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Neutrino and *CP*-even Higgs boson masses in a nonuniversal $U(1)'$ extension

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We propose a new anomaly-free and family nonuniversal $U(1)'$ extension of the standard model with the addition of two scalar singlets and a new scalar doublet. The quark sector is extended by adding three exotic quark singlets, while the lepton sector includes two exotic charged lepton singlets, three right-handed neutrinos, and three sterile Majorana leptons to obtain the fermionic mass spectrum of the standard model. The lepton sector also reproduces the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and the squared-mass differences data from neutrino oscillation experiments. Also, analytical relations of the PMNS matrix are derived via the inverse seesaw mechanism, and numerical predictions of the parameters in both normal and inverse order scheme for the mass of the phenomenological neutrinos are obtained. We employed a simple seesawlike method to obtain analytical mass eigenstates of the *CP*-even 3×3 mass matrix of the scalar sector.

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I. INTRODUCTION

Despite all its success, the standard model (SM) of Glashow, Weinberg and Salam [1] has some unexplained features, which has motivated many models and extensions. In particular, the observed fermion mass hierarchies, their mixing and the three family structure are not explained in the SM. From the phenomenological point of view, it is possible to describe some features of the mass hierarchy by assuming zero-texture Yukawa matrices [2]. Models with spontaneously broken flavor symmetries may also produce hierarchical mass structures. For example, in models with gauge symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, the electroweak doublets exhibit a discrete symmetry after the spontaneous symmetry breaking, obtaining Fritzsch zero-texture mass matrices [3] in the basis $\mathbf{U} = (u_0, c_0, t_0)$ of the form:

$$-\langle \mathcal{L}_{Y,U} \rangle_0 = \overline{\mathbf{U}_L} \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix} \mathbf{U}_R + \text{H.c.} \tag{1}$$

The zero-texture of the above matrix can describe the mass spectrum in the quark sector and the *CP* violation phase observed in the experiments. This mass structure can also be obtained in the lepton sector, as shown by Fukugita, Tanimoto y Yanagida [4], where very small mass values are predicted through a seesaw mechanism. In addition, these type of models contain Majorana neutrinos which induce matter-antimatter asymmetry through leptogenesis [5].

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Another issue that the SM can not explain is the observation of neutrino oscillations. These observations have been confirmed by many experiments from four different sources: solar neutrinos as in Homestake [6], SAGE [7], GALLEX & GNO [8], SNO [9], Borexino [10], and Super-Kamiokande [11] experiments, atmospheric neutrinos as in IceCube [12], neutrinos from reactors as KamLAND [13], CHOOZ [14], Palo Verde [15], Daya Bay [16], RENO [17], and SBL [18], and from accelerators as in MINOS [19], T2K [20], and NOνA [21]. The experimental data are compatible with the hypothesis that at least two species of neutrinos have mass, where the left-handed flavor neutrino fields are linear combinations of mass eigenstates,

$$|\nu_L^a\rangle = \sum_{i=1,2,3} U_{ai} |\nu_L^i\rangle, \qquad a = e, \mu, \tau, \tag{2}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which can be parametrized as a function of three mixing angles and one *CP* violating phase [22,23]. However, the experiments cannot determine the true nature of the active neutrinos (Majorana or Dirac) nor the absolute values of their mass. Table I shows the parameters from references [22,24] and available at NuFIT 3.0 [25], where two hierarchies are assumed: normal ordering (NO), where the squared mass difference between the third and first species accomplish $\Delta m_{31}^2 > 0$, and inverted ordering (IO), where $\Delta m_{32}^2 < 0$ between the second and third species.

On the other hand, in order to obtain tiny neutrino masses, two methods can be used: radiative corrections and the seesaw mechanism. The latter scheme has been studied in the literature and is considered as one of the most traditional schemes for the explanation of smallness of

TABLE I. Three-flavor oscillation parameter values at 1σ reported by [22,24]. $\ell = 1$ for NO and 2 for IO.

	Normal ordering (NO)	Inverted ordering (IO)
$\sin^2\theta_{12}$	$0.308^{+0.013}_{-0.012}$	$0.308^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.440^{+0.023}_{-0.019}$	$0.584^{+0.018}_{-0.022}$
$\sin^2\theta_{13}$	$0.02163^{+0.00074}_{-0.00074}$	$0.02175^{+0.00075}_{-0.00074}$
δ_{CP}	289^{+38}_{-51}	269^{+39}_{-45}
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.17}$	$7.49^{+0.19}_{-0.17}$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.526^{+0.039}_{-0.037}$	$-2.518^{+0.038}_{-0.037}$

neutrino masses. The seesaw mechanism implies the addition of a lepton-number-violating high-energy scale (M), which gives masses to light neutrinos as $m_\nu = v_w^2/M$. There are some basic ways to implement this mechanism: a heavy right-handed Majorana neutrino ν_R mixed to the corresponding left-handed neutrino ν_L via the SM scalar doublet (type I seesaw), a heavy scalar triplet bosons (type II), or a heavy fermionic triplet (type III). Since the new scale M associated with the new fields is high ($\sim 10^{12}$ GeV), this mechanism cannot be tested in experiments. However, there is another possibility: the inverse seesaw mechanism (ISS), where a very light Majorana neutrino N_R is incorporated, such that in the basis $(\nu_L, \nu_R{}^c, N_R{}^c)$ the mass matrix has the form of the Fristzsch zero-texture:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_\nu^{\mathrm{T}} & 0 \\ m_\nu & 0 & m_N^{\mathrm{T}} \\ 0 & m_N & M_N \end{pmatrix}, \tag{3}$$

where the submatrix m_N has components of the order of the TeV scale, while M_N is of the order of the KeV scale, in order to obtain active neutrinos at the sub-eV scale. The inverse seesaw mechanism was proposed in [26]. This mechanism has also been implemented in the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ models in order to study the $\mu \to e\gamma$ decay [27].

On the other hand, the discovery of the Higgs boson at ATLAS [28] and CMS [29] whose mass is 125 GeV opens the window to propose other scalar fields. A new scalar sector is considered as extension to the SM in order to explain some phenomenological aspects. One of the most studied SM extension is the two-Higgs-doublet-model (2HDM) which proposes the existence of two scalar doublets whose scalar potential mixes them together obtaining two charged scalar bosons $H^\pm$, a CP-odd pseudoscalar A^0 and two CP-even scalar bosons h and H [30]. This model was motivated in order to give masses to uplike and downlike quarks [31] where vacuum expectation values (VEV) v_2 and v_1 are related to the electroweak VEV by $v^2 = v_2^2 + v_1^2$.

There are also extensions to the 2HDM adding a new scalar singlet χ, as in the Next-to-Minimal 2HDM (N2HDM) [32]. In some cases, this additional singlet implement the spontaneous symmetry breaking (SSB) of an additional $U(1)'$ gauge symmetry through the acquisition of non-vanishing VEV v_χ, and consequently its imaginary part become in the would-be Goldstone boson eaten by the corresponding gauge boson of $U(1)'$ [33]. Furthermore, if this SSB happens at a higher scale than the electroweak ($v \ll v_\chi$), the CP-even mass matrix exhibits an internal hierarchy which allows us to employ a perturbative seesawlike method in order to obtain analytical expressions for the mass eigenvalues and angles of the corresponding mixing matrix.

Models with extra $\mathrm{U}(1)'$ symmetry are one of the most studied extensions of the SM, which implies many phenomenological and theoretical advantages including flavor physics [34], neutrino physics [35], dark matter [36], among other effects [37]. A complete review of the above possibilities can be found in reference [38]. In particular, family nonuniversal $\mathrm{U}(1)'$ symmetry models have many well-established motivations. For example, they provide hints for solving the SM flavor puzzle, where even though all the fermions acquire masses at the same scale, $v = 246$ GeV, experimentally they exhibit very different mass values. These models also imply a new Z' neutral boson, which contains a large number of phenomenological consequences at low and high energies [39]. In addition to the new neutral gauge boson Z', an extended fermion spectrum is necessary in order to obtain an anomaly-free theory. Also, the new symmetry requires an extended scalar sector in order to (i) generate the breaking of the new Abelian symmetry and (ii) obtain heavy masses for the new Z' gauge boson and the extra fermion content. A nonuniversal $\mathrm{U}(1)'$ model in the quark sector was proposed in [33], obtaining zero-texture quark mass matrices with hierarchical structures, where three quarks [up, down and strange] acquire masses at the MeV scale, and three quarks [charm, bottom and top] exhibit masses at the GeV scale. Additional phenomenological consequences of this model were studied in [40–42] including effects on scalar DM.

The main purpose of this paper is to construct an anomaly-free and family nonuniversal $\mathrm{U}(1)'$ symmetry model in both the quark and leptonic sector, with extra lepton and quark singlets, two scalar doublets, and two scalar singlets. The leptonic sector includes new charged and right-handed neutral leptons, and sterile Majorana neutrinos in order to reproduce the PMNS matrix and the observed mass structure of the leptons. In Sec. II, we describe the spectrum and most important properties of the model. We also show the scalar and gauge Lagrangians, including rotations into mass eigenvectors. In Sec. III we show how mass structures in the fermion sector are predicted in the model, first for the quark sector in subsection III A, and later for the leptonic sector in subsection III B. Sec. IV is devoted to obtain some phenomenological parameters from neutrino oscillation data at 1σ. Finally, the Sec. V outlines the main results of the article.

II. NONUNIVERSAL MODEL WITH EXTRA $U(1)_X$ SYMMETRY

The model proposes the existence of a new nonuniversal gauge group $U(1)'$ whose gauge boson and coupling constant are Z'_μ and g_X, respectively. It brings the following triangle anomaly equations:

$$[SU(3)_C]^2 U(1)_X \to A_C = \sum_Q X_{Q_L} - \sum_Q X_{Q_R}, \tag{4}$$

$$[SU(2)_L]^2 U(1)_X \to A_L = \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L}, \tag{5}$$

$$[U(1)_Y]^2 U(1)_X \to A_{Y^2} = \sum_{\ell,Q} [Y^2_{\ell_L} X_{\ell_L} + 3Y^2_{Q_L} X_{Q_L}] - \sum_{\ell,Q} [Y^2_{\ell_R} X_{L_R} + 3Y^2_{Q_R} X_{Q_R}], \tag{6}$$

$$U(1)_Y [U(1)_X]^2 \to A_Y = \sum_{\ell,Q} [Y_{\ell_L} X^2_{\ell_L} + 3Y_{Q_L} X^2_{Q_L}] - \sum_{\ell,Q} [Y_{\ell_R} X^2_{\ell_R} + 3Y_{Q_R} X^2_{Q_R}], \tag{7}$$

$$[U(1)_X]^3 \to A_X = \sum_{\ell,Q} [X^3_{\ell_L} + 3X^3_{Q_L}] - \sum_{\ell,Q} [X^3_{\ell_R} + 3X^3_{Q_R}], \tag{8}$$

$$[\mathrm{Grav}]^2 U(1)_X \to A_G = \sum_{\ell,Q} [X_{\ell_L} + 3X_{Q_L}] - \sum_{\ell,Q} [X_{\ell_R} + 3X_{Q_R}], \tag{9}$$

where the sums in Q run over quarks while ℓ runs over leptons with nontrivial $U(1)_X$ values. Y is the corresponding weak hypercharge. The fermion content compatible with the above conditions is composed by ordinary SM particles but also new exotic non-SM particles, as shown in Table II, where column X contains the quantum numbers of the extra $U(1)_X$ and the $\mathbf{Z}_2$ column presents their corresponding Z_2-parity under a new Z_2 discrete symmetry. Some properties of this spectrum are outlined below:

1. The $U(1)_X$ symmetry is only nonuniversal in the left-handed SM quark sector: the first family 1 has $X = 1/3$ while the last two 2,3 have $X = 0$. Leptons exhibit nonuniversal charges in both left- and right-handed sectors: $X = 0$ for the left-handed components e, μ and $X = -1$ for τ, while for the right-handed components $X = -4/3$ for e, τ and $X = -1/3$ for μ. We use the following assignation for the phenomenological families:

$$U^{1,2,3} = (u,c,t), \qquad D^{1,2,3} = (d,s,b),$$
$$e^{e,\mu,\tau} = (e,\mu,\tau), \qquad \nu^{e,\mu,\tau} = (\nu^e,\nu^\mu,\nu^\tau). \tag{10}$$

2. In order to ensure cancellation of the gauge chiral anomalies, the model includes extra isospin singlets. The quark sector has an up T and two down $J^{1,2}$ quarks. For the lepton sector, three right-handed neutrinos $\nu_R^{e,\mu,\tau}$ and two charged leptons E and $\mathcal{E}$ are added with nontrivial $U(1)_X$ charges, as shown in Table II.
3. The most natural way to obtain massive neutrinos, according to neutrino oscillations, is through a seesaw mechanism, which requires the introduction of extra Majorana neutrinos. Thus, for obtaining a realistic model compatible with massive neutrinos, three sterile Majorana neutrinos $N_R^{e,\mu,\tau}$ are included.

TABLE II. Nonuniversal X quantum number and $\mathbf{Z}_2$ parity for SM and non-SM fermions.

Quarks	X	$\mathbf{Z}_2$	Leptons	X	$\mathbf{Z}_2$
SM fermionic isospin doublets					
$q^1_L = \begin{pmatrix} U^1 \\ D^1 \end{pmatrix}_L$	$+1/3$	$+$	$\ell^e_L = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0	$+$
$q^2_L = \begin{pmatrix} U^2 \\ D^2 \end{pmatrix}_L$	0	$-$	$\ell^\mu_L = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	0	$+$
$q^3_L = \begin{pmatrix} U^3 \\ D^3 \end{pmatrix}_L$	0	$+$	$\ell^\tau_L = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1	$+$
SM fermionic isospin singlets					
$U_R^{1,3}$	$+2/3$	$+$	$e_R^{e,\tau}$	$-4/3$	$-$
U_R^2	$+2/3$	$-$	e_R^μ	$-1/3$	$-$
$D_R^{1,2,3}$	$-1/3$	$-$			
Non-SM quarks			Non-SM leptons		
T_L	$+1/3$	$-$	$\nu_R^{e,\mu,\tau}$	$1/3$	$-$
T_R	$+2/3$	$-$	$N_R^{e,\mu,\tau}$	0	$-$
$J_L^{1,2}$	0	$+$	E_L, $\mathcal{E}_R$	-1	$+$
$J_R^{1,2}$	$-1/3$	$+$	$\mathcal{E}_L$, E_R	$-2/3$	$+$

The scalar sector of the model is shown in Table III, which exhibits the following properties:

1. Two scalar doublets $\phi_{1,2}$ are included with $U(1)_X$ charges $+2/3$ and $+1/3$ respectively, whose vacuum expectation values (VEVs) are related to the electroweak VEV by $v = \sqrt{v_1^2 + v_2^2}$. The internal $\mathbf{Z}_2$ symmetry is introduced in order to obtain adequate zero texture matrices.
2. An extra scalar singlet χ with VEV v_χ is required for the SSB of $U(1)_X$ and also to generate masses to

TABLE III. Nonuniversal X quantum number for Higgs fields.

Scalar bosons	X	$\mathbf{Z}_2$
Higgs doublets		
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	2/3	+
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	1/3	−
Higgs singlets		
$\chi = \frac{\xi_\chi+v_\chi+i\zeta_\chi}{\sqrt{2}}$	−1/3	+
σ	−1/3	−

exotic isospin singlets. We assume that it happens at a larger scale $v_\chi \gg v$ than electroweak.

3. Another scalar singlet σ is introduced. Since it is not essential for the symmetry breaking mechanisms, we may choose $v_\sigma = 0$ for its VEV.

Finally, in the vector sector, an extra gauge boson Z'_μ is required to obtain a local $\mathrm{U}(1)_X$ symmetry. The covariant derivative of the model is

$$D_\mu = \partial_\mu - igW^\alpha_\mu T_\alpha - ig'\frac{Y}{2}B_\mu - ig_X X Z'_\mu, \qquad (11)$$

where $2T^\alpha$ corresponds to the Pauli matrices for isospin doublets and $T^\alpha = 0$ for isospin singlets. The electric charge is defined by the Gell-Mann-Nishijima relation:

$$Q = T_3 + \frac{Y}{2}. \qquad (12)$$

A. Scalar masses

The scalar potential of the model is

$$\begin{aligned} V = {} & \mu_1^2\phi_1^\dagger\phi_1 + \mu_2^2\phi_2^\dagger\phi_2 + \mu_\chi^2\chi^*\chi + \mu_\sigma^2\sigma^*\sigma + \frac{f}{\sqrt{2}}(\phi_1^\dagger\phi_2\chi^* + \mathrm{H.c.}) + \frac{f'}{\sqrt{2}}(\phi_1^\dagger\phi_2\sigma^* + \mathrm{H.c.}) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 \\ & + \lambda_3(\chi^*\chi)^2 + \lambda_4(\sigma^*\sigma)^2 + \lambda_5(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda'_5(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (\phi_1^\dagger\phi_1)[\lambda_6(\chi^*\chi) + \lambda'_6(\sigma^*\sigma)] \\ & + (\phi_2^\dagger\phi_2)[\lambda_7(\chi^*\chi) + \lambda'_7(\sigma^*\sigma)] + \lambda_8(\chi^*\chi)(\sigma^*\sigma) + \lambda'_8[(\chi^*\sigma)(\chi^*\sigma) + \mathrm{H.c.}]. \end{aligned} \qquad (13)$$

After symmetry breaking, the mass matrices for the scalar sector are found. For the charged scalar bosons, the mass matrix is obtained in the basis $(\phi_1^\pm, \phi_2^\pm)$

$$M_{\mathrm{C}}^2 = \frac{1}{4}\begin{pmatrix} -f\frac{v_\chi v_2}{v_1} - \lambda'_5 {v_2}^2 & f v_\chi + \lambda'_5 v_1 v_2 \\ f v_\chi + \lambda'_5 v_1 v_2 & -f\frac{v_\chi v_1}{v_2} - \lambda'_5 {v_1}^2 \end{pmatrix}, \qquad (14)$$

which is diagonalized by

$$R_{\mathrm{C}} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \qquad (15)$$

where $\tan\beta = s_\beta/c_\beta = v_1/v_2$. The mass matrix has the eigenvalues

$$m^2_{G_W^\pm} = 0, \qquad m^2_{H^\pm} = -\frac{1}{4}\frac{f v_\chi}{s_\beta c_\beta} - \frac{1}{4}\lambda'_5 v^2, \qquad (16)$$

yielding the Goldstone bosons $G_W^\pm$, which provide the mass to the physical $W_\mu^\pm$ gauge bosons, and two physical charged Higgs bosons $H^\pm$.

Regarding the neutral scalar sector, the mass matrix of the CP-odd sector in the basis $(\eta_1, \eta_2, \zeta_\chi)$ is:

$$M_{\mathrm{I}}^2 = -\frac{f}{4}\begin{pmatrix} \frac{v_2 v_\chi}{v_1} & -v_\chi & v_2 \\ -v_\chi & \frac{v_1 v_\chi}{v_2} & -v_1 \\ v_2 & -v_1 & \frac{v_1 v_2}{v_\chi} \end{pmatrix}, \qquad (17)$$

which can be diagonalized by the following transformation,

$$R_{\mathrm{I}} = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_\gamma & 0 & s_\gamma \\ 0 & 1 & 0 \\ -s_\gamma & 0 & c_\gamma \end{pmatrix}, \qquad (18)$$

where γ describes the doublet-singlet mixing $\tan\gamma = s_\gamma/c_\gamma = v_\chi/v s_\beta c_\beta$. When R_{I} acts on M_{I}^2, the following eigenvalues are obtained,

$$\begin{aligned} m^2_{G_Z^0} &= 0, \\ m^2_{G_{Z'}^0} &= 0, \\ m^2_{A^0} &= -\frac{1}{4}\frac{f v_\chi}{s_\beta c_\beta s_\gamma^2}, \end{aligned} \qquad (19)$$

where the first two are the would-be Goldstone bosons of the neutral vector bosons Z_μ and Z'_μ, respectively, while the latter is a physical CP-odd pseudoscalar boson A^0.

On the other hand, the *CP*-even scalar mass matrix is

$$M_{\mathrm{R}}^2=\begin{pmatrix} \lambda_1 v_1^2-\frac{1}{4}\frac{f v_\chi v_2}{v_1} & \hat{\lambda}_5 v_1 v_2+\frac{1}{4}f v_\chi & \frac{1}{4}\lambda_6 v_1 v_\chi+\frac{1}{4}f v_2 \\ \hat{\lambda}_5 v_1 v_2+\frac{1}{4}f v_\chi & \lambda_2 v_2^2-\frac{1}{4}\frac{f v_\chi v_1}{v_2} & \frac{1}{4}\lambda_7 v_2 v_\chi+\frac{1}{4}f v_1 \\ \frac{1}{4}\lambda_6 v_1 v_\chi+\frac{1}{4}f v_2 & \frac{1}{4}\lambda_7 v_2 v_\chi+\frac{1}{4}f v_1 & \lambda_3 v_\chi^2-\frac{1}{4}\frac{f v_1 v_2}{v_\chi} \end{pmatrix}, \tag{20}$$

where $\hat{\lambda}_5=(\lambda_5+\lambda_5')/2$. Since this matrix exhibits a characteristic third order polynomial with nontrivial eigenvalues, it is convenient to use another approximation in order to obtain the eigenvalues and mixing angles. We propose a seesawlike mechanism by assuming a hierarchy of VEVs through the condition $|f|v_\chi, v_\chi^2 \gg v^2$ in the matrix elements. Thus, the matrix (20) can be written in blocks as

$$M_{\mathrm{R}}^2=\begin{pmatrix} \mathcal{M}_1 & \mathcal{M}_{12}^{\mathrm{T}} \\ \mathcal{M}_{12} & \mathcal{M}_2 \end{pmatrix}, \tag{21}$$

where

$$\mathcal{M}_1=\begin{pmatrix} \lambda_1 v_1^2-\frac{1}{4}\frac{f v_\chi v_2}{v_1} & \hat{\lambda}_5 v_1 v_2+\frac{1}{4}f v_\chi \\ \hat{\lambda}_5 v_1 v_2+\frac{1}{4}f v_\chi & \lambda_2 v_2^2-\frac{1}{4}\frac{f v_\chi v_1}{v_2} \end{pmatrix},$$
$$\mathcal{M}_{12}^{\mathrm{T}}=\begin{pmatrix} \frac{\lambda_6 v_1 v_\chi}{4}+\frac{f v_2}{4} \\ \frac{\lambda_7 v_2 v_\chi}{4}+\frac{f v_1}{4} \end{pmatrix}\approx\begin{pmatrix} \frac{\lambda_6 v_1 v_\chi}{4} \\ \frac{\lambda_7 v_2 v_\chi}{4} \end{pmatrix},$$
$$\mathcal{M}_2=\lambda_3 v_\chi^2-\frac{1}{4}\frac{f v_1 v_2}{v_\chi}\approx\lambda_3 v_\chi^2. \tag{22}$$

According to the block diagonalization procedure shown in Appendix A, the mass matrix (21) can be decoupled into two independent blocks through a unitary transformation as

$$R_{\mathrm{S}}^{\mathrm{T}} M_{\mathrm{R}}^2 R_{\mathrm{S}}=\begin{pmatrix} M_{hH}^2 & 0 \\ 0 & m_{H_\chi}^2 \end{pmatrix}, \tag{23}$$

where the transformation matrix can be approximately written as

$$R_{\mathrm{S}}=\begin{pmatrix} 1 & F_{\mathrm{R}}^T \\ -F_{\mathrm{R}} & 1 \end{pmatrix}, \tag{24}$$

with

$$F_{\mathrm{R}}\approx\mathcal{M}_2{}^{-1}\mathcal{M}_{12},$$
$$m_{H_\chi}^2\approx\mathcal{M}_2=\lambda_3 v_\chi^2,$$
$$M_{hH}^2\approx\mathcal{M}_1-\mathcal{M}_{12}^{\mathrm{T}}\mathcal{M}_2{}^{-1}\mathcal{M}_{12}^{\mathrm{T}}, \tag{25}$$

and

$$M_{hH}^2=\begin{pmatrix} \tilde{\lambda}_1 v^2 s_\beta^2-\frac{1}{4}\frac{f v_\chi}{t_\beta} & \tilde{\lambda}_5 v^2 s_\beta^2 c_\beta^2+\frac{1}{4}f v_\chi \\ \tilde{\lambda}_5 v^2 s_\beta^2 c_\beta^2+\frac{1}{4}f v_\chi & \tilde{\lambda}_2 v^2 c_\beta^2-\frac{1}{4}f v_\chi t_\beta \end{pmatrix}, \tag{26}$$

where the new tilde constants are

$$\tilde{\lambda}_1=\lambda_1-\frac{\lambda_6^2}{4\lambda_3}-\frac{\lambda_7^2}{4\lambda_3 t_\beta^2},$$
$$\tilde{\lambda}_2=\lambda_2-\frac{\lambda_6^2 t_\beta^2}{4\lambda_3}-\frac{\lambda_7^2}{4\lambda_3},$$
$$\tilde{\lambda}_5=\hat{\lambda}_5-\frac{\lambda_6^2 t_\beta}{2\lambda_3}-\frac{\lambda_7^2}{2\lambda_3 t_\beta}. \tag{27}$$

In order to obtain the largest eigenvalue of M_{hH}^2, we neglect nondominant terms from the condition that $f v_\chi \gg v_2^2,\ v_1^2,\ v_2 v_1$, which leads us to

$$M_{hH}^2\approx-\frac{1}{4}f v_\chi\begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix}. \tag{28}$$

Due to this approximation, the new matrix has null determinant and its trace is of the order of the largest eigenvalue:

$$m_H^2\approx\mathrm{Tr}[M_{hH}^2]\approx-\frac{1}{4}\frac{f v_\chi}{s_\beta c_\beta}. \tag{29}$$

The lightest mass eigenvalue can be calculated through the ratio of the determinant and the trace of (26), i.e.,

$$\frac{\mathrm{Det}[M_{hH}^2]}{\mathrm{Tr}[M_{hH}^2]}=\frac{m_h^2 m_H^2}{m_h^2+m_H^2}\approx m_h^2, \tag{30}$$

obtaining

$$m_h^2\approx(\tilde{\lambda}_1 s_\beta^2+2\tilde{\lambda}_5 c_\beta s_\beta+\tilde{\lambda}_2 c_\beta^2)v^2, \tag{31}$$

which we associate with the observed 125 GeV Higgs boson. The mixing angle associated with (26) is defined as $t_{2\alpha}=\tan 2\alpha$, where

$$t_{2\alpha}=\frac{f v_\chi+2\tilde{\lambda}_5 s_\beta c_\beta v^2}{f v_\chi+2t_{2\beta}(s_\beta^2\tilde{\lambda}_1-c_\beta^2\tilde{\lambda}_2)v_2}t_{2\beta}. \tag{32}$$

Finally, the diagonalization of the *CP*-even matrix (20) is achieved by R_{R} parametrized by a CKM-like matrix

$$R_{\rm R}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}\\ 0 & 1 & 0\\ -s_{13} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_\alpha & s_\alpha & 0\\ -s_\alpha & c_\alpha & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{33}$$

where $t_\alpha = s_\alpha/c_\alpha$ and

$$s_{13}=\frac{1}{2}\frac{\lambda_6 v s_\beta}{\lambda_3 v_\chi}, \qquad s_{23}=\frac{1}{2}\frac{\lambda_7 v c_\beta}{\lambda_3 v_\chi}, \tag{34}$$

whose corresponding cosines are approximated as $c_{13}\approx 1-s_{13}^2/2$ and $c_{23}\approx 1-s_{23}^2/2$. In fact, the $R_{\rm S}$ matrix which block-diagonalizes $M_{\rm R}^2$ is the product of the former two rotation matrices with mixing angles θ_{23} and θ_{13}.

In conclusion, the scalar spectrum of the model is

(i) Four would-be Goldstone bosons: $G_W^\pm$, G_Z^0 y $G_{Z'}^0$.

(ii) Three scalar CP-even h, H y H_χ fields with mass

$$\begin{aligned} m_h^2 &\approx (\tilde{\lambda}_1 c_\beta^4 + 2\tilde{\lambda}_5 c_\beta^2 s_\beta^2 + \tilde{\lambda}_2 s_\beta^4) v^2,\\ m_H^2 &\approx -\frac{f v_\chi}{4 s_\beta c_\beta},\\ m_{H_\chi}^2 &\approx \lambda_3 v_\chi^2. \end{aligned} \tag{35}$$

(iii) A pseudoscalar CP-odd A^0 whose mass is

$$m_{A^0}^2 = -\frac{1}{4}\frac{f v_\chi}{s_\beta c_\beta s_\gamma^2}. \tag{36}$$

(iv) Two charged scalar bosons $H^\pm$ with mass

$$m_{H^\pm}^2 = -\frac{1}{4}\frac{f v_\chi}{s_\beta c_\beta} - \frac{1}{4}\lambda_5' v^2. \tag{37}$$

B. Gauge boson masses

The kinetic terms of the scalar fields are

$$\mathcal{L}_{\rm kin} = \sum_i (D_\mu S)^\dagger (D^\mu S). \tag{38}$$

After the symmetry breaking, the charged bosons $W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$ acquire masses $M_W = gv/2$, while the masses for neutral gauge bosons are obtained from the following squared mass matrix in the basis (W_μ^3, B_μ, Z_μ'),

$$M_0^2 = \frac{1}{4}\begin{pmatrix} g^2 v^2 & -g g' v^2 & -\frac{2}{3} g g_X v^2 (1+c_\beta^2)\\ * & g'^2 v^2 & \frac{2}{3} g' g_X v^2 (1+c_\beta^2)\\ * & * & \frac{4}{9} g_X^2 v_\chi^2 [1+(1+3c_\beta^2)\epsilon^2] \end{pmatrix}, \tag{39}$$

where $\epsilon = v/v_\chi$. Taking into account $\epsilon^2 \ll 1$, the matrix can be diagonalized with only two angles, obtaining the following mass eigenstates,

$$\begin{pmatrix} A_\mu\\ Z_{1\mu}\\ Z_{2\mu}\end{pmatrix} \approx R_0 \begin{pmatrix} W_\mu^3\\ B_\mu\\ Z_\mu' \end{pmatrix}, \tag{40}$$

with

$$R_0 = \begin{pmatrix} s_W & c_W & 0\\ c_W c_Z & -s_W c_Z & s_Z\\ -c_W s_Z & s_W s_Z & c_Z \end{pmatrix}, \tag{41}$$

where $\tan\theta_W = s_W/c_W = g'/g$ defines the Weinberg angle, and $s_Z = \sin\theta_Z$ is a small mixing angle between the SM neutral gauge boson Z and the $\mathrm{U}(1)_X$ gauge boson Z' such that in the limit $s_Z \to 0$, $Z_1 = Z$ and $Z_2 = Z'$. This mixing angle is approximately

$$s_Z \approx (1+c_\beta^2)\frac{2 g_X c_W}{3g}\left(\frac{M_Z}{M_{Z'}}\right)^2, \tag{42}$$

where the neutral masses are

$$M_Z \approx \frac{gv}{2c_W}, \qquad M_{Z'} \approx \frac{g_X v_\chi}{3}. \tag{43}$$

III. FERMION MASSES

A. Quark sector

We find the Yukawa Lagrangian compatible with the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_X$ gauge symmetry. For the quark sector, we obtain

$$\begin{aligned} -\mathcal{L}_Q = {}& \overline{q_L^1}(\tilde{\phi}_2 h_2^U)_{1j} U_R^j + \overline{q_L^a}(\tilde{\phi}_1 h_1^U)_{aj} U_R^j + \overline{q_L^1}(\phi_1 h_1^D)_{1j} D_R^j + \overline{q_L^a}(\phi_2 h_2^D)_{aj} D_R^j + \overline{q_L^1}(\phi_1 h_1^J)_{1m} J_R^m + \overline{q_L^a}(\phi_2 h_2^J)_{am} J_R^m\\ & + \overline{q_L^1}(\tilde{\phi}_2 h_2^T)_1 T_R + \overline{q_L^a}(\tilde{\phi}_1 h_1^T)_a T_R + \overline{T_L}(\sigma h_\sigma^U + \chi h_\chi^U)_j U_R^j + \overline{T_L}(\sigma h_\sigma^T + \chi h_\chi^T) T_R + \overline{J_L^n}(\sigma^* h_\sigma^D + \chi^* h_\chi^D)_{nj} D_R^j\\ & + \overline{J_L^n}(\sigma^* h_\sigma^J + \chi^* h_\chi^J)_{nm} J_R^m + \text{H.c.}, \end{aligned} \tag{44}$$

where $\tilde{\phi}_{1,2} = i\sigma_2\phi^*_{1,2}$ are conjugate fields, $a = 2, 3$ label the second and third quark doublets and $n(m) = 1, 2$ is the index of the exotic $J^{n(m)}$ quarks. A sum over the indices i, a, and n is understood. We can see in the quark Lagrangian that due to the nonuniversality of the $\mathrm{U}(1)_X$ symmetry, not all couplings between quarks and scalars are allowed by the gauge symmetry, which leads us to specific zero-texture Yukawa matrices. However, these structures are not inherited by the mass matrices of the quarks, due to the interactions of the four scalar fields ϕ_1, ϕ_2, σ_0, and χ_0 that couple simultaneously to all quark flavors. In order to reproduce the observed mass spectrum, we must restrict further the number of couplings in the Lagrangian, which can be done by assuming the $\boldsymbol{Z_2}$ discrete symmetries shown in Tables II and III. Assuming these discrete symmetries, the Lagrangian (44) after the symmetry breaking leads us to the following mass terms at tree level:

$$-\langle\mathcal{L}_Q\rangle = \overline{U^i_L}(M_U)_{ij}U^j_R + \overline{D^i_L}(M_D)_{ij}D^j_R + \overline{T_L}(M_T)T_R + \overline{J^n_L}(M_J)_{nm}J^m_R + \overline{T_L}(M_{TU})_jU^j_R + \overline{U^i_L}(M_{UT})_iT_R + \overline{D^i_L}(M_{DJ})_{im}J^m_R + \mathrm{H.c.}, \tag{45}$$

where the mass matrices generate the following zero structures:

$$M_U = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \upsilon_2 a_{12} & 0 \\ 0 & \upsilon_1 a_{22} & 0 \\ \upsilon_1 a_{31} & 0 & \upsilon_1 a_{33}\end{pmatrix}, \qquad M_D = \frac{\upsilon_2}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ B_{31} & B_{32} & B_{33}\end{pmatrix},$$
$$M_J = \frac{\upsilon_\chi}{\sqrt{2}}\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22}\end{pmatrix}, \qquad M_T = \frac{\upsilon_\chi}{\sqrt{2}}h^T_\chi,$$
$$M_{TU} = \frac{\upsilon_\chi}{\sqrt{2}}(0, c_2, 0), \qquad M_{UT} = \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_2 y_1 \\ \upsilon_1 y_2 \\ 0\end{pmatrix}$$
$$M_{DJ} = \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_1 j_{11} & \upsilon_1 j_{12} \\ \upsilon_2 j_{21} & \upsilon_2 j_{22} \\ 0 & 0\end{pmatrix}, \qquad M_{JD} = 0, \tag{46}$$

which leads us to the following extended mass matrices:

$$M'_U = \left(\begin{array}{c|c} M_U & M_{UT} \\ \hline M_{TU} & M_T\end{array}\right) = \frac{1}{\sqrt{2}}\left(\begin{array}{ccc|c} 0 & \upsilon_2 a_{12} & 0 & \upsilon_2 y_1 \\ 0 & \upsilon_1 a_{22} & 0 & \upsilon_1 y_2 \\ \upsilon_1 a_{31} & 0 & \upsilon_1 a_{33} & 0 \\ \hline 0 & \upsilon_\chi c_2 & 0 & \upsilon_\chi h^T_\chi\end{array}\right),$$
$$M'_D = \left(\begin{array}{c|c} M_D & M_{DJ} \\ \hline M_{JD} & M_J\end{array}\right) = \frac{1}{\sqrt{2}}\left(\begin{array}{ccc|cc} 0 & 0 & 0 & \upsilon_1 j_{11} & \upsilon_1 j_{12} \\ 0 & 0 & 0 & \upsilon_2 j_{21} & \upsilon_2 j_{22} \\ \upsilon_2 B_{31} & \upsilon_2 B_{32} & \upsilon_2 B_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & \upsilon_\chi k_{11} & \upsilon_\chi k_{12} \\ 0 & 0 & 0 & \upsilon_\chi k_{21} & \upsilon_\chi k_{22}\end{array}\right). \tag{47}$$

After diagonalization, the above structures leads us to hierarchies of the phenomenological quarks, as detailed below.

1. Up sector

First, we consider the up-type matrix M'_U in Eq. (47). We obtain its symmetrical quadratic form as

$$\mathbb{M}_U^2 = M'_U(M'_U)^T = \frac{1}{2}\begin{pmatrix} v_2^2(a_{12}^2+y_1^2) & v_1v_2(a_{12}a_{22}+y_1y_2) & 0 & | & v_2v_\chi(a_{12}c_2+y_1h_\chi^T) \\ v_1v_2(a_{12}a_{22}+y_1y_2) & v_1^2(a_{22}^2+y_2^2) & 0 & | & v_1v_\chi(a_{22}c_2+y_2h_\chi^T) \\ 0 & 0 & v_1^2(a_{31}^2+a_{33}^2) & | & 0 \\ — & — & — & — & — \\ v_2v_\chi(a_{12}c_2+y_1h_\chi^T) & v_1v_\chi(a_{22}c_2+y_2h_\chi^T) & 0 & | & v_\chi^2(c_2^2+h_\chi^{T2}) \end{pmatrix}. \tag{48}$$

The above mass matrix can be written as

$$\mathbb{M}_U^2 = \begin{pmatrix} A & C \\ C^T & D \end{pmatrix}, \tag{49}$$

which has the same structure as the general form of Eq. (A1) in the Appendix A, where each block is

$$A = \frac{1}{2}\begin{pmatrix} v_2^2(a_{12}^2+y_1^2) & v_1v_2(a_{12}a_{22}+y_1y_2) & 0 \\ v_1v_2(a_{12}a_{22}+y_1y_2) & v_1^2(a_{22}^2+y_2^2) & 0 \\ 0 & 0 & v_1^2(a_{31}^2+a_{33}^2) \end{pmatrix},$$
$$C = \frac{1}{2}\begin{pmatrix} v_2v_\chi(a_{12}c_2+y_1h_\chi^T) \\ v_1v_\chi(a_{22}c_2+y_2h_\chi^T) \\ 0 \end{pmatrix},$$
$$D = \frac{1}{2}v_\chi^2(c_2^2+h_\chi^{T2}). \tag{50}$$

We can see that each block are of the order $A \sim v_{1,2}^2$, $C \sim v_{1,2}v_\chi$ and $D \sim v_\chi^2$, respectively, obeying the hierarchy from Eq. (A2). Thus, according to Appendix A, the mass matrix (49) can be block diagonalized as

$$\mathfrak{m}_U^2 = (V_L^{(U)})^T \mathbb{M}_U^2 V_L^{(U)} = \begin{pmatrix} m_U^2 & 0 \\ 0 & m_T^2 \end{pmatrix}, \tag{51}$$

where:

$$m_U^2 \approx A - CD^{-1}C^T, \qquad m_T^2 \approx D, \tag{52}$$

and the rotation matrix has the approximated form:

$$V_L^{(U)} \approx \begin{pmatrix} I & F_U \\ -F_U^T & I \end{pmatrix}, \qquad F_U \approx CD^{-1}. \tag{53}$$

Since the block D is just a number [see Eq. (50)], from (52) we obtain directly the mass of the heavy T quark:

$$m_T^2 \approx \frac{1}{2}v_\chi^2(c_2^2+h_\chi^{T2}). \tag{54}$$

On the other hand, from the matrices in (50), and after some algebra, the matrix m_U^2 in (52), which contains the SM sector, can be put into the form:

$$m_U^2 \approx \frac{1}{2}\begin{pmatrix} v_2^2r_1^2 & v_1v_2r_1r_2 & 0 \\ v_1v_2r_1r_2 & v_1^2r_2^2 & 0 \\ 0 & 0 & v_1^2(a_{31}^2+a_{33}^2) \end{pmatrix}, \tag{55}$$

where:

$$r_1 = \frac{(a_{12}h_\chi^T - y_1c_2)}{\sqrt{c_2^2+h_\chi^{T2}}},$$
$$r_2 = \frac{(a_{22}h_\chi^T - y_2c_2)}{\sqrt{c_2^2+h_\chi^{T2}}}. \tag{56}$$

We see that the 33 component of (55) appears decoupled, which corresponds to one of the eigenvalues. We associate this component to the top quark:

$$m_t^2 = \frac{1}{2}v_1^2(a_{31}^2+a_{33}^2), \tag{57}$$

which leaves us with the 2×2 submatrix

$$m^2_{uc}\approx\frac{1}{2}\begin{pmatrix} v_2^2r_1^2 & v_1v_2r_1r_2\\ v_1v_2r_1r_2 & v_1^2r_2^2\end{pmatrix}. \tag{58}$$

It is evident that the above matrix has null determinant, which leads us to at least one null eigenvalue. In fact, this structure produces one massless quark, which we associate to the lightest quark: the up quark (u), while the other eigenvalue, associated to the charm quark, corresponds to the trace of the matrix:

$$m_c^2=\mathrm{Tr}[m^2_{uc}]=\frac{1}{2}(v_1^2r_2^2+v_2^2r_1^2)\approx\frac{1}{2}v_1^2r_2^2, \tag{59}$$

Since the mass of the top quark in (57) depends only on v_1, we take $v_2\ll v_1$, which leads us to the approximation in Eq. (59).

In order to generate mass to the u quark, we consider the one-loop radiative correction shown in figure 1(a). This contribution add an input into the 11 component in the original 4×4 matrix M'_U in (47), which produces the one-loop quadratic mass matrix

$$\mathbb{M}^2_{U(1)}=\mathbb{M}^2_U+\Delta\mathbb{M}^2_U, \tag{60}$$

where the small one-loop contribution is:

$$\Delta\mathbb{M}^2_U=\frac{1}{2}\left(\begin{array}{ccc|c} v_1^2\Sigma_{11}^2 & 0 & v_1^2a_{31}\Sigma_{11} & 0\\ 0 & 0 & 0 & 0\\ v_1^2a_{31}\Sigma_{11} & 0 & 0 & 0\\ \hline 0 & 0 & 0 & 0\end{array}\right), \tag{61}$$

and Σ_{11} the value of the diagram in Fig. 1(a) which obey the following analytical expression:

$$\Sigma_{11}=\frac{-1}{16\pi^2}\frac{f'(h^U_\sigma)_1(h^T_2)_1}{\sqrt{2}M_T}C_0\left(\frac{M_2}{M_T},\frac{M_\sigma}{M_T}\right), \tag{62}$$

where

$$C_0(x_1,x_2)=\frac{1}{(1-x_1^2)(1-x_2^2)(x_1^2-x_2^2)}\times\left[x_1^2x_2^2\ln\left(\frac{x_1^2}{x_2^2}\right)-x_1^2\ln x_1^2+x_2^2\ln x_2^2\right], \tag{63}$$

and M_2 is a characteristic mass derived from the internal ϕ_2 line as linear combinations of mass eigenvalues. The new one-loop contribution only has effect on the 3×3 block matrix m_U^2 in (55), which change into the one-loop mass matrix

$$m^2_{U(1\text{-loop})}\approx\frac{1}{2}\begin{pmatrix} v_2^2r_1^2+v_1^2\Sigma_{11}^2 & v_1v_2r_1r_2 & v_1^2a_{31}\Sigma_{11}\\ v_1v_2r_1r_2 & v_1^2r_2^2 & 0\\ v_1^2a_{31}\Sigma_{11} & 0 & 2m_t^2\end{pmatrix}, \tag{64}$$

where m_t is the top mass at tree level obtained in (57). The new 13 component emerged from the 1 loop diagram will correct the top mass. However, we will neglect this correction, which leads us again to a 2×2 matrix

$$m^2_{uc(1\text{-loop})}\approx\frac{1}{2}\begin{pmatrix} v_2^2r_1^2+v_1^2\Sigma_{11}^2 & v_1v_2r_1r_2\\ v_1v_2r_1r_2 & v_1^2r_2^2\end{pmatrix}, \tag{65}$$

which exhibits determinant different from zero. The trace of the matrix corresponds to the sum of the eigenvalues, i.e.:

$$\mathrm{Tr}[m^2_{uc(1\text{-loop})}]=m_u^2+m_c^2=\frac{1}{2}(v_1^2r_2^2+v_2^2r_1^2)+\frac{1}{2}v_1^2\Sigma_{11}^2. \tag{66}$$

If we approximate the mass of the charm quark according to (59), we obtain for the quark u that

$$m_u^2=\frac{1}{2}v_1^2\Sigma_{11}^2. \tag{67}$$

2. Down sector

For the down-type matrix M'_D in (47), for simplicity we take in the heavy sector, proportional to v_χ, a diagonal form, i.e., $k_{ij}=0$ for $i\neq j$. In this scenery, its quadratic form can also be put in the block form

$$\mathbb{M}^2_D=\begin{pmatrix} A & C\\ C^T & D\end{pmatrix}, \tag{68}$$

where

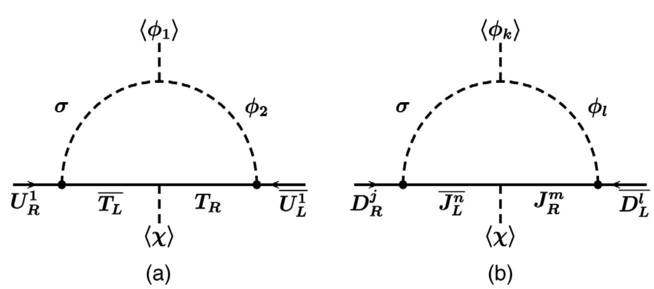


FIG. 1. Mass one-loop correction for (a) up and (b) down sector, where k, l, m, $n=1$, 2 and $j=1$, 2, 3.

$$A = \frac{1}{2}\begin{pmatrix} v_1^2(j_{11}^2 + j_{12}^2) & v_1 v_2(j_{11}j_{21} + j_{12}j_{22}) & 0 \\ v_1 v_2(j_{11}j_{21} + j_{12}j_{22}) & v_2^2(j_{21}^2 + j_{22}^2) & 0 \\ 0 & 0 & v_2^2(B_{31}^2 + B_{32}^2 + B_{33}^2) \end{pmatrix},$$
$$C = \frac{1}{2}\begin{pmatrix} v_1 v_\chi j_{11}k_{11} & v_1 v_\chi j_{12}k_{22} \\ v_2 v_\chi j_{21}k_{11} & v_2 v_\chi j_{22}k_{22} \\ 0 & 0 \end{pmatrix},$$
$$D = \frac{v_\chi^2}{2}\begin{pmatrix} k_{11}^2 & 0 \\ 0 & k_{22}^2 \end{pmatrix}. \tag{69}$$

After block diagonalization, the matrix becomes

$$\mathbb{m}_D^2 = (V_L^{(D)})^T \mathbb{M}_D^2 V_L^{(D)} = \begin{pmatrix} m_D^2 & 0 \\ 0 & m_J^2 \end{pmatrix}, \tag{70}$$

where

$$m_D^2 \approx A - CD^{-1}C^T, \qquad m_J^2 \approx D, \tag{71}$$

with

$$V_L^{(D)} \approx \begin{pmatrix} I & F_D \\ -F_D^T & I \end{pmatrix}, \qquad F_D \approx CD^{-1}. \tag{72}$$

First, since the matrix D appears diagonal, we obtain directly the mass of the heavy down-type quarks:

$$m_{J^1}^2 = \frac{1}{2}v_\chi^2 k_{11}^2, \qquad m_{J^2}^2 = \frac{1}{2}v_\chi^2 k_{22}^2. \tag{73}$$

Second, for the SM down sector, the matrix m_D^2 in (71) gives

$$m_D^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_2^2(B_{31}^2 + B_{32}^2 + B_{33}^2) \end{pmatrix}, \tag{74}$$

which exhibits two massless quarks: the down (d) and strange (s) quarks, and one massive quark associated to the bottom (b):

$$m_b^2 = \frac{1}{2}v_2^2(B_{31}^2 + B_{32}^2 + B_{33}^2). \tag{75}$$

In order to obtain mass for d and s, we again consider the one-loop contribution shown in Fig. 1(b), which produces new entrances different from zero in (74) as follows:

$$m_{D(1-\mathrm{loop})}^2 = \frac{1}{2}\begin{pmatrix} v_2^2(\Sigma_{11}^2 + \Sigma_{12}^2 + \Sigma_{13}^2) & v_1 v_2(\Sigma_{11}\Sigma_{21} + \Sigma_{12}\Sigma_{22} + \Sigma_{13}\Sigma_{23}) & v_2^2(\Sigma_{11}B_{31} + \Sigma_{12}B_{32} + \Sigma_{13}B_{33}) \\ * & v_1^2(\Sigma_{21}^2 + \Sigma_{22}^2 + \Sigma_{23}^2) & v_1 v_2(\Sigma_{21}B_{31} + \Sigma_{22}B_{32} + \Sigma_{23}B_{33}) \\ * & * & 2m_b^2 \end{pmatrix}, \tag{76}$$

where the one-loop correction is

$$\Sigma_{lj} = \frac{-1}{16\pi^2}\frac{f'(h_l^J)_{lm}(h_\sigma^D)_{nj}}{\sqrt{2}M_J}C_0\left(\frac{M_l}{M_J}, \frac{M_\sigma}{M_J}\right). \tag{77}$$

If the matrix in (76) is grouped as

$$m_{D(1\text{-loop})}^2 = \begin{pmatrix} m_1^2 & n \\ n^T & 2m_b^2 \end{pmatrix}, \tag{78}$$

where the bottom mass is dominant, we can block diagonalize it as

$$R_L^T m_{D(1\text{-loop})}^2 R_L \approx \begin{pmatrix} m_{ds}^2 & 0 \\ 0 & 2m_b^2 \end{pmatrix}, \tag{79}$$

with

$$m_{ds}^2 = m_1^2 - \frac{nn^T}{2m_b^2}$$
$$= \frac{1}{2m_b^2}\begin{pmatrix} s_{11}v_2^2 & s_{12}v_1 v_2 \\ s_{12}v_1 v_2 & s_{22}v_1^2 \end{pmatrix}, \tag{80}$$

and

$$
\begin{aligned}
s_{11} &= (\Sigma_{11}B_{32}-\Sigma_{12}B_{31})^2+(\Sigma_{11}B_{33}-\Sigma_{13}B_{31})^2+(\Sigma_{12}B_{33}-\Sigma_{13}B_{32})^2,\\
s_{22} &= (\Sigma_{21}B_{32}-\Sigma_{22}B_{31})^2+(\Sigma_{21}B_{33}-\Sigma_{23}B_{31})^2+(\Sigma_{22}B_{33}-\Sigma_{23}B_{32})^2,\\
s_{12} &= B_{31}^2(\Sigma_{13}\Sigma_{23}+\Sigma_{12}\Sigma_{32})+B_{32}^2(\Sigma_{11}\Sigma_{12}+\Sigma_{13}\Sigma_{23})+B_{33}^2(\Sigma_{11}\Sigma_{21}+\Sigma_{12}\Sigma_{22})\\
&\quad - B_{31}B_{32}(\Sigma_{12}\Sigma_{21}+\Sigma_{11}\Sigma_{22})-B_{31}B_{33}(\Sigma_{11}\Sigma_{23}+\Sigma_{13}\Sigma_{21})-B_{32}B_{33}(\Sigma_{13}\Sigma_{22}+\Sigma_{12}\Sigma_{23}). \qquad (81)
\end{aligned}
$$

The eigenvalues of m^2_{ds} in (80) will lead us to the down and strange masses. For example, if the mixing component s_{12} is null, we obtain

$$m_d^2 \approx \frac{s_{11}v_2^2}{2m_b^2}, \qquad m_s^2 \approx \frac{s_{22}v_1^2}{2m_b^2}. \qquad (82)$$

B. Lepton sector

The nonuniversal $U(1)_X$ also forbids some Yukawa couplings between leptons and scalar bosons. The allowed couplings are shown below for neutral and charged leptons, respectively:

$$
\begin{aligned}
-\mathcal{L}_{Y,N} &= h^{\nu e}_{2e}\overline{\ell^e_L}\tilde{\phi}_2\nu^e_R + h^{\nu\mu}_{2e}\overline{\ell^e_L}\tilde{\phi}_2\nu^\mu_R + h^{\nu\tau}_{2e}\overline{\ell^e_L}\tilde{\phi}_2\nu^\tau_R\\
&\quad + h^{\nu e}_{2\mu}\overline{\ell^\mu_L}\tilde{\phi}_2\nu^e_R + h^{\nu\mu}_{2\mu}\overline{\ell^\mu_L}\tilde{\phi}_2\nu^\mu_R + h^{\nu\tau}_{2\mu}\overline{\ell^\mu_L}\tilde{\phi}_2\nu^\tau_R\\
&\quad + h^{\nu j}_{\chi i}\overline{\nu^{iC}_R}\chi^* N_R + \frac{1}{2}\overline{N^{iC}_R}M^{ij}_N N^j_R + \text{H.c.}, \qquad (83)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{Y,E} &= \eta\overline{\ell^e_L}\phi_2 e^\mu_R + h\overline{\ell^\mu_L}\phi_2 e^\mu_R + \zeta\overline{\ell^\tau_L}\phi_2 e^e_R + H\overline{\ell^\tau_L}\phi_2 e^\tau_R\\
&\quad + q_{11}\overline{\ell^e_L}\phi_1 E_R + q_{21}\overline{\ell^\mu_L}\phi_1 E_R + h^E_{\sigma e}\overline{E_L}\sigma e^e_R\\
&\quad + h^{\mathcal{E}}_{\sigma\mu}\overline{\mathcal{E}_L}\sigma^* e^\mu_R + h^E_{\sigma\tau}\overline{E_L}\sigma e^\tau_R + H_1\overline{E_L}\chi E_R\\
&\quad + H_2\overline{\mathcal{E}_L}\chi^*\mathcal{E}_R + \text{H.c.} \qquad (84)
\end{aligned}
$$

Since the Higgs doublet ϕ_2 has the discrete symmetry $\phi_2 \to -\phi_2$, all the right-handed leptons except E_R and $\mathcal{E}_R$ also have $\mathbf{Z}_2$ negative parities in order to obtain the adequate zero textures, i.e.,

$$e_R^{e,\mu,\tau} \to -e_R^{e,\mu,\tau}, \quad \nu_R^{e,\mu,\tau} \to -\nu_R^{e,\mu,\tau}, \quad N_R^{e,\mu,\tau} \to -N_R^{e,\mu,\tau}. \qquad (85)$$

1. Neutral leptons

Evaluating in the VEVs, the terms obtained from (83) can be written in the following mass term using the basis $\mathbf{N}_L = (\nu_L^{e,\mu,\tau}, (\nu_R^{e,\mu,\tau})^C, (N_R^{e,\mu,\tau})^C)^{\mathrm{T}}$ for the neutral sector

$$-\mathcal{L}_{Y,N} = \frac{1}{2}\overline{\mathbf{N}^C_L}\mathbb{M}_\nu\mathbf{N}_L, \qquad (86)$$

where the mass matrix is

$$\mathbb{M}_\nu = \begin{pmatrix} 0 & m_D^{\mathrm{T}} & 0 \\ m_D & 0 & M_D^{\mathrm{T}} \\ 0 & M_D & M_M \end{pmatrix}, \qquad (87)$$

with $M_D = h^\nu_\chi v_\chi/\sqrt{2}$ being a Dirac mass between ν^c_R and N_R, where $h_{N\chi}$ is a 3×3 matrix, and

$$m_D = \frac{v_2}{\sqrt{2}}\begin{pmatrix} h^{\nu e}_{2e} & h^{\nu\mu}_{2e} & h^{\nu\tau}_{2e} \\ h^{\nu e}_{2\mu} & h^{\nu\mu}_{2\mu} & h^{\nu\tau}_{2\mu} \\ 0 & 0 & 0 \end{pmatrix} \qquad (88)$$

is a Dirac mass matrix between ν_L and ν_R. M_M is the mass of the Majorana neutrino N_R.

Considering that $M_M \ll m_D$ and M_D, the matrix $\mathbb{M}_\nu$ can be diagonalized through the inverse seesaw mechanism [26,27]. If the following blocks are defined,

$$
\begin{aligned}
\mathcal{M}_\nu &= \begin{pmatrix} m_D \\ 0 \end{pmatrix},\\
\mathcal{M}_N &= \begin{pmatrix} 0 & M_D^{\mathrm{T}} \\ M_D & M_M \end{pmatrix}, \qquad (89)
\end{aligned}
$$

the mass matrix becomes

$$\mathbb{M}_\nu = \begin{pmatrix} 0 & \mathcal{M}_\nu^{\mathrm{T}} \\ \mathcal{M}_\nu & \mathcal{M}_N \end{pmatrix}, \qquad (90)$$

which has the same form as the block matrix (A1) from Appendix A in the limit with $A = 0$. Thus, we define the rotations

$$\mathbb{W}_{\mathrm{SS}}{}^{\mathrm{T}}\mathbb{M}_\nu\mathbb{W}_{\mathrm{SS}} = \begin{pmatrix} m_{\mathrm{light}} & 0 \\ 0 & m_{\mathrm{heavy}} \end{pmatrix}, \qquad (91)$$

with

$$
\begin{aligned}
\mathbb{W}_{\mathrm{SS}} &\approx \begin{pmatrix} I & F^N \\ -(F^N)^T & I \end{pmatrix},\\
F^N &\approx (\mathcal{M}_N)^{-1}\mathcal{M}_\nu, \qquad (92)
\end{aligned}
$$

and

$$m_{\mathrm{light}} \approx -\mathcal{M}_\nu^{\mathrm{T}}\mathcal{M}_N^{-1}\mathcal{M}_\nu, \qquad (93)$$

$$m_{\text{heavy}} \approx \mathcal{M}_N. \tag{94}$$

Since

$$\mathcal{M}_N^{-1} = \begin{pmatrix} -(M_D)^{-1}M_M(M_D^{\mathrm{T}})^{-1} & M_D^{-1} \\ (M_D^{\mathrm{T}})^{-1} & 0 \end{pmatrix}, \tag{95}$$

the light mass term is

$$m_{\text{light}} = m_D^{\mathrm{T}}(M_D)^{-1}M_M(M_D^{\mathrm{T}})^{-1}m_D. \tag{96}$$

Now, a unitary matrix $\mathbb{V}$ is considered which diagonalizes the 3×3 block matrix $\mathcal{M}_N$ [27]:

$$\mathbb{V}^{\mathrm{T}}\mathcal{M}_N\mathbb{V} = \mathbb{V}^{\mathrm{T}}\begin{pmatrix} 0 & M_D \\ M_D^{\mathrm{T}} & M_N \end{pmatrix}\mathbb{V} = \begin{pmatrix} V_1^* M_1^{\text{diag}} V_1^\dagger & 0 \\ 0 & V_2^* M_2^{\text{diag}} V_2^\dagger \end{pmatrix}, \tag{97}$$

with V_1 and V_2 subrotation matrices. $\mathbb{V}$ may be formally expressed as [27]

$$\mathbb{V} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1-\frac{SS^\dagger}{2} & S \\ -S^\dagger & 1-\frac{S^\dagger S}{2} \end{pmatrix}. \tag{98}$$

Using (97), and assuming that $M_D = M_D^{\mathrm{T}}$, $M_M S^\dagger = S^{\mathrm{T}} M_M$, $M_M S = S^* M_M$, $M_D S^\dagger = S^{\mathrm{T}} M_D$ and $M_D S = S^* M_D$, from the off-diagonal elements, we find

$$S = S^\dagger = -\frac{1}{4}M_D^{-1}M_M, \tag{99}$$

and substituting for the diagonal elements, we get the mass matrices

$$V_1^* M_1^{\text{diag}} V_1^\dagger = \frac{M_M}{2} - M_D - \frac{1}{8}M_M M_D^{-1} M_M \approx -M_D, \tag{100}$$

$$V_2^* M_2^{\text{diag}} V_2^\dagger = \frac{M_M}{2} + M_D + \frac{1}{8}M_M M_D^{-1} M_M \approx M_D. \tag{101}$$

The mass eigenstates $\mathbf{n}_L$ are constructed as

$$\mathbf{N}_L = \mathbb{U}_N \mathbf{n}_L, \tag{102}$$

with $\mathbf{n}_L = (\nu_L^{1,2,3}, N_{1L}^{1,2,3}, N_{2L}^{1,2,3})$, and the rotation matrix as

$$\mathbb{U}_N = \mathbb{W}_{\text{SS}}\mathbb{W}_{\text{H}}\mathbb{W}_{\text{B}}, \tag{103}$$

with $\mathbb{W}_{\text{SS}}$ the seesaw matrix rotation from (92),

$$\mathbb{W}_{\text{H}} = \begin{pmatrix} 1 & 0 \\ 0 & \mathbb{V} \end{pmatrix}, \tag{104}$$

the matrix rotation of the heavy neutrinos, and

$$\mathbb{W}_{\text{B}} = \text{block diag}(U_\nu, V_1, V_2) \tag{105}$$

the matrices that diagonalize each 3×3 block.

2. Charged leptons

For the charged sector in the flavor basis $\mathbf{E} = (e^e, e^\mu, e^\tau, E)$, the mass terms obtained from (84) after the symmetry breaking are

$$-\mathcal{L}_{Y,E} = \overline{\mathbf{E}_L}\mathbb{M}_E\mathbf{E}_R + \frac{H_2 v_\chi}{\sqrt{2}}\overline{\mathcal{E}_L}\mathcal{E}_R + \text{H.c.}, \tag{106}$$

where the lepton mass matrix $\mathbb{M}_E$ has the following form,

$$\mathbb{M}_E = \frac{v_2}{\sqrt{2}}\left(\begin{array}{ccc|c} 0 & \eta & 0 & q_{11}t_\beta \\ 0 & h & 0 & q_{21}t_\beta \\ \zeta & 0 & H & 0 \\ \hline 0 & 0 & 0 & H_1 v_\chi/v_2 \end{array}\right), \tag{107}$$

which exhibits one massless lepton (the electron). To obtain a massive electron, we include the one-loop correction shown in Fig. 2, which adds a new term,

$$\mathbb{M}_{E(1)} = \mathbb{M}_E + \Delta\mathbb{M}_E, \tag{108}$$

with

$$\Delta\mathbb{M}_E = \frac{v_2}{2}\left(\begin{array}{ccc|c} \Sigma_{11} & 0 & \Sigma_{13} & 0 \\ \Sigma_{21} & 0 & \Sigma_{23} & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right). \tag{109}$$

Since $\mathbb{M}_{E(1)}$ is not hermitian, there are two rotation matrices $\mathbb{V}_L^E$ and $\mathbb{V}_R^E$ for left- and right-handed electrons.

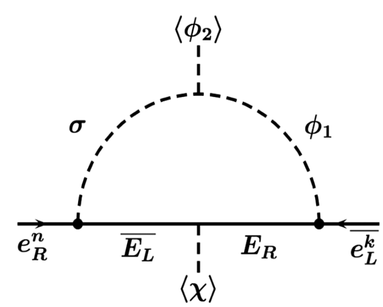


FIG. 2. Mass one-loop correction for charged leptons, where $n = e, \tau$ and $k = e, \mu$.

Hence, the left-handed rotation is obtained by diagonalizing $\mathbb{M}_E\mathbb{M}_E^\dagger$ obtaining the corresponding eigenvalues

$$\begin{aligned} m_e^2 &= \frac{h^2\Sigma_{11}^2 v_2^2}{2(\eta^2+h^2)} \approx \frac{v_2^2}{2}\Sigma_{11}^2, \\ m_\mu^2 &= \frac{v_2^2}{2}(\eta^2+h^2) \approx \frac{v_2^2}{2}h^2, \\ m_\tau^2 &= \frac{v_2^2}{2}(\zeta^2+H^2) \approx \frac{v_2^2}{2}H^2, \\ m_E^2 &= \frac{H_1^2 v_\chi^2}{2}. \end{aligned} \tag{110}$$

In addition, the flavor eigenstates are related to mass eigenstates $\mathbf{e} = (e,\mu,\tau,E')^{\mathrm{T}}$ by

$$\mathbf{E}_L = \mathbb{V}_L^E \mathbf{e}_L, \tag{111}$$

where the corresponding left-handed rotation matrix can be expressed as

$$\mathbb{V}_L^E = \mathbb{V}_{\mathrm{SS},L}^E \mathbb{V}_{\mathrm{SM},L}^E, \tag{112}$$

which diagonalizes as

$$\mathbb{M}_E\mathbb{M}_E^\dagger = \frac{1}{2}\begin{pmatrix} \mathcal{M}_{ee}^2 & \mathcal{M}_{eE}^2 \\ \mathcal{M}_{eE}^{2\mathrm{T}} & \mathcal{M}_{EE}^2 \end{pmatrix}, \tag{113}$$

whose blocks are

$$\begin{aligned} \mathcal{M}_{ee}^2 &= \frac{v_2^2}{2}\begin{pmatrix} q_{11}^2 t_\beta^2 + \eta^2 + \Sigma_{11}^2 + \Sigma_{13}^2 & q_{11}q_{21}t_\beta^2 + h\eta + \Sigma_{11}\Sigma_{21} + \Sigma_{13}\Sigma_{23} & \zeta\Sigma_{11} + H\Sigma_{13} \\ * & q_{21}^2 t_\beta^2 + h^2 + \Sigma_{21}^2 + \Sigma_{23}^2 & \zeta\Sigma_{21} + H\Sigma_{23} \\ * & * & H^2 + \zeta^2 \end{pmatrix}, \\ \mathcal{M}_{eE}^2 &= \frac{v_1 v_\chi}{2} H_1 \begin{pmatrix} q_{11} \\ q_{21} \\ 0 \end{pmatrix}, \\ \mathcal{M}_{EE}^2 &= \frac{v_\chi^2 H_1}{2}. \end{aligned} \tag{114}$$

The angle $\alpha_{e\mu}$ is defined by $t_{\alpha_{e\mu}} = \tan\alpha_{e\mu} \approx \eta/h$, which is a free parameter of the model as shown below.

The former matrix $\mathbb{V}_{\mathrm{SS},L}^E$ is

$$\mathbb{V}_{\mathrm{SS},L}^E = \begin{pmatrix} I & F^E \\ -F^{E\dagger} & I \end{pmatrix}, \tag{115}$$

with $F^E = \mathcal{M}_{eE}^2(\mathcal{M}_{EE}^2)^{-1}$. The latter rotation is

$$\mathbb{V}_{\mathrm{SM},L}^E = \begin{pmatrix} V_{\mathrm{SM},L}^E & 0 \\ 0 & 1 \end{pmatrix}, \tag{116}$$

where the top-left block diagonalizes the SM charged lepton masses,

$$V_{\mathrm{SM},L}^E = \begin{pmatrix} c_{\alpha_{e\mu}} & s_{\alpha_{e\mu}} & \frac{\Sigma_{13}}{H} \\ -s_{\alpha_{e\mu}} & c_{\alpha_{e\mu}} & \frac{\Sigma_{23}}{H} \\ -\frac{\Sigma_{13}}{H} & -\frac{\Sigma_{23}}{H} & 1 \end{pmatrix}. \tag{117}$$

IV. PMNS MATRIX

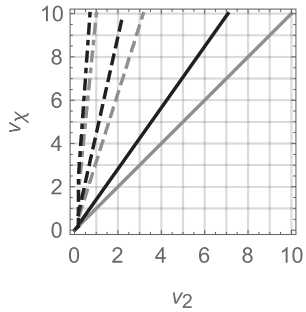
To explore some phenomenological consequences of the above structures, we assume for simplicity that M_D is diagonal and M_M is proportional to the identity

$$M_D = \begin{pmatrix} h_{N\chi 1} & 0 & 0 \\ 0 & h_{N\chi 2} & 0 \\ 0 & 0 & h_{\chi N3} \end{pmatrix} \frac{v_\chi}{\sqrt{2}} \tag{118}$$

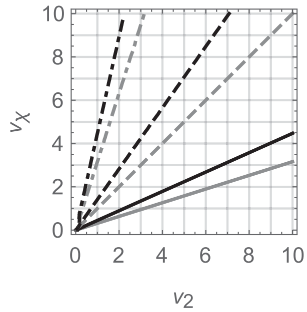
$$M_M = \mu_N \mathbb{I}_{3\times 3}. \tag{119}$$

Thus, $V_1 = V_2 = \mathbb{I}_{3\times 3}$ in (97). On the other hand, replacing the Dirac matrix from (88) into the light mass eigenvalues in (96), we obtain

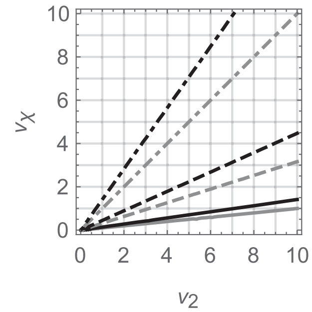
$$m_{\mathrm{light}} = \frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 v_\chi^2}\begin{pmatrix} (h_{2e}^{\nu e})^2 + (h_{2\mu}^{\nu e})^2\rho^2 & h_{2e}^{\nu e}h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e}h_{2\mu}^{\nu\mu}\rho^2 & h_{2e}^{\nu e}h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e}h_{2\mu}^{\nu\tau}\rho^2 \\ h_{2e}^{\nu e}h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e}h_{2\mu}^{\nu\mu}\rho^2 & (h_{2e}^{\nu\mu})^2 + (h_{2\mu}^{\nu\mu})^2\rho^2 & h_{2e}^{\nu\mu}h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu}h_{2\mu}^{\nu\tau}\rho^2 \\ h_{2e}^{\nu e}h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e}h_{2\mu}^{\nu\tau}\rho^2 & h_{2e}^{\nu\mu}h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu}h_{2\mu}^{\nu\tau}\rho^2 & (h_{2e}^{\nu\tau})^2 + (h_{2\mu}^{\nu\tau})^2\rho^2 \end{pmatrix}, \tag{120}$$



(a) NO: $h_{N\chi 1}{}^2 = 0.01$.



(b) NO: $h_{N\chi 1}{}^2 = 0.10$.



(c) NO: $h_{N\chi 1}{}^2 = 1.00$.

FIG. 3. Contour plots of v_χ vs v_2 from Eq. (128) for different values of $h_{N\chi 1}{}^2$ and μ_N. From below to above, there are the corresponding contour plots for the following values of μ_N: 500 eV (gray, line), 1 keV (black, line), 5 keV (gray, dashed), 10 keV (black, dashed), 50 keV (gray, dot-dashed), and 100 keV (black, dot-dashed).

where $\rho = h_{N\chi 1}/h_{N\chi 2}$. The matrix $m_{\rm light}$ has zero determinant, obtaining at least one massless neutrino. The above matrix is diagonalized through

$$U_\nu^{\rm T} m_{\rm light} U_\nu = m_{\rm light}^{\rm diag}, \tag{121}$$

where U_ν contains the mixing angles that transform the weak eigenstates $\nu_L^{e,\mu,\tau}$ into mass eigenstates $\nu_L^{1,2,3}$. The PMNS matrix is defined as the product of the above rotation matrix and the rotation matrix of the charged sector $V_{{\rm SM},L}^E$,

$$U_{\rm PMNS} = (V_{{\rm SM},L}^E)^\dagger U_\nu. \tag{122}$$

We use the following parametrization for the PMNS matrix [43]:

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{123}$$

The mixing angles can be obtained from some matrix components as

$$s_{13}^2 = |U_{e3}|^2, \quad s_{23}^2 = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}, \quad s_{12}^2 = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}. \tag{124}$$

A. Parameter values

In order to have a model consistent with neutrino oscillation data [22], the values of the Yukawa parameters $h_{2e}^{\nu e}$, $h_{2e}^{\nu\mu}$, $h_{2e}^{\nu\tau}$, $h_{2\mu}^{\nu e}$, $h_{2\mu}^{\nu\mu}$, $h_{2\mu}^{\nu\tau}$, and $\alpha_{e\mu}$ must be properly adjusted. To achieve this, we implement a Monte Carlo method to generate random numbers in the parameter space, where only the numbers which match up the mass matrix to experimental data are accepted, while the others are rejected. It is worth mentioning that the other two rotation parameters described by Σ_{13}/H and Σ_{23}/H were approximated to m_e/m_τ, while $h_{2e}^{\nu\mu}$ was chosen null to simplify the search.

On the other hand, the appropriate mass scale and mass ordering can be obtained by adjusting the outer factor of the mass matrix and the ratio ρ. For NO, the Yukawa coupling can be set by

$$h_{N\chi 1}{}^2 = 0.02, \qquad \rho^2 = 0.5, \tag{125}$$

while for IO,

$$h_{N\chi 1}{}^2 = 0.025, \qquad \rho^2 = 0.625. \tag{126}$$

TABLE IV. Yukawa coupling domain which fulfil at 1σ neutrino oscillation data for NO reported by [22]. $h_{2\mu}^{\nu e} = 0$ for simplifying the Monte Carlo search.

	$\alpha_{e\mu} = 0°$	$\alpha_{e\mu} = 15°$	$\alpha_{e\mu} = 30°$
$h_{2e}^{\nu e}$	$0.264 \to 0.278$	$0.285 \to 0.299$	$0.237 \to 0.270$
$h_{2e}^{\nu\mu}$	$-0.707 \to -0.244$	$-0.726 \to -0.335$	$-0.796 \to -0.547$
$h_{2\mu}^{\nu\mu}$	$-0.491 \to -0.190$	$-0,464 \to -0.173$	$-0.342 \to -0.039$
$h_{2e}^{\nu\tau}$	$0.267 \to 0.748$	$0.313 \to 0.677$	$0.140 \to 0.355$
$h_{2\mu}^{\nu\tau}$	$0.130 \to 0.462$	$0.196 \to 0.460$	$0.440 \to 0.510$

TABLE V. Yukawa coupling domain which fulfils at 1σ neutrino oscillation data for IO reported by [22]. $h_{2\mu}^{\nu e} = 0$ for simplifying the Monte Carlo search.

	$\alpha_{e\mu} = 0°$	$\alpha_{e\mu} = 1°$	$\alpha_{e\mu} = 2°$
$h_{2e}^{\nu e}$	$1.094 \to 1.107$	$1.091 \to 1.105$	$1.090 \to 1.103$
$h_{2e}^{\nu\mu}$	$-0.122 \to -0.106$	$-0.127 \to -0.113$	$-0.128 \to -0.118$
$h_{2\mu}^{\nu\mu}$	$0.970 \to 1.060$	$0.980 \to 1.070$	$1.010 \to 1.080$
$h_{2e}^{\nu\tau}$	$0.110 \to 0.127$	$0.122 \to 0.138$	$0.135 \to 0.149$
$h_{2\mu}^{\nu\tau}$	$0.930 \to 1.030$	$0.920 \to 1.010$	$0.910 \to 0.980$

In the same way, the mass scale is set by

$$v_2 = 7\ \text{GeV},\quad v_\chi = 7\ \text{TeV},\quad \mu_N = 1\ \text{keV}. \tag{127}$$

The above values fix the outer factor of the mass matrix (120) at 50 meV, which yields to the correct squared-mass differences. Nevertheless, there exist other possible values for the parameters μ_N, $h_{N1\chi}$, v_χ, and $\tan\beta$ that lead us to the factor at 50 meV.

If the following constraint is assumed,

$$\frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 {v_\chi}^2} = 50\ \text{meV}, \tag{128}$$

contour plots can be done for different values of μ_N in the v_χ vs v_2 plane, as shown in Fig. 3.

Tables IV and V and V show regions where the neutrino Yukawa couplings and the angle $\alpha_{e\mu}$ make consistent this model with neutrino oscillation data reported by [22] at 3σ.

The Yukawa coupling $h_{\chi N3}$ is not fixed by oscillations of the light neutrinos; however, they may contribute to the total rotation matrix $\mathbb{U}_N$ in (103). Thus, the neutral spectrum of the model is composed by three active light neutrinos $\nu_L^{1,2,3}$ and six quasidegenerated sterile neutrinos $N_{1L}^{1,2,3}$ and $N_{2L}^{1,2,3}$ at the TeV scale.

V. CONCLUSIONS

Abelian nonuniversal gauge extensions of the SM are very well-motivated models which involve a wide number of theoretical aspects. In this work, by requiring nonuniversality in the left-handed quark sector and in the lepton sector, we propose a new $G_{\rm SM} \times U(1)'$ gauge model. We obtained a free-anomaly theory with invariant Yukawa interactions, predicting hierarchical mass structures in the quark and charged lepton sector with few free parameters

For the quark sector, we identify three energy scales. First, at the breaking scale of the $U(1)_X$ symmetry, we obtain heavy masses to the extra heavy quarks J^n and T, with $M_{J^n} \approx M_T \sim v_X$. Second, at tree level, we obtain masses at the electroweak scale for the c, t and b quarks, with $M_{c,t,u} \sim v_{1,2}$. Finally, at one-loop level, we obtain light masses for the u, d and s quarks, with $M_{u,d,s} \sim v_{1,2}^2/v_\chi$. For the leptonic sector, we also obtain the same hierarchical structure, where the extra leptons E and $\mathcal{E}$ acquire masses at the v_χ scale, the μ and τ have masses at the electroweak scale, and the electron obtain masses at one-loop, which is suppressed as $v_{1,2}^2/v_\chi$.

On the other hand, with the addition of extra Majorana neutrinos, we found that neutrinos may acquire tiny masses via the inverse seesaw mechanism. The selection of a small Majorana mass term (from eV to KeV scale) and the experimental limits on observables from neutrino oscillations allows us to perform numerical adjustment for the values of the Yukawa couplings of neutrinos in NO and IO scenarios. In addition, because the nonuniversal $U(1)_X$ charges, the electron remains massless at tree level but a nonvanishing mass term emerges at one-loop corrections which gives a viable explanation for its small mass compared to the electroweak scale.

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APPENDIX: BLOCK DIAGONALIZATION

Let us take a generic matrix with arbitrary dimension of the form

$$\mathbb{M}^2 = \begin{pmatrix} A & C \\ C^T & D \end{pmatrix}, \tag{A1}$$

with A, D, and C submatrices, whose elements obey the hierarchy

$$A \ll C \ll D. \tag{A2}$$

The matrix (A1), as shown in Ref. [44], can be block diagonalized approximately by a unitary rotation of the form

$$V = \begin{pmatrix} I & F \\ -F^T & I \end{pmatrix}, \tag{A3}$$

where I is an identity matrix, and F a small subrotation with $F \ll 1$. Keeping only up to linear terms on F, the rotation gives

$$V^T \mathbb{M}^2 V = \begin{pmatrix} A - CF^T - FC^T & C + AF - FD \\ C^T + F^T A - DF^T & D + C^T F + F^T C \end{pmatrix}, \tag{A4}$$

which, by definition, must lead us to a diagonal block form,

$$\mathbb{m}^2 = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}, \tag{A5}$$

with a and d nondiagonal matrices, and 0 the null matrix. By matching the upper right nondiagonal block in (A4) and (A5), we obtain that $C + AF - FD = 0$. Taking into account the hierarchy in (A2), we may neglect the term with A, finding the following approximate solution:

$$F \approx CD^{-1}. \tag{A6}$$

On the other hand, if we match the diagonal blocks in (A4) and (A5), and use the solution (A6), we can obtain the form of the submatrices a and b in terms of the original blocks A, C, and D. We obtain at dominant order that

$$a \approx A - CD^{-1}C^T \qquad b \approx D. \tag{A7}$$

The above matrices can be diagonalized independently.

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