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Hadronic weak decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+, \frac{3}{2}^+) + V$

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It is shown that for the effective Lagrangian with the factorization ansatz considered here, in the two-body hadronic decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+, \frac{3}{2}^+) + V$, with $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the representation $\bar{3}$, the only allowed decay channel is $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + V$, where $\mathcal{B}(\frac{1}{2}^+)$ belongs to the representation 8 of $SU(3)$. However, for $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the sextet representation 6, the allowed decay channels are $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+, \frac{3}{2}^+) + V$, where $\mathcal{B}(\frac{1}{2}^+)$ and $\mathcal{B}(\frac{3}{2}^+)$ belong to the octet representation $8'$ and the decuplet 10 of $SU(3)$, respectively. The decay channel $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + V$ is analyzed in detail. The decay rate (Γ) and the asymmetry parameters $\alpha, \alpha', \beta, \gamma$, and γ' are expressed in terms of four amplitudes. In particular, for the decay $\Lambda_b \to \Lambda + J/\psi$ it is shown that within the factorization framework, using heavy quark spin symmetry, the decay rate and the asymmetry parameters can be expressed in terms of two form factors F_1 and F_2/F_1, which are to be evaluated in some model. By using the values of these form factors calculated in a quark model, the branching ratio and the asymmetry parameters α and α' are calculated numerically. For other heavy quarks belonging to the triplet and sextet representations, the results can be easily obtained by using $SU(3)$ symmetry and a phase-space factor. Finally, the decay $\Omega_b^- \to \Omega^- + J/\psi$ is analyzed within the factorization framework. It is shown that the asymmetry parameter α in this particular decay is zero. The branching ratio obtained in the first approximation is compared with the experimental value.

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I. INTRODUCTION

Heavy flavor physics is of topical interest. New data for decays of b hadrons will be forthcoming from the LHCb. In 2013 the LHCb Collaboration performed an angular analysis of the decay $\Lambda_b \to \Lambda + J/\Psi$, where the Λ_b's are produced in proton-proton (pp) collisions at a center of mass energy $\sqrt{s} = 7$ TeV at the LHC [1]. By fitting several asymmetry parameters in the cascade decay distribution of $\Lambda_b \to \Lambda(\to p\pi^-) + J/\Psi(\to \ell^+\ell^-)$, the collaboration has reported the relative magnitude of helicity amplitudes in $\Lambda_b \to \Lambda + J/\Psi$ decay and also the transverse polarization of Λ_b relative to the production plane.

Theoretically, the nonleptonic decay $\Lambda_b \to \Lambda + J/\Psi$ is quite attractive because only the factorizable tree diagram contributes to the decay and there is no contribution due to W exchange diagrams [2]. In the b baryon sector, the decay $\Lambda_b \to \Lambda + J/\Psi$ has been studied theoretically in the quark model by using the factorization hypothesis [3–11] and the results of some of these calculations have been compared to the new experimental results by the LHCb Collaboration. The results of the branching fraction of $\Lambda_b \to \Lambda + J/\Psi$ decay given by the Particle Data Group, $\mathcal{B}r(\Lambda_b \to \Lambda + J/\Psi) \times \mathcal{B}r(b \to \Lambda_b^0) = (5.8 \pm 0.8) \times 10^{-5}$ [12], are deduced from the measurements by the CDF [13] and D0 collaborations [14]. The result for the branching fraction from the LHCb is still missing for this decay. In the present study, we give a general formalism for $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + V$, especially with $V = J/\psi$. Using this formalism, we analyze $\Lambda_b \to \Lambda + J/\Psi$ decay in detail.

Heavy baryons with $J^P = \frac{1}{2}^+$ belong to either representation $\bar{3}$ or the sextet 6, whereas $J^P = \frac{3}{2}^+$ belongs only to the sextet representation of the $SU(3)$ [15]:

$$\begin{aligned} \bar{3}:\ A_{ij} &= \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i) Q \chi_{MA}, \\ 6:\ S_{ij} &= \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i) Q \chi_{MS}, \\ 6:\ S^*_{ij} &= \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i) Q \chi_{S}, \end{aligned} \tag{1}$$

where q_i, q_j are u, d, s; $Q = b$ or c; and the χ's are the spin wave functions [15]. In Eq. (1) A_{ij}, S_{ij}, and S^*_{ij} correspond to $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, respectively. The triplet of heavy baryons is

$$(A_{12}, A_{13}, A_{23}):\ (\Lambda^{0,+}_{b,c}, \Xi^{0,+}_{b,c}, \Xi^{-,0}_{b,c}), \tag{2}$$

whereas the sextet is

$$(S_{11}, S_{12}, S_{22}){:}\ (\sqrt{2}\Sigma_{b,c}^{+,++}, \Sigma_{b,c}^{0,+}, \sqrt{2}\Sigma_{b,c}^{-,0}),$$
$$(S_{13}, S_{23}){:}\ (\Xi_{b,c}^{\prime 0,+}, \Xi_{b,c}^{\prime -,0}),$$
$$S_{33}{:}\ \sqrt{2}\Omega_{b,c}^{-,0}. \qquad (3)$$

In the Standard Model (SM), two-body hadronic decays of heavy flavor mesons and baryons are analyzed in terms of the effective Lagrangian or Hamiltonian. Here, we take the Hamiltonian

$$H_{\mathrm{eff}} = V_{cb}V_{cs}^{*}[a_1(\bar{s}c)_{V-A}(\bar{c}b)_{V-A} + a_2(\bar{c}c)_{V-A}(\bar{s}b)_{V-A}], \qquad (4)$$

where $a_1 = C_1 + \zeta C_2$ and $a_2 = C_2 + \zeta C_1$, with ζ being the parameter for the possible number of colors. In terms of the diagrams, a_1 and a_2 correspond to the contribution from tree and color-suppressed tree diagrams, respectively.

In the factorization ansatz, for the tree diagram and color-suppressed tree diagram, the relevant matrix elements are $\langle \mathcal{B}_c|(\bar{c}b)_{V-A}|\mathcal{B}_b\rangle$ and $\langle \mathcal{B}_s|(\bar{s}b)_{V-A}|\mathcal{B}_b\rangle$, respectively. First, one can notice that $\bar{c}b$ is a $SU(3)$ singlet, whereas $\bar{s}b$ is a $SU(3)$ triplet. Now

$$3\times\bar{3} = 8+1,$$
$$3\times 6 = 10+8'. \qquad (5)$$

Hence the possible decay modes for $\mathcal{B}_b(\frac{1}{2}^+)$ for the first term in Eq. (4) are

$$\bar{3}{:}\ (\Lambda_b^0, \Xi_b^0, \Xi_b^-) \to (\Lambda_c^+, \Xi_c^+, \Xi_c^0)(D_s^-)^*,$$
$$6{:}\ (\Sigma_b^+, \Sigma_b^0, \Sigma_b^-) \to (\Sigma_c^{++}, \Sigma_c^+, \Sigma_c^0)^*(D_s^-)^*,$$
$$(\Xi_b^{\prime 0}, \Xi_b^{\prime -}) \to (\Xi_c^{+\prime}\Xi_c^0)^*(D_s^-)^*,$$
$$\Omega_b^- \to \Omega_c^0(\Omega_c^{*0})(D_s^-)^*. \qquad (6)$$

Some of the decays given in Eq. (6) have been studied in Ref. [16]. The main focus of the present study is the heavy to light decays of b baryons.

For the color-suppressed tree diagram, as noted in Eq. (5), for $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the representation $\bar{3}$ the possible decay mode is

$$\mathcal{B}_b\left(\frac{1^+}{2}\right) \to \mathcal{B}\left(\frac{1^+}{2}\right)J/\Psi, \qquad (7)$$

where $B(\frac{1}{2}^+)$ belongs to the octet representation 8 of $SU(3)$. However, for $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the sextet representation, we have two possible decay modes:

$$\mathcal{B}_b\left(\frac{1^+}{2}\right) \to \mathcal{B}\left(\frac{1^+}{2}\right)J/\Psi,$$
$$\mathcal{B}_b\left(\frac{1^+}{2}\right) \to \mathcal{B}\left(\frac{3^+}{2}\right)J/\Psi. \qquad (8)$$

For this case, $\mathcal{B}(\frac{1}{2}^+)$ and $\mathcal{B}(\frac{3}{2}^+)$ belong to the octet representation $8'$ and decuplet representation 10 of $SU(3)$, respectively. For the decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+)J/\Psi$, the decay channels are

$$\bar{3}{:}\ (\Lambda_b, \Xi_b^0, \Xi_b^-) \to (\Lambda, \Xi^0, \Xi^-)J/\Psi,$$
$$6{:}\ (\Sigma_b^0, \Sigma_b^-, \Sigma_b^+) \to (\Sigma^0, \Sigma^-, \Sigma^+)J/\Psi,$$
$$(\Xi_b^{0\prime}, \Xi_b^{-\prime}) \to (\Xi^0, \Xi^-)J/\Psi, \qquad (9)$$

where Λ, Ξ^0, Ξ^- are members of the octet representation 8 and $\Sigma^0, \Sigma^-, \Sigma^+, \Xi^0, \Xi^-$ are members of the octet representation $8'$. This study focuses on the analysis of $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+)V$ decays.

For the decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{3}{2}^+)J/\Psi$, where $\mathcal{B}_b(\frac{1}{2}^+)$ belongs to the representation 6, the decay channels are

$$(\Sigma_b^0, \Sigma_b^-, \Sigma_b^+) \to (\Sigma^{*0}, \Sigma^{*-}, \Sigma^{*+})J/\Psi,$$
$$(\Xi_b^{0\prime}, \Xi_b^{-\prime}) \to (\Xi^{*0}, \Xi^{*-})J/\Psi,$$
$$\Omega_b^- \to \Omega^- J/\Psi, \qquad (10)$$

where the last decay is the most interesting in this category.

II. HADRONIC WEAK DECAY OF THE BARYON $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+)V$: A GENERAL FORMALISM

For the decay

$$\mathcal{B}_b\left(\frac{1^+}{2}\right)(p) \to \mathcal{B}\left(\frac{1^+}{2}\right)(p') + V(k,\epsilon), \qquad (11)$$

where $p = p' + k$ and $k\cdot\epsilon = 0$, the Lorentz structure of the T matrix is given by

$$T = \frac{1}{(2\pi)^{9/2}}\sqrt{\frac{mm'}{2p_0p_0'k_0}}\bar{u}(p')[\gamma\cdot\epsilon(A(s)+B(s)\gamma_5)$$
$$+ i\epsilon^{\mu}\sigma_{\mu\nu}k^{\nu}(C(s)+D(s)\gamma_5)]u(p). \qquad (12)$$

In Eq. (12) the amplitudes A, B, C, and D are functions of the square of the momentum transfer, i.e., $s = (p-p')^2$. In the rest frame of baryon $\mathcal{B}_b$

$$m = p_0' + k_0,$$
$$\vec{p}' = -\vec{k} = -|\vec{k}|\vec{n}. \qquad (13)$$

In this particular frame, one can write

$$T = \chi_f^\dagger M \chi_i, \tag{14}$$

where

$$M = \frac{1}{(2\pi)^{9/2}} \frac{1}{\sqrt{2k_0}} [i f_1 \vec{\sigma} \cdot (\vec{n} \times \vec{\epsilon}) + g_1 \vec{\sigma} \cdot \vec{\epsilon} + f_2 \vec{n} \cdot \vec{\epsilon} + g_2 (\vec{n} \cdot \vec{\epsilon})(\vec{\sigma} \cdot \vec{n})], \tag{15}$$

where $\vec{\sigma}$ are the Pauli matrices. The amplitudes $f_{1,2}$, $g_{1,2}$, and h can be written in terms of A, B, C, and D:

$$f_1 = \frac{|\vec{k}|}{\sqrt{2p_0'(p_0' + m')}} [A(s) - C(s)(m + m')], \tag{16}$$

$$g_1 = -\frac{1}{\sqrt{2p_0'(p_0' + m')}} \times [B(s)(p_0' + m') + D(s)(k_0(m + m') - m_V^2)], \tag{17}$$

$$f_2 = \frac{1}{\sqrt{2p_0'(p_0' + m')}} \frac{|\vec{k}|}{k_0} [A(s)(m + m') - C(s) m_V^2], \tag{18}$$

$$g_2 = \frac{1}{\sqrt{2p_0'(p_0' + m')}} \frac{|\vec{k}|^2}{k_0} [-B(s) + D(s)(m + m')], \tag{19}$$

$$h = g_1 + g_2 = \frac{-1}{\sqrt{2p_0'(p_0' + m')}} \times \frac{1}{k_0} [B(s)((m + m')k_0 - m_V^2) + D(s) m_V^2 (p_0' + m')]. \tag{20}$$

Under space reflection, $\vec{\sigma} \to \vec{\sigma}$, $\vec{n} \to -\vec{n}$, and $\vec{\epsilon} \to -\vec{\epsilon}$, and thus f_1 and f_2 are the parity-conserving, i.e., p-wave amplitudes, whereas g_1 and g_2 are the parity-violating s-wave amplitudes. We also note that for the transverse polarization of the V meson, only f_1 and g_1 are relevant, whereas for the longitudinal polarization the relevant amplitudes are f_2 and h. The decay width of the above mode is given by

$$d\Gamma = (2\pi)^7 \delta^4 (p - p' - k) \left[\frac{1}{2} \mathrm{Tr}(M M^\dagger)\right] d^3 p' d^3 k, \tag{21}$$

which gives

$$\Gamma = \frac{|\vec{k}| p_0'}{2\pi m} \left[2(|f_1)|^2 + |g_1|^2) + \frac{k_0^2}{m_V^2} (f_2)|^2 + |h|^2)\right]. \tag{22}$$

The first term on the left-hand side of Eq. (22) corresponds to the transverse polarization and the second term to the longitudinal one.

Let $\vec{S}$ and $\vec{s}$ be the polarizations (spins) of $\mathcal{B}_b$ and $\mathcal{B}$, respectively. The decay probability in terms of these polarization vectors is given by

$$dW = (2\pi)^7 \delta^4 (p - p' - k) \times \frac{1}{2} \mathrm{Tr}[(1 + \vec{\sigma} \cdot \vec{s}) M (1 + \vec{\sigma} \cdot \vec{S}) M^\dagger] d^3 p' d^3 k. \tag{23}$$

Hence, the transition rate is

$$\frac{dW}{\Gamma} = \frac{d\Omega_S d\Omega_s}{(4\pi)^2} [1 + \alpha \vec{S} \cdot \vec{n} + \alpha' \vec{s} \cdot \vec{n} + \beta \vec{s} \cdot (\vec{S} \times \vec{n}) + ((\vec{s} \cdot \vec{n})(\vec{S} \cdot \vec{n}))(-1 + \gamma') + \gamma \vec{s} \cdot (\vec{n} \times (\vec{S} \times \vec{n}))], \tag{24}$$

where

$$\alpha = 2\mathrm{Re}\left[-2 f_1^* g_1 + \left(\frac{k_0}{m_V}\right)^2 f_2^* h\right] p_0' \frac{|\vec{k}|}{2\pi m \Gamma}, \tag{25}$$

$$\alpha' = 2\mathrm{Re}\left[2 f_1^* g_1 + \left(\frac{k_0}{m_V}\right)^2 f_2^* h\right] p_0' \frac{|\vec{k}|}{2\pi m \Gamma}, \tag{26}$$

$$\beta = 2\mathrm{Im}[f_2^* h] p_0' \frac{|\vec{k}|}{2\pi m \Gamma}, \tag{27}$$

$$\gamma = \left(\frac{k_0}{m_V}\right)^2 [|f_2|^2 - |h|^2] p_0' \frac{|\vec{k}|}{2\pi m \Gamma}, \tag{28}$$

$$\gamma' = \left(\frac{k_0}{m_V}\right)^2 [|f_2|^2 + |h|^2] p_0' \frac{|\vec{k}|}{2\pi m \Gamma}. \tag{29}$$

A few comments are in order. For the transverse polarization, the asymmetry parameters are

$$\alpha = -\frac{4\mathrm{Re}[f_1^* g_1] p_0' k_0}{2\pi m \Gamma} = -\alpha', \tag{30}$$

whereas in the case of the longitudinal polarization

$$\alpha = \left(\frac{k_0}{m_V}\right)^2 \frac{2\mathrm{Re}[f_2^* h] p_0' k_0}{2\pi m \Gamma} = \alpha'. \tag{31}$$

It is clear from Eqs. (27)–(29), that β, γ, and γ' are nonzero only for the longitudinal polarization. For the longitudinal polarization, we get exactly the same result as that in the nonleptonic decay of the $\mathcal{B}$ baryon, when V is replaced by the pseudoscalar meson P [17].

III. FACTORIZATION: BARYON FORM FACTORS

In the factorization framework, the effective Hamiltonian for the decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + J/\psi$ is given by

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 \langle 0|\bar{c}\gamma^\mu (1-\gamma_5) c|J/\Psi\rangle \times \langle \mathcal{B}|\bar{s}\gamma_\mu (1-\gamma_5) b|\mathcal{B}_b\rangle. \quad (32)$$

The relevant matrix elements are

$$\langle 0|\bar{c}\gamma^\mu (1-\gamma_5) c|J/\Psi\rangle = \left(\frac{1}{2\pi}\right)^{3/2} \frac{1}{\sqrt{2k_0}} F_{J/\Psi} m_{J/\Psi} \epsilon^\mu, \quad (33)$$

$$\langle \mathcal{B}|\bar{s}\gamma_\mu (1-\gamma_5) b|\mathcal{B}_b\rangle = \left(\frac{1}{2\pi}\right)^3 \sqrt{\frac{mm'}{p_0 p'_0}} \bar{u}(p')[(g_V(k^2) - g_A(k^2)\gamma_5)\gamma_\mu - i(f_V(k^2) + h_A(k^2)\gamma_5)\sigma_{\mu\nu} k^\nu - (h_V(k^2) - f_A(k^2)\gamma_5) k_\mu u(p)], \quad (34)$$

where $f_V(k^2), g_V(k^2), f_A(k^2), g_A(k^2), h_V(k^2)$, and $h_A(k^2)$ are the form factors. Now, using Eqs. (33) and (34) in Eq. (32), the T matrix can be written as

$$T = \frac{1}{(2\pi)^{9/2}} \sqrt{\frac{mm'}{2k_0 p_0 p'_0 k_0}} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 F_{J/\Psi} m_{J/\Psi} \bar{u}(p') \times [\gamma\cdot\epsilon (g_V(k^2) + g_A(k^2)\gamma_5) - i\epsilon^\mu \sigma_{\mu\nu} k^\nu (f_V(k^2) + h_A(k^2)\gamma_5)] u(p). \quad (35)$$

Hence, comparing Eq. (12) and Eq. (35), one gets

$$\begin{aligned} A &= G' m_{J/\psi} F_{J/\psi} g_V(k^2), \\ B &= G' m_{J/\psi} F_{J/\psi} g_A(k^2), \\ C &= -G' m_{J/\psi} F_{J/\psi} f_V(k^2), \\ D &= -G' m_{J/\psi} F_{J/\psi} h_A(k^2), \end{aligned} \quad (36)$$

where

$$G' = V_{cb} V_{cs}^* \frac{G_F}{\sqrt{2}} (C_2 + \zeta C_1). \quad (37)$$

The short-distance QCD effects are taken care of in the Wilson coefficients C_1 and C_2. The long-distance interactions are shifted to the form factors g_V, g_A, f_V, and h_A, which need to be evaluated in some model. Using the heavy quark spin symmetry, one can relate the different form factors [18] for which there are two choices:

$$\begin{aligned} (i):\ & g_V(k^2) = g_A(k^2) = F_1(k^2) + \frac{m_b}{m} F_2(k^2), \\ & f_V(k^2) = h_A(k^2) = \frac{1}{m} F_2(k^2), \\ (ii):\ & g_V(k^2) = -g_A(k^2) = F_1(k^2) + \frac{m_b}{m} F_2(k^2), \\ & f_V(k^2) = -h_A(k^2) = \frac{1}{m} F_2(k^2), \end{aligned} \quad (38)$$

where $m = m_{\Lambda_b}$ and m_b is the mass of the b quark, which in this work is taken to be 4.65 GeV. Thus, in terms of the form factors $F_1(k^2)$ and $F_2(k^2)$, we can write

$$\begin{aligned} A &= G' m_{J/\psi} F_{J/\psi} F_1(k^2)\left(1 + \frac{m_b}{m}\frac{F_2(k^2)}{F_1(k^2)}\right) = \pm B, \\ C &= -G' m_{J/\psi} F_{J/\psi} F_1(k^2) \frac{1}{m}\frac{F_2(k^2)}{F_1(k^2)} = \pm D, \end{aligned} \quad (39)$$

where the $\pm$ sign in Eq. (39) corresponds to the choices (i) and (ii), respectively. We need the form factors at $k^2 = m_{J/\psi}^2$:

$$F_1(m_{J/\psi}^2) \equiv F_1, \frac{F_2(m_{J/\psi}^2)}{F_1(m_{J/\psi}^2)} \equiv \frac{F_2}{F_1}. \quad (40)$$

From Eqs. (16)–(19) and by using Eqs. (39) and (40), we can express the amplitudes f_1, g_1, f_2, and h in terms of the form factors F_1 and F_2/F_1 as

$$f_1 = R\frac{|\vec{k}|}{\sqrt{2p'_0(p'_0+m')}} F_1\left[1 + (m_b + (m+m'))\frac{1}{m}\frac{F_2}{F_1}\right], \quad (41)$$

$$g_1 = R\frac{p'_0+m'}{\sqrt{2p'_0(p'_0+m')}} F_1\left[\mp 1 + \left(\frac{\mp m_b(p'_0+m') \pm k_0(m+m') - m_{J/\Psi}^2}{(p'_0+m')}\right)\frac{1}{m}\frac{F_2}{F_1}\right], \quad (42)$$

$$f_2 = R\frac{|\vec{k}|}{k_0}\frac{m+m'}{\sqrt{2p'_0(p'_0+m')}} F_1\left[1 + \frac{m_b(m+m') + m_{J/\Psi}^2}{m+m'}\frac{1}{m}\frac{F_2}{F_1}\right], \quad (43)$$

$$h = R\frac{(m+m')k_0 - m_{J/\Psi}^2}{\sqrt{2p'_0(p'_0+m')}} F_1\left[\mp 1 + \left(\frac{\mp m_b(k_0(m+m'-m_{J/\Psi}^2) \pm m_{J/\Psi}^2(p'_0+m'))}{(k_0(m+m') - m_{J/\Psi}^2)}\right)\frac{1}{m}\frac{F_2}{F_1}\right], \quad (44)$$

with $R = G' m_{J/\Psi} F_{J/\Psi}$ which is a dimensionless parameter.

TABLE I. Numerical values of the amplitudes for $\Lambda_b \to \Lambda + J/\psi$ for $F_2/F_1 \approx 0.169$. Here $R^2 = (V_{cb}V_{cs}^* \frac{G_F}{\sqrt{2}}(C_2 + \zeta C_1) m_{J/\Psi} F_{J/\Psi})^2 \approx 18.97 \times 10^{-14}(C_2 + \zeta C_1)^2$. The values of the masses are from Ref. [12]. Here, the − and + signs are for the choices (i) and (ii), respectively.

Amplitudes	Numerical Values
f_1	$RF_1(0.644)$
g_1	$RF_1(\mp 0.880)$
f_2	$RF_1(1.075)$
h	$RF_1(\mp 1.197)$

We now consider the decay $\Lambda_b \to \Lambda + J/\psi$ which is of experimental interest. In order to calculate this decay, various models to evaluate the form factors have been considered in the literature [9–11]. In Ref. [6], form factors were evaluated in a quark model, and their values were $F_1 \approx -0.219$ and $F_2/F_1 \approx 0.169$. We put $F_2/F_1 \approx 0.169$ and other input parameters into Eqs. (41)–(44), and the numerical values of the amplitudes are given in Table I. These results can be extended for other baryons by using physical masses for the relevant parameters and $SU(3)$ symmetry.

Making use of the values of the amplitudes outlined in Table I, the value of the branching ratio for $\Lambda_b \to \Lambda + J/\psi$ decay is [cf. Eq. (22)]

$$\mathcal{B}_r \approx 1.18 \times 10^{-2}(C_2 + \zeta C_1)^2, \tag{45}$$

where $\zeta = \frac{1}{N_c}$, where N_c is the effective number of colors. As noted in Ref. [6], there are two regimes, viz. $N_c < 1/3$ [Eq. (46)] and the large-N_c limit. In Table II, we show the values for the branching ratio using the Wilson coefficients $C_2 = -0.257$ and $C_1 = 1.009$ [19] and for different values of ζ that correspond to the large-N_c limit. One can see that for $\zeta = 0$, our results for the branching ratio are comparable with the value 8.9×10^{-4} that was obtained in Ref. [11].

Similarly, for the values of ζ that correspond to the small-N_c limit, the values of the branching ratios are given in Table III. The experimental value of the branching ratio [12] is

TABLE II. The values of the branching ratio for $\Lambda_b \to \Lambda + J/\psi$ for different values of ζ that correspond to the large-N_c limit.

$\Lambda_b \to \Lambda + J/\psi$	$\zeta = 0$	$\zeta = 0.01$	$\zeta = 0.05$
Br	7.8×10^{-4}	6.1×10^{-4}	5.0×10^{-4}

$$\mathcal{B}_r(\Lambda_b \to \Lambda J/\psi) \times \mathcal{B}_r(b \to \Lambda_b^0) = (5.8 \pm 0.8) \times 10^{-5}.$$

Using $\mathcal{B}(b \to \text{baryon}) \approx 9.29 \times 10^{-3}$, the experimental value of the branching ratio for $\Lambda_b \to \Lambda J/\psi$ is $\mathcal{B}_r = (6.2 \pm 0.8) \times 10^{-4}$, which is comparable to our value 6.1×10^{-4} when $\zeta = 0.48$ as well as for $\zeta = 0.01$.

The values of the asymmetry parameters for $\Lambda_b \to \Lambda + J/\psi$ decay are obtained from Eqs. (30) and (31):

$$\begin{aligned} &\alpha \approx \mp 0.19, \quad &\alpha_T \approx \pm 0.39, \quad &\alpha_L \approx \mp 0.58, \\ &\alpha' \approx \mp 0.98, \quad &\alpha'_T \approx \mp 0.39, \quad &\alpha'_L \approx \mp 0.58. \end{aligned} \tag{46}$$

The experimental value of the asymmetry parameter $\alpha = 0.18 \pm 0.13$. With our choice (i) of the form factors given in Eq. (38), the value of the asymmetry parameter $\alpha = -0.19$ is comparable to the values obtained in Refs. [3–9]. However, for choice (ii) of the form factors, the value of the asymmetry parameter $\alpha = 0.19$ is comparable to the experimental value $\alpha = 0.18 \pm 0.13$.

We have discussed $\Lambda_b \to \Lambda + J/\Psi$ decay in detail and with this in hand, for a heavy baryon belonging to representations $\bar{3}$ and 6, the branching ratio can be easily obtained by using $SU(3)$ symmetry, taking into account the phase space for each baryon decay. For the decays $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+)J/\Psi$, $SU(3)$ gives the relation

$$\bar{3}\colon\ (\Xi_b^-, \Xi_b^0, \Lambda_b) \to (\Xi^-, \Xi^0, \Lambda)J/\Psi\colon (1, 1, \sqrt{2/3}) \tag{47}$$

for $\mathcal{B}_b(\frac{1}{2}^+)$ belong to representation $\bar{3}$ and $\mathcal{B}(\frac{1}{2}^+)$ belonging to the octet representation. In the case of $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the sextet representation and $\mathcal{B}(\frac{1}{2}^+)$ belonging to the representation $8'$, $SU(3)$ gives

$$\begin{aligned} &(\Sigma_b^+, \Sigma_b^0 \Sigma_b^-) \to (\Sigma^+, \Sigma^0, \Sigma^-)J/\Psi\colon\ \sqrt{2}(-1, 1, 1), \\ &(\Xi_b'^-, \Xi_b'^0) \to \Xi^-, \Xi^0)J/\Psi\colon\ (1, 1). \end{aligned} \tag{48}$$

IV. THE DECAY $\Omega_b \to \Omega^- + J/\Psi$

In the factorization ansatz corresponding to the effective Hamiltonian given in Eq. (4), the matrix element for $\Omega_b \to \Omega^- + J/\Psi$ decay is

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^*(C_2 + \zeta C_1)\langle 0|\bar{c}\gamma^\mu(1-\gamma^5)c|J/\Psi\rangle\langle\Omega^-|\bar{s}\gamma_\mu \times (1-\gamma^5)b|\Omega_b^-\rangle. \tag{49}$$

We can write

TABLE III. The values of the branching ratio for $\Lambda_b \to \Lambda + J/\psi$ for different values of ζ that correspond to the small-N_c limit.

$\Lambda_b \to \Lambda + J/\psi$	$\zeta = 1/3$	$\zeta = 0.40$	$\zeta = 0.45$	$\zeta = 0.48$	$\zeta = 0.50$
Br	0.74×10^{-4}	2.6×10^{-4}	4.6×10^{-4}	6.1×10^{-4}	7.2×10^{-4}

$$\langle\Omega^-|\bar{s}\gamma_\mu(1-\gamma^5)b|\Omega_b^-\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{mm'}{p_0 p_0'}}[(F_1^V-\gamma^5 F_1^A)(\bar{u}_\mu(p')u(p))+\cdots], \quad (50)$$

where the dots denote the contribution from other form factors which are suppressed by a factor of $\frac{1}{m_{\Omega_b}}$ compared to F_1^V and F_1^A and hence will be neglected. From Eqs. (49) and (50) along with Eq. (38), we get

$$|\mathcal{M}|^2 = G' F_{J/\Psi}^2 m_{J/\Psi}^2 \epsilon^\mu \epsilon^\nu \times [u_\nu(p')\bar{u}_\mu(p')(F_1^V-\gamma^5 F_1^A)u(p)\bar{u}(p) \times (F_1^{*V}+\gamma^5 F_1^{*A})], \quad (51)$$

where $G' = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_2+\zeta C_1)$. Now

$$\sum_{\text{Polarization}} \epsilon^\mu(k)\epsilon^\nu(k) = \left(-\eta^{\mu\nu}+\frac{k^\mu k^\nu}{m_{J/\Psi}^2}\right),$$
$$\sum_{\text{Spin}} u_\nu(p')\bar{u}_\mu(p') = -\frac{\gamma\cdot p'+m'}{2m} \times \left[\eta_{\nu\mu}-\gamma_\nu\gamma_\mu+\frac{i}{3m'}(\gamma_\nu p'_\mu - p'_\nu\gamma_\mu) - \frac{2}{3m'^2}p'_\nu p'_\mu\right],$$
$$\bar{\sum_{\text{spin}}} u(p)\bar{u}(p) = \frac{1}{2}\frac{\gamma\cdot p+m}{2m}. \quad (52)$$

Using the above equations, the decay rate is given by

$$\Gamma = \frac{1}{2\pi m}|\vec{k}|(G' F_{J/\Psi} m_{J/\Psi})^2\left(1+\frac{1}{3}\frac{m^2}{m'^2}\frac{|\vec{k}|^2}{m_{J/\Psi}^2}\right) \times [|F_1^V|^2(p_0'+m')+|F_1^A|^2)(p_0'-m')](C_2+\zeta C_1)^2. \quad (53)$$

In particular, for $\Omega_b^- \to \Omega^- + J/\Psi$, we have $m = m_{\Omega_b}$ and $m' = m_\Omega$. Now

$$|\Omega^-\rangle = \frac{1}{\sqrt{3}}(s^\uparrow s^\uparrow s^\downarrow + s^\uparrow s^\downarrow s^\uparrow + s^\downarrow s^\uparrow s^\uparrow),$$
$$|\Omega_b^-\rangle = -\frac{1}{\sqrt{6}}|s^\uparrow s^\downarrow b^\uparrow + s^\downarrow s^\uparrow b^\uparrow - 2 s^\uparrow s^\uparrow b^\downarrow\rangle.$$

In nonrelativistic quark model, the relevant operators for $\mathcal{O}(v^2/c^2)$ are (for details, see Ref. [20]) β and $\beta\sigma_i$ with $i = z$. Using $\beta|b\rangle = |s\rangle$, we have

$$\beta|\Omega_b^-,\frac{1}{2}\rangle = -\frac{1}{\sqrt{6}}|(s^\uparrow s^\downarrow s^\uparrow + s^\downarrow s^\uparrow s^\uparrow - 2 s^\uparrow s^\uparrow s^\downarrow)\rangle$$

and

$$\beta\sigma_z|\Omega_b^-,\frac{1}{2}\rangle = -\frac{1}{\sqrt{6}}|(s^\uparrow s^\downarrow s^\uparrow + s^\downarrow s^\uparrow s^\uparrow + 2 s^\uparrow s^\uparrow s^\downarrow)\rangle.$$

This gives $F_1^V = 0$ and $F_1^A(0) = -\frac{2\sqrt{2}}{3}$ [21]. Thus,

$$\Gamma(\Omega_b^- \to \Omega^- + J/\Psi) \approx 1.76\times 10^{-14}|F_1^A|^2(C_2+\zeta C_1)^2\ \text{GeV}. \quad (54)$$

Hence, the branching ratio

$$Br(\Omega_b^- \to \Omega^- + J/\Psi) = \frac{\tau_{\Omega_b}}{\hbar}\Gamma(\Omega_b^- \to \Omega^- + J/\Psi) = 2.94\times 10^{-2}|F_1^A|^2(C_2+\zeta C_1)^2, \quad (55)$$

where $F_1^A = F_1^A(m_{J/\Psi}^2)$ and τ_{Ω_b} is the decay time of Ω_b. Now, using

$$F_1^A = \frac{1}{m_{J/\Psi}}\frac{m_b m_s}{m_b+m_s}F_1^A(0) \approx \frac{m_s}{m_{J/\Psi}}F_1^A(0) \approx 0.152, \quad (56)$$

the form factor F_1^A at $m_{J/\psi}$ is expected to be smaller than $F_1^A(0)$. For this purpose, we have introduced a dimensionless phenomenological factor $(\frac{1}{m_{J/\psi}})(\frac{m_b m_s}{m_b+m_s})$, where the second factor is the reduced mass of the constituents of Ω_b^-. Using $F_1^A \approx 0.152$, the branching ratio is

$$\mathcal{B}_r \approx 6.8\times 10^{-4}(C_2+\zeta C_1)^2.$$

The values of the branching ratio corresponding to different values of ζ are given in Table IV.

The experimental $\mathcal{B}r(\Omega_b^- \to \Omega^- + J/\Psi)\times\mathcal{B}r(b\to\Omega_b) = (2.9^{+1.1}_{-0.8})\times 10^{-6}$ with $\mathcal{B}(b\to \text{baryon}) \approx 9.29\times 10^{-3}$ [12] gives

$$\mathcal{B}r(\Omega_b^- \to \Omega^- J/\Psi) = (3.12^{+1.1}_{-0.8})\times 10^{-5}. \quad (57)$$

TABLE IV. The values of the branching ratio for $\Omega_b \to \Omega + J/\psi$ for different values of ζ.

$\Omega_b \to \Omega + J/\psi$	$\zeta = 0$	$\zeta = 0.01$	$\zeta = 0.05$	$\zeta = 1/3$	$\zeta = 0.40$	$\zeta = 0.44$
Br	4.5×10^{-5}	4.1×10^{-5}	2.9×10^{-5}	0.8×10^{-5}	1.8×10^{-5}	3.0×10^{-5}

Finally, in this model, the asymmetry parameter is

$$\alpha(\Omega_b^- \to \Omega^- J/\Psi) = 0. \quad (58)$$

To conclude: using the effective Lagrangian together with the factorization ansatz, the two-body hadronic decay $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+, \frac{3}{2}^+) + V$ was calculated. In the case of $\mathcal{B}_b(\frac{1}{2}^+)$ belonging to the representation $\bar{3}$, the only allowed decay channel is $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + V$, where $\mathcal{B}(\frac{1}{2}^+)$ belongs to the representation 8 of $SU(3)$. However, if $\mathcal{B}_b(\frac{1}{2}^+)$ belongs to the sextet representation 6, the allowed decay channels are $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+, \frac{3}{2}^+) + V$, where $\mathcal{B}(\frac{1}{2}^+)$ and $\mathcal{B}(\frac{3}{2}^+)$ belong to the octet representation $8'$ and the decuplet 10 of $SU(3)$, respectively. We have analyzed the decay channel $\mathcal{B}_b(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + V$ in detail, where the decay rate Γ and the asymmetry parameters $\alpha, \alpha', \beta, \gamma$, and γ' were expressed in terms of four amplitudes. These amplitudes were written in terms of the transverse and longitudinal polarizations of V. This general formalism was then applied to the decay $\Lambda_b \to \Lambda J/\psi$. It was shown that within the factorization framework, using heavy quark spin symmetry, the decay rate and asymmetry parameters can be expressed in terms of two form factors F_1 and F_2/F_1, which nonperturbative quantities that need to be evaluated in some model. Here, by taking the values of these form factors calculated in the quark model [6], the branching ratio and asymmetry parameters α and α' were obtained numerically. By taking the color factor $\zeta = 0.01$ or $\zeta = 0.48$, the branching ratio for the decay $\Lambda_b \to \Lambda + J/\psi$ was matched to the corresponding experimental value. Having worked out $\Lambda_b \to \Lambda + J/\psi$ decay, this formalism can be easily applied to other heavy quarks belonging to the triplet and sextet representations by using $SU(3)$ symmetry and the phase-space factor. Finally, the decay $\Omega_b^- \to \Omega^- + J/\psi$ was analyzed within the factorization framework. It was shown that the asymmetry parameter α in this particular decay is zero. The branching ratio obtained in the first approximation was compared with the experimental value.

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