

**Drell-Yan-like processes and duality**I. V. Anikin,<sup>1,\*</sup> L. Szymanowski,<sup>2</sup> O. V. Teryaev,<sup>1</sup> and N. Volchanskiy<sup>1,3</sup><sup>1</sup>*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*<sup>2</sup>*National Centre for Nuclear Research (NCBJ), 00-999 Warsaw, Poland*<sup>3</sup>*Research Institute of Physics, Southern Federal University, 344090 Rostov-on-Don, Russia*

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We calculate the gauge invariant Drell-Yan-like hadron tensors. In connection with new COMPASS results, we predict the new single spin asymmetry which probes gluon poles together with chiral-odd and time-odd functions. The relevant pion production as a particular case of the Drell-Yan-like process has been discussed. For the meson-induced Drell-Yan (DY) process, we model an analog of the twist-3 distribution function, which is a collinear function in inclusive channel, by means of two noncollinear distribution amplitudes which are associated with exclusive channel. This modeling demonstrates the fundamental duality between different factorization regimes.

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The current studies of hadron structure involve both semi-inclusive and exclusive processes. They are described by transverse momentum dependent (TMDs) and generalized parton distributions (GPDs), respectively. The transitions, duality and matching between these regimes are of much importance for the coherent QCD description of hadron structure. Here we concentrate on the manifestation of such effects in the pion-nucleon Drell-Yan process at large  $x_F$ , when pion is described by wave functions and distribution amplitudes rather than parton distributions.

It has been shown long ago that the specific effects related to the high-twist corrections lead to the sizable nonscaling and nonfactorizing contributions to the unpolarized cross sections for the Drell-Yan-like processes in the well-defined kinematic regimes of a large fraction  $x$  [1,2]. Also, the inclusive production of dimuons from the hard scattering of pions on an unpolarized nuclear target and the similar process with longitudinally polarized protons have been analyzed in [3]. In both cases, it has demonstrated the role played by the pion bound state in terms of the pion distribution amplitudes. Moreover, in [4] it is shown that the angular distribution is rather sensitive to the shape of the pion distribution amplitude in the kinematic region where one of the pion constituents is off shell. In the kinematic regime where the photon has a large longitudinal momentum fraction, the cross section and the single spin asymmetry for the dimuon production with taking into account pion bound state effects are calculated in [5]. The predictions of [5] are directly proportional to the pion distribution amplitude. Therefore, the measurement of the polarized Drell-Yan cross section can determine the shape of the pion distribution amplitude.

To the present day the study of hadron (in particular nucleon) composite structure is the most important subject of hadron physics. From the experimental viewpoint, one of

the widespread and useful instruments for such investigations is the single spin asymmetry (SSA). Especially, the single transverse spin asymmetry opens the access to the three-dimensional nucleon structure owing to the nontrivial connection between the transverse spin and the parton transverse momentum dependence (see, for example, [6–11]). There are several experimental programs which pursue the measurements with Drell-Yan-like processes, RHIC [12], COMPASS [13,14] and future NICA [15,16].

In the paper, we study the Drell-Yan and pion production hadron tensors related to the meson-baryon processes with the essential spin transversity and “primordial” transverse momenta. The main attention has been paid for the gluon poles which are manifested in the corresponding distribution functions or/amplitudes.

Inspired by the recent experimental studies by COMPASS [14], we propose an approach to calculate the gauge invariant meson-induced Drell-Yan hadron tensor which finally gives a prediction for the single transverse spin asymmetry. We focus on the case where one of the pion (or meson) distribution amplitudes has been projected onto the chiral-odd combination. In turn, the pion chiral-odd distribution amplitudes also separate the chiral-odd tensor combination in the nucleon matrix element. The access to the single spin asymmetries, in particular the angular dependence, induced by chiral-odd and time-odd distribution functions/amplitudes has been opened only owing to the gluon pole presence. In other words, the angular dependence of SSA predicted in the paper can give implicit evidence for the gluon pole observation in COMPASS experiments.

Moreover, even in the collinear collision of hadron beams it is possible to get experimentally some evidence for the leading role of the transverse parton movements inside hadrons. Indeed, thanks for the frame independency, within the so-called Collins-Soper frame we can study the angular dependence of SSA as a function of  $\varphi \sim \vec{S}_\perp \wedge \vec{P}_\perp$  provided the gluon poles presence. The nontrivial angular dependence of SSA can be treated as a signal for the

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transverse primordial momentum dependence. Thus, we impose the single spin asymmetries which are reachable in COMPASS and can probe simultaneously gluon poles, duality, chiral-odd and time-odd functions.

Further, let us go over to kinematics, we study the meson-induced Drell-Yan process and semi-inclusive deep-inelastic scattering with pion production where baryons are transversely polarized:  $M(P_1) + N^{(\uparrow\downarrow)}(P_2) \rightarrow \gamma^*(q) + \bar{q}(K) + X(P_X) \rightarrow \bar{q}(K) + \ell(l_1) + \bar{\ell}(l_2) + X(P_X)$  and  $N^{(\uparrow\downarrow)}(P_2) + \ell(l_1) \rightarrow M(P_1) + \ell(l_2) + \bar{q}(K) + X(P_X)$ . The virtual photon producing the lepton pair ( $l_1 + l_2 = q$ ) has a large mass squared ( $q^2 = Q^2$ ) while the transverse momenta are small and integrated out.

The Sudakov decompositions take the forms (for the sake of shortness, we omit the four-dimension indices)

$$P_1 \approx \frac{Q}{x_B \sqrt{2}} n^* + P_{1\perp}, \quad P_2 \approx \frac{Q}{y_B \sqrt{2}} n + P_{2\perp}, \quad (1)$$

$$S \approx \frac{\lambda}{M_2} P_2 + S_{\perp} \quad (2)$$

for the hadron momenta and spin vector;

$$q = \frac{Q}{\sqrt{2}} n^* + \frac{Q}{\sqrt{2}} n + q_{\perp}, \quad q_{\perp}^2 \ll Q^2, \quad (3)$$

for the photon momentum. The hadron momenta  $P_1$  and  $P_2$  have the plus and minus dominant light-cone components, respectively. Accordingly, the dominant quark and gluon momenta  $k_1$  and  $\ell$  lie along the plus direction while the dominant antiquark momentum  $k_2$  is along the minus direction.

We also define the Collins-Soper (CS) frame as [17]

$$\hat{t}^{\mu} = \frac{q^{\mu}}{Q}, \quad \hat{x}^{\mu} = \frac{q_{\perp}^{\mu}}{Q_{\perp}}, \quad \hat{z}^{\mu} = \frac{x_B}{Q} \tilde{P}_1^{\mu} - \frac{y_B}{Q} \tilde{P}_2^{\mu}, \quad (4)$$

where  $\tilde{P}_1 = P_1 - q/(2x_B)$  and  $\tilde{P}_2 = P_2 - q/(2y_B)$ . In what follows we are not so precise about writing the covariant and contravariant vectors in any kinds of definitions and summations over the four-dimensional vectors, except the cases where this notation may lead to misunderstanding. We can also write down that

$$\sqrt{2}n^* = \hat{t} + \hat{z} - \frac{Q_{\perp}}{Q} \hat{x}, \quad \sqrt{2}n = \hat{t} - \hat{z} - \frac{Q_{\perp}}{Q} \hat{x}. \quad (5)$$

With minor modifications this reference frame is suitable for the direct pion production as well.

For the processes we consider, we deal with a large  $Q^2$  and, therefore, we apply the factorization theorem to get the corresponding hadron tensor factorized in the form of convolution:

$$\text{Hadron tensor} = \{\text{Hard (pQCD)}\} \otimes \{\text{Soft (npQCD)}\}. \quad (6)$$

Based on DY kind of processes, it is natural to study the role of twist-3 by exploring of different kinds of asymmetries. In particular, the left-right asymmetry which means that the transverse momenta of the leptons or/and hadrons are correlated with the directions  $\vec{S}_{\perp} \wedge \hat{z}$  and  $\vec{S}_{\perp} \wedge \hat{x}$ , see Eq. (4).

The single spin asymmetries (SSAs) under our consideration is given by

$$A = (d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)}) / (d\sigma^{(\uparrow)} + d\sigma^{(\downarrow)}), \quad (7)$$

with (see [11])

$$\frac{d\sigma^{(\uparrow\downarrow)}}{d^4 q d\Omega} = \frac{\alpha_{em}^2}{2j q^4} \mathcal{L}_{\mu\nu} H_{\mu\nu}, \quad (8)$$

where  $j$  is the standard flux factor,  $d\Omega$  specifies the frame angle orientations. In Eq. (8),  $\mathcal{L}_{\mu\nu}$  implies the unpolarized lepton tensor and  $H_{\mu\nu}$  stands for the hadronic tensor. Since we dwell on the unpolarized lepton case which leads to the real lepton tensor, the hadron tensor  $H_{\mu\nu}$  has to be a real one as well. Moreover, as a rule, the hadron tensor includes at least two nonperturbative blobs which are associated with two different dominant (the light-cone plus and minus) directions.

Following [18–20], we continue to study the gluon pole influence on the different asymmetries. We now focus on the cases where the upper nonperturbative blob depicted in Figs. 1 and 2 corresponds to the matrix elements, first, with the spin transversity and, second, with the primordial hadron transverse momentum. The corresponding matrix elements can be parametrized by either the chiral-odd or time-odd twist-2 functions, i.e. [see below Eqs. (16) and (18)]

$$\begin{aligned} \langle P_2, S_{\perp} | \bar{\psi} \sigma^{-\perp} \psi | S_{\perp}, P_2 \rangle &\stackrel{\mathcal{F}}{\sim} \varepsilon^{-\perp S^{\perp} P_2} \bar{h}_1(y), \\ \langle P_2, S_{\perp} | \bar{\psi} \gamma^{-} \psi | S_{\perp}, P_2 \rangle &\stackrel{\mathcal{F}}{\sim} i \varepsilon^{-\perp S_{\perp} P_2^{\perp}} \bar{f}_T(y). \end{aligned} \quad (9)$$

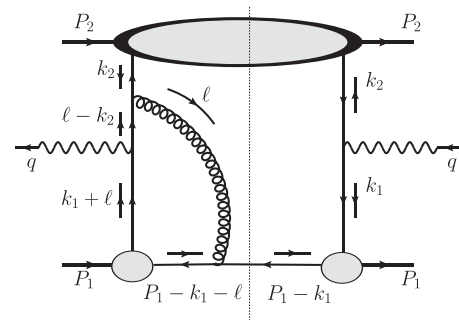


FIG. 1. The Feynman diagrams which contribute to the polarized Drell-Yan hadron tensor: the standard diagram.

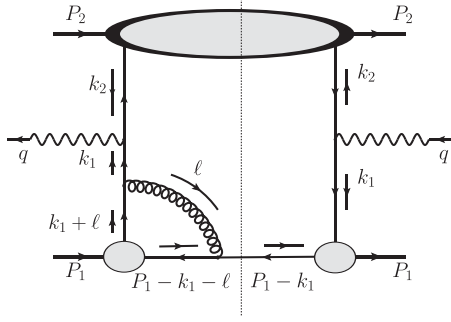


FIG. 2. The Feynman diagrams which contribute to the polarized Drell-Yan hadron tensor: the nonstandard diagram.

We remind that for the upper blob the dominant light-cone direction is a minus direction.

On the other hand, for the lower nonperturbative blob we replace the twist-3  $B^V$ -function of [18] with two distribution amplitudes with twist-2 and -3, see Figs. 1 and 2. Schematically, this can be demonstrated as

$$\begin{aligned} & \langle P_1, S | [\bar{\psi} A_{\perp}^{\alpha} \psi]^{\text{tw-3}} | S, P_1 \rangle |_{[18]} \\ & \Rightarrow \mathcal{D}^{\alpha\beta} \langle 0 | [\bar{\psi} \psi]^{\text{tw-2}} | S, P_1 \rangle \gamma_{\beta} \langle P_1, S | [\bar{\psi} \psi]^{\text{tw-3}} | 0 \rangle |_{\text{this work}}, \end{aligned} \quad (10)$$

where  $\mathcal{D}^{\alpha\beta}$  stands for the gluon propagator. Here, all spinor indices in the corresponding matrix element combinations are open. Notice that the replacement shown in Eq. (10) gives us the possibility to study the so-called gluon poles in the most explicit form. Indeed, as it is shown below [see Eq. (22)], the longitudinal dominant part of the gluon propagator finally generates the gluon pole with the certain complex prescription (arisen from the contour gauge) which compensates the complexity of (9).

We are now in position to discuss the derivation of the hadron tensor. We begin with the hadron tensor that relates to the standard diagram, see Fig. 1. Throughout the paper, we adhere the terminology and the *collinear* factorization procedure as described in [18–20]. The so-called standard diagram (which exists even if the  $B^V$ -function in [18] is real) implies the diagram depicted in Fig. 1.

Before factorization, the standard diagram gives the hadron tensor (all prefactors are included in the integration measures)

$$\begin{aligned} \mathcal{W}_{\mu\nu}^{(\text{stand})} &= \int (d^4 k_1) (d^4 k_2) \delta^{(4)}(k_1 + k_2 - q) \\ & \times \int (d^4 \ell) \mathcal{D}_{\alpha\beta}(\ell) \text{tr}[\gamma_{\nu} \Gamma \gamma_{\alpha} S(\ell - k_2) \gamma_{\mu} \Gamma_1 \gamma_{\beta} \Gamma_2] \\ & \times \bar{\Phi}^{[\Gamma]}(k_2) \Phi_{(2)}^{[\Gamma_1]}(k_1; \ell) \Phi_{(1)}^{[\Gamma_2]}(k_1) \delta((P_1 - k_1)^2), \end{aligned} \quad (11)$$

where

$$\begin{aligned} \bar{\Phi}^{[\Gamma]}(k_2) &= \sum_X \int (d^4 \eta_2) e^{-ik_2 \eta_2} \\ & \times \langle P_2, S_{\perp} | \text{tr}[\psi(0) | P_X \rangle \langle P_X | \bar{\psi}(\eta_2) \Gamma] | S_{\perp}, P_2 \rangle \end{aligned} \quad (12)$$

and

$$\Phi_{(2)}^{[\Gamma_1]}(k_1; \ell) = \int (d^4 \eta_1) e^{i(P_1 - \ell - k_1) \eta_1} \langle 0 | \bar{\psi}(\eta_1) \Gamma_1 \psi(0) | P_1 \rangle, \quad (13)$$

$$\Phi_{(1)}^{[\Gamma_2]}(k_1) = \int (d^4 \xi) e^{-ik_1 \xi} \langle P_1 | \bar{\psi}(\xi) \Gamma_2 \psi(0) | 0 \rangle. \quad (14)$$

In Eq. (11), we write explicitly the  $\delta$ -function which shows that the quark operator with  $P_1 - k_1$  corresponds to the on-shell fermion.

For the cases under our consideration, we choose the  $\gamma$ -structure in Eqs. (11), (13), and (14) to be

$$\begin{aligned} \Gamma \otimes \bar{\Phi}^{[\Gamma]} &\Rightarrow \gamma^+ \otimes \bar{\Phi}^{[\gamma^-]} \oplus \sigma^{+\perp} \otimes \bar{\Phi}^{[\sigma^{-\perp}]}, \\ \Gamma_1 \otimes \Phi_{(2)}^{[\Gamma_1]} &\Rightarrow \gamma^-(\gamma_5) \otimes \Phi_{(2)}^{[\gamma^+(\gamma_5)]}, \\ \Gamma_2 \otimes \Phi_{(1)}^{[\Gamma_2]} &\Rightarrow \gamma_{\rho}^{\perp}(\gamma_5) \otimes \Phi_{(1)}^{[\gamma_{\rho}^{\perp}(\gamma_5)]} \oplus \sigma^{+-}(\gamma_5) \otimes \Phi_{(1)}^{[\sigma^{+-}(\gamma_5)]} \end{aligned} \quad (15)$$

which correspond to the nucleon parton distribution of twist-2 and the pion or/and rho-meson distribution amplitudes of twist-2 and -3, respectively. However, the concrete type of hadrons do not play a crucial role in our consideration.

The next item is to perform the factorization procedure for the hadron tensor. We are not going to stop on all of the factorization stages (the comprehensive description of factorization can be found in many papers, see, for example, [21–24]). Instead, we dwell on the corresponding parton functions parametrizing the nonperturbative matrix elements which appear after the collinear factorization procedure. For the twist-2 distribution function which characterizes the upper blob, we have

$$\begin{aligned} \bar{\Phi}^{[\sigma^{-\perp}]}(y) &\stackrel{\text{def}}{=} \int (d^4 k_2) \delta(y - k_2^- / P_2^-) \bar{\Phi}^{[\sigma^{-\perp}]}(k_2) \\ &= \varepsilon^{-\perp S^{\perp} P_2} \bar{h}_1(y), \end{aligned} \quad (16)$$

$$\begin{aligned} \bar{\Phi}^{[\gamma^-]}(y) &\stackrel{\text{def}}{=} \int (d^4 k_2) \delta(y - k_2^- / P_2^-) \bar{\Phi}^{[\gamma^-]}(k_2) \\ &= i \varepsilon^{-+ S_{\perp} P_2} \bar{f}_T(y). \end{aligned} \quad (17)$$

The other object is an analog of  $B^V$ -function expressed through the gluon propagator and functions  $\Phi_{(1)}$ ,  $\Phi_{(2)}$ . We have (here,  $x_{21} = x_2 - x_1$ )

$$\begin{aligned} \mathbb{B}_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2) & \stackrel{\text{def}}{=} \int (d^4 k_1) \delta(x_1 - k_1^+ / P_1^+) \int (d^4 \ell) \delta(x_{21} - \ell^+ / P_1^+) \\ & \times \frac{d_{\alpha\beta}(\ell)}{2\ell^+ \ell^- - \vec{\ell}_\perp^2 + i0} \Phi_{(2)}^{[\Gamma_1]}(k_1; \ell) \Phi_{(1)}^{[\Gamma_2]}(k_1) \delta((P_1 - k_1)^2), \end{aligned} \quad (18)$$

where the gluon propagator has been written in the explicit form with the causal prescription and  $d_{\alpha\beta}(\ell)$  given by the axial (contour) gauge  $A^+ = 0$  (see [25]).

Since we are interested in the gluon pole decoded in Eq. (18) we have to keep only the transverse gluon contributions to the gluon propagator. Indeed, the gluon propagator generated by  $\langle A_\perp^i A^- \rangle$  is associated with the essential transverse component of gluon momentum  $\ell_\perp^i$ . This leads to the case where there are not any sources for the gluon pole at  $x_1 = x_2$  [20,26]. On the other hand, the  $\langle A_\perp^i A^- \rangle$  part of the gluon propagator is gauge dependent and disappears ultimately in the physical observables and hadron tensor.

Therefore, Eq. (18) can be rewritten in the following form:

$$\mathbb{B}_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2) = \frac{g_{\alpha\beta}^\perp}{2P_1^+(x_2 - x_1)} \Phi_{(1)}^{[\Gamma_2]}(x_1) \overset{\mathbf{k}_1^+}{\otimes} \Phi_{(2)}^{[\Gamma_1]}(\bar{x}_2), \quad (19)$$

where integration over  $dk_1^-$  with  $\delta((P_1 - k_1)^2)$  has been done and we introduce the notations

$$\begin{aligned} F \overset{\mathbf{k}_1^+}{\otimes} G &= \int (d^2 \vec{k}_1^+) F(\vec{k}_1^+) \\ & \times \int (d\ell^- d^2 \vec{\ell}_\perp) \Delta(\ell^-, \vec{\ell}_\perp) G(\vec{k}_1^+; \ell^-, \vec{\ell}_\perp), \end{aligned} \quad (20)$$

$$\Delta(\ell^-, \vec{\ell}_\perp) = \frac{1}{\ell^- - \vec{\ell}_\perp^2 / (2x_{21} P_1^+) + i \text{sign}(x_{21}) 0}. \quad (21)$$

In Eq. (19), the function  $\mathbb{B}_{\alpha\beta}^{[\Gamma_2, \Gamma_1]}(x_1, x_2)$  (which is expressed, generally speaking, through two noncollinear pion distribution amplitudes) is an analog of the function  $B^V(x_1, x_2)$  appeared in the inclusive channel [18]. The similarity of these two functions takes place provided the small invariant mass of spectators in the pion sector. In other words, the mentioned similarity can be understood as the manifestation of duality between exclusive and inclusive channels. Besides, if we have the restriction somewhat of  $|\vec{\ell}_\perp| \gg |\mathbf{k}_1^+|$ , the approximation can be implemented by two collinear distribution amplitudes.

Let us discuss the pole at  $x_1 = x_2$  which is factorized as a prefactor in Eq. (19). This is exactly the gluon pole which has to be treated within the contour gauge frame as described in [18–20], i.e.

$$\frac{1}{x_2 - x_1} \stackrel{\text{c.g.}}{\Rightarrow} \frac{1}{x_2 - x_1 - i\epsilon}. \quad (22)$$

Notice that the complex prescription emanates from the corresponding integral representation of the theta function (see [19] for details).

Having performed the Lorentz decomposition in the corresponding matrix elements, in the case we consider the  $B^V$ -function analog takes the form

$$\mathbb{B}_{\alpha\beta}^{[\Gamma_2]}(x_1, x_2) = \frac{1}{2} \frac{g_{\alpha\beta}^\perp V^{[\Gamma_2]}}{x_2 - x_1 - i\epsilon} \Phi_{(1)}^{\text{tw-3}}(x_1) \overset{\mathbf{k}_1^+}{\otimes} \Phi_{(2)}^{\text{tw-2}}(x_2), \quad (23)$$

where the Lorentz tensor  $V^{[\Gamma_2]}$  means  $P_1^\perp$  or  $\vec{n}^- P_1^+$  for the pion-to-vacuum matrix elements and  $e^\perp$  for the rho-meson-to-vacuum matrix element. In Eq. (23),  $P_1^+$  in the numerator which originates from the parametrization of  $\Phi_{(2)}^{[\Gamma_1]}$  cancels the same component coming from the denominator of the gluon propagator [see Eq. (19)].

With these, after factorization the standard diagram hadron tensor reads

$$\begin{aligned} \mathcal{W}_{\mu\nu}^{(\text{stand})} &= i \int (dx_1)(dy) \delta^{(4)}(x_1 P + y P_2 - q) \bar{\Phi}^{[\Gamma]}(y) \\ & \times \int (dx_2) \text{tr} \left[ \gamma_\nu \Gamma \gamma_\alpha \frac{x_{21} \hat{P}_1^+}{-x_{21} y S + i0} \gamma_\mu \gamma^- \gamma_\beta \Gamma_2 \right] \\ & \times \frac{1}{2} \frac{g_{\alpha\beta}^\perp V^{[\Gamma_2]}}{x_2 - x_1 - i\epsilon} \Phi_{(1)}^{\text{tw-3}}(x_1) \overset{\mathbf{k}_1^+}{\otimes} \Phi_{(2)}^{\text{tw-2}}(x_2). \end{aligned} \quad (24)$$

Here and below, for the sake of convenience, we single out the complex  $i$  which emanates from either the parametrization of the upper blob, Eq. (16), or from the parametrization of the pion-to-vacuum matrix element of the twist-2 operator in the lower blob.

The next object of our discussion is the additional diagram, see Fig. 2, which contributes to the hadron tensor. According to [18], this is the so-called nonstandard diagram which does not exist if the  $B^V$ -function is real.

In principle, derivation of this part of the hadron tensor is similar to what we present for the standard diagram contribution. Before factorization, we have

$$\begin{aligned} \mathcal{W}_{\mu\nu}^{(\text{nonstand})} &= \int (d^4 k_1)(d^4 k_2) \delta^{(4)}(k_1 + k_2 - q) \\ & \times \int (d^4 \ell) \mathcal{D}_{\alpha\beta}(\ell) \text{tr} [\gamma_\nu \Gamma \gamma_\mu S(k_1) \gamma_\alpha \Gamma_1 \gamma_\beta \Gamma_2] \\ & \times \bar{\Phi}^{[\Gamma]}(k_2) \Phi_{(2)}^{[\Gamma_1]}(k_1; \ell) \Phi_{(1)}^{[\Gamma_2]}(k_1) \delta((P_1 - k_1)^2). \end{aligned} \quad (25)$$

Then, we again perform the factorization procedure and, finally, the nonstandard diagram hadron tensor is given by



$$\begin{aligned} \mathcal{W}_{\mu\nu}^{(\text{nonstand})} &= i \int (dx_1)(dy) \delta^{(4)}(x_1 P_1 + y P_2 - q) \bar{\Phi}^{[\Gamma]}(y) \\ &\times \text{tr} \left[ \gamma_\nu \Gamma \gamma_\mu \frac{\gamma^+}{2x_1 P_1^+ + i0} \gamma_\alpha \gamma^- \gamma_\beta \Gamma_2 \right] \frac{1}{2} \int (dx_2) \\ &\times \frac{g_{\alpha\beta}^\perp V^{[\Gamma_2]}}{x_2 - x_1 - i\varepsilon} \Phi_{(1)}^{\text{tw-3}}(x_1) \otimes \Phi_{(2)}^{\text{tw-2}}(x_2). \end{aligned} \quad (26)$$

Last but not least, to get the gauge invariant expression for the hadron tensor we sum the contributions of Eqs. (24) and (26). The sum reads

$$\begin{aligned} \overline{\mathcal{W}}_{\mu\nu} &= \int d^2 \vec{q}_\perp \mathcal{W}_{\mu\nu} \\ &= i \int (dx_1)(dy) \delta(x_1 P_1^+ - q^+) \delta(y P_2^- - q^-) \\ &\times \bar{F}(y) \int (dx_2) \tilde{B}(x_1, x_2) \frac{T^\nu}{P_1 \cdot P_2} \left[ \frac{P_1^\mu}{y} - \frac{P_2^\mu}{x_1} \right], \end{aligned} \quad (27)$$

where

$$\bar{F}(y) = \begin{pmatrix} \bar{f}_T(y) \\ \bar{h}_1(y) \end{pmatrix} \quad T^\nu = \begin{pmatrix} \varepsilon^{P_1 + S_\perp P_2^\perp} V_\perp^\nu \\ \varepsilon^{\nu S_\perp P_2 P_1} \end{pmatrix} \quad (28)$$

and

$$\tilde{B}(x_1, x_2) = \frac{1}{2} \frac{\Phi_{(1)}^{\text{tw-3}}(x_1) \otimes \Phi_{(2)}^{\text{tw-2}}(x_2)}{x_2 - x_1 - i\varepsilon}. \quad (29)$$

In Eq. (29), the pion distribution amplitudes of twist-2 and -3 include the different dimensionful prefactors. Notice that the derived gauge invariant hadron tensor, see Eq. (27), coincides formally with the results obtained in [18] for the usual Drell-Yan process.

We now calculate the single spin asymmetry, see Eq. (7). Within the CS frame [11,17], calculating the imaginary part of  $\tilde{B}(x_1, x_2)$  and contracting the leptonic and hadron tensors, we obtain (here,  $\mathcal{L}_{\mu\nu}$  implies the unpolarized lepton tensor)

$$\begin{aligned} \mathcal{L}_{\mu\nu} \Im \overline{\mathcal{W}}_{\mu\nu} &= \bar{F}(y_B) \Phi_{(1)}^{\text{tw-3}}(x_B) \otimes \Phi_{(2)}^{\text{tw-2}}(x_B) \\ &\times \frac{2}{x_B y_B} (\ell_1 \cdot \hat{z}) \left( \frac{(\ell_1^\perp \cdot V^\perp) \vec{S}_\perp \wedge \vec{P}_{2\perp}}{\varepsilon^{\ell_1 S_\perp P_2 P_1}} \right), \end{aligned} \quad (30)$$

where  $(\ell_1 \cdot \hat{z}) = -\frac{Q^2}{2} \cos\theta$ ,  $\varepsilon^{\ell_1 S_\perp P_1 P_2} = -\frac{sQ}{4} |\vec{S}_\perp| \sin\theta \sin\phi_S$ .

Therefore, for the chiral-odd contributions, we predict a new asymmetry which, in terms of [14], reads

$$\mathcal{A}_T = \frac{S_\perp}{Q} \frac{D_{[\sin 2\theta]} \sin\phi_S B_{UT}^{\sin\phi_S}}{\bar{f}_1(y_B) H_1(x_B)}, \quad D_{[\sin 2\theta]} = \frac{\sin 2\theta}{1 + \cos^2\theta}, \quad (31)$$

where  $B_{UT}^{\sin\phi_S} = 2\bar{h}_1(y_B) \Phi_{(1)}^{\text{tw-3}}(x_B) \otimes \Phi_{(2)}^{\text{tw-2}}(x_B)$ ;  $\bar{f}_1(y_B)$  and  $H_1(x_B)$  stem from the unpolarized cross section and they parametrize the following matrix elements:

$$\begin{aligned} \langle P_2 | \bar{\psi} \gamma^- \psi | P_2 \rangle &\stackrel{\mathcal{F}}{\sim} P_2^- \bar{f}_1(y), \\ \langle P_1 | \bar{\psi}_+ | q(K) \rangle \langle q(K) | \psi_+ | P_1 \rangle &\stackrel{\mathcal{F}}{\sim} P_1^+ H_1(x), \end{aligned} \quad (32)$$

where

$$H_1(x) = \frac{1}{2\bar{x}_1 P_1^+} \int (d^2 \vec{k}_1^\perp) \Phi_{(2)}^{[y^+(\gamma_S)]}(\bar{x}_1, \vec{k}_1^\perp) \Phi_{(1)}^{[0(\gamma_S)]}(x_1, \vec{k}_1^\perp). \quad (33)$$

Indeed, the leading twist Siverson asymmetry  $A_{UT}^{\sin\phi_S}$ , which formally stands at the similar tensor combination  $\varepsilon^{\ell_1^\perp S_\perp P_1 P_2}$ , appears only together with the depolarization factor  $D_{[1+\cos^2\theta]}$ . In its turn, the higher twist asymmetries  $A_{UT}^{\sin(\phi_S \pm \phi)}$  at  $D_{[\sin 2\theta]}$  correspond to the different tensor structures,  $\varepsilon^{q S_\perp P_1^\perp P_2} \sim \sin(\phi_S \pm \phi)$ .

In conclusion, we derive the gauge invariant meson-induced DY hadron tensor with the essential spin transversity and primordial transverse momenta. Our calculation includes both the standard-like, which is well known, and nonstandard-like diagram, which is first found in [18], contributions. The latter contribution plays a crucial role for the gauge invariance.

In the paper, we focus on the case where one of the pion distribution amplitudes has been projected onto the chiral-odd combination. The latter singles out the chiral-odd parton distribution inside nucleons. The chiral-odd tensor combinations are very relevant for the future experiments implemented by COMPASS [14]. We predict new single transverse spin asymmetries to be measured experimentally which are associated with the spin transversity and with the nontrivial  $\varphi$ -angular dependence. The latter asymmetry can eventually be treated as a signal for the transverse primordial momentum dependence which probes both gluon poles and time-odd functions. In contrast to [4], our SSA is given by the interference of two amplitudes rather than the square of a given amplitude. We emphasize that the possibility to study different SSAs reachable in COMPASS experiments and induced by chiral-odd and time-odd distribution functions/amplitudes appears only thanks for the gluon pole presence. Thus, the proposed angular dependence of SSA can give implicit evidences for the gluon pole observation in COMPASS experiments.

We model the analog of the collinear parton distribution function  $B^V(x_1, x_2)$ -function of [18] determined in the inclusive channel by means of the gluon propagator and two noncollinear exclusive meson distribution amplitudes of twist-2 and -3. For this modeling, the invariant masses of undetected spectators in the pion sector have to be considered as relatively small. Our model demonstrates

the manifestation of duality between different factorization regimes in exclusive and inclusive channels (see [27]).

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