masses and widths of $\Omega_c = css$ candidates reported by the LHCb Collaboration [1]. The proposed values of spin parity J^P are ours. An alternative set of assignments is shown in parentheses.

State	Mass (MeV)[a]	Width (MeV)	Proposed J^P
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1$	$4.5 \pm 0.6 \pm 0.3$	$1/2^-$ $(3/2^-)$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1$	$0.8 \pm 0.2 \pm 0.1$ <1.2 MeV, 95% C.L.	$1/2^-$ $(3/2^-)$
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3$	$3.5 \pm 0.4 \pm 0.2$	$3/2^-$ $(5/2^-)$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5$	$8.7 \pm 1.0 \pm 0.8$	$3/2^-$ $(1/2^+)$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9$	$1.1 \pm 0.8 \pm 0.4$ <2.6 MeV, 95% CL	$5/2^-$ $(3/2^+)$

[a]Additional common error of $+0.3$, -0.5 MeV from $M(\Xi_c^+)$ uncertainty.

*marek@proton.tau.ac.il
†rosner@hep.uchicago.edu

Two states with the same J^P could interfere with one another, but the line shapes of the resonances do not reflect significant interference effects. To test possible interference effects, LHCb added an extra phase between any pair of close peaks under the assumption they have the same quantum numbers. The effect of the interference turned out to be negligible [8].

The narrowness of the states could be a signal that it is hard to pull apart the two s quarks in a diquark. One s quark has to go into the K^-, and the other into the Ξ_c^+. It is also possible that the three states which have to decay by a D-wave are narrower than the other two. We shall find that our preferred J^P assignments only partially conform to this expectation, while an alternative assignment is consistent with it.

If indeed the narrowness of these Ω_c states is due to the difficulty of pulling apart the two quarks in an (ss) diquark, then perhaps this can also explain the narrowness of some excited ordinary Ξ baryons. The analogy is as follows:

$$\begin{array}{lcccc} & \Omega_c & \rightarrow & \Xi_c^+ & K^- \\ & c-(ss) & & (csu) & (s\bar{u}) \\ \text{Replacing } c \text{ with } u: & & & & \\ & u-(ss) & & (usu) & (s\bar{u}) \\ & \Xi^0 & \rightarrow & \Sigma^+ & K^- \end{array} \quad (1)$$

There are several excited Ξ baryons whose decay channels include $\Sigma\bar{K}$ and $\Lambda\bar{K}$, and which have quite narrow widths, even though some of them have large phase space available for the decay [9]:
$\Xi(1690)$, $\Gamma < 30$ MeV (J^P unknown),
$\Xi(1820)$, $\Gamma = 24^{+16}_{-10}$ MeV ($J^P = 3/2^-$),
$\Xi(1950)$, $\Gamma = 60 \pm 20$ MeV (J^P unknown),
$\Xi(2030)$, $\Gamma = 20^{+15}_{-5}$ MeV (J^P unknown).

The analogy is only partially correct, because in the csq system there is no analogue of the relatively light Λ which is 77 MeV lighter than Σ^0. The Λ contains a $ud\, I = 0$ spin-0 diquark which is significantly lighter than the $I = 1$ ud spin-1 diquark in Σ^0. Under $c \leftrightarrow u$, $\Lambda(sud)$ is replaced by $(scd) = \Xi_c^0$. In the latter, the corresponding spin-0 (cd) diquark has no reason to be light. Perhaps this explains why the Ω_c states are significantly more narrow than the Ξ states.

In this context, note that the *only* way for $c(ss)$ states below a certain mass to decay hadronically is to rip apart the two s quarks in an ss diquark.[1] The alternative is kinematically forbidden: if the two s quarks remain together, than the decay is $c(ss) \rightarrow q(ss)(c\bar{q})$; i.e., the final state is $\Xi D^{(*)}$. The lightest among these is $\Xi^0 D^0$ at 3180 MeV, which is 61 MeV above the heaviest of the narrow states, $\Omega_c(3119)$.

[1]Isospin-violating decay into $\Omega_c \pi^0$ is possible but highly suppressed, as discussed in Sec. VI.

III. SPIN DEPENDENCE OF MASSES

We recapitulate the discussion in Ref. [7], replacing the spin-1, isospin-1 (uu, ud, dd) diquark with a spin-1, isospin-0 doubly strange diquark (ss). We adopt the notation of Ref. [10], which has predictions for the masses of these states that we shall discuss presently. The spin-dependent potential between a heavy quark Q and the (ss) spin-1 diquark is

$$\begin{aligned} V_{SD} = {} & a_1 \boldsymbol{L} \cdot \boldsymbol{S}_{ss} + a_2 \boldsymbol{L} \cdot \boldsymbol{S}_Q \\ & + b[-\boldsymbol{S}_{ss} \cdot \boldsymbol{S}_Q + 3(\boldsymbol{S}_{ss} \cdot \boldsymbol{r})(\boldsymbol{S}_Q \cdot \boldsymbol{r})/r^2] \\ & + c \boldsymbol{S}_{ss} \cdot \boldsymbol{S}_Q, \end{aligned} \quad (2)$$

where the first two terms are spin-orbit forces, the third is a tensor force, and the last describes hyperfine splitting. If $a_1 = a_2$, the spin-orbit force becomes proportional to $\boldsymbol{L} \cdot (\boldsymbol{S}_{ss} + \boldsymbol{S}_Q) = \boldsymbol{L} \cdot \boldsymbol{S}$, where $\boldsymbol{S}$ is the total spin, so states may be classified as $^{2S+1}P_J = {}^2P_{1/2}$, $^2P_{3/2}$, $^4P_{1/2}$, $^4P_{3/2}$, and $^4P_{5/2}$. When $a_1 \neq a_2$, the states with the same J but different S mix with one another and are eigenstates of 2×2 matrices, involving a correction to Ref. [7] (see also Ref. [11]). Details of this calculation are given in Appendix A.

$$\Delta\mathcal{M}_{1/2} = \begin{bmatrix} \frac{1}{3}a_2 - \frac{4}{3}a_1 & \frac{\sqrt{2}}{3}(a_2 - a_1) \\ \frac{\sqrt{2}}{3}(a_2 - a_1) & -\frac{5}{3}a_1 - \frac{5}{6}a_2 \end{bmatrix} + b \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad (3)$$

$$\Delta\mathcal{M}_{3/2} = \begin{bmatrix} \frac{2}{3}a_1 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) \\ \frac{\sqrt{5}}{3}(a_2 - a_1) & -\frac{2}{3}a_1 - \frac{1}{3}a_2 \end{bmatrix} + b \begin{bmatrix} 0 & -\sqrt{5}/10 \\ -\sqrt{5}/10 & \frac{4}{5} \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad (4)$$

$$\Delta\mathcal{M}_{5/2} = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c. \quad (5)$$

The spin-weighted sum of these mass shifts is zero:

$$\sum_J (2J+1)\Delta\mathcal{M}_J = 0, \quad (6)$$

implying one linear relation among the mass shifts. For any given assignment of the five states to two values of $J^P = 1/2^-$, two of $J^P = 3/2^-$, and one of $J^P = 5/2$, there should, in principle, exist exactly one solution for the four parameters a_1, a_2, b, and c. In practice, as we discuss below, only one solution in which all states are P-waves gives reasonable values of these parameters, and it is the one shown in Table I.

Although m_c is not much larger than the (ss) diquark mass (which we shall evaluate presently), it will be helpful to quote a linearized version of the mixing using lowest-order perturbation theory in the inverse of m_c [7]. For this purpose one replaces the (ss) diquark spin $S_{(ss)} = 1$ and the orbital angular momentum $L = 1$ to a light-quark total angular momentum $j = 0$, 1, 2. The states with definite J, j can be expressed in terms of those with definite J, S via Clebsch-Gordan coefficients. Details are given in Appendix B. Expanding in definite-j eigenfunctions of the $\boldsymbol{L}\cdot\boldsymbol{S}_{ss}$ term, the result is

$$\Delta M\left(J=\frac{1}{2}, j=0\right) = -2a_1, \tag{7}$$

$$\Delta M\left(J=\frac{1}{2}, j=1\right) = -a_1 - \frac{1}{2}a_2 - b - \frac{1}{2}c, \tag{8}$$

$$\Delta M\left(J=\frac{3}{2}, j=1\right) = -a_1 + \frac{1}{4}a_2 + \frac{1}{2}b + \frac{1}{4}c, \tag{9}$$

$$\Delta M\left(J=\frac{3}{2}, j=2\right) = a_1 - \frac{3}{4}a_2 + \frac{3}{10}b - \frac{3}{4}c, \tag{10}$$

$$\Delta M\left(J=\frac{5}{2}, j=2\right) = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c. \tag{11}$$

This expresses five mass shifts in terms of four parameters. One linear relation among them is the vanishing of their spin-weighted sum, as before. But here, a_2 and c always occur in the combination $a_2 + c$, so that the five mass shifts are expressed in terms of the three free parameters a_1, $a_2 + c$, and b. Hence, the mass shifts satisfy one additional linear relation, which is convenient to write as two separate ones:

$$2\Delta M(1/2,1) + 4\Delta M(3/2,1) = 3\Delta M(1/2,0), \tag{12}$$

$$4\Delta M(3/2,2) + 6\Delta M(5/2,2) = -5\Delta M(1/2,0). \tag{13}$$

Here the first number refers to J and the second to j. These two relations imply Eq. (6). They are not well satisfied by our favored assignment, implying a shortcoming of the $1/m_c$ expansion for such a heavy "light diquark" (ss). We shall label states by their total J and their heavy quark limit j, even when mixed [Eqs. (3) and (4)].

An initial effort to assign J^P values to the five states made use of the linearized equations (7)–(11). With a_1 and a_2 extrapolated from Ref. [7], it was shown that $M(1/2,0) < M(1/2,1) < M(3/2,2) < M(5/2,2)$ for all reasonable values of the tensor-force parameter b, while $M(3/2,1)$ could lie below all, three, or two of the above four. Although the pattern should be somewhat different for the css system, this greatly simplified the search for a reasonable permutation of J^P assignments. The criteria for "reasonable" included the following:

(i) The hyperfine splitting parameter c should be small, as it depends on a P-wave wave function near the origin.

(ii) The parameter a_2 should be close to that estimated in Ref. [7] from the Λ_c system, $a_2 = 23.9$ MeV, as it refers to the matrix element of a term $\boldsymbol{L}\cdot\boldsymbol{S}_Q$.

(iii) The parameter a_1 should be positive but smaller than the value of 55.1 MeV estimated in Ref. [7] as the coefficient of the $\boldsymbol{L}\cdot\boldsymbol{S}_{(uu)}$ term. Naive scaling by the ratio of diquark masses would yield for the Ω_c system $a_1 = (783/1095)\cdot 55.1 = 39.4$ MeV, where the (uu) diquark mass was evaluated in Ref. [7], and the (ss) diquark mass is evaluated in the next section.

With these criteria, all $5! = 120$ *a priori* possible assignments of P-wave states were examined. The assignment in Table I was favored, corresponding to the parameter choices

$$\boxed{a_1 = 26.95\ \text{MeV},\quad a_2 = 25.74\ \text{MeV},\quad b = 13.52\ \text{MeV},\quad c = 4.07\ \text{MeV}.} \tag{14}$$

This assignment of spins and parities is superposed on the LHCb $M(\Xi_c^+ K^-)$ spectrum [1] in Fig. 1. With the assignment in Table I, the spin-averaged mass is

$$\bar{M} = (1/18)\sum_J (2J+1)M(J) = 3079.94\ \text{MeV}. \tag{15}$$

However, the sum rules (12) and (13) are poorly obeyed, showing the shortcoming of the linear approximation for the $c(ss)$ system.

One other plausible assignment consists of interchanging the states at 3050 and 3066 MeV, giving rise to a parameter set

$$a_1 = 21.40\ \text{MeV},\qquad a_2 = 40.75\ \text{MeV},$$
$$b = 5.67\ \text{MeV},\qquad c = 0.45\ \text{MeV}. \tag{16}$$

Here a_1 and a_2 are both farther from the expected values. One additional possibility involves the identification of $M(1/2,0)$, $M(1/2,1)$, $M(3/2,1)$, $M(3/2,2)$, $M(5/2,2)$ with the respective states at 3000, 3050, 3066, 3119, and 3090 MeV. This gives rise to a parameter set

$$a_1 = 21.51\ \text{MeV},\qquad a_2 = -2.81\ \text{MeV},$$
$$b = 38.42\ \text{MeV},\qquad c = 2.30\ \text{MeV}, \tag{17}$$

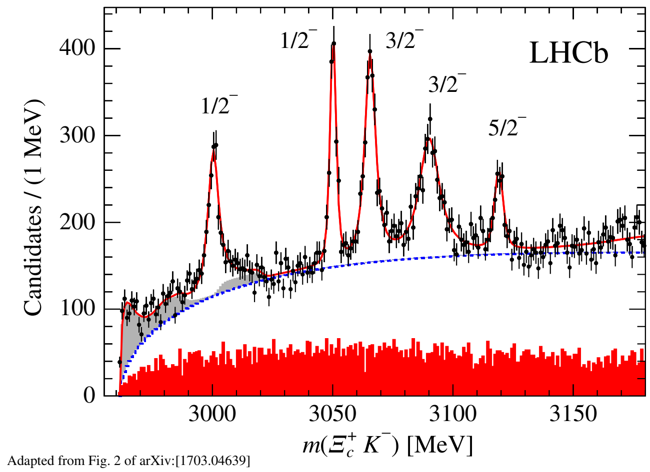

FIG. 1. Proposed assignment of spins and parities of excited $\Omega_c = c(ss)$ states observed by the LHCb Collaboration if all five are P-wave excitations of the (ss) diquark with respect to the charmed quark. Adapted from a zoom-in on Fig. 2 of Ref. [1].

with a_2 very far from expectations. All the other 5! permutations lead to no solution or to ones with negative (unacceptable) signs of a_1 and a_2.

One might have speculated that the $J = 3/2$ and $J = 5/2$ states, decaying to $\Xi_c^+ K^-$ via a D-wave, would be narrower that those with $J = 1/2$. With our assignments, the state at 3050 MeV, assigned by us to $J = 1/2$, is seemingly the narrowest of all. But given the large statistical error in the width of $\Omega(3119)$, LHCb cannot currently rule out the possibility that $\Omega(3119)$ is narrower than $\Omega(3050)$ [8]. No permutation of assignments which assigns the widest states, those at 3000 and 3090 MeV, to $J = 1/2$ leads to an acceptable set of parameters. Hence, some other source of suppression of the width of the state at 3050 MeV must be found if it really has $J = 1/2$.

IV. ENERGY COST OF A *P*-WAVE EXCITATION OF THE (ss) DIQUARK RELATIVE TO *c*

The experimental S-P splitting, given the preferred spin assignments in Table I, is calculated from the mean $c(ss)$ P-wave mass, $\bar{M} = 3079.94$ MeV, minus the spin-weighted average of the S-wave masses [9]:

$$(1/3)[M(\Omega_c) + 2M(\Omega_c^*)] = (1/3)[(2695.2) + 2(2765.9)] = 2742.33\ \text{MeV}, \tag{18}$$

or $\Delta E_{PS}(\Omega_c) = 337.6$ MeV. One may ask if this is a reasonable value.

In Ref. [7], the corresponding splitting for Σ_c states was estimated in Table III to be 290.7 MeV. The reduced mass in the $c(uu)$ system, where (uu) denotes the nonstrange spin-1, isospin-1 diquark, was found to be 536.8 MeV. The S-P splitting is expected to be a monotonically decreasing function of reduced mass. For an (ss) diquark, using parameters from Table I of Ref. [12], one calculates the mass of the (ss) diquark to be $M_{(ss)} = 2m_s^b + a/(m_s^b)^2 = 2\cdot 536.3 + 49.3\cdot(363.7/536.3)^2 = 1095$ MeV, and hence the reduced mass (using $m_c = 1709$ MeV) is 667 MeV. Using Fig. 1 of Ref. [7], one would then estimate $\Delta E_{PS}(\Omega_c) \simeq 240$ MeV, or nearly 100 MeV below the observed value. If that were the case, at least some of the states we predict would not correspond to the five observed by LHCb, but would lie below the $\Xi_c^+ K^-$ threshold. There is, in fact, some hint in the LHCb data, just near the threshold, of some activity exceeding phase space [1].[2]

A value of $\Delta E_{PS}(\Omega_c)$ larger than 240 MeV is estimated by comparison with the observed S-P splitting in the Ξ_c states. The light S-wave, color 3^* diquarks sq can exist in both the flavor-antisymmetric spin-0 state $[sq]$ and the flavor-symmetric spin-1 state (sq). This classification ignores small mixing effects due to flavor-SU(3) breaking. The S-wave positive-parity ground states and candidates for their P-wave partners are summarized in Table II.

Only three of the five expected $c(sq)$ states are firmly established, and we do not have spins for any of them, so we cannot use them to estimate the S-P splitting. However, the mass difference between the ground-state $\Xi_c = c[sq]$ at an isospin-averaged mass of 2469.4 MeV and the spin-weighted average of the $\Xi_c(2790)$ and $\Xi_c(2815)$ masses,

$$\bar{M}(c[sq], L = 1) = (1/3)\cdot(2789.1 + 2\cdot 2816.6) = 2807.4\ \text{MeV}, \tag{19}$$

is 338 MeV. The corresponding spin-0 and spin-1 sq diquark masses are

$$\begin{aligned} M[sq] &= m_s^b + m_q^b - \frac{3a}{m_s^b m_q^b} \\ &= 536.3 + 363.7 - 3(49.3)(363.7/536.3) \\ &= 800\ \text{MeV}, \\ M(sq) &= m_s^b + m_q^b + \frac{a}{m_s^b m_q^b} = 933\ \text{MeV}, \end{aligned} \tag{20}$$

implying a reduced mass of 545 MeV for $c[sq]$. According to Fig. 1 of Ref. [7], this would lead to the prediction $\Delta E_{PS} \simeq 280$ MeV, nearly 60 MeV below the observed value. So it is quite possible that S-P splittings for baryons with one heavy quark and at least one strange quark have been underestimated using the method of Ref. [7].

Some recent data from Belle [13,14] on excited Ξ_c states may help the identification of the spins and parities of the last three states listed in Table II. The width of the state at

[2]LHCb currently interpret the threshold enhancement as a feed-down from $\Omega_c(3066) \to \Xi_c' K \to \Xi_c \gamma K$ with the γ not reconstructed, but alternative interpretations, such as additional states, are not ruled out [8].

TABLE II. Lowest-lying Ξ_c states classified in Ref. [9] with three stars (***).

State	Mass (MeV) (PDG fit or average)	Light diquark	Candidate J^P
Ξ_c^+	$2467.93^{+0.28}_{-0.40}$	$[sq]$	$1/2^+$
Ξ_c^0	$2470.85^{+0.28}_{-0.40}$	$[sq]$	$1/2^+$
Average	2469.4	$[sq]$	$1/2^+$
$(\Xi')_c^+$	2575.7 ± 3.0	(sq)	$1/2^+$
$(\Xi')_c^0$	2577.9 ± 2.9	(sq)	$1/2^+$
Average	2576.8	(sq)	$1/2^+$
Ξ_c^*	2645.9 ± 0.5	(sq)	$3/2^+$
$\Xi_c(2790)$	2789.1 ± 3.2	$[sq]$	$1/2^-$
$\Xi_c(2815)$	2816.6 ± 0.9	$[sq]$	$3/2^-$
$\Xi_c(2970)$[a]	2970.2 ± 2.2	(sq)	$?^-$
$\Xi_c(3055)$[a]	3055.1 ± 1.7	(sq)	$?^-$
$\Xi_c(3080)$[a]	3076.94 ± 0.28	(sq)	$?^-$

[a]The parity of these states is not yet verified experimentally. In addition, PDG quotes single-* Ξ_c candidates at $2931 \pm 3 \pm 5$ and $3122.9 \pm 1.3 \pm 0.3$ MeV, which could account for the remaining (sq) candidates.

2970 MeV is measured to be about 30 MeV, while those for the states at 3055 and 3080 MeV are seen to be about 7 MeV and less than 6.3 MeV, respectively. That suggests $J^P = 1/2^-$ for the state at 2970 MeV and two $3/2^-$ assignments, or one of $3/2^-$ and one of $5/2^-$ for the two higher-mass states.

V. PREDICTIONS FOR $\Omega_b = b(ss)$ STATES

The proposed identification of the five LHCb excited Ω_c states allows us to speculate upon the properties of a similar system consisting of a b quark and a spin-1 (ss) diquark. Here the large mass of the b quark implies that the linear approximation to the masses in Eqs. (7)–(11) should be much better, so we shall use it with the following inputs:

(i) The hyperfine parameter c is set to zero.

(ii) The parameter a_1 is kept as in the $c(ss)$ system, as it expresses the coefficient of $\boldsymbol{L}\cdot\boldsymbol{S}_{(ss)}$: $a_1[b(ss)] = a_1[c(ss)] = 26.95$ MeV.

(iii) The parameter a_2 is rescaled by the ratio of heavy-quark masses, $a_2[b(ss)] = (1708.8/5041.8)(25.74) = 8.72$ MeV, where we have taken the charm- and bottom-quark masses from Ref. [7].

(iv) The parameter b is taken to have a range of ± 20 MeV around zero, as in Ref. [7].

(v) The S-P splitting is taken as unknown, given that the reduced mass of the $b(ss)$ system, about 900 MeV, is outside the range for which we feel comfortable making an estimate. It should be *roughly* of the order of 300 MeV.

These assumptions lead to the following mass shifts $\Delta M(J,j)$ in MeV (see Fig. 2):

$$\Delta M(1/2,0) = -53.9, \tag{21}$$

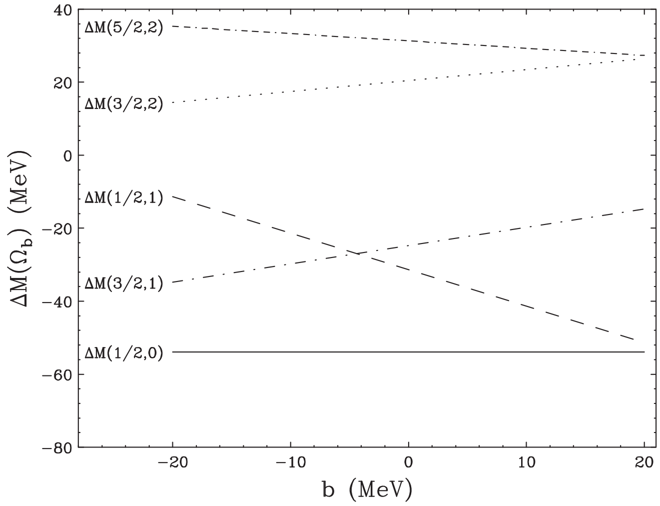

FIG. 2. Masses of P-wave Ω_b states $b(ss)$ as functions of tensor force parameter b in the scenario with all five peaks observed by LHCb, corresponding to P-wave excitations of the (ss) diquark with respect to the charmed quark.

$$\Delta M(1/2,1) = -31.3 - b, \tag{22}$$

$$\Delta M(3/2,1) = -24.8 + \frac{1}{2}b, \tag{23}$$

$$\Delta M(3/2,2) = 20.4 + \frac{3}{10}b, \tag{24}$$

$$\Delta M(5/2,2) = 31.3 - \frac{1}{5}b. \tag{25}$$

The order of the states is similar to that for the $c(ss)$ system, with only the shift $\Delta M(3/2,1)$ in indeterminate position with regard to the shifts $\Delta M(1/2,0)$ and $\Delta M(1/2,1)$. As found in Ref. [7] for the P-wave Σ_b states, for moderate b there is a clear separation between the three lowest masses with $j = 0$, 1 and the two highest with $j = 2$.

The Ω_b^* $(J^P = 3/2^+)$ partner of $\Omega_b(6046.4 \pm 1.9)$ should have a mass about $(m_c/m_b)\Delta M(\Omega_c) \simeq (1/3)(71\ \text{MeV}) \simeq 24$ MeV above Ω_b, so the spin-weighted average S-wave mass is about 6062 MeV. The spin-weighted average of the five $b(ss)$ states should then be about 6362 MeV $+ \Delta E_{PS}(\Omega_b) - 300$ MeV.

VI. ALTERNATIVE INTERPRETATIONS

Predictions of $\sim 3000 \pm 40$ MeV for the negative-parity Ω_c states, and an analogous range for the Ω_b states, have been made by several authors [10,15–27]. [Reference [27] treats only $2S$ levels, identifying the states at 3066, 3119 MeV as candidates for $J^P = 1/2^+$, $3/2^+$, respectively. In addition, there have recently appeared works which also identified the five observed Ω_c states as $1P$ excitations of the (ss) diquark with respect to the charmed quark [2–6], and interpretations based on pentaquarks [28–30]]. The authors of Ref. [10] predict $M(1/2, 1/2, 3/2, 3/2, 5/2) = (3055, 2966, 3054, 3029, 3051)$ MeV for the P-wave excited Ω_c states, and (6339,6330,6340,6331, 6334) MeV for the P-wave excited Ω_b states. They, too, consider only excitations in which the (ss) diquark remains intact, with orbital angular momentum 1 with respect to the heavy quark. Most of the other excited Ω_c predictions mentioned above are clustered somewhat below the spin-weighted average based on our assignments in Table I.

The possibility thus must be considered that not all of the states reported by LHCb are P-wave excitations of the (ss) diquark with respect to the charmed quark [10,31–34]). Indeed, Ref. [10] predicts candidates for the 2S $c(ss)$ states at 3088 MeV ($J^P = 1/2^+$) and 3123 MeV ($J^P = 3/2^+$), not far from the two highest masses (3090 and 3119 MeV) reported by LHCb. This leaves the states at 3000, 3050, and 3066 MeV to be identified as three out of the five expected P-waves. Where are the other two?

One possibility is that two of the observed peaks, though they appear consistent with a single resonance, are actually composed of two, as suggested by the near degeneracies predicted in Ref. [10]. A spin-parity analysis of the LHCb data should resolve this question.

Another possibility is that one or both of the missing states are below the $\Xi_c^+ K^-$ threshold ($\simeq$2962 MeV). Such states would then be expected to decay either by an electric dipole transition to $\Omega_c\gamma$ or via isospin violation/mixing to $\Omega_c\pi^0$ [in the manner of $D_s(2317)$ decay]. The $\Omega_c\gamma$ spectrum has been studied by *BABAR* [35] and Belle [36] in reporting the existence of the Ω_c^{*0}, a candidate for the $J^P = 3/2^+$ partner of the Ω_c^0. The *BABAR* spectrum shows no peak above the Ω_c^{*0}, up to a mass of 3 GeV, while Belle only presents a spectrum up to an excitation energy of 0.2 GeV, again showing no peak besides the Ω_c^{*0}. Still, it might be interesting to examine the $\Omega_c\gamma$ and $\Omega_c\pi^0$ spectra in the forthcoming operation of Belle II.

In a specific realization of this scenario, the states at 3000, 3050, and 3066 MeV are narrow because they decay via D-waves. They then correspond to the two states with $J^P = 3/2^-$ and the one with $J^P = 5/2^-$. The two $J^P = 1/2^-$ states would be more elusive, because either they are broader or they are below the $\Xi_c^+ K^-$ threshold. To test this possibility, we choose parameters motivated by the estimates in Sec. III: $a_1 = 39.4$ MeV [item (iii)] and $a_2 = 23.9$ MeV [item (ii)]. We vary b, c, and $\bar{M}$ in a least-squares fit to the masses $M(3/2, 1) = 3000.4$ MeV, $M(3/2, 2) = 3065.6$ MeV, and $M(5/2, 2) = 3119.1$ MeV. We find $b = 27.85$ MeV, $c = -0.42$ MeV, and $\bar{M} = 3020.03$ MeV, giving rise to the predictions $M(1/2, 0) = 2904.2$ MeV, $M(1/2, 1) = 2978.0$ MeV. (We thank Nilmani Mathur for informing us that this does not seem to be a valid option in Ref. [2].) The corresponding alternative J^P assignments of the LHCb peaks are shown in Fig. 3. A fit with $M(3/2, 2)$ and $M(5/2, 2)$ interchanged gives rise to an unphysically large negative value of c; other permutations of the three negative-parity states do not result in a successful fit.

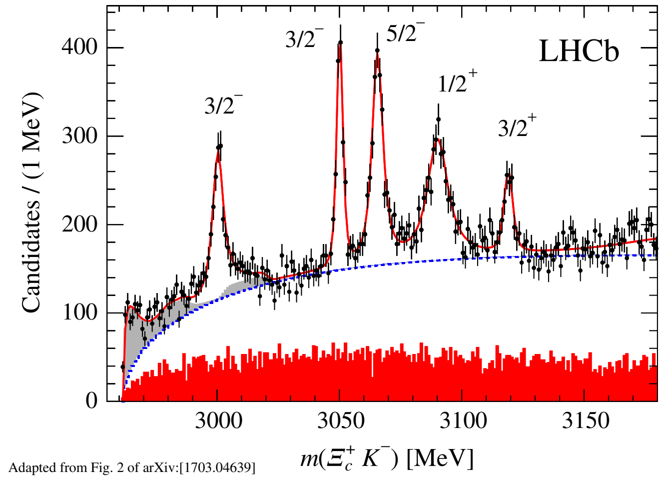

FIG. 3. Proposed assignment of spins and parities of excited $\Omega_c = c(ss)$ states observed by the LHCb Collaboration if the lowest three are P-wave excitations of the (ss) diquark with respect to the charmed quark, having $J^P = 3/2^-, 3/2^-, 5/2^-$, and the upper two are $2S$ excitations with $J^P = 1/2^+, 3/2^+$. Adapted from a zoom-in on Fig. 2 of Ref. [1].

The favored solution's spin-averaged mass $\bar{M} = 3020.03$ MeV is 278 MeV above the S-wave spin-averaged value in Eq. (18). This is considerably closer to our estimate of about 240 MeV than the value of about 338 MeV taking all five LHCb states as P-waves in Eqs. (15) and (18).

The prediction of a state around 2904 MeV should be easy to confirm or refute by studying the $\Omega_c\gamma$ and $\Omega_c\pi^0$ spectra. As mentioned, these were studied by Belle [36], but only up to a mass of about 2900 MeV, and by *BABAR* [35], with no evidence for a signal.

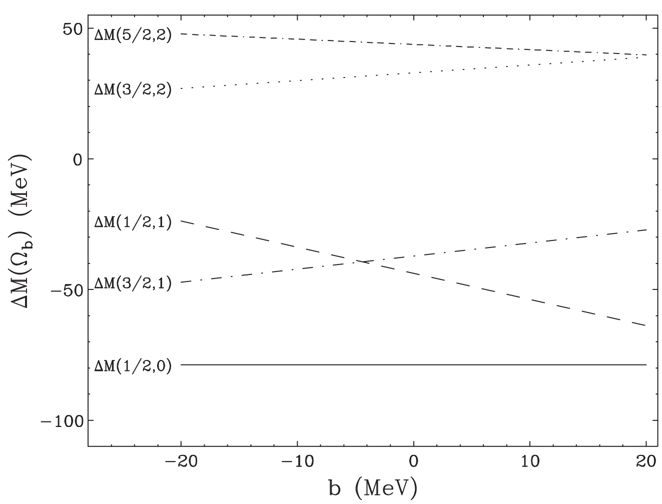

FIG. 4. Masses of P-wave Ω_b states $b(ss)$ as functions of tensor force parameter b in the scenario in which the LHCb peaks have $J^P = 3/2^-, 3/2^-, 5/2^-, 1/2^+, 3/2^+$ in ascending order of mass.

The predictions of the previous section for P-wave Ω_b mass splittings are altered in the present scenario, where we take $a_1 = 39.4$ MeV instead of 26.95 MeV [item (ii) of Sec. V]. The constants in Eqs. (21)–(25) are replaced by −78.8, −43.8, −37.2, 32.9, 43.8 MeV, respectively, with the same dependence on b. The corresponding pattern of mass shifts qualitatively resembles that of Fig. 2, with the $J^P = 5/2^-$ state and one of the $J^P = 3/2^-$ states close to one another and significantly heavier than the other three P-waves. (See Fig. 4.)

VII. CONCLUSIONS

The new excited Ω_c states observed by the LHCb Collaboration are a spectroscopist's delight because of their high significance and narrow widths, leading to well-defined and prominent signals. We have interpreted these five states in terms of the five states expected when a spin-1 (ss) diquark is excited with respect to the charm quark by one unit of orbital angular momentum. In our interpretation, the masses of the states are monotonically increasing with their total spin. This pattern remains to be confirmed. If the two highest states instead are $2S$ with $J^P = 1/2^+$ and $3/2^+$, the three lower states are likely on the basis of their narrow widths to be two with $J^P = 3/2^-$ and one with $J^P = 5/2^-$. Then two predicted $J^P = 1/2^-$ states remain to be identified, one around 2904 MeV decaying to $\Omega_c\gamma$ and/or $\Omega_c\pi^0$, and the other around 2978 MeV decaying to $\Xi_c^+ K^-$ in an S-wave. We have also provided a template for mass shifts in the corresponding $b(ss)$ system. It is not clear whether some or all of the predicted P-wave states lie below the $\Xi_b K$ threshold, in which case they may be hard to identify, requiring identification in the $\Omega_b\gamma$ or $\Omega_b\pi^0$ channel.

ACKNOWLEDGMENTS

We thank Simon Eidelman, Avraham Gal, Shmuel Nussinov, Antimo Palano, Marco Pappagallo, and Tomasz Skwarnicki for helpful comments; Nilmani Mathur for alerting us to Ref. [2]; and Richard Lebed for pointing out an error in an earlier version of this manuscript. The work of J. L. R. was supported by the U.S. Department of Energy, Division of High Energy Physics, Grant No. DE-FG02-13ER41958.

APPENDIX A: SPIN-DEPENDENT MASS SHIFTS

An error in Eq. (A.1) of Ref. [7] affects the calculation of the tensor force. The correct expression for $S_{12}/2$ [twice the contribution to Eq. (2)],

$$\frac{S_{12}}{2} \equiv \langle 6(\boldsymbol{S}_{ss}\cdot\boldsymbol{r})(\boldsymbol{S}_Q\cdot\boldsymbol{r})/r^2 - 2\boldsymbol{S}_{ss}\cdot\boldsymbol{S}_Q\rangle, \tag{A1}$$

is *not* equal to

$$\langle 3(\boldsymbol{S}\cdot\boldsymbol{r})(\boldsymbol{S}\cdot\boldsymbol{r})/r^2 - \boldsymbol{S}^2\rangle, \tag{A2}$$

because a term in the latter expression quadratic in S_{ss} does not vanish. Instead, one has

$$\frac{S_{12}}{2} = \langle 3(\boldsymbol{S}\cdot\boldsymbol{r})(\boldsymbol{S}\cdot\boldsymbol{r})/r^2 - \boldsymbol{S}^2\rangle - \mathcal{C}, \quad \text{where} \tag{A3}$$

$$\mathcal{C} = \langle 3(\boldsymbol{S}_{ss}\cdot\boldsymbol{r})(\boldsymbol{S}_{ss}\cdot\boldsymbol{r})/r^2 - \boldsymbol{S}_{ss}^2\rangle. \tag{A4}$$

We now evaluate the correction term. In Eqs. (A.4) and (A.5) of Ref. [7], we substitute $S \to S_{ss}$ and $J \to j$, with the result for $(j = 0, 1, 2)$ that $\mathcal{C} = (-2, 1, -1/5)$. We want the matrix elements of $\mathcal{C}$ between states $^{2S+1}L_J$, so we need the inverse of the Clebsch-Gordan relations (A.15)–(A.18) of Ref. [7]:

$$|^2P_{1/2}\rangle = \sqrt{1/3}|j=0\rangle + \sqrt{2/3}|j=1\rangle, \tag{A5}$$

$$|^4P_{1/2}\rangle = \sqrt{2/3}|j=0\rangle - \sqrt{1/3}|j=1\rangle, \tag{A6}$$

$$|^2P_{3/2}\rangle = \sqrt{1/6}|j=1\rangle + \sqrt{5/6}|j=2\rangle, \tag{A7}$$

$$|^4P_{3/2}\rangle = \sqrt{5/6}|j=1\rangle - \sqrt{1/6}|j=2\rangle. \tag{A8}$$

$$\langle^2P_{1/2}|\mathcal{C}|^2P_{1/2}\rangle = \frac{1}{3}(-2) + \frac{2}{3}(1) = 0, \tag{A9}$$

$$\langle^2P_{1/2}|\mathcal{C}|^4P_{1/2}\rangle = \sqrt{2}\left(-\frac{2}{3} - \frac{1}{3}\right) = -\sqrt{2}, \tag{A10}$$

$$\langle^4P_{1/2}|\mathcal{C}|^2P_{1/2}\rangle = -\sqrt{2}, \tag{A11}$$

$$\langle^4P_{1/2}|\mathcal{C}|^4P_{1/2}\rangle = \frac{2}{3}(-2) + \frac{1}{3}(1) = -1, \tag{A12}$$

$$\langle^2P_{3/2}|\mathcal{C}|^2P_{3/2}\rangle = \frac{1}{6}(1) + \frac{5}{6}\left(-\frac{1}{5}\right) = 0, \tag{A13}$$

$$\langle^2P_{3/2}|\mathcal{C}|^4P_{3/2}\rangle = \sqrt{5}\left(\frac{1}{6} + \frac{1}{30}\right) = \sqrt{5}/5, \tag{A14}$$

$$\langle^4P_{3/2}|\mathcal{C}|^2P_{3/2}\rangle = \sqrt{5}/5, \tag{A15}$$

$$\langle^4P_{3/2}|\mathcal{C}|^4P_{3/2}\rangle = \frac{5}{6}(1) - \frac{1}{30} = \frac{4}{5}. \tag{A16}$$

In addition, a correction term

$$\langle^4P_{5/2}|\mathcal{C}|^4P_{5/2}\rangle = -\frac{1}{5} \tag{A17}$$

affects the contribution of the tensor force to $\mathcal{M}_{5/2}$. The corrected mass operators are as shown in Sec. III.

For completeness, we describe here an alternative method of computing the tensor term. Denoting $\boldsymbol{n} \equiv \boldsymbol{r}/r$, we have

$$\begin{aligned}\hat{B} &\equiv \langle -\boldsymbol{S}_{ss}\cdot\boldsymbol{S}_Q + 3(\boldsymbol{S}_{ss}\cdot\boldsymbol{n})(\boldsymbol{S}_Q\cdot\boldsymbol{n})\rangle \\ &= 3\left\langle n^i S^i_{ss} n^j S^j_Q - \frac{1}{3}\delta_{ij} S^i_{ss} S^j_Q \right\rangle \\ &= 3\left\langle n^i n^j - \frac{1}{3}\delta_{ij}\right\rangle S^i_{ss} S^j_Q. \end{aligned} \tag{A18}$$

Using the formula in Ref. [11],

$$\left\langle n^i n^j - \frac{1}{3}\delta_{ij}\right\rangle = a\left[L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij} L(L+1)\right], \quad a = -1/[(2L-1)(2L+3)], \tag{A19}$$

for $L = 1$ we have

$$\left\langle n^i n^j - \frac{1}{3}\delta_{ij}\right\rangle = -\frac{1}{5}\left(L_i L_j + L_j L_i - \frac{4}{3}\delta_{ij}\right) \tag{A20}$$

so that

$$\begin{aligned}\hat{B} &= -\frac{3}{5}\left(L_i L_j + L_j L_i - \frac{4}{3}\delta_{ij}\right) S^i_{ss} S^j_Q \\ &= -\frac{3}{5}\left(L_i L_j S^i_{ss} S^j_Q + L_j L_i S^i_{ss} S^j_Q - \frac{4}{3}\boldsymbol{S}_{ss}\cdot\boldsymbol{S}_Q\right) \\ &= -\frac{3}{5}\left[(\boldsymbol{L}\cdot\boldsymbol{S}_{ss})(\boldsymbol{L}\cdot\boldsymbol{S}_Q) + (\boldsymbol{L}\cdot\boldsymbol{S}_Q)(\boldsymbol{L}\cdot\boldsymbol{S}_{ss}) - \frac{4}{3}\boldsymbol{S}_{ss}\cdot\boldsymbol{S}_Q\right], \end{aligned} \tag{A21}$$

where the last step is possible because $[\boldsymbol{L}, \boldsymbol{S}_{ss}] = [\boldsymbol{L}, \boldsymbol{S}_Q] = [\boldsymbol{S}_{ss}, \boldsymbol{S}_Q] = 0$. Next, we want to compute matrix elements of $\hat{B}$ between states of $J = 1/2$, $J = 3/2$, and $J = 5/2$. This can easily be done in terms of the known matrix elements of the three other operators, $\hat{A}^1 \equiv \boldsymbol{L}\cdot\boldsymbol{S}_{ss}$, $\hat{A}^2 \equiv \boldsymbol{L}\cdot\boldsymbol{S}_Q$, and $\hat{C} \equiv \boldsymbol{S}_{ss}\cdot\boldsymbol{S}_Q$. For example, the matrix elements of $(\boldsymbol{L}\cdot\boldsymbol{S}_{ss})(\boldsymbol{L}\cdot\boldsymbol{S}_Q)$ can be computed by inserting a complete set of states between $\boldsymbol{L}\cdot\boldsymbol{S}_{ss}$ and $\boldsymbol{L}\cdot\boldsymbol{S}_Q$:

$$\langle\alpha|(\boldsymbol{L}\cdot\boldsymbol{S}_{ss})(\boldsymbol{L}\cdot\boldsymbol{S}_Q)|\beta\rangle = \langle\alpha|(\boldsymbol{L}\cdot\boldsymbol{S}_{ss})|\gamma\rangle\langle\gamma|(\boldsymbol{L}\cdot\boldsymbol{S}_Q)|\beta\rangle. \tag{A22}$$

Then,

$$\hat{B}_J = -\frac{3}{5}\left(\hat{A}^1_J\cdot\hat{A}^2_J + \hat{A}^2_J\cdot\hat{A}^1_J - \frac{4}{3}\hat{C}_J\right). \tag{A23}$$

Explicitly,

$$\begin{aligned}\hat{B}_{1/2} &= -\frac{3}{5}\left(\begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3}\end{bmatrix}\begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6}\end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6}\end{bmatrix}\begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3}\end{bmatrix} - \frac{4}{3}\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2}\end{bmatrix}\right) \\ &= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1\end{bmatrix}, \end{aligned} \tag{A24}$$

$$\begin{aligned}\hat{B}_{3/2} &= -\frac{3}{5}\left(\begin{bmatrix} \frac{2}{3} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{2}{3}\end{bmatrix}\begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3}\end{bmatrix} + \begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3}\end{bmatrix}\begin{bmatrix} \frac{2}{3} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{2}{3}\end{bmatrix} - \frac{4}{3}\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2}\end{bmatrix}\right) \\ &= \begin{bmatrix} 0 & -\frac{\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{10} & \frac{4}{5}\end{bmatrix}, \end{aligned} \tag{A25}$$

$$\hat{B}_{5/2} = -\frac{3}{5}\left(1\cdot\frac{1}{2} + \frac{1}{2}\cdot 1 - \frac{4}{3}\cdot\frac{1}{2}\right) = -\frac{1}{5}. \tag{A26}$$

APPENDIX B: LINEARIZED APPROXIMATION

The linearized approximation for the mass shift can be derived starting from the exact expressions for $J = 1/2$, $J = 3/2$, and $J = 5/2$:

$$\Delta\mathcal{M}_{1/2} = \begin{bmatrix} \frac{1}{3}a_2 - \frac{4}{3}a_1 & \frac{\sqrt{2}}{3}(a_2 - a_1) \\ \frac{\sqrt{2}}{3}(a_2 - a_1) & -\frac{5}{3}a_1 - \frac{5}{6}a_2 \end{bmatrix} + b\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1\end{bmatrix} + c\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2}\end{bmatrix}, \tag{B1}$$

$$\Delta\mathcal{M}_{3/2} = \begin{bmatrix} \frac{2}{3}a_1 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) \\ \frac{\sqrt{5}}{3}(a_2 - a_1) & -\frac{2}{3}a_1 - \frac{1}{3}a_2 \end{bmatrix} + b\begin{bmatrix} 0 & -\sqrt{5}/10 \\ -\sqrt{5}/10 & \frac{4}{5}\end{bmatrix} + c\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2}\end{bmatrix}, \tag{B2}$$

$$\Delta\mathcal{M}_{5/2} = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c. \tag{B3}$$

States of definite J and j can be expressed as linear combinations of states with definite J and S:
for $J = 1/2$

$$|J = 1/2, j = 0\rangle = \sqrt{\frac{1}{3}}|^2P_{1/2}\rangle + \sqrt{\frac{2}{3}}|^4P_{1/2}\rangle, \tag{B4}$$

$$|J = 1/2, j = 1\rangle = \sqrt{\frac{2}{3}}|^2P_{1/2}\rangle - \sqrt{\frac{1}{3}}|^4P_{1/2}\rangle; \tag{B5}$$

for $J = 3/2$

$$|J=3/2, j=1\rangle = \sqrt{\frac{1}{6}}|^2P_{3/2}\rangle + \sqrt{\frac{5}{6}}|^4P_{3/2}\rangle, \tag{B6}$$

$$|J=3/2, j=2\rangle = \sqrt{\frac{5}{6}}|^2P_{3/2}\rangle - \sqrt{\frac{1}{6}}|^4P_{3/2}\rangle; \tag{B7}$$

and for $J = 5/2$

$$|J=5/2, j=2\rangle = |^4P_{5/2}\rangle. \tag{B8}$$

Then,

$$\Delta M\left(J=\frac{1}{2}, j=0\right) = \left\langle J=\frac{1}{2}, j=0\middle|\Delta\mathcal{M}_{1/2}\middle|J=\frac{1}{2}, j=0\right\rangle = -2a_1, \tag{B9}$$

$$\Delta M\left(J=\frac{1}{2}, j=1\right) = \left\langle J=\frac{1}{2}, j=1\middle|\Delta\mathcal{M}_{1/2}\middle|J=\frac{1}{2}, j=1\right\rangle = -a_1 - \frac{1}{2}a_2 - b - \frac{1}{2}c, \tag{B10}$$

$$\Delta M\left(J=\frac{3}{2}, j=1\right) = \left\langle J=\frac{3}{2}, j=1\middle|\Delta\mathcal{M}_{3/2}\middle|J=\frac{3}{2}, j=1\right\rangle = -a_1 + \frac{1}{4}a_2 + \frac{1}{2}b + \frac{1}{4}c, \tag{B11}$$

$$\Delta M\left(J=\frac{3}{2}, j=2\right) = \left\langle J=\frac{3}{2}, j=2\middle|\Delta\mathcal{M}_{3/2}\middle|J=\frac{3}{2}, j=2\right\rangle = a_1 - \frac{3}{4}a_2 + \frac{3}{10}b - \frac{3}{4}c, \tag{B12}$$

$$\Delta M\left(J=\frac{5}{2}, j=2\right) = \left\langle J=\frac{5}{2}, j=2\middle|\Delta\mathcal{M}_{5/2}\middle|J=\frac{5}{2}, j=2\right\rangle = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c. \tag{B13}$$

[1] R. Aaij *et al.* (LHCb Collaboration), Observation of Five New Narrow Ω_c^0 States Decaying to $\Xi_c^+K^-$, Phys. Rev. Lett. **118**, 182001 (2017).

[2] M. Padmanath and N. Mathur, Quantum numbers of recently discovered Ω_c^0 baryons from lattice QCD, arXiv:1704:00259.

[3] W. Wang and R.-L. Zhu, Interpretation of the newly observed Ω_c^0 resonances, arXiv:1704.00179.

[4] Z. G. Wang, Analysis of the $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$ with QCD sum rules, arXiv:1704.01854.

[5] B. Chen and X. Liu, Six Ω_c^0 states discovered by LHCb as new members of $1P$ and $2S$ charmed baryons, arXiv:1704.02583.

[6] T. M. Aliev, S. Bilmis, and M. Savci, Are the new excited Ω_c baryons negative parity states? arXiv:1704.03439.

[7] M. Karliner and J. L. Rosner, Prospects for observing the lowest-lying odd-parity Σ_c and Σ_b baryons, Phys. Rev. D **92**, 074026 (2015).

[8] Marco Pappagallo (private communication).

[9] C. Patrignani *et al.* (Particle Data Group), Review of particle physics, Chin. Phys. C **40**, 100001 (2016).

[10] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, Phys. Rev. D **84**, 014025 (2011).

[11] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics (Nonrelativistic Theory)*, 3rd ed. (Pergamon Press, Oxford, 1977).

[12] M. Karliner, S. Nussinov, and J. L. Rosner, $QQ\bar{Q}\,\bar{Q}$ states: Masses, production, and decays, Phys. Rev. D **95**, 034011 (2017).

[13] Y. Kato *et al.* (Belle Collaboration), Studies of charmed strange baryons in the ΛD final state at Belle, Phys. Rev. D **94**, 032002 (2016).

[14] J. Yelton *et al.* (Belle Collaboration), Study of excited Ξ_c states decaying into Ξ_c^0 and Ξ_c^+ baryons, Phys. Rev. D **94**, 052011 (2016).

[15] K. Maltman and N. Isgur, Baryons with strangeness and charm in a quark model with chromodynamics, Phys. Rev. D **22**, 1701 (1980).

[16] D. Ebert, R. N. Faustov, and V. O. Galkin, Masses of excited heavy baryons in the relativistic quark model, Phys. Lett. B **659**, 612 (2008).

[17] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A **23**, 2817 (2008).

[18] H. Garcilazo, J. Vijande, and A. Valcarce, Faddeev study of heavy baryon spectroscopy, J. Phys. G **34**, 961 (2007).

[19] S. Migura, D. Merten, B. Metsch, and H. R. Petry, Charmed baryons in a relativistic quark model, Eur. Phys. J. A **28**, 41 (2006).

[20] A. Valcarce, H. Garcilazo, and J. Vijande, Towards an understanding of heavy baryon spectroscopy, Eur. Phys. J. A **37**, 217 (2008).

[21] Y. Yamaguchi, S. Ohkoda, A. Hosaka, T. Hyodo, and S. Yasui, Heavy quark symmetry in multihadron systems, Phys. Rev. D **91**, 034034 (2015).

[22] P. Pérez-Rubio, S. Collins, and G. S. Bali, Charmed baryon spectroscopy and light flavor symmetry from lattice QCD, Phys. Rev. D **92**, 034504 (2015).

[23] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, and K. Sadato, Spectrum of heavy baryons in the quark model, Phys. Rev. D **92**, 114029 (2015).

[24] Z. Shah, K. Thakkar, A. K. Rai, and P. C. Vinodkumar, Mass spectra and Regge trajectories of Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0 baryons, Chin. Phys. C **40**, 123102 (2016).

[25] Z. Y. Wang, K. W. Wei, J. J. Qi, and X. H. Guo, Spectra of charmed and bottom baryons with hyperfine interaction, arXiv:1701.04524.

[26] Z. Zhao, D. D. Ye, and A. Zhang, Hadronic decay properties of newly observed Ω_c baryons, arXiv:1704.02688.

[27] S. S. Agaev, K. Azizi, and H. Sundu, On the nature of the newly discovered Ω_c^0 states, arXiv:1703.07091.

[28] G. Yang and J. Ping, The structure of pentaquarks Ω_c^0 in the chiral quark model, arXiv:1703.08845.

[29] H. C. Kim, M. V. Polyakov, and M. Praszałowicz, On a possibility of charmed exotica, arXiv:1704.04082.

[30] H. Huang, J. Ping, and F. Wang, Investigating the excited Ω_c^0 states through $\Xi_c K$ and $\Xi'_c K$ decay channels, arXiv:1704.01421.

[31] H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, Decay properties of P-wave charmed baryons from light-cone QCD sum rules, Phys. Rev. D **95**, 094008 (2017).

[32] K. L. Wang, L. Y. Xiao, X. H. Zhong, and Q. Zhao, Understanding the newly observed Ω_c states through their decays, arXiv:1703.09130.

[33] H. Y. Cheng and C. W. Chiang, Quantum numbers of Ω_c states and other charmed baryons, Phys. Rev. D **95**, 094018 (2017).

[34] S. S. Agaev, K. Azizi, and H. Sundu, Interpretation of the new Ω_c^0 states via their mass and width, arXiv:1704.04928.

[35] B. Aubert *et al.* (*BABAR* Collaboration), Observation of an excited charm baryon Ω_c^* decaying to $\Omega_c^0\gamma$, Phys. Rev. Lett. **97**, 232001 (2006).

[36] E. Solovieva *et al.* (Belle Collaboration), Study of Ω_c^0 and Ω_c^{*0} baryons at Belle, Phys. Lett. B **672**, 1 (2009).