Yang-Baxter σ-models, conformal twists, and noncommutative Yang-Mills theory

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The Yang-Baxter σ-model is a systematic way to generate integrable deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5$. We recast the deformations as seen by open strings, where the metric is undeformed $\mathrm{AdS}_5 \times \mathrm{S}^5$ with constant string coupling, and all information about the deformation is encoded in the noncommutative (NC) parameter Θ. We identify the deformations of AdS_5 as twists of the conformal algebra, thus explaining the noncommutativity. We show that the unimodularity condition on r-matrices for supergravity solutions translates into Θ being divergence-free. Integrability of the σ-model for unimodular r-matrices implies the existence and planar integrability of the dual NC gauge theory.

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I. INTRODUCTION

Integrable models have been key to enriching our knowledge of condensed matter systems, field theory, and string theory. Within string theory, considerable attention has focused on integrable structures underlying the AdS/Conformal Field Theory (CFT) correspondence [1]. The most studied example is a duality between superstrings on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N} = 4$ super Yang-Mills (sYM). Remarkably, the two-dimensional (2D) string world sheet σ-model on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is classically integrable [2]; it has an infinite set of conserved charges.

There is immense interest in identifying integrable structures beyond the maximally symmetric setting of $\mathrm{AdS}_5 \times \mathrm{S}^5$, or equivalently sYM on $\mathbb{R}^{1,3}$. It is curious that the earliest integrability preserving deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$ [3–5] was inspired by noncommutative (NC) spacetimes, which are ubiquitous in string theory [6,7] (see [8] for a review). In hindsight, we understand these deformations as T-duality shift T-duality (TsT) transformations in the string and gravity side [9–11].

Recently, Yang-Baxter (YB) deformations of the σ-model [12–15] were generalized to the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [16,17]. We now understand TsT transformations as part of a larger class of YB deformations of the σ-model [18–31], which are defined by r-matrices satisfying the classical Yang-Baxter equation (cYBE). A further unimodularity condition ensures the YB deformation has a valid string theory (supergravity) description [32]. It has been conjectured [33] (for the proof of the boson part and the result for the supersymmetric case, see [34]) that homogeneous YB deformations [15,17] may all be realized through non-Abelian duality transformations [35–39].

In this article, we retrace TsT transformations to NC deformations of quantum field theories (QFTs). We encounter a number of surprises. First, irrespective of the YB deformation, for r-matrix solutions to the homogeneous cYBE, there is a universal description in open string parameters. Concretely, we show that the open string metric [7] is always the original *undeformed* $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric with constant open string coupling, and all information about the YB deformation is encoded in a NC parameter Θ. This in particular implies that all YB string theory σ-models of $\mathrm{AdS}_5 \times \mathrm{S}^5$ have a NC gauge theory dual on $\mathbb{R}^{1,3}$ where integrability of the σ-model has direct bearing on planar integrability.

For our second result, sharpening an earlier conjecture [24], we confirm that YB deformations of AdS_5 are simply Drinfeld twists of the conformal algebra. To better understand this fact, we recall that in NC spacetimes the coordinate operators $\hat{x}^\mu$ satisfy the commutation relation,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} \quad (\mu, \nu = 0, \ldots, 3), \tag{1}$$

where $\Theta^{\mu\nu}$ is in general an x-dependent antisymmetric matrix. For twists of Poincaré algebra, the x-dependence of Θ is fixed to be constant, linear, or quadratic [40–42]. As we will argue, however, for twists in the conformal algebra we can also have cubic and quartic dependence. In fact, the homogeneous YB deformations studied to date [18–28,30–32] provide predictions for NC parameters that arise from twists of the *full* conformal algebra. We establish by exhaustion that the NC parameters and r-matrices are directly related [43],

$$\Theta^{MN} = -2\eta r^{MN} \quad (M, N = 0, \ldots, 3, z), \tag{2}$$

where η is the deformation parameter, z is the radial direction of AdS_5, and r^{MN} is the r-matrix expressed as differential operators on AdS_5.

Finally, non-unimodular YB deformations lead to geometries that solve generalized supergravity equations, specified through a modification given by a Killing vector field I [44,45]; setting $I = 0$, we recover usual supergravity. We show Θ and I are related through the equation,

$$\nabla_M \Theta^{MN} = I^N, \tag{3}$$

evaluated with an open string metric. This remarkable result, which marries open and closed string descriptions, is a requirement of the Λ-symmetry [46,47] of the string σ-model. Under Λ-symmetry the Neveu-Schwarz-Neveu-Schwarz (NSNS) two-form B-field is transformed by $\mathrm{d}\Lambda$, which in the presence of D-branes (open strings) must be supplemented by a shift of the gauge field on the brane by a one-form Λ. This novel observation provides the first explanation of the unimodularity condition [32] from a symmetry principle. Observe, for supergravity solutions, Θ^{MN} is divergence-free.

II. CLOSED STRING PICTURE

In an effort to make this article self-contained, we review the essentials of the YB σ-model, following the presentation of Ref. [25]. Here, we restrict ourselves to deformations of AdS_5 by considering the coset space $SO(4,2)/SO(4,1)$ and the homogeneous cYBE. Furthermore, to avoid unnecessary technicalities, we suppress the Ramond-Ramond (RR) sector, which does not affect any of our results. The corresponding YB σ-model action is [15,17]

$$\mathcal{L} = \mathrm{Tr}\left[AP^{(2)}\circ\frac{1}{1-2\eta R_g\circ P^{(2)}}A\right], \tag{4}$$

with a deformation parameter η and $R_g(X) \equiv g^{-1}R(gXg^{-1})g$. Here $A = -g^{-1}\mathrm{d}g$, $g \in SO(4,2)$, is a left-invariant current, while $P^{(2)}$ is a projector onto the coset space $\mathfrak{so}(4,2)/\mathfrak{so}(4,1)$, spanned by the generators $\mathbf{P}_m (m = 0, \ldots, 4)$, which satisfy $\mathrm{Tr}[\mathbf{P}_m\mathbf{P}_n] = \eta_{mn} = \mathrm{diag}(-++++)$. Details, such as matrix representations, are given in [25]. $P^{(2)}$ may be expressed as

$$P^{(2)}(X) = \eta^{mn}\mathrm{Tr}[X\mathbf{P}_m]\mathbf{P}_n, \qquad X \in \mathfrak{so}(4,2). \tag{5}$$

Above, R is an antisymmetric operator satisfying the homogeneous cYBE

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = 0, \tag{6}$$

with $X, Y \in \mathfrak{so}(4,2)$. In turn, the operator R can be written in terms of an r-matrix as

$$R(X) = \mathrm{Tr}_2[r(1\otimes X)] = \sum_{i,j} r^{ij} b_i \mathrm{Tr}[b_j X], \tag{7}$$

where $r \in \mathfrak{so}(4,2)\otimes\mathfrak{so}(4,2)$ is

$$r = \frac{1}{2}\sum_{i,j} r^{ij} b_i \wedge b_j, \quad \text{with} \quad b_i \in \mathfrak{so}(4,2). \tag{8}$$

The r-matrix is called Abelian if $[b_i, b_j] = 0$ and unimodular if it satisfies the following condition [32]:

$$r^{ij}[b_i, b_j] = 0. \tag{9}$$

Note i, j range over the generators of $\mathfrak{so}(4,2)$, but expressed as differential operators on AdS_5, one finds r^{MN}.

To determine the YB deformed geometry, we adopt the following parametrization for $g \in SO(4,2)$:

$$g = \exp[x^\mu P_\mu]\exp[(\log z)D], \tag{10}$$

where $P_\mu (\mu = 0, \ldots, 3)$, D, respectively, denote translation and dilatation generators and are related to $\mathbf{P}_m$ [25]. In terms of these coordinates, we define

$$r = \frac{1}{2}r^{MN}\partial_M\wedge\partial_N, \qquad \partial_M \in \{\partial_\mu, \partial_z\}. \tag{11}$$

Then, the YB deformed metric $g_{MN} (M, N = 0, \ldots, 4)$, NSNS two-form B_{MN}, and dilaton Φ (in string frame) can be expressed as [25]

$$g_{MN} = e_M^m e_N^n k_{(mn)}, \qquad B_{MN} = e_M^m e_N^n k_{[nm]}, \tag{12}$$

$$\mathrm{e}^\Phi = g_s(\det_5 k)^{-1/2}, \qquad k_{mn} = k_{(mn)} + k_{[mn]}, \tag{13}$$

where e_M^m is the AdS_5 vielbein, and we have defined

$$k_m{}^n \equiv (\delta^m{}_n - 2\eta\lambda^m{}_n)^{-1}, \tag{14}$$

$$\lambda_m{}^n \equiv \eta^{nl}\mathrm{Tr}[\mathbf{P}_l R_g(\mathbf{P}_m)]. \tag{15}$$

It is useful to exemplify the deformation for the simplest case of the Abelian r-matrix [19],

$$r = \frac{1}{2}P_2\wedge P_3, \tag{16}$$

corresponding to the closed string background [3,4],

$$\mathrm{d}s^2 = \frac{1}{z^2}[-\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + h(z)(\mathrm{d}x_2^2 + \mathrm{d}x_3^2) + \mathrm{d}z^2],$$
$$B_{23} = \eta h(z)/z^4, \qquad \mathrm{e}^{2\Phi} = g_s^2 h(z), \tag{17}$$

where $h^{-1} = 1 + \eta^2 z^4$. The above together with S^5 and the RR-fields constitute a supergravity solution, which is obtained simply via TsT from $\mathrm{AdS}_5\times\mathrm{S}^5$ [3,4].

In passing, we comment that while we focus on AdS_5, following [18], similar arguments apply equally to S^5. In particular, the case of β [9] or γ-deformations [10] is related to Abelian twists of $SO(6)$, and via AdS/CFT, to marginal deformations of $\mathcal{N}=4$ sYM [48].

III. OPEN STRING PICTURE

Given closed string parameters (g_{MN}, B_{MN}, g_s), the open string metric G_{MN}, NC parameter Θ^{MN}, and coupling G_s are defined as [7]

$$G_{MN} = (g - Bg^{-1}B)_{MN}, \tag{18}$$

$$\Theta^{MN} = -((g+B)^{-1}B(g-B)^{-1})^{MN}, \tag{19}$$

$$G_s = g_s e^{\Phi}\left(\frac{\det(g+B)}{\det g}\right)^{\frac{1}{2}}. \tag{20}$$

For YB deformations of AdS_5 (12), we find

$$G^{MN} + \Theta^{MN} = e^M_m e^N_n(\eta^{mn} + 2\eta\lambda^{mn}), \tag{21}$$

where e^M_m denotes the inverse vielbein. As λ^{mn} is antisymmetric, it is easy to separate the components, getting

$$G^{MN} = e^M_m e^N_n \eta^{mn}, \qquad \Theta^{MN} = 2\eta e^M_m e^N_n \lambda^{mn}. \tag{22}$$

Inverting G^{MN}, it is clear that the open string metric is precisely the original AdS_5 metric. Moreover, inserting (12) and (13) into (20), we get $G_s = g_s = \mathrm{const}$. That is, all the information about the YB deformation, as viewed by open strings, is sitting in Θ^{MN}, while the geometry is undeformed AdS_5 [49].

For the example (17), the open string parameters are

$$\begin{aligned} ds^2_{\mathrm{open}} &= \frac{1}{z^2}(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2), \\ \Theta^{23} &= -\eta, \qquad G_s = g_s. \end{aligned} \tag{23}$$

While the closed string metric (17) has a severely deformed causal and boundary structure [3–5], the spacetime as seen by the open strings is the usual $\mathrm{AdS}_5 \times \mathrm{S}^5$ with $\mathbb{R}^{1,3}$ boundary, indicating that the dual gauge theory description is a Θ-deformed sYM.

IV. CONFORMAL TWISTS AND NC GAUGE THEORY

One can formulate the QFT on the NC spacetime specified by Θ (1). Let us start with the constant Θ case, relevant to the example (17). The NCQFT may be obtained by replacing the usual product of functions, or fields in QFT, with the Moyal star product, $f(x)g(x) \to (f\star g)(x)$, such that

$$(f\star g)(x) = f(x) e^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}} g(x). \tag{24}$$

The Moyal bracket of two functions is defined to be

$$[f,g]_\star := f\star g - g\star f = i\Theta^{\mu\nu}\partial_\mu f\partial_\nu g + \mathcal{O}(\partial^3 f, \partial^3 g). \tag{25}$$

It is worth noting that $f(x) = x^\mu$, $g(x) = x^\nu$ reproduces the commutator (1). It has been shown that the introduction of the Moyal $\star$-product is equivalent to using the coproducts with a Drinfeld twist element [40],

$$\mathcal{F} = e^{-2i\eta r} = e^{\frac{i}{2}\Theta^{\mu\nu}P_\mu\wedge P_\nu}. \tag{26}$$

This is a special case of an Abelian Poincaré twist, and the r-matrix satisfies the cYBE [19]. Abelian twists have the remarkable property that they do not affect the Poincaré algebra $\mathcal{P}$ [40], but instead deform the coproduct of $\mathcal{U}(P)$ [51], where $\mathcal{U}(P)$ is the universal enveloping algebra of the Poincaré algebra.

In (26), we have considered the simplest twist, with constant Θ. However, for other solutions to the cYBE, the NC parameter need not be a constant. Indeed, including Lorentz generators $M_{\mu\nu}$, the cYBE has solutions $r \sim P \wedge M$ and $r \sim M \wedge M$, which, respectively, lead to linear and quadratic Θ [42]. For example, for $r = \frac{1}{2}M_{01}\wedge M_{23}$, modulo a convention dependent sign in the twist (26), the NC parameter has components [42]

$$\begin{aligned} \Theta^{02} &= -2\sinh\frac{\eta}{2}\cdot x^1x^3, \qquad \Theta^{03} = 2\sinh\frac{\eta}{2}\cdot x^1x^2, \\ \Theta^{12} &= -2\sinh\frac{\eta}{2}\cdot x^0x^3, \qquad \Theta^{13} = 2\sinh\frac{\eta}{2}\cdot x^0x^2. \end{aligned} \tag{27}$$

We recover the same result (at leading order) from the YB prescription (22).

This example shows that the open string parameter Θ knows about the Moyal bracket, which may be derived from twists of the Poincaré algebra. One can repeat the YB analysis for *all* r-matrices of the *conformal algebra* and show that (2) holds once the r-matrix is expressed in terms of differential operators [43]. Note, (2) generalizes existing results [24,30] from the Poincaré to conformal algebra.

In support of our claim, we present two examples,

$$\begin{aligned} r_1 &= \frac{1}{2}D\wedge K_1, \\ r_2 &= \frac{1}{2}(P_0 - P_3)\wedge(D + M_{03}), \end{aligned} \tag{28}$$

which involve scale D and special conformal symmetries K_μ. Note, the first is non-unimodular and the second appears in the classification of unimodular r-matrices [32]. The NC parameter in each case can easily be calculated from (15) and (22). For r_1, we find

$$\Theta^{1\mu} = \eta x^\mu(x_\nu x^\nu + z^2), \qquad \Theta^{1z} = \eta z(x_\nu x^\nu + z^2), \tag{29}$$

where $\mu \neq 1$, while for r_2, we get

$$\Theta^{-+} = -4\eta x^{+}, \quad \Theta^{-i} = -2\eta x^{i}, \quad \Theta^{-z} = -2\eta z, \qquad (30)$$

where $i = 1, 2$ and we have employed $x^{\pm} = x^0 \pm x^3$. One recovers the same results from conformal twists of the dual CFT [43]. We interpret this mathematical agreement as evidence in support of our claim that YB deformations based on unimodular r-matrices are dual to NC deformations of $\mathcal{N} = 4$ sYM. We establish this through an almost exhausting set of examples in our upcoming work [43].

Some comments and remarks are in order:

(1) In both cases one can confirm that Eq. (2) holds.

(2) One generically encounters cubic and quartic terms from conformal twists.

(3) Not only are there nonzero $\Theta^{z\mu}$ components, they also have nontrivial z-dependence. Nonetheless, it can be shown in general that $\Theta^{z\mu}$ components vanish at the AdS boundary at $z = 0$, where the dual field theory resides. Viewing Eq. (3) as a first order equation for Θ^{MN}, the z-components and dependence can be recovered from the $\Theta^{\mu\nu}$; no information is lost in the dual field theory side.

(4) For YB deformations corresponding to unimodular r-matrices, there is a well-defined string theory picture. Following the usual reasoning of AdS/CFT, wherever the decoupling limit exists, closed string theory on these deformed AdS_5 backgrounds is expected to be dual to NC deformations of sYM with noncommutativity $\Theta^{\mu\nu} = -2\eta r^{\mu\nu}$. Particular examples are discussed in [3–5]. However, we note that the existence of a decoupling limit, where the open string theory is reduced to its low energy limit of NC sYM, is not trivial [5] (see also [24,30] for related discussion). For the cases with $\Theta^{\mu\nu}\Theta_{\mu\nu} < 0$, so-called "electric" noncommutativity, it has been argued that the open string theory does not reduce to NC sYM. In these cases we are dealing with the noncritical NC open string theory (NCOS) [52–55] which is related to NC sYM at strong coupling.

V. UNIMODULARITY AND Λ-SYMMETRY

Our statements about the universal open string description are true, irrespective of unimodularity. Here, we address the origin of unimodularity in terms of string theory and its symmetry.

The key to our explanation is Λ-symmetry [46,47]. It is known that closed string theory (supergravity) is invariant under $B \to B + \mathrm{d}\Lambda$, where B is the NSNS two-form and Λ is an arbitrary one-form. Upon introduction of open strings with Dirichlet boundary conditions, this symmetry survives, since B appears in the brane Dirac-Born-Infeld (DBI) action only through the combination $B + F$, where $F = \mathrm{d}A$ is the field strength of the brane gauge field A [56], and one can compensate by shifting $A \to A - \Lambda$. Therefore, the action of the system, which is the sum of the supergravity and DBI actions, maintains the Λ-symmetry.

Open string parameters (18), (19), and (20), however, are defined in a particular Λ-gauge, where the expectation (or background) value of F is set to zero. So, the expression for Θ^{MN} (19) is not necessarily Λ-invariant [7,47]. In fact, recalling that when F is set to zero [7],

$$\frac{1}{G_s}\sqrt{\det G} = \frac{\mathrm{e}^{\Phi}}{g_s}\sqrt{\det(g+B)},$$

one can readily see that the variation of the DBI action with respect to Λ-symmetry is $\nabla_M\Theta^{MN}$, where the divergence is computed with respect to open string metric G_{MN}. So, invariance of the full action for the unimodular cases where the supergravity part is Λ-invariant on its own leads to $\nabla_M\Theta^{MN} = 0$. See also [57] for related arguments.

For the non-unimodular cases, where we encounter generalized supergravity equations with Killing vector I, one can show that these equations are Λ-symmetric. However, the presence of the isometry direction I would modify the DBI action by an $I^M A_M$ term, which is not Λ-invariant [43]. Therefore, to restore Λ-symmetry, the NC parameter should satisfy (3). As an example consider r_1 in (28), which is known to be non-unimodular with $I = K_1$. One can then explicitly check that Θ given in (29) satisfies (3).

VI. OUTLOOK

Our observations and results have broad implications. It is imperative to revisit Poincaré twists [40–42] and extend them to conformal twists [43], thus testing our claim that the conformal twists can be described as YB deformations. While we considered only bosonic deformations of AdS_5, one can easily repeat for different coset spaces, in different dimensions, or extend the analysis to the fermionic sector of the $AdS_5 \times S^5$ σ-model, where one will encounter fermionic T-duality [58,59], or potentially a non-Abelian generalization of it.

We recall that the homogeneous YB deformations may be described as non-Abelian T-duality [33]. In principle, a careful treatment of the Θ parameter for non-Abelian T-duals supported by RR flux [38,39] may elucidate the dual theory [60]. It is interesting that the open string, via Λ-symmetry, knows about generalized supergravity through I. Since the latter is reproducible from the double field theory description, it may be interesting to push this connection by following [63–65].

The $AdS_5 \times S^5$ YB σ-model integrability has implications for the dual gauge theory and the dual open strings. The fact that open strings reside in an undeformed $AdS_5 \times S^5$ geometry prompts the proposal of integrability of the corresponding open string σ-model. The effects of the deformation should then appear in Θ which is expected to affect only open string end point dynamics (which end on the $AdS_5 \times S^5$ boundary). This open string integrability dovetails with the fact that some of the deformed backgrounds can be obtained through TsT transformations and that T-duality is a symmetry of the world sheet theory. Establishing this open string integrability proposal, however, requires a thorough analysis of the boundary conditions.

Integrability of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ σ-model is intimately connected with the planar integrability of the corresponding dual $\mathcal{N} = 4$ sYM. With the same token, one would expect the associated NC sYM to be planar integrable. Some preliminary analysis and results for a special case have already appeared [66]. This is a highly nontrivial statement and extends the important sYM integrability to a big list of NC gauge theories. In the same line, one would expect that Drinfeld twists and Drinfeld doubles of the original Yangian, which underlies the integrability of sYM, to be at work for the NC cases.

It is known that the constant magnetic NC sYM at strong coupling flows to the NCOS [52–55]. It is interesting to check if the same feature extends to more general x-dependent twist elements. Recalling the S-duality of type IIB supergravity, this is expected to be the case. It is also interesting to explore the direct consequences of the twisted conformal symmetry on the corresponding NCOS and in particular features like Hagedorn transition [67]. One may also explore extending these considerations about the S-duality and NCOS to the non-unimodular cases and to generalized IIB supergravity.

VII. DATA MANAGEMENT

No additional research data beyond the data presented and cited in this work are needed to validate the research findings in this work.

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APPENDIX

1. Four-dimensional conformal algebra

We record the conformal algebra $\mathfrak{so}(4,2)$ employed in this work,

$$\begin{aligned} &[D, P_\mu] = P_\mu, \qquad [D, K_\mu] = -K_\mu, \\ &[P_\mu, K_\nu] = 2(\eta_{\mu\nu} D + M_{\mu\nu}), \\ &[M_{\mu\nu}, P_\rho] = -2\eta_{\rho[\mu}P_{\nu]}, \qquad [M_{\mu\nu}, K_\rho] = -2\eta_{\rho[\mu}K_{\nu]}, \\ &[M_{\mu\nu}, M_{\rho\sigma}] = -\eta_{\mu\rho}M_{\nu\sigma} + \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\sigma}M_{\mu\rho}. \end{aligned} \tag{A1}$$

The algebra can be realized in terms of differential operators as

$$\begin{aligned} &P_\mu = -\partial_\mu, \qquad K_\mu = -(x_\nu x^\nu + z^2)\partial_\mu + 2x_\mu(x^\nu\partial_\nu + z\partial_z), \\ &D = -x^\mu\partial_\mu - z\partial_z, \qquad M_{\mu\nu} = x_\mu\partial_\nu - x_\nu\partial_\mu. \end{aligned} \tag{A2}$$

2. Duality between YB σ-models and NC sYM for conformal twists

In the body of this article, we determined Θ^{MN} for two r-matrices r_1 and r_2. We have conjectured for unimodular r-matrices, for example r_2, that the YB deformation is dual to a NC deformation of $\mathcal{N} = 4$ sYM. To support this claim, we now show that Eq. (30), evaluated at $z = 0$, agrees with the resulting NC parameter from the corresponding conformal twist. The analysis presented here generalizes known Drinfeld twists with respect to the Poincaré subalgebra to the conformal algebra.

Let us recall the r-matrix,

$$r_2 = \frac{1}{2}(P_0 - P_3) \wedge (D + M_{03}). \tag{A3}$$

We introduce null coordinates, $x^\pm = x^0 \pm x^3$, so that the four-dimensional Minkowski metric is

$$\mathrm{d}s^2 = -\mathrm{d}x^+\mathrm{d}x^- + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2. \tag{A4}$$

It is worth noting that $\eta_{+-} = -\frac{1}{2}$, $\eta^{+-} = -2$. In these coordinates, the generators correspond to differential operators,

$$P_0 - P_3 = -2\partial_-, \quad D + M_{03} = -2x^+\partial_+ - x^1\partial_1 - x^2\partial_2. \tag{A5}$$

Note, there is no z-dependence, and the operators are essentially the AdS_5 Killing vectors evaluated at $z = 0$. Following the standard procedure, we introduce the twist element, which acts on the commutative algebra $\mathcal{A}$ of functions, $f(x)$, $g(x)$, in Minkowski space,

$$\mathcal{F} = \mathrm{e}^{-2i\eta r_2} = \mathrm{e}^{-i\eta(P_0 - P_3)\wedge(D + M_{03})}. \tag{A6}$$

The star product then takes the form

$$\begin{aligned} f(x)\star g(x) &= m\circ\mathcal{F}(f(x)\otimes g(x)) \\ &= m\circ e^{-i\eta(P_0-P_3)\wedge(D+M_{03})}(f(x)\otimes g(x)) \\ &= m\circ e^{-i\eta\partial_-\wedge(2x^+\partial_++x^1\partial_1+x^2\partial_2)}(f(x)\otimes g(x)), \end{aligned} \tag{A7}$$

where m denotes the operation of commutative multiplication, $m(f(x)\otimes g(x)) \coloneqq f(x)g(x)$. Taking $f(x)=x^\mu$, $g(x)=x^\nu$, $\mu,\ \nu=+,\ -,\ 1,\ 2$, while expanding to first order, one finds

$$\begin{aligned} x^\mu\star x^\nu &= x^\mu x^\nu - \frac{i}{2}\eta(x^+\eta^{\mu+}\eta^{\nu-} - x^1\eta^{\mu+}\eta^{\nu 1} \\ &\quad - x^2\eta^{\mu+}\eta^{\nu 1} - \mu\leftrightarrow\nu), \\ x^\nu\star x^\mu &= x^\nu x^\mu - \frac{i}{2}\eta(x^+\eta^{\nu+}\eta^{\mu-} - x^1\eta^{\nu+}\eta^{\mu 1} \\ &\quad - x^2\eta^{\nu+}\eta^{\mu 1} - \nu\leftrightarrow\mu). \end{aligned} \tag{A8}$$

Therefore, the Moyal bracket is

$$\begin{aligned} [x^\mu, x^\nu]_\star &= x^\mu\star x^\nu - x^\nu\star x^\mu \\ &= -i\eta(x^+\eta^{\mu+}\eta^{\nu-} - x^1\eta^{\mu+}\eta^{\nu 1} - x^2\eta^{\mu+}\eta^{\nu 1} - \mu\leftrightarrow\nu). \end{aligned}$$

At this stage, it is easy to read off the nonzero components of $\Theta^{\mu\nu}$,

$$\begin{aligned} \Theta^{-+} &= -4\eta x^+, \\ \Theta^{-1} &= -2\eta x^1, \\ \Theta^{-2} &= -2\eta x^2. \end{aligned} \tag{A9}$$

This precisely agrees with Eq. (30), which was derived from the open string description and evaluated at $z=0$.

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