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# Exploring the resonances $X(4140)$ and $X(4274)$ through their decay channels

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Investigation of the resonances $X(4140)$ and $X(4274)$, which were recently confirmed by the LHCb Collaboration [1], is carried out by treating them as the color triplet and sextet $[cs][\bar{c}\,\bar{s}]$ diquark-antidiquark states with the spin-parity $J^P = 1^+$, respectively. We calculate the masses and meson-current couplings of these tetraquarks in the context of the QCD two-point sum rule method by taking into account the quark, gluon, and mixed vacuum condensates up to eight dimensions. We also study the vertices $X(4140)J/\psi\phi$ and $X(4274)J/\psi\phi$ and evaluate corresponding strong couplings $g_{X(4140)J/\psi\phi}$ and $g_{X(4274)J/\psi\phi}$ by means of the QCD light-cone sum rule method and a technique of the soft-meson approximation. In turn, these couplings contain required information to determine the width of the $X(4140) \to J/\psi\phi$ and $X(4274) \to J/\psi\phi$ decay channels. We compare our results for the masses and decay widths of the $X(4140)$ and $X(4274)$ resonances with the LHCb data and alternative theoretical predictions.

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## I. INTRODUCTION

Recently, the LHCb Collaboration presented results of the analysis of the exclusive decays $B^+ \to J/\psi\phi K^+$ and confirmed existence of the resonances $X(4140)$ and $X(4274)$ in the $J/\psi\phi$ invariant mass distribution [1]. It also reported on the observation of the heavy resonances $X(4500)$ and $X(4700)$ in the same $J/\psi\phi$ channel. The measured masses and decay widths of these resonances (hereafter, $X(4140) \Rightarrow X_1$, $X(4274) \Rightarrow X_2$, $X(4500) \Rightarrow X_3$ and $X(4700) \Rightarrow X_4$, respectively) read

$$\begin{aligned}
X_1&: M = 4146 \pm 4.5^{+4.6}_{-2.8}\ \text{MeV}, \qquad \Gamma = 83 \pm 21^{+21}_{-14}\ \text{MeV},\\
X_2&: M = 4273 \pm 8.3^{+17.2}_{-3.6}\ \text{MeV}, \qquad \Gamma = 56 \pm 11^{+8}_{-11}\ \text{MeV},\\
X_3&: M = 4506 \pm 11^{+12}_{-15}\ \text{MeV}, \qquad \Gamma = 92 \pm 21^{+21}_{-20}\ \text{MeV},\\
X_4&: M = 4704 \pm 10^{+14}_{-24}\ \text{MeV}, \qquad \Gamma = 120 \pm 31^{+42}_{-33}\ \text{MeV}.
\end{aligned} \tag{1}$$

The LHCb determined the spin parities of these resonances, as well. It turned out that $X_1$ and $X_2$ are axial-vector states with $J^{PC} = 1^{++}$, while the quantum numbers of $X_3$ and $X_4$ are $J^{PC} = 0^{++}$.

The resonances $X_1$ and $X_2$ are old members of the XYZ family of exotic states: They were observed by the CDF Collaboration [2] in the decay processes $B^\pm \to J/\psi\phi K^\pm$ and later confirmed by the CMS [3] and D0 collaborations [4], respectively. The states $X_3$ and $X_4$ are heavier than $X_1$, $X_2$ and were found for the first time. All of the $X$ resonances may belong to a class of the hidden-charm exotic states. From production mechanisms and decay channels, it is clear that, as tetraquark candidates, they should contain the strange quark-antiquark pair $s\bar{s}$. In other words, the quark content of the $X$ states is $c\bar{c}s\bar{s}$.

The unconventional hadrons, such as glueballs, hybrid resonances, exotic four-quark systems, and pentaquarks already attracted the interest of physicists [5–12]. Besides the general theoretical problems of the multiparton states, in some of these works their parameters were calculated, as well. The $X$ resonances as the four-quark states can be treated within the diquark-antidiquark [13,14] or molecular pictures suggested to explain their internal organization. In fact, in theoretical investigations of $X_1$ and $X_2$, both of these models were used: The resonances $X_1$ and $X_2$ were considered as the meson molecules in Refs. [15–23], while in Ref. [24,25] they were treated in the framework of the diquark-antidiquark model. There are also alternative approaches analyzing them as dynamically generated resonances [26,27] or coupled-channel effects [28]. The recent comprehensive review of the various theoretical models, achieved progress, and existing problems in the physics of multiquark resonances can be found in Ref. [29].

The experimental situation, stabilized after the LHCb report, imposes new constraints on possible models of $X$ resonances. Indeed, an analysis carried out by the LHCb Collaboration in Ref. [1] on the basis of the collected experimental information ruled out an explanation of the $X_1$ as $0^{++}$ or $2^{++}$ $D_s^{*+}D_s^{*-}$ molecular states. The LHCb also emphasized that molecular bound states or cusps cannot account for $X_2$.

Therefore, in order to explain the experimental data, new models and ideas are suggested. First of all, there are traditional attempts to describe the $X$ resonances as excited states of the conventional charmonium or as dynamical effects. Indeed, by analyzing experimental information of the Belle and *BABAR* collaborations (see Refs. [30,31]) on the $B \to K\chi_{c1}\pi^+\pi^-$ and $B \to KD\bar{D}$ decays, in Ref. [32] the author identified the resonances $X_1$ and $Y(4080)$ with the P-wave excited charmonium states $\chi_{c1}(3^3P_1)$ and $\chi_{c0}(3^3P_0)$, respectively,

The contribution of the rescattering effects to the process $B^+ \to J/\psi\phi K^+$ was studied in Ref. [33] aiming to determine whether or not they can simulate the observed $X_1$, $X_2$, $X_3$, and $X_4$ resonances. It was found that the $D_s^{*+}D_s^-$ and $\psi'\phi$ rescatterings via meson loops may simulate the structures $X_1$ and $X_4$, respectively. But, description of the $X_2$ and $X_3$ states as rescattering effects seem problematic, which implies that they could be real four-quark resonances. Nevertheless, on the basis of some other arguments (for details, see Ref. [33]), the author did not exclude treating $X_2$ as the excited $\chi_{c1}(3^3P_1)$ state of the conventional charmonium.

The diquark-antidiquark and moleculelike models prevail in other pictures and form a theoretical basis for numerous calculations to account for available information on the $X$ resonances [34–38]. Thus, the masses of the axial-vector $J^P = 1^+$ diquark-antidiquark $[cs][\bar{c}\,\bar{s}]$ states with the triplet and sextet color structures were calculated in Ref. [34]. Recently, in the light of the experimental data of the LHCb Collaboration, they were interpreted as the $X_1$ and $X_2$ resonances, respectively [35]. Within the same approach, the $X_3$ and $X_4$ states were considered as the $D$-wave excitations of their light counterparts $X_1$ and $X_2$ [35].

In the context of the tetraquark models, the resonances $X_1$ and $X_2$ were studied in Refs. [36,37], as well. In accordance with Ref. [36], the light $X_1$ resonance cannot be considered as the diquark-antidiquark compact state. The similar conclusion was made in respect to $X_2$, which was examined as an octet-octet-type, moleculelike state: The mass of the $X_2$ resonance found there was in agreement with the LHCb data, but its decay width overshot considerably the experimental result [37]. The scalar resonance $X_3$ was considered as the first radial excitation of the axial-vector diquark-antidiquark $X(3915)$ state, while $X_4$ was analyzed as the ground state of the $[cs][\bar{c}\,\bar{s}]$ tetraquark built of the vector diquark and antidiquark [38]. Here some comments about $X(3915)$ are in order. It was registered by the Belle Collaboration as a resonance in the $J/\psi\omega$ invariant mass distribution at the exclusive decay $B \to J/\psi\omega K$ [39] and also seen in the reaction $\gamma\gamma \to J/\psi\omega$ [40]. This resonance was confirmed by the *BABAR* Collaboration in the same $B \to J/\psi\omega K$ process [41]. The $X(3915)$ was traditionally interpreted as the scalar $c\bar{c}$ meson $\chi_{c0}(2^3P_0)$, but a lack of its expected $\chi_{c0}(2P) \to D\bar{D}$ decay modes gave rise to other conjectures. Thus, an alternative assumption concerning the $X(3915)$ resonance was made in Ref. [42], where it was identified with the lightest scalar $[cs][\bar{c}\,\bar{s}]$ tetraquark state. Namely, this resonance was considered in Ref. [38] as the ground state of $X_3$. Calculations seem to confirm suggestions made on the nature of the $X_3$ and $X_4$ resonances [38].

An abundance of the observed charmoniumlike resonances necessitated spectroscopic analysis of the diquark-antidiquark states, which resulted in the suggestion of various multiplets to systemize the discovered tetraquarks (see Refs. [43–45]). The $X$ resonances were included in the $1S$ and $2S$ multiplets of color triplet $[cs]_{s=0,1}[\bar{c}\,\bar{s}]_{\bar{s}=0,1}$ tetraquarks [44]. Thus, $X_1$ was identified with the $J^{PC} = 1^{++}$ level of the $1S$ ground-state multiplet. The $X_2$ resonance is, supposedly, a linear superposition of two states with $J^{PC} = 0^{++}$ and $J^{PC} = 2^{++}$. This suggestion was made because, in the multiplet of the color triplet tetraquarks, only one state can bear the quantum numbers $J^{PC} = 1^{++}$. The heavy resonances $X_3$ and $X_4$ are included into the $2S$ multiplet as its $J^{PC} = 0^{++}$ members. But apart from the color triplet multiplets, there may exist a multiplet of the color sextet tetraquarks [43], which also contains a state with $J^{PC} = 1^{++}$. In other words, the multiplet of the color sextet tetraquarks doubles a number of the states with the same spin-parity [43], and the $X_2$ resonance may be identified with its $J^{PC} = 1^{++}$ member.

Even from this brief survey, it is evident that, in the context of the diquark-antidiquark model, there exist different, sometimes contradictory, suggestions concerning the internal structure of the $X$ resonances. Moreover, in almost all of these investigations, the spectroscopic parameters of newly discovered states were found by means of the QCD two-point sum rule method. Predictions of the sum rules for the parameters of the exotic states extracted by using various assumptions on the interpolating currents, within theoretical errors, are consistent with the experimental data. In most cases, results of various works are in accord with each other, as well. In other words, the static parameters of the exotic states, such as their masses and meson-current couplings are not enough to verify existing models by confronting them with experimental data and/or alternative theoretical models. The additional information useful in such cases can be gained from investigation of decay channels of the exotic states.

The QCD sum rule is the powerful nonperturbative method to explore the exclusive hadronic processes and calculate the parameters of the hadrons, including the width of their strong decays [46]. The width of the decay channels can be computed by applying either the three-point sum rule approach or the light-cone sum rule (LCSR) method [47]. The tetraquark states dominantly decay to two conventional mesons. In the present work, we will study, namely, such decay modes of the $X_1$ and $X_2$ resonances. Calculation of the couplings corresponding to strong vertices of a tetraquark and two mesons in the context of the LCSR method requires the use of additional technical tools. The reasons for a distinct treatment of the vertices with tetraquarks is very simple: Because these states are composed of four valence quarks, the light-cone expansion of the relevant nonlocal correlation function in terms of meson distribution amplitudes unavoidably reduces to expressions with local matrix elements of the same meson. As a result, conservation of the four-momentum in such a strong vertex is fulfilled only if the four-momentum of this meson is set equal to zero. The situation that emerges can be handled by invoking the soft-meson approximation [48,49]. For investigation of the diquark-antidiquark states, the soft-meson approximation was adapted in Ref. [50] and successfully applied to analyze decays some of the tetraquarks in Refs. [51–53].

In the present work, we explore the properties of the $X_1$ and $X_2$ resonances in the context of the QCD sum rule method. We are going to interpolate $X_1$ and $X_2$, as in Ref. [34], by the spin-parity $J^{PC} = 1^{++}$ currents with the antisymmetric and symmetric color structures, respectively. By accepting this scheme, we suggest that there exist two different ground-state multiplets of triplet-triplet- and sextet-sextet-type tetraquarks, and the $X_1$ and $X_2$ resonances are their members with the same $J^{PC} = 1^{++}$. The correctness of this hypothesis can be checked by computing the masses of the $X_1$ and $X_2$ states, and, more importantly, their decay widths $\Gamma(X_1 \to J/\psi\phi)$ and $\Gamma(X_2 \to J/\psi\phi)$. The masses and meson-current couplings of $X_1$ and $X_2$ will be computed by utilizing the two-point QCD sum rule approach. We will also analyze the vertices $X_1 J/\psi\phi$, $X_2 J/\psi\phi$ and calculate the strong couplings $g_{X_1 J/\psi\phi}$ and $g_{X_2 J/\psi\phi}$ by means of the light-cone sum rule method employing the soft-meson technique. The obtained results will enable us to find the widths of the $X_1 \to J/\psi\phi$ and $X_2 \to J/\psi\phi$ decays.

This work is structured in the following manner. In Sec. II, we calculate the masses and meson-current couplings of the $X_1$ and $X_2$ resonances. In Sec. III, we find the strong couplings corresponding to the vertices $X_1 J/\psi\phi$ and $X_2 J/\psi\phi$ and calculate the widths of the decay channels $X_1 \to J/\psi\phi$ and $X_2 \to J/\psi\phi$. In Sec. IV, we compare our results with LHCb data and predictions obtained in other works. It also contains our concluding remarks. The explicit expressions of the quark propagators used in sum rule calculations are moved to the Appendix.

## II. PARAMETERS OF THE $X(4140)$ AND $X(4274)$ RESONANCES

The QCD two-point sum rules for calculation of the masses and meson-current couplings of the $X_1$ and $X_2$ resonances can be obtained from analysis of the correlation function,

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq\cdot x} \langle 0|\mathcal{T}\{J_\mu(x) J_\nu^\dagger(0)\}|0\rangle, \tag{2}$$

where $J_\mu(x)$ is the interpolating current of the $X$ state with the quantum numbers $J^{PC} = 1^{++}$.

In accordance with the approach defended in Refs. [34,35], the $X_1$ and $X_2$ resonances have the same quantum numbers, but different internal color organization. We follow their assumptions and study the $X_1$ and $X_2$ states within the QCD two-point sum rule method using different interpolating currents. Namely, we suggest that the current

$$\begin{aligned} J_\mu^1 &= s_a^T C\gamma_5 c_b (\bar{s}_a \gamma_\mu C \bar{c}_b^T - \bar{s}_b \gamma_\mu C \bar{c}_a^T) \\ &\quad + s_a^T C\gamma_\mu c_b (\bar{s}_a \gamma_5 C \bar{c}_b^T - \bar{s}_b \gamma_5 C \bar{c}_a^T), \end{aligned} \tag{3}$$

which has the antisymmetric $[\bar{3}_c]_{cs} \otimes [3_c]_{\bar{c}\bar{s}}$ color structure, presumably describes the resonance $X_1$, while

$$\begin{aligned} J_\mu^2 &= s_a^T C\gamma_5 c_b (\bar{s}_a \gamma_\mu C \bar{c}_b^T + \bar{s}_b \gamma_\mu C \bar{c}_a^T) \\ &\quad + s_a^T C\gamma_\mu c_b (\bar{s}_a \gamma_5 C \bar{c}_b^T + \bar{s}_b \gamma_5 C \bar{c}_a^T), \end{aligned} \tag{4}$$

with the symmetric $[6_c]_{cs} \otimes [\bar{6}_c]_{\overline{cs}}$ color organization corresponding to the tetraquark $X_2$. In Eqs. (3) and (4), $a$ and $b$ are color indices, and $C$ is the charge conjugation matrix.

In order to derive the required sum rules, we find, as usual, the expression of the correlator in terms of the physical parameters of the $X$ state. To this end, we saturate the correlation function with a complete set of states with the quantum numbers of $X$ and perform in Eq. (2) an integration over $x$ to get

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = \frac{\langle 0|J_\mu|X(q)\rangle\langle X(q)|J_\nu^\dagger|0\rangle}{m_X^2 - q^2} + \cdots, \tag{5}$$

with $m_X$ being the mass of the $X$ state. Here, the dots indicate contributions to the correlation function arising from the higher resonances and continuum states. We introduce the meson-current coupling $f_X$ by means of the matrix element

$$\langle 0|J_\mu|X(q)\rangle = f_X m_X \varepsilon_\mu, \tag{6}$$

where $\varepsilon_\mu$ is the polarization vector of the $X$ resonance. Then, in terms of $m_X$ and $f_X$, the correlation function can be recast to the form

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = \frac{m_X^2 f_X^2}{m_X^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_X^2}\right) + \cdots. \tag{7}$$

By applying the Borel transformation to Eq. (7), we get

$$\mathcal{B}_{q^2}\Pi_{\mu\nu}^{\mathrm{Phys}}(q) = m_X^2 f_X^2 e^{-m_X^2/M^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_X^2}\right) + \cdots. \tag{8}$$

The QCD side of the sum rule has to be calculated by employing the quark-gluon degrees of freedom. For this purpose, we contract the $c$- and $s$-quark fields and find, for the correlation function $\Pi_{\mu\nu}^{\mathrm{QCD}}(q)$, the following expression (for definiteness, below we provide explicit expression for the current $J_\mu^1$):

$$\Pi_{\mu\nu}^{\rm QCD}(q) = -i\int d^4xe^{iqx}\epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}'\{{\rm Tr}[\gamma_\mu\tilde{S}_c^{n'n}(-x)\times\gamma_\nu S_s^{m'm}(-x)]{\rm Tr}[\gamma_5\tilde{S}_s^{aa'}(x)\gamma_5 S_c^{bb'}(x)]+{\rm Tr}[\gamma_\mu\tilde{S}_c^{n'n}(-x)\gamma_5 S_s^{m'm}(-x)]{\rm Tr}[\gamma_\nu\tilde{S}_s^{aa'}(x)\times\gamma_5 S_c^{bb'}(x)]+{\rm Tr}[\gamma_5\tilde{S}_c^{n'n}(-x)\gamma_\nu S_s^{m'm}(-x)]\times{\rm Tr}[\gamma_5\tilde{S}_s^{aa'}(x)\gamma_\mu S_c^{bb'}(x)]+{\rm Tr}[\gamma_5\tilde{S}_c^{n'n}(-x)\times\gamma_5 S_s^{m'm}(-x)]{\rm Tr}[\gamma_\nu\tilde{S}_s^{aa'}(x)\gamma_\mu S_c^{bb'}(x)]\}, \quad (9)$$

where $\epsilon = \epsilon^{cab}$, $\tilde{\epsilon} = \epsilon^{cmn}$ and $\epsilon' = \epsilon^{c'a'b'}$, $\tilde{\epsilon}' = \epsilon^{c'm'n'}$. In Eq. (9), $S_s^{ab}(x)$ and $S_c^{ab}(x)$ are the $s$- and $c$-quark propagators, respectively (see the Appendix). Here we also use the notation

$$\tilde{S}_{s(c)}(x) = CS_{s(c)}^T(x)C. \quad (10)$$

The QCD sum rule can be obtained by isolating the same Lorentz structures in both $\Pi_{\mu\nu}^{\rm Phys}(q)$ and $\Pi_{\mu\nu}^{\rm QCD}(q)$. We work with the terms $\sim g_{\mu\nu}$. The invariant amplitude $\Pi^{\rm QCD}(q^2)$ corresponding to this structure can be written down as the dispersion integral,

$$\Pi^{\rm QCD}(q^2) = \int_{4(m_c+m_s)^2}^{\infty}\frac{\rho^{\rm QCD}(s)}{s-q^2}ds + \cdots, \quad (11)$$

where $\rho^{\rm QCD}(s)$ is the two-point spectral density. By applying the Borel transformation to $\Pi^{\rm QCD}(q^2)$, equating the obtained expression with the relevant part of the function $\mathcal{B}_{q^2}\Pi_{\mu\nu}^{\rm Phys}(q)$, and subtracting the continuum contribution, we find the final sum rule. The mass of the $X$ state can be evaluated from the sum rule,

$$m_X^2 = \frac{\int_{4(m_c+m_s)^2}^{s_0} ds s\rho^{\rm QCD}(s)e^{-s/M^2}}{\int_{4(m_c+m_s)^2}^{s_0} ds\rho(s)e^{-s/M^2}}, \quad (12)$$

while to find the meson-current coupling $f_X$, we employ the expression

$$f_X^2 m_X^2 e^{-m_X^2/M^2} = \int_{4(m_c+m_s)^2}^{s_0} ds\rho^{\rm QCD}(s)e^{-s/M^2}. \quad (13)$$

The methods for deriving the spectral density $\rho^{\rm QCD}(s)$ were presented in the literature (see, for example, Ref. [50].) Therefore, we do not concentrate here on details of these standard and rather routine calculations.

The expressions for the mass and meson-current coupling given by Eqs. (12) and (13) contain the input parameters, the numerical values of which are collected in Table I. The sum rules depend also on the auxiliary parameters $M^2$ and $s_0$. In general, the physical quantities extracted from the sum rules should not depend on the Borel parameter and continuum threshold, but in real calculations we can only minimize their effect on the obtained results. They have also to obey the standard requirements imposed by the sum rule calculations. Thus, in the working regions of these parameters, a prevalence of the pole contribution to the sum rules and convergence of the operator product expansion (OPE) have to be satisfied. Namely, these restrictions, and the stability of the obtained predictions determine ranges within which the parameters $M^2$ and $s_0$ can be varied. Results of our analysis are collected in Table II, where we provide both the working windows for the parameters $M^2$ and $s_0$, as well as the sum rule's results for the mass and meson-current couplings of the $X(4140)$ and $X(4274)$ resonances. In the working ranges of the parameters, the pole contributions equal 23% of the whole result, which is typical for the sum rule calculations involving four-quark systems. In order to control the convergence of OPE, we evaluate the contribution arising from each term of the fixed dimension: in the ranges shown in Table II, the convergence of OPE is fulfilled: It is enough to note that the contribution of the dimension-eight term to the final result does not exceed 1% of its value.

As seen from Figs. 1 and 2, the mass and meson-current coupling of the $X(4140)$ state are sensitive to the parameters $M^2$ and $s_0$. While their effects on the mass $m_X$ are mild, the dependence of the meson-current coupling $f_X$ on the chosen values of the Borel and continuum threshold

TABLE I. Parameters used in sum rule calculations.

| Parameters | Values |
|---|---|
| $m_{J/\psi}$ | $(3096.900 \pm 0.006)$ MeV |
| $f_{J/\psi}$ | 405 MeV |
| $m_\phi$ | $(1019.461 \pm 0.019)$ MeV |
| $f_\phi$ | $215 \pm 5$ MeV |
| $m_c$ | $(1.27 \pm 0.03)$ GeV |
| $m_s$ | $96_{-4}^{+8}$ MeV |
| $\langle\bar{q}q\rangle$ | $-(0.24 \pm 0.01)^3$ GeV$^3$ |
| $\langle\bar{s}s\rangle$ | $0.8\langle\bar{q}q\rangle$ |
| $m_0^2$ | $(0.8 \pm 0.1)$ GeV$^2$ |
| $\langle\bar{s}g_s\sigma Gs\rangle$ | $m_0^2\langle\bar{s}s\rangle$ |
| $\langle\frac{\alpha_s G^2}{\pi}\rangle$ | $(0.012 \pm 0.004)$ GeV$^4$ |
| $\langle g_s^3G^3\rangle$ | $(0.57 \pm 0.29)$ GeV$^6$ |

TABLE II. The masses and meson-current couplings of the $X(4140)$ and $X(4274)$ tetraquarks.

| $X$ | $X(4140)$ | $X(4274)$ |
|---|---|---|
| $M^2$(GeV$^2$) | 4–6 | 4–6 |
| $s_0$(GeV$^2$) | 20–22 | 21–23 |
| $m_X$(MeV) | $4183 \pm 115$ | $4264 \pm 117$ |
| $f_X$(GeV$^4$) | $(0.94 \pm 0.16) \times 10^{-2}$ | $(1.51 \pm 0.21) \times 10^{-2}$ |

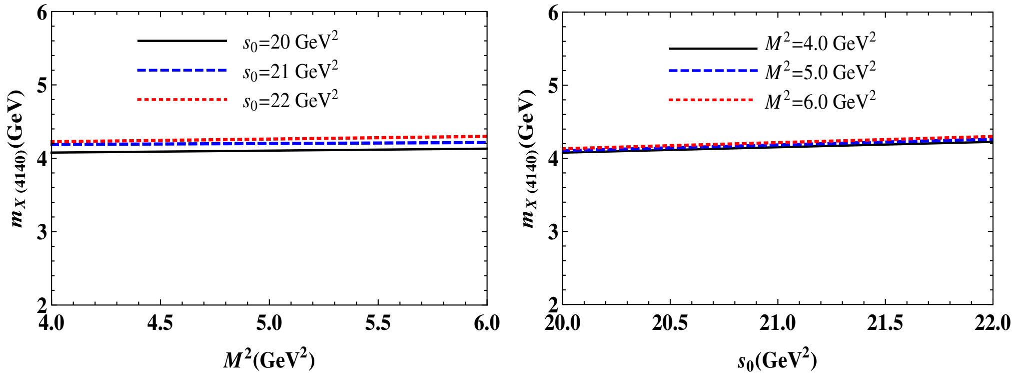


FIG. 1. The mass of the $X(4140)$ state as a function of the Borel parameter $M^2$ at fixed $s_0$ (left panel) and as a function of the continuum threshold $s_0$ at fixed $M^2$ (right panel).

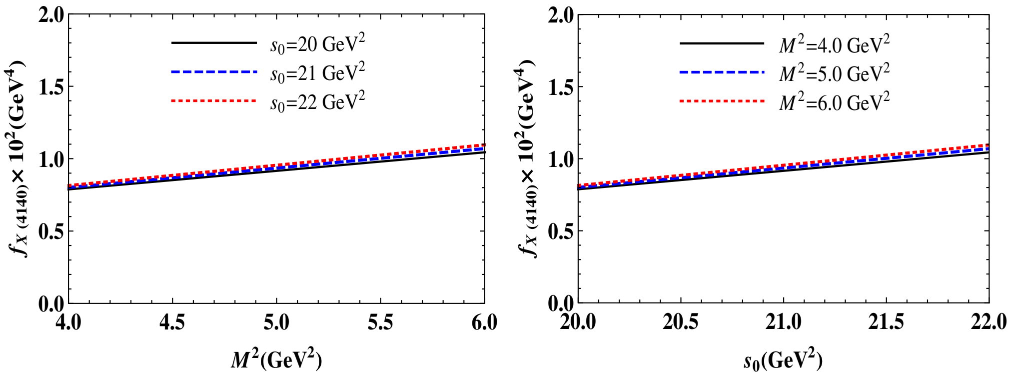


FIG. 2. The dependence of the meson-current coupling $f_X$ of the $X(4140)$ resonance on the Borel parameter at chosen values of $s_0$ (left panel), and on the $s_0$ at fixed $M^2$ (right panel).

parameters is noticeable. These effects combined with the ambiguities of the input parameters generate the theoretical errors in the sum rule calculations, which are their unavoidable feature. The errors of the calculations are also presented in Table II. Similar estimations are valid for the $X(4274)$ state, as well (see Figs. 3 and 4).

The masses of the $X(4140)$ and $X(4274)$ found in the present work are in nice agreement with the LHCb data. At this stage of our investigation, we can conclude that $X(4140)$ and $X(4274)$ are the diquark-antidiquark $J^{PC} = 1^{++}$ states of the color triplet and sextet multiplets, respectively.

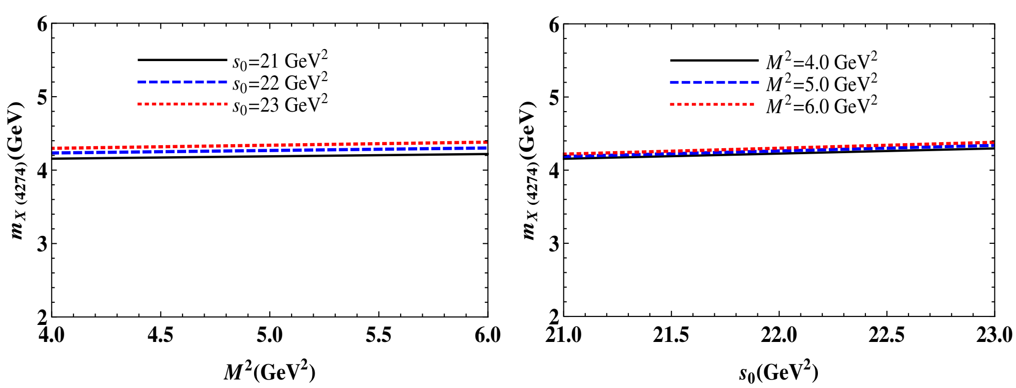


FIG. 3. The mass of the $X(4274)$ resonance as a function of the Borel parameter $M^2$ at fixed $s_0$ (left panel) and as a function of the continuum threshold $s_0$ at fixed $M^2$ (right panel).

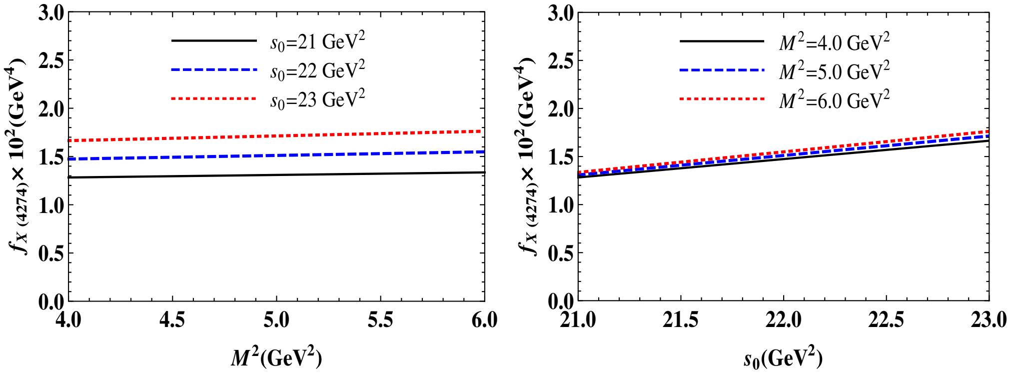


FIG. 4. The meson-current coupling $f_X$ of the $X(4140)$ resonance as a function of the Borel parameter $M^2$ at chosen values of $s_0$ (left panel) and as a function of $s_0$ at fixed $M^2$ (right panel).

## III. WIDTH OF THE $X(4140) \to J/\psi\phi$ AND $X(4274) \to J/\psi\phi$ DECAYS

The $X_1$ and $X_2$ states were observed as resonances in the $J/\psi\phi$ invariant mass distribution. Therefore, the processes $X_1 \to J/\psi\phi$ and $X_2 \to J/\psi\phi$ may be considered as their main decay channels. In this section, we are going to concentrate namely on these two decay processes. We will outline steps necessary to analyze the vertex $XJ/\psi\phi$, where $X$ is one of the $X_1$ and $X_2$ states, and calculate the strong coupling $g_{XJ/\psi\phi}$ and width of the decay $X \to J/\psi\phi$.

Within the sum rule method, the strong vertex $XJ/\psi\phi$ can be studied using the correlation function

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{ipx}\langle\phi(q)|\mathcal{T}\{J_\mu^{J/\psi}(x)J_\nu^\dagger(0)\}|0\rangle, \quad (14)$$

where $J_\nu$ and $J_\mu^{J/\psi}$ are the interpolating currents of the $X$ state and $J/\psi$ meson, respectively. The current $J_\nu$ is defined by one of the Eqs. (3) and (4), whereas $J/\psi$ has the form

$$J_\mu^{J/\psi}(x) = \bar{c}_l(x)\gamma_\mu c_l(x). \quad (15)$$

We calculate $\Pi_{\mu\nu}(p,q)$ employing the QCD sum rule on the light-cone and soft approximation. To this end, at the first stage of calculations, one has to express this function in terms of the physical quantities, namely in terms of the masses, decay constants of the involved particles, and the strong coupling $g_{XJ/\psi\phi}$ itself. For $\Pi_{\mu\nu}^{\rm Phys}(p,q)$, we get

$$\Pi_{\mu\nu}^{\rm Phys}(p,q) = \frac{\langle 0|J_\mu^{J/\psi}|J/\psi(p)\rangle}{p^2 - m_{J/\psi}^2}\langle J/\psi(p)\phi(q)|X(p')\rangle \times \frac{\langle X(p')|J_\nu^\dagger|0\rangle}{p'^2 - m_X^2} + \cdots, \quad (16)$$

where $p$, $q$ are the momenta of the $J/\psi$ and $\phi$ mesons, respectively, and by $p' = p + q$ we denote the momentum of the $X$ state.

We define the matrix element of the $J/\psi$ meson in the form

$$\langle 0|J_\mu^{J/\psi}|J/\psi(p)\rangle = f_{J/\psi}m_{J/\psi}\varepsilon_\mu(p),$$

where $m_{J/\psi}$, $f_{J/\psi}$, and $\varepsilon_\mu(p)$ are its mass, decay constant and polarization vector, respectively. We introduce also the matrix element corresponding to the vertex

$$\langle J/\psi(p)\phi(q)|X(p')\rangle = ig_{XJ/\psi\phi}\epsilon_{\alpha\beta\gamma\delta}\varepsilon_\alpha^*(p)\varepsilon_\beta(p')\varepsilon_\gamma^*(q)p_\delta. \quad (17)$$

Here $\varepsilon_\gamma^*(q)$ is the polarization vector of the $\phi$ meson. Then the contribution coming from the ground state takes the form

$$\Pi_{\mu\nu}^{\rm Phys}(p,q) = i\frac{f_{J/\psi}f_X m_{J/\psi}m_X g_{XJ/\psi\phi}}{(p'^2 - m_X^2)(p^2 - m_{J/\psi}^2)} \times \left(\epsilon_{\mu\nu\gamma\delta}\varepsilon_\gamma^*(p)p_\delta - \frac{1}{m_X^2}\epsilon_{\mu\beta\gamma\delta}\varepsilon_\gamma^*(p)p_\delta p'_\beta p'_\nu\right) + \cdots. \quad (18)$$

In the soft limit $p = p'$ (see a discussion below and Ref. [50]), and only the term $\sim i\epsilon_{\mu\nu\gamma\delta}\varepsilon_\gamma^*(p)p_\delta$ survives in Eq. (18).

In the soft-meson approximation we employ the one-variable Borel transformation on $p^2$. Then, an invariant amplitude $\Pi^{\rm Phys}(p^2)$ depends on the variable $p^2$

$$\Pi^{\rm Phys}(p^2) = \frac{f_{J/\psi}f_X m_{J/\psi}m_X g_{XJ/\psi\phi}}{(p^2 - m^2)^2}, \quad (19)$$

where $m^2 = (m_X^2 + m_{J/\psi}^2)/2$. Additionally, we apply to both sides of the sum rule the operator

$$\left(1 - M^2\frac{d}{dM^2}\right)M^2 e^{m^2/M^2}, \quad (20)$$

which eliminates effects of unsuppressed terms in $\Pi^{\rm Phys}(p^2)$ appeared in the soft limit [48,49].

The QCD expression for the correlation function $\Pi_{\mu\nu}^{\rm QCD}(p,q)$ is calculated employing the quark propagators. For the current $J_\mu^1$, it takes the following form,

$$\Pi_{\mu\nu}^{\rm QCD}(p,q) = i\int d^4x e^{ipx}\epsilon^{ijk}\epsilon^{imn} \times \{[\gamma_\nu \tilde{S}_c^{ak}(x)\gamma_\mu \tilde{S}_c^{na}(-x)\gamma_5] - [\gamma_5 \tilde{S}_c^{ak}(x)\gamma_\mu \tilde{S}_c^{na}(-x)\gamma_\nu]\}_{\alpha\beta}\langle\phi(q)|\bar{s}_\alpha^j s_\beta^m|0\rangle, \tag{21}$$

with $\alpha$ and $\beta$ being the spinor indices.

To proceed, we employ the replacement,

$$\bar{s}_\alpha^j s_\beta^m \to \frac{1}{4}\Gamma_{\beta\alpha}^k(\bar{s}^j\Gamma^k s^m), \tag{22}$$

where $\Gamma^k$ is the full set of Dirac matrices, and carry out the color summation. Then we substitute Eq. (A2) into the expression obtained after the color summation and perform four-dimensional integration over $x$. This integration leads to the appearance of the Dirac delta $\delta^4(p'-p)$ in the integrand. The correlation function does not contain the $s$-quark propagator; therefore, the argument of the Dirac delta depends only on the four-momenta of the $X$ state and $J/\psi$ meson. The next operation, i.e. an integration over $p$ or $p'$ inevitably equates $p=p'$, which is the result of the conservation of the four-momentum at the vertex $XJ/\psi\phi$. In other words, to conserve the four-momentum in the tetraquark-meson-meson vertex, one should set $q=0$, which in the full LCSR is known as the soft-meson approximation [49]. At vertices of conventional mesons, in general $q\neq 0$, and only in the soft-meson approximation does one set $q$ equal to zero, while the tetraquark-meson-meson vertex can be treated in the context of the LCSR method only if $q=0$. Nevertheless, an important observation made in Ref. [49] is that both the soft-meson approximation and full LCSR treatment of the ordinary mesons' vertices lead to very close numerical results for the strong couplings.

In the soft limit, only the matrix element,

$$\langle 0|\bar{s}(0)\gamma_\mu s(0)|\phi(p,\lambda)\rangle = f_\phi m_\phi \epsilon_\mu^{(\lambda)}, \tag{23}$$

of the $\phi$ meson contributes to the correlation function, where $m_\phi$ and $f_\phi$ are its mass and decay constant, respectively. The soft-meson limit also reduces possible Lorentz structures in $\Pi_{\mu\nu}^{\rm QCD}(p,q)$ to the term $\sim i\epsilon_{\mu\nu\gamma\delta}\varepsilon_\gamma^*(p)p_\delta$, which matches with the corresponding structure in $\Pi_{\mu\nu}^{\rm Phys}(p,q=0)$.

The relevant invariant amplitude can be written down as a dispersion integral in terms of the spectral density $\rho_c^{\rm QCD}(s)$. We omit details of the calculations and provide the final expression for $\rho_c^{\rm QCD}(s)$, which reads

$$\rho_c^{\rm QCD}(s) = \frac{f_\phi m_\phi m_c}{4}\left[\frac{\sqrt{s(s-4m_c^2)}}{\pi^2 s} + \mathrm{F}^{\rm n-pert}(s)\right]. \tag{24}$$

The nonperturbative component of $\rho_c^{\rm QCD}(s)$, i.e. $\mathrm{F}^{\rm n-pert}(s)$ is given by the following formula,

$$\mathrm{F}^{\rm n-pert}(s) = \left\langle\frac{\alpha_s G^2}{\pi}\right\rangle\int_0^1 f_1(z,s)dz + \langle g_s^3G^3\rangle \times\int_0^1 f_2(z,s)dz + \left\langle\frac{\alpha_s G^2}{\pi}\right\rangle^2\int_0^1 f_3(z,s)dz, \tag{25}$$

where the terms proportional to $\langle\alpha_s G^2/\pi\rangle$, $\langle g_s^3G^3\rangle$, and $\langle\alpha_s G^2/\pi\rangle^2$ are nonperturbative contributions to the spectral density and have four, six, and eight dimensions, respectively. The explicit form of the functions $f_1(z,s)$, $f_2(z,s)$, and $f_3(z,s)$ are

$$f_1(z,s) = \frac{1}{18r^2}\{-(2+3r(3+2r))\delta^{(1)}(s-\Phi) + (1+2r)[m_c^2 - sr]\delta^{(2)}(s-\Phi)\}, \tag{26}$$

$$f_2(z,s) = \frac{(1-2z)}{2^7\cdot 9\pi^2 r^5}\{2r[3r(1+rR)\delta^{(2)}(s-\Phi) + [3sr^2(1+r) - 2m_c^2(1+rR)]\delta^{(3)}(s-\Phi)] + [s^2r^4 - 2sm_c^2r^2(1+r) + m_c^4(1+rR)] \times\delta^{(4)}(s-\Phi)\}, \tag{27}$$

$$f_3(z,s) = \frac{m_c^2\pi^2}{2^2\cdot 3^4 r^2}[\delta^{(4)}(s-\Phi) - s\delta^{(5)}(s-\Phi)], \tag{28}$$

where we introduce the short-hand notations,

$$r = z(z-1), \qquad R = 3+r, \qquad \Phi = \frac{m_c^2}{z(1-z)}, \tag{29}$$

and $\delta^{(n)}(s-\Phi)$ is defined as

$$\delta^{(n)}(s-\Phi) = \frac{d^n}{ds^n}\delta(s-\Phi). \tag{30}$$

For the interpolating current $J_\mu^2$, we find

$$\Pi_{\mu\nu}^{\rm QCD}(p,q) = i\int d^4x e^{ipx}\{[\gamma_\nu \tilde{S}_c^{ib}(x)\gamma_\mu \tilde{S}_c^{ai}(-x)\gamma_5 - \gamma_5 \tilde{S}_c^{ib}(x)\gamma_\mu \tilde{S}_c^{ai}(-x)\gamma_\nu]_{\alpha\beta}\langle\phi(q)|\bar{s}_\alpha^a s_\beta^b|0\rangle + [\gamma_\nu \tilde{S}_c^{ib}(x)\gamma_\mu \tilde{S}_c^{bi}(-x)\gamma_5 - \gamma_5 \tilde{S}_c^{ib}(x)\gamma_\mu \tilde{S}_c^{bi}(-x)\gamma_\nu]_{\alpha\beta} \times\langle\phi(q)|\bar{s}_\alpha^a s_\beta^a|0\rangle\}. \tag{31}$$

TABLE III. The strong coupling $g_{XJ/\psi\phi}$ and decay width $\Gamma(X \to J/\psi\phi)$.

| $X$ | $X(4140)$ | $X(4274)$ |
|---|---|---|
| $M^2(\mathrm{GeV}^2)$ | 5–7 | 5–7 |
| $s_0(\mathrm{GeV}^2)$ | 20–22 | 21–23 |
| $g_{XJ/\psi\phi}$ | $2.34 \pm 0.89$ | $3.41 \pm 1.21$ |
| $\Gamma(X \to J/\psi\phi)$ (MeV) | $80 \pm 29$ | $272 \pm 81$ |

The corresponding spectral density is

$$\rho_c^{(2)\mathrm{QCD}}(s) = 2\rho_c^{(1)\mathrm{QCD}}(s), \tag{32}$$

where $\rho_c^{(1)\mathrm{QCD}}(s)$ is given by Eq. (24).

The final expression for the strong coupling $g_{XJ/\psi\phi}$ has the form

$$g_{XJ/\psi\phi} = \frac{1}{f_{J/\psi} f_X m_{J/\psi} m_X} \left(1 - M^2 \frac{d}{dM^2}\right) M^2 \times \int_{4m_c^2}^{s_0} ds e^{(m^2 - s)/M^2} \rho_c^{\mathrm{QCD}}(s). \tag{33}$$

The width of the decay $X \to J/\psi\phi$ is given by the formula

$$\Gamma(X \to J/\psi\phi) = \frac{\lambda(m_X, m_{J/\psi}, m_\phi)}{48\pi m_X^4 m_\phi^2} g_{XJ/\psi\phi}^2 [(m_X^2 + m_\phi^2) \times m_{J/\psi}^4 + (m_X^2 - m_\phi^2)^2 (m_X^2 + m_\phi^2 - 2m_{J/\psi}^2) + 4m_X^2 m_{J/\psi}^2 m_\phi^2], \tag{34}$$

where $\lambda(a, b, c)$ is the standard function:

$$\lambda(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)}}{2a}.$$

The results of the numerical computations for the strong couplings and decay widths are collected in Table III. Here we also show the working ranges for the parameters $M^2$ and $s_0$, where the predictions for the couplings $g_{X_1 J/\psi\phi}$ and $g_{X_2 J/\psi\phi}$ are obtained. Within these ranges, the sum rules satisfy all requirements typical for such kind of calculations. Indeed, the pole contribution to the sum rule on the average amounts to ~44% of the result. The convergence of OPE is fulfilled, too. Thus, the dimension-eight contribution constitutes only 1% of the sum rule.

In Fig. 5, we plot the couplings $g_{X_1 J/\psi\phi}$ and $g_{X_2 J/\psi\phi}$ as functions of the Borel parameter at fixed $s_0$. One can see that the couplings are sensitive to the choice of the auxiliary parameters $M^2$ and $s_0$. This sensitivity is a main source of the theoretical ambiguities of the performed analysis, numerical estimates of which can be found in Table III.

Comparing the theoretical predictions and LHCb data, one sees that the width of the decay $X(4140) \to J/\psi\phi$ is in accord with the experimental data, whereas $\Gamma(X(4274) \to J/\psi\phi)$ considerably exceeds and does not explain them.

## IV. DISCUSSION AND CONCLUDING REMARKS

In the present work, we have calculated the masses of the resonances $X(4140)$ and $X(4274)$ and the width of the decay channels $X(4140) \to J/\psi\phi$ and $X(4274) \to J/\psi\phi$. We have treated these resonances as the $1^{++}$ states in the multiplet of the color triplet and sextet diquark-antidiquarks, respectively. As seen from Table IV, our predictions for the masses of $X(4140)$ and $X(4274)$, obtained using the two-point QCD sum rule method, are in nice agreement with recent measurements of the LHCb Collaboration [1].

The $X(4140)$ and $X(4274)$ states were previously studied in Refs. [17,34–37]. Thus, the resonance $X(4140)$ was treated in Ref. [17] as a moleculelike bound state with $J^{PC} = 0^{++}$ built of the mesons $D_s^* \bar{D}_s^*$. The calculation of its mass, performed there using the two-point QCD sum rule method and relevant interpolating current, gives a result, which correctly describes the experimental data. Nevertheless, the LHCb Collaboration has excluded interpretation of the $X(4140)$ resonance as a moleculelike state.

As we have noted above, the masses of the $X(4140)$ and $X(4274)$ resonances in the context of the two-point sum rule method were computed also in Ref. [34]. The obtained

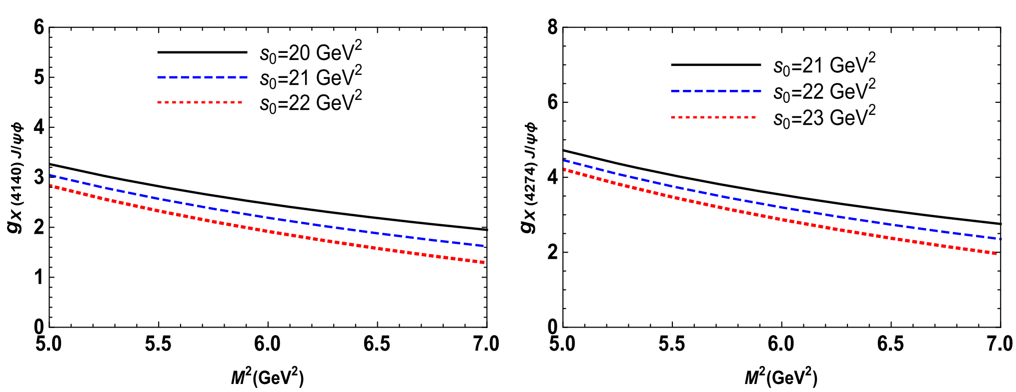


FIG. 5. The strong coupling $g_{X_1 J/\psi\phi}$ (left) and $g_{X_2 J/\psi\phi}$ (right) as functions of the Borel parameter.

TABLE IV. The LHCb data and theoretical predictions for the mass and decay width of the resonances $X(4140)$ and $X(4274)$.

| | $m_{X_1}$ (MeV) | $\Gamma_{X_1}$ (MeV) | $m_{X_2}$ (MeV) | $\Gamma_{X_2}$ (MeV) |
|---|---|---|---|---|
| LHCb | $4146 \pm 4.5^{+4.6}_{-2.8}$ | $83 \pm 21^{+21}_{-14}$ | $4273 \pm 8.3^{+17.2}_{-3.6}$ | $56 \pm 11^{+8}_{-11}$ |
| Our w. | $4183 \pm 115$ | $80 \pm 29$ | $4264 \pm 117$ | $272 \pm 81$ |
| [17] | $4140 \pm 90$ | – | – | – |
| [34] | $4070 \pm 100$ | – | $4220 \pm 100$ | – |
| [36] | $3950 \pm 90$ | – | – | – |
| | $5000 \pm 100$ | – | – | – |
| [37] | – | – | $4270 \pm 90$ | 1800 |

predictions within errors explain the LHCb data [35]. Let us note that the $X(4140)$ and $X(4274)$ resonances were treated in Refs. [34,35] as the axial-vector states with triplet and sextet color structures, respectively.

The investigations carried out in Ref. [36] using a sum rule approach and two types of interpolating currents, excluded interpretation of the $X(4140)$ resonance as a diquark-antidiquark state. The reason was that $m_{X_1}$ extracted from the corresponding sum rules either lay below the LHCb data or overshot it (see Table IV).

The $X(4274)$ was explored as a moleculelike color octet state [37], and its mass $m_{X_2}$ was estimated as

$$m_{X_2} = 4.27 \pm 0.09 \text{ GeV}. \tag{35}$$

But the width of the decay $X(4274) \to J/\psi\phi$,

$$\Gamma(X(4274) \to J/\psi\phi) = 1.8 \text{ GeV}, \tag{36}$$

evaluated in the framework of the three-point QCD sum rule approach, considerably exceeded the LHCb value; therefore, the author ruled out the suggested interpretation of the $X(4274)$ state.

We have calculated the widths of the $X(4140/4274) \to J/\psi\phi$ decays, as well. The obtained predictions are collected in Table IV. It is evident that our results for the mass and width of the $X(4140)$ resonance allow us to consider it as a serious candidate to the color triplet $J^{PC} = 1^{++}$ diquark-antidiquark state. At the same time, interpretation of $X(4274)$ as a pure color sextet tetraquark, which is, in accordance with our results, a "wide" resonance in light of the LHCb data, seems problematic: LHCb specifies it as a narrow state. Perhaps $X(4274)$ is an admixture of the color sextet tetraquark and a conventional charmonium. But this and alternative suggestions on the nature of the $X(4274)$ resonance require further investigation.

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## APPENDIX: THE S− AND C-QUARK PROPAGATORS

The light and heavy quark propagators are the important quantities for finding the QCD side of the correlation functions in both the mass and strong coupling calculations. We employ the $s$-quark propagator $S_s^{ab}(x)$, which is given by the following formula:

$$\begin{aligned} S_s^{ab}(x) &= i\delta_{ab}\frac{\not x}{2\pi^2 x^4} - \delta_{ab}\frac{m_s}{4\pi^2 x^2} - \delta_{ab}\frac{\langle \bar{s}s\rangle}{12} \\ &\quad + i\delta_{ab}\frac{\not x m_s\langle \bar{s}s\rangle}{48} - \delta_{ab}\frac{x^2}{192}\langle \bar{s}g_s\sigma G s\rangle + i\delta_{ab}\frac{x^2\not x m_s}{1152} \\ &\quad \times \langle \bar{s}g_s\sigma G s\rangle - i\frac{g_s G_{ab}^{\alpha\beta}}{32\pi^2 x^2}[\not x\sigma_{\alpha\beta} + \sigma_{\alpha\beta}\not x] \\ &\quad - i\delta_{ab}\frac{x^2\not x g_s^2\langle \bar{s}s\rangle^2}{7776} - \delta_{ab}\frac{x^4\langle \bar{s}s\rangle\langle g_s^2 G^2\rangle}{27648} + \cdots. \end{aligned} \tag{A1}$$

For the $c$-quark propagator $S_c^{ab}(x)$, we employ the well-known expression

$$\begin{aligned} S_c^{ab}(x) = i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\Bigg\{&\frac{\delta_{ab}(\not k + m_c)}{k^2 - m_c^2} \\ &-\frac{g_s G_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not k + m_c) + (\not k + m_c)\sigma_{\alpha\beta}}{(k^2 - m_c^2)^2} \\ &+\frac{g_s^2 G^2}{12}\delta_{ab} m_c\frac{k^2 + m_c\not k}{(k^2 - m_c^2)^4} + \frac{g_s^3 G^3}{48}\delta_{ab}\frac{(\not k + m_c)}{(k^2 - m_c^2)^6} \\ &\times[\not k(k^2 - 3m_c^2) + 2m_c(2k^2 - m_c^2)](\not k + m_c) + \cdots\Bigg\}. \end{aligned} \tag{A2}$$

In Eqs. (A1) and (A2), we adopt the notations

$$\begin{aligned} G_{ab}^{\alpha\beta} &= G_A^{\alpha\beta}t_{ab}^A, \qquad G^2 = G_{\alpha\beta}^A G_{\alpha\beta}^A, \\ G^3 &= f^{ABC}G_{\mu\nu}^A G_{\nu\delta}^B G_{\delta\mu}^C, \end{aligned} \tag{A3}$$

with $a$, $b = 1$, 2, 3 being the color indices, and $A, B, C = 1, 2\ldots 8$. In Eq. (A3) $t^A = \lambda^A/2$, $\lambda^A$ are the Gell-Mann matrices, and the gluon field strength tensor $G_{\alpha\beta}^A \equiv G_{\alpha\beta}^A(0)$ is fixed at $x = 0$.

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